Title: Spin-Peierls instability of the U(1) Dirac spin liquid

Speakers: Urban Seifert

Series: Quantum Matter

Date: September 26, 2023 - 11:00 AM

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Abstract: The presence of many competing classical ground states in frustrated magnets implies that quantum fluctuations may stabilize quantum spin liquids (QSL), which are characterized by fractionalized excitations and emergent gauge fields. A paradigmatic example is the U(1) Dirac spin liquid (DSL), which at low-energies is described by emergent quantum electrodynamics in 2+1 dimensions (QED3), a strongly interacting field theory with conformal symmetry. While the DSL is believed to be intrinsically stable, its robustness against various other couplings has been largely unexplored and is a timely question, also given recent experiments on triangular-lattice rare-earth oxides. In this talk, using complementary perturbation theory and scaling arguments as well as results from numerical DMRG simulations, I will show that a symmetry-allowed coupling between (classical) finite-wavevector lattice distortions and monopole operators of the U(1) Dirac spin liquid generally induces a spin-Peierls instability towards a (confining) valence-bond solid state. Away from the limit of static distortions, I will argue that the phonon energy gap establishes a parameter regime where the spin liquid is expected to be stable.

Zoom link https://pitp.zoom.us/j/96764903405?pwd=Y0gyU3hGSC9va0hzWnZRZFBOVmRCZz09

Spin-Peierls instability of U(1) Dirac spin liquids

Urban F. P. Seifert

with ***Josef Willsher, Markus Drescher, Frank Pollmann***
and ***Johannes Knolle (TU Munich)***

Kavli Institute for Theoretical Physics
University of California, Santa Barbara

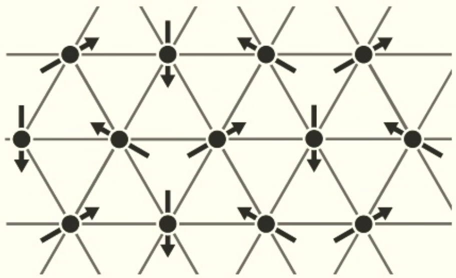
Frustrated (classical) magnets

Frustration :⇔ Cannot minimize all contributions to energy simultaneously

⇒ Non-collinear magnetic order, ground-state degeneracies

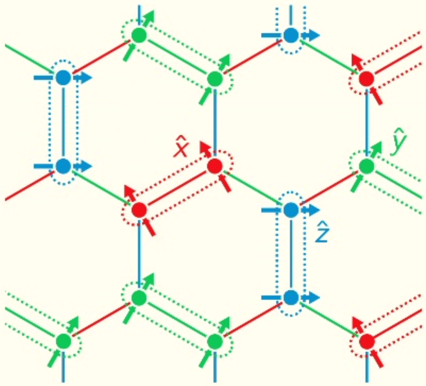
Triangular lattice Heisenberg AFM

$$H = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j, \quad J > 0$$



Kitaev's honeycomb model

$$H_K = -K^x \sum_{\langle ij \rangle_x} S_i^x S_j^x - K^y \sum_{\langle ij \rangle_y} S_i^y S_j^y - K^z \sum_{\langle ij \rangle_z} S_i^z S_j^z$$



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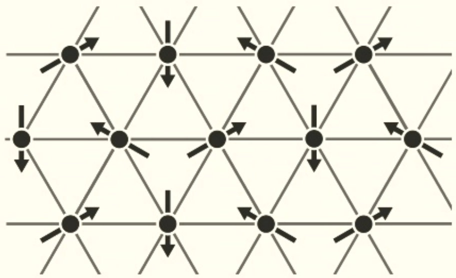

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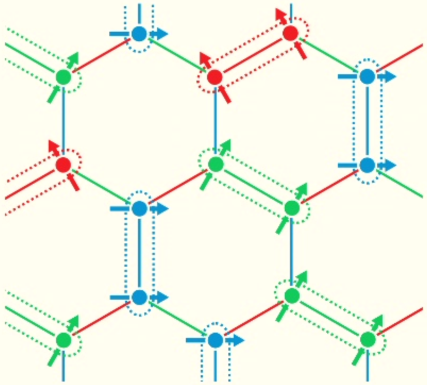

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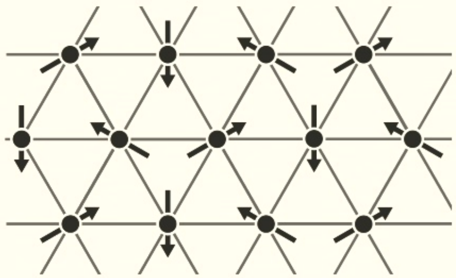

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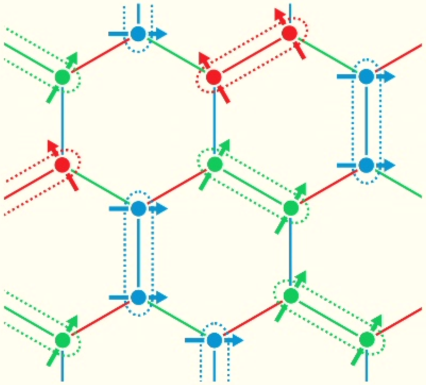


Quantum fluctuations can have a strong impact:
stabilize new "quantum" phases of matter

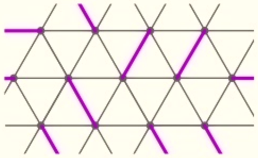
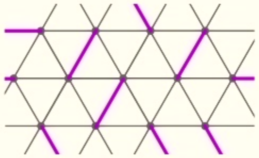
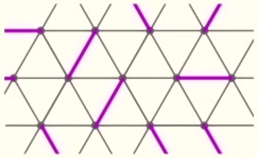
2

Quantum spin liquids in frustrated magnets

⇒ Consider quantum magnet $\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$

Quantum spin liquids — prototypical quantum matter

Resonating valence bonds: example for a quantum spin liquid

$|\text{RVB}\rangle =$  $+$  $+$  $+ \ldots$

 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Non-trivial state without symmetry-breaking (no order parameter), with **long-range entanglement** and **fractionalized excitations**

X.-G. Wen, OUP 2006

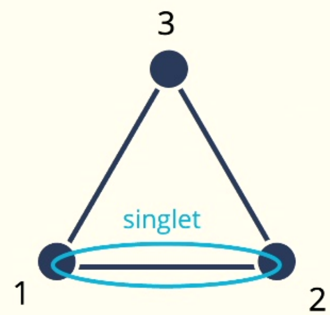
Frustrated magnets

But: large ground-state degeneracies ⇔ **strong sensitivity** to perturbations, *in particular* quenched disorder (impurities) and lattice degrees of freedom

Toy model — Jahn-Teller distortion of a single triangle

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\left(\frac{1}{2} \otimes \frac{1}{2}\right) \otimes \frac{1}{2} = (0 \oplus 1) \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$$



4 degenerate ground states

Frustrated magnets

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Toy model — Jahn-Teller distortion of a single triangle

$$H = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j + \sum_{\langle ij\rangle} \frac{\partial J}{\partial \mathbf{r}} \delta\mathbf{r}_{ij} \vec{S}_i \cdot \vec{S}_j + V(\delta\mathbf{r}_{ij})$$

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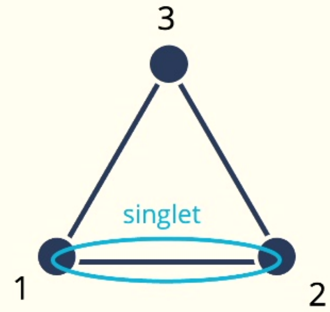

Consider distortions $J(\mathbf{r}_{ij} + \delta\mathbf{r}_{ij}) \approx J_0 + \frac{\partial J}{\partial \mathbf{r}} \delta\mathbf{r}_{ij}$ with potential $V \simeq K|\delta\mathbf{r}_{ij}|^2$

Frustrated magnets

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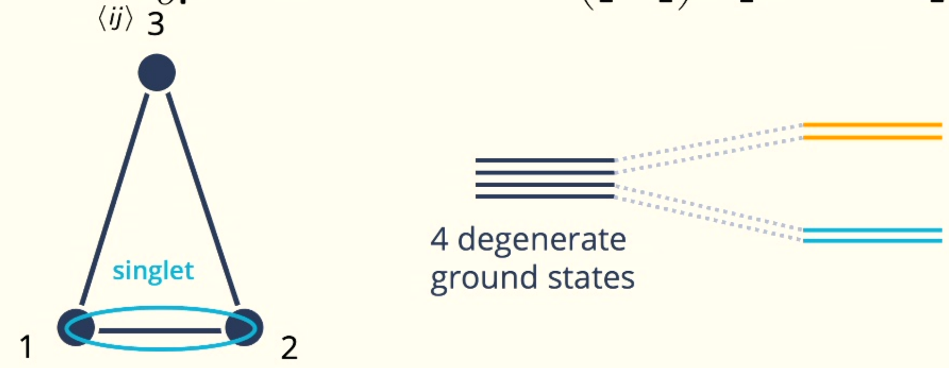

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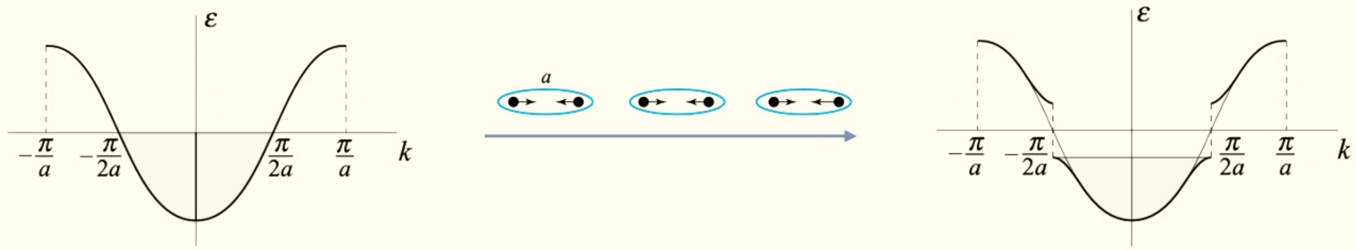
Gain in exchange energy wins over elastic potential: Lift degeneracies!

Peierls instability of gapless systems (1d)

Peierls instability of gapless (free) fermions in one dimension:



$$E(u) = \frac{K}{2}u^2 - (\text{const.}) \times u^2 \ln u$$

Gain in energy outcompetes elastic energy!

$$E(u) = \frac{K}{2}u^2 - (\text{const.}) \times u^\chi$$

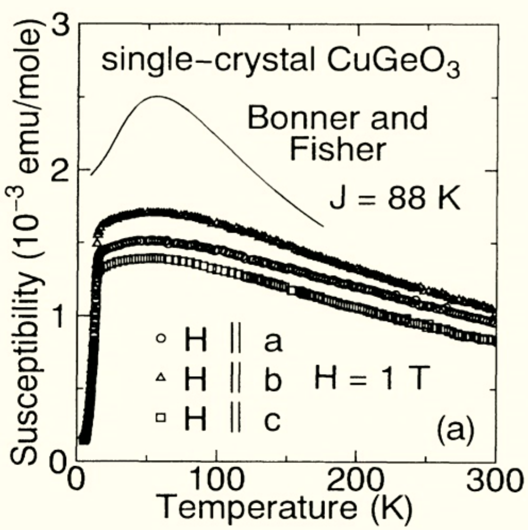
Interactions enhance instability,
e.g. $\chi = 4/3$ for antiferromagnetic spin chain

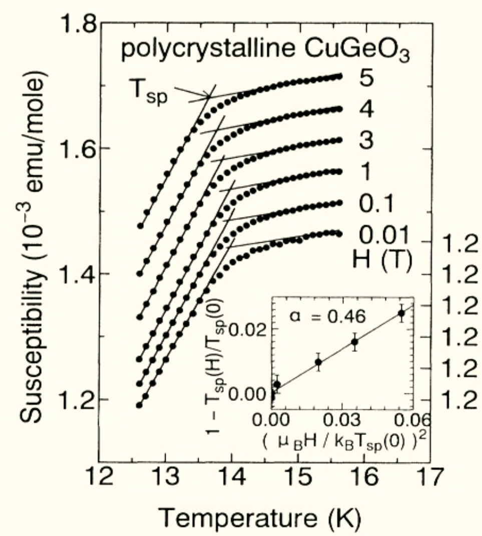
Spin-Peierls instability in $CuGeO_3$

VOLUME 70, NUMBER 23 PHYSICAL REVIEW LETTERS 7 JUNE 1993

Observation of the Spin-Peierls Transition in Linear Cu^{2+} (Spin-$\frac{1}{2}$) Chains in an Inorganic Compound $CuGeO_3$

Masashi Hase, Ichiro Terasaki, and Kunimitsu Uchinokura
Department of Applied Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-Ku, Tokyo 113, Japan
(Received 4 January 1993)





Today

Fate of **gapless 2d quantum spin liquids** under **coupling to lattice**?

Outline

1. Dirac spin liquids and low-energy conformal field theory
2. Coupling to classical distortion modes and analysis
3. Numerical results
4. Quantum-mechanical phonons
5. Conclusion

Spin liquids — Parton constructions

In frustrated spin systems, quantum fluctuations can stabilize exotic ground states: **emergent gauge fields** and **fractionalized excitations**

$$\mathcal{H} = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Parton construction: $\vec{S}_i = \frac{1}{2} f^{\dagger}_{i,\alpha} \vec{\sigma}_{\alpha\beta} f_{i,\beta}$

Local constraint: $f^{\dagger}_{i,\uparrow} f_{i,\uparrow} + f^{\dagger}_{i,\downarrow} f_{i,\downarrow} = 1 \ \forall i$

$$\Rightarrow \quad \mathcal{H} = -\sum_{\langle ij \rangle} \chi_{ij} f^{\dagger}_{i,\alpha} f_{j,\alpha} + \text{h.c.}$$

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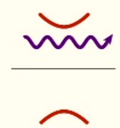
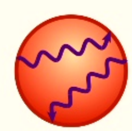
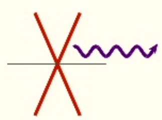
Local $\mathrm{U}(1)$ gauge redundancy: $f_{j,\alpha} \to e^{i\phi_j} f_{j,\alpha}$ with $\chi_{ij} \to e^{i(\phi_i - \phi_j)} \chi_{ij}$

- Gapped (chiral) spin liquid 
- Spinon Fermi surface with U(1) gauge bosons 
- Dirac Fermions: Quantum electrodynamics in 2+1 dims. 

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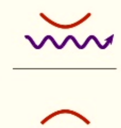
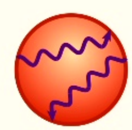
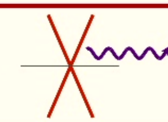
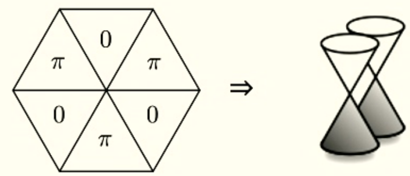
- Gapped (chiral) spin liquid 
- Spinon Fermi surface with U(1) gauge bosons 
- Dirac Fermions: Quantum electrodynamics in 2+1 dims. 

Dirac spin liquids on the triangular lattice

Focus on Dirac spin liquid (staggered flux)



Low-energy theory: N=4 Quantum electrodynamics

$$\mathcal{L}_{\mathrm{QED}_3} = \sum_{i=1}^{N=4} \left[-\bar{\psi}_i \gamma^\mu (\partial_\mu - \mathrm{i} a_\mu) \psi_i \right] + \frac{1}{g^2} f_{\mu\nu} f^{\mu\nu} + \ldots$$

enlarged symmetry in the IR

$\mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{SU}(2)_{\mathrm{valley}} \to \mathrm{SU}(4)$

$[g^2] = 1$

Theory flows to **strong coupling** in the infrared!

Precise phase structure (spontaneous symmetry breaking?) under debate

Applequist *et al.*, PRD 1987; Bashir *et al.*, PRC 2008; Grover, PRL 2008; Braun *et al.*, PRD 2014 ...

⇒ Here: Assume that QED_3 flows to **conformal fixed point** (justified for large N)

QED_3 as a conformal field theory

$$\mathcal{L}_{\mathrm{QED}_3} = \sum_{i=1}^{N=4} \left[-\bar{\psi}_i \gamma^\mu (\partial_\mu - \mathrm{i} a_\mu)\psi_i\right] + \frac{1}{g^2} f_{\mu\nu} f^{\mu\nu} + \ldots$$

Built-in conservation of $\mathrm{U}(1)_{\mathrm{top}}$ $\quad j^\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu a_\rho \quad \Rightarrow \partial_\mu j^\mu = 0$

But: Gauge field is compact ⇒ **Monopoles** can tunnel 2π flux

No explicit representation in terms of a_μ or ψ_i !

Exploit that QED_3 is a **conformal field theory:**

Scaling operators $\quad \Phi_a(bx) = b^{-\Delta_\Phi}\Phi_a(x)$

Correlation functions $\quad \langle \Phi_a^\dagger(x)\Phi_b(y)\rangle_{\mathrm{QED}_3} = \frac{\delta_{a,b}}{|x-y|^{2\Delta_\Phi}}$

Monopoles Φ_a strongly relevant ($\Delta_\Phi \approx 1.02$)

Bootstrap results: Albayrak *et al.*, arXiv 2021

Monopole operators — Quantum numbers and stability

Song, Wang, Vishwanath, He, Nat. Comms. 2018

Dirac fermions with π-flux: zero modes ⇒ half-fill for gauge invariance

⇒ Monopoles transform as $SO(6)=SU(4)/Z_2$ vectors, $\Phi_a^\dagger \sim f_\alpha^\dagger f_\beta^\dagger \mathcal{M}_{2\pi}^\dagger$

Embedding of **lattice symmetries** in the IR

	T_1	T_2	R	C_6	$\mathcal{T}$
$\Phi_1^\dagger$	$e^{-i\frac{\pi}{3}}\Phi_1^\dagger$	$e^{i\frac{\pi}{3}}\Phi_1^\dagger$	$-\Phi_3^\dagger$	Φ_2	Φ_1
$\Phi_2^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_2^\dagger$	$e^{i\frac{\pi}{3}}\Phi_2^\dagger$	$\Phi_2^\dagger$	$-\Phi_3$	Φ_2
$\Phi_3^\dagger$	$e^{-i\frac{\pi}{3}}\Phi_3^\dagger$	$e^{-i\frac{2\pi}{3}}\Phi_3^\dagger$	$-\Phi_1^\dagger$	$-\Phi_1$	Φ_3
$\Phi_{4/5/6}^\dagger$	$e^{i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$e^{-i\frac{2\pi}{3}}\Phi_{4/5/6}^\dagger$	$\Phi_{4/5/6}^\dagger$	$-\Phi_{4/5/6}$	$-\Phi_{4/5/6}$

Song *et al.*, Nat. Comms. 2018

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Song *et al.*, Nat. Comms. 2018

Pure U(1) Dirac spin liquid (triangular lattice) is **stable:**
Adding relevant operators **forbidden by lattice (UV) symmetries**

Monopole operators — Quantum numbers and stability

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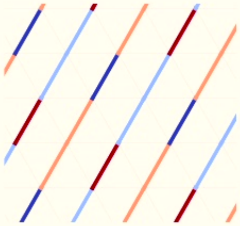
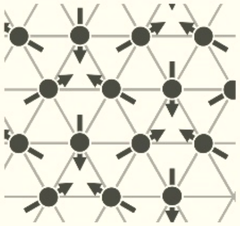
Song *et al.*, Nat. Comms. 2018

VBS order parameters
(momentum $\mathbf{k}_a = -\mathbf{K}_a/2$)
$\vec{S}_j \cdot \vec{S}_{j+\delta_a} \sim \mathrm{Re}\left[\Phi_a(\mathbf{r}_i)e^{i\mathbf{k}_a\cdot\mathbf{r}_i}\right]$



e.g. $\langle\Phi_2\rangle \neq 0$

Néel order parameters
(120° order)
$\vec{S}_j \sim \mathrm{Re}\left[ie^{i\mathbf{K}_1\cdot\mathbf{r}_j}\vec{\Phi}(\mathbf{r}_j)\right]$



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Song *et al.*, Nat. Comms. 2018

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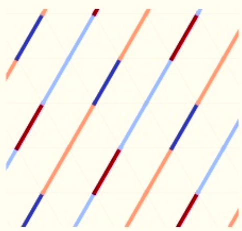

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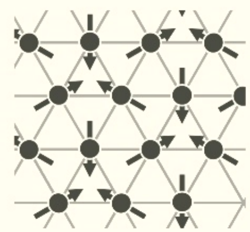
Néel order parameters
(120° order)
$\vec{S}_j \sim \mathrm{Re}\left[ie^{i\mathbf{K}_1\cdot\mathbf{r}_j}\vec{\Phi}(\mathbf{r}_j)\right]$



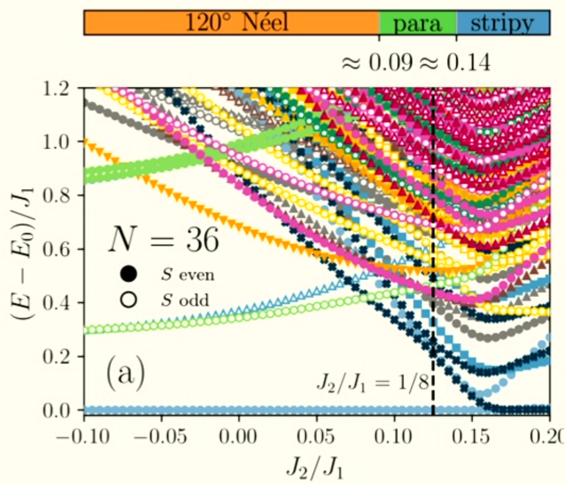
Monopole proliferation ⇒ ordered (confined) states
(Dirac spin liquid as the "Mother of competing orders")

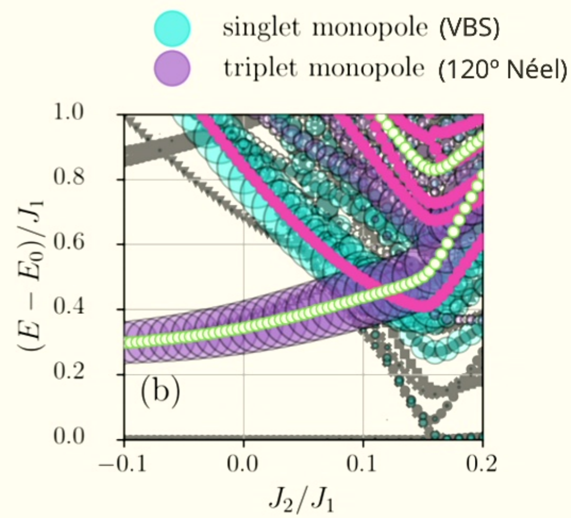
Hermele, Senthil, Fisher, PRB 2005

QED_3 — spectrum of the triangular lattice antiferromagnet

Wietek, Capponi, Läuchli, arXiv:2303.01585

Consider $H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$ on the triangular lattice





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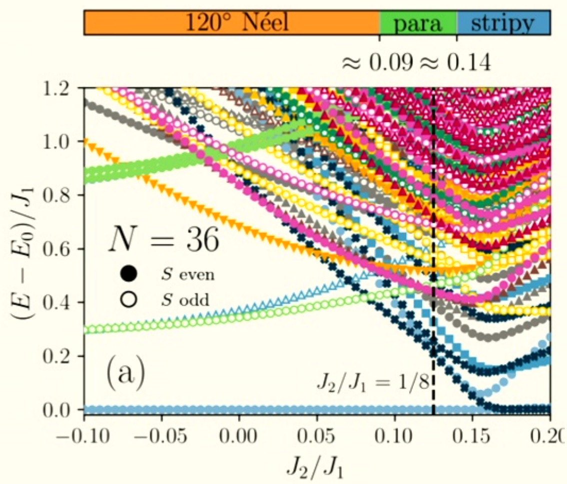
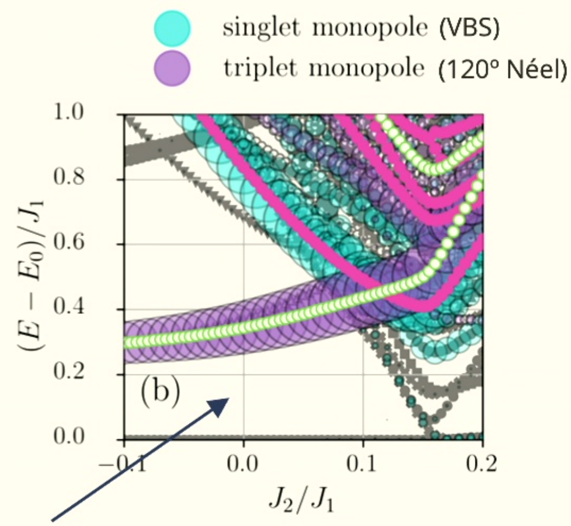


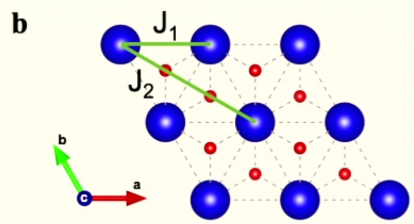


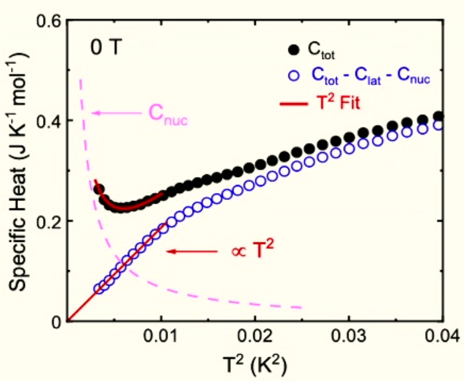
Overlaps of ED levels with $|\Phi\rangle = \mathcal{P}_G\,|\text{Dirac Sea} + \pi\text{-Flux}\rangle$

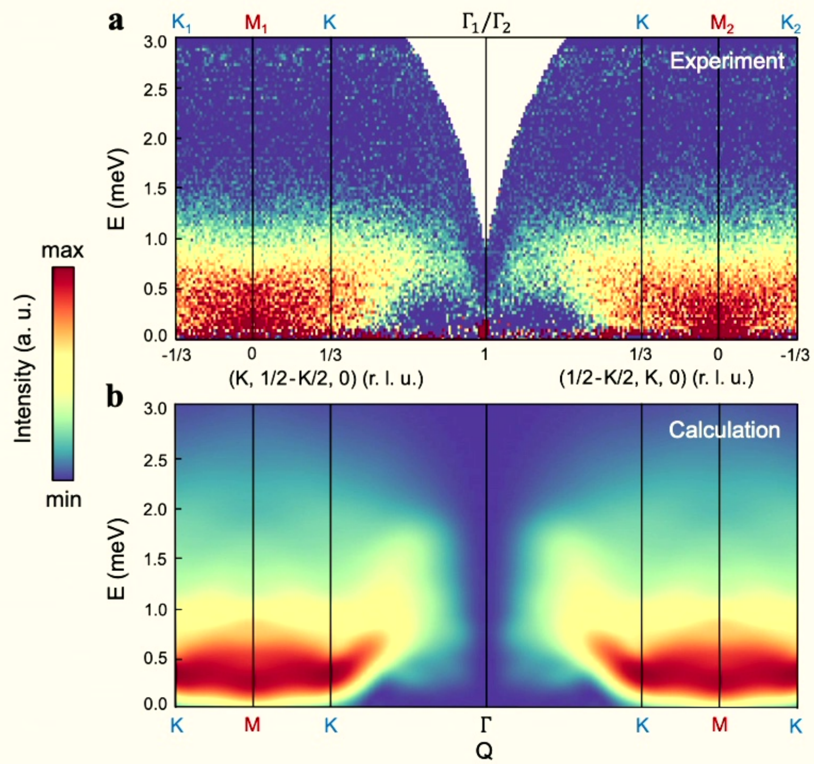
Candidate materials — Rare-earth oxides

Xu, ..., Moore, Haravifard, arXiv:2305.20040

$YbMgGaO_4$: Strong disorder effects ... New candidate: $YbZn_2GaO_5$





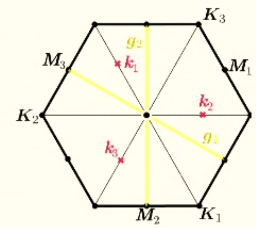


Coupling the U(1) Dirac spin liquid to lattice distortions

Monopole-lattice coupling

UFPS, Willsher, Drescher, Pollmann, Knolle, arXiv:2307.12295

Symmetry-allowed, but irrelevant:

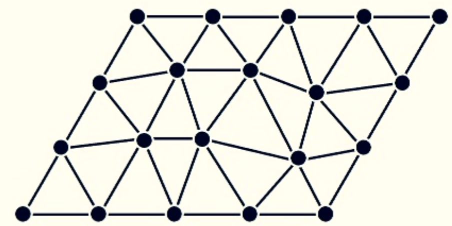


$$\mathcal{H} \sim \left(e^{i\mathbf{k}_1 \cdot \mathbf{R}} \Phi_1 + e^{i\mathbf{k}_2 \cdot \mathbf{R}} \Phi_2 - e^{i\mathbf{k}_3 \cdot \mathbf{R}} \Phi_3 + \text{h.c.}\right)$$

Consider distortions (Eulerian coordinates):

$$\mathbf{R} \to \mathbf{x} = \mathbf{R} + \mathbf{u}(\mathbf{x})$$

global (lab) coordinates — **classical** distortion field



Small distortions: Expand in $|\mathbf{u}(\mathbf{x})| \ll |\mathbf{k}_a|^{-1} \sim a_0$

$$\mathcal{H}_g[\mathbf{u}(\mathbf{x})] = g \sum_{a=1,2,3} s_a \left[(i\mathbf{k}_a \cdot \mathbf{u}(\mathbf{x}))\, e^{i\mathbf{k}_a \cdot \mathbf{x}} \Phi_a + \text{h.c.} \right]$$

Monopole-lattice coupling

$$\mathcal{H}_g[\mathbf{u}(\mathbf{x})] = g \sum_{a=1,2,3} s_a \left[(\mathrm{i}\mathbf{k}_a \cdot \mathbf{u}(\mathbf{x}))\, e^{\mathrm{i}\mathbf{k}_a \cdot \mathbf{x}} \Phi_a + \mathrm{h.c.} \right]$$

↑

Fourier expansion $\mathbf{u}(\mathbf{x}) = \sum_{\mathbf{G}} e^{\mathrm{i}\mathbf{G}\cdot\mathbf{x}} \mathbf{u}_{\mathbf{G}}$

Relevant *and* **symmetry-allowed** interaction between VBS monopoles and lattice distortion modes with lattice momenta $\mathbf{k}_a = -\mathbf{K}_a/2$:

$$\mathcal{H}_g[\mathbf{u}(\mathbf{x})] \approx g \sum_{a=1,2,3} s_a \left[(\mathrm{i}\mathbf{k}_a \cdot \mathbf{u}^*_{\mathbf{k}_a})\, \Phi_a + \mathrm{h.c.} \right]$$

Effective action for distortion fields

Competition between elastic energy and energy of distorted Dirac spin liquid.

Goal: find saddlepoints of $E[\mathbf{u}] = E_{\text{elast.}}[\mathbf{u}] + E_{\text{DSL}}[\mathbf{u}]$

$$E_{\text{elast.}}[\mathbf{u}] = \kappa \sum_{a=1,2,3} |\mathbf{u}_{\mathbf{k}_a}|^2 \qquad E_{\text{DSL}}[\mathbf{u}] = -\lim_{\beta\to\infty} \frac{1}{\beta V} \ln \left[\int_{\beta,V} \mathcal{D}[\ldots] e^{-S_{\text{QED}_3} - S_g[\mathbf{u}]} \right]$$

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Tractable limits:

1. Perturbation theory in $g a_0^{2-\Delta_\Phi} |\mathbf{u}_\mathbf{k}| \ll 1$
2. Scaling ansatz

Effective action for distortion fields

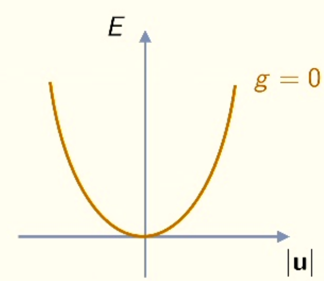
Expand $E_{\mathrm{DSL}}[\mathbf{u}] = -\lim_{\beta\to\infty} \frac{1}{\beta V} \ln\left[\int_{\beta,V} \mathcal{D}[\ldots] e^{-S_{\mathrm{QED}_3} - S_g[\mathbf{u}]}\right]$ to second order:

$$E_{\mathrm{DSL}}[\mathbf{u}] - E_{\mathrm{DSL}}[0] \sim -g^2 \int_0^\beta d\tau \int d^2\mathbf{x} \sum_a |\mathbf{k}_a \cdot \mathbf{u}_{\mathbf{k}_a}|^2 \langle \Phi_a^\dagger(\tau,\mathbf{x}) \Phi_a(0) \rangle_{\mathrm{QED}_3}$$

$$= \frac{1}{|\tau^2 + \mathbf{x}^2|^{\Delta_\Phi}}$$ by conformal symmetry

Integral is IR divergent if $\Delta_\Phi < 3/2$! ⇒ Regulate with temperature $\beta = 1/T < \infty$

$$E_{\mathrm{DSL}}[\mathbf{u}] \sim \lim_{\beta\to\infty} \left(\kappa - c_0 g^2 \beta^{3-2\Delta_\Phi} |\mathbf{k}|^2\right) |\mathbf{u}_{\mathbf{k}}|^2$$



Effective action for distortion fields

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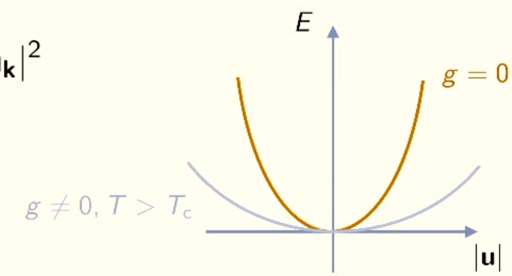

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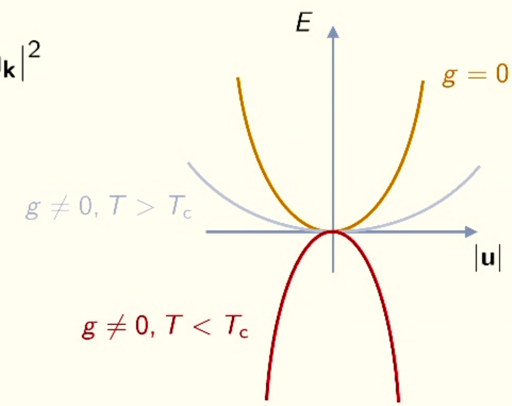
Unbounded potential for $T \to 0$ ⇒ **Instability!**

Extract critical temperature scaling: $T_c \sim \left(\frac{g^2}{\kappa a_0^2}\right)^{\frac{1}{3-2\Delta_\Phi}}$



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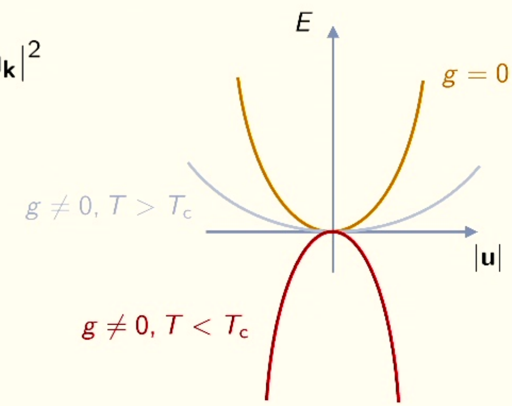


Scaling ansatz

Goal: find effective action for **u** after "integrating out" QED_3 with deformation

$$S_g[\mathbf{u}] = \sum_{a=1,2,3} \int \mathrm{d}\tau \int \mathrm{d}^2\mathbf{x}\ [s_a g(\mathrm{i}\mathbf{k}_a \cdot \mathbf{u}^*_{\mathbf{k}_a})\Phi_a + \text{h.c.}]$$

Scaling analysis: $\Phi \sim \ell^{-\Delta_\Phi} \quad \Rightarrow \ h_a \equiv s_a g(\mathrm{i}\mathbf{k}_a \cdot \mathbf{u}^*_{\mathbf{k}_a}) \sim \ell^{-(3-\Delta_\Phi)}$

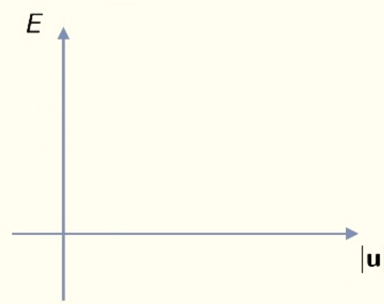
Allowed by symmetry and consistent with scaling:

$$S_{\text{eff.}}[\mathbf{u}] \sim -\int \mathrm{d}\tau \int \mathrm{d}^2\mathbf{x}\ \frac{2C_0}{\chi}|\vec{h}|^{3/(3-\Delta_\Phi)}$$

$\chi \equiv 3/(3-\Delta_\Phi) < 2$ if $\Delta_\Phi < 3/2$

Total energy of the system:

$$\mathcal{E}[\mathbf{u}] = \mathcal{E}_{\text{elast.}} + \mathcal{E}_{\text{eff.}} \sim \kappa|\vec{u}|^2 - \text{const.} \times g^\chi \mathrm{a}_0^\chi |\vec{u}|^\chi$$



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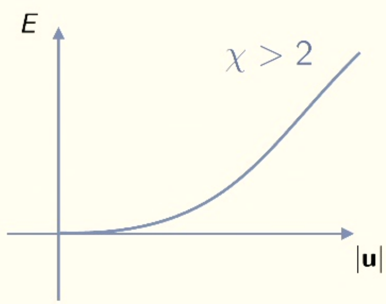

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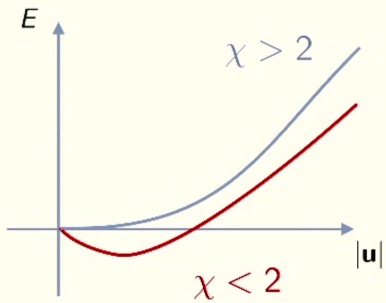

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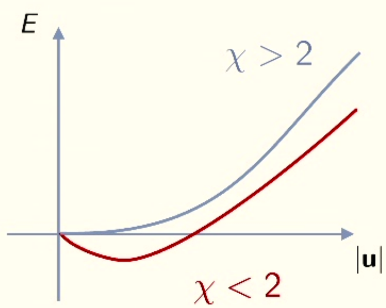


For any $g \neq 0$: local minimum at $|\mathbf{u}| \neq 0 \;\Rightarrow$ instability!

Lifting of unphysical continuous symmetry

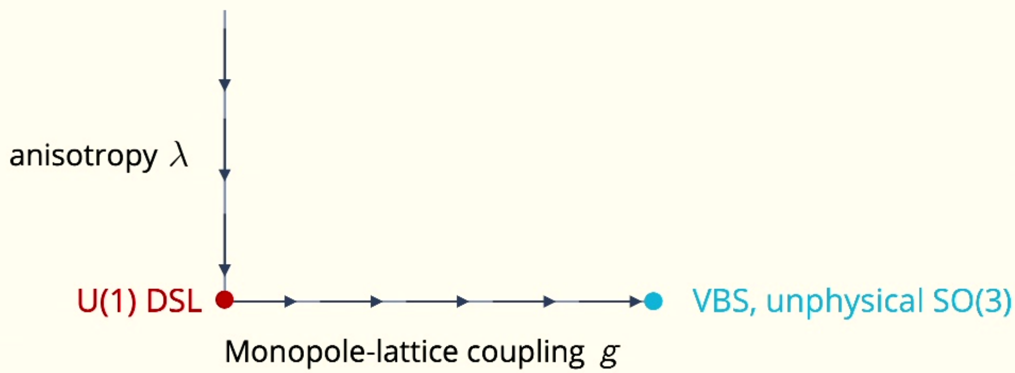
Both effective actions depend only on $\sum_{a=1,2,3} \mathbf{u}_{\mathbf{k}_a} \cdot \mathbf{k}_a = |\mathbf{k}| \left(|u_1|^2 + |u_2|^2 + |u_3|^2 \right)$

longitudinal modes $u_i = \mathbf{u}_{\mathbf{k}_i} \cdot \mathbf{k}_i / |\mathbf{k}|$

⇒ **Unphysical** SO(3) degeneracy of distortion with different lattice momenta!

Lift by symmetry-allowed, but irrelevant coupling

$$S^3 \sim \lambda \int \mathrm{d}\tau \mathrm{d}^2\mathbf{x} \left[\Phi_1 \Phi_2 \Phi_3 + \mathrm{h.c.} \right] \quad \Rightarrow \quad S_{\mathrm{eff}} \sim \lambda u_1 u_2 u_3 + \mathrm{h.c.}$$



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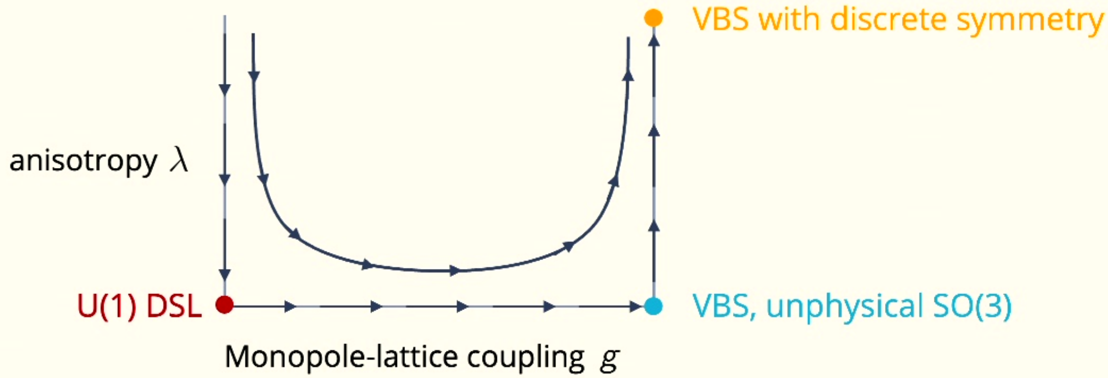
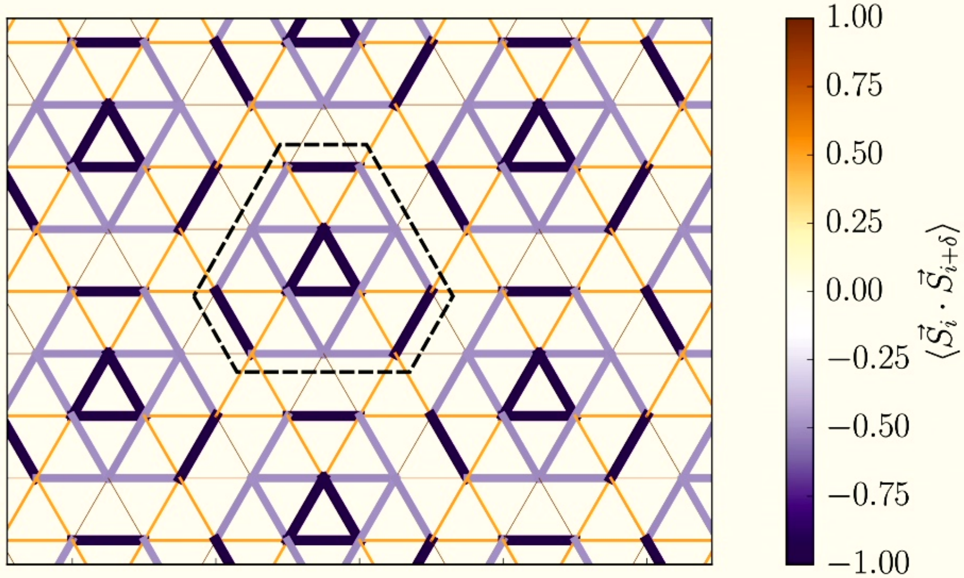


Predicted VBS order

Pick $\mathbf{u}_{\mathbf{k}_i}$ that minimize effective action, which induces finite $\langle \Phi_a \rangle \neq 0$

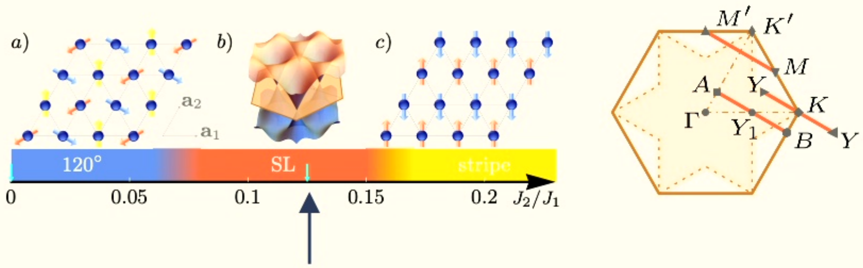
⇒ obtain spin correlations using $\vec{S}_j \cdot \vec{S}_{j+\delta_a} \sim \mathrm{Re}\left[\Phi_a(\mathbf{r}_i) e^{i\mathbf{k}_a \cdot \mathbf{r}_i}\right]$

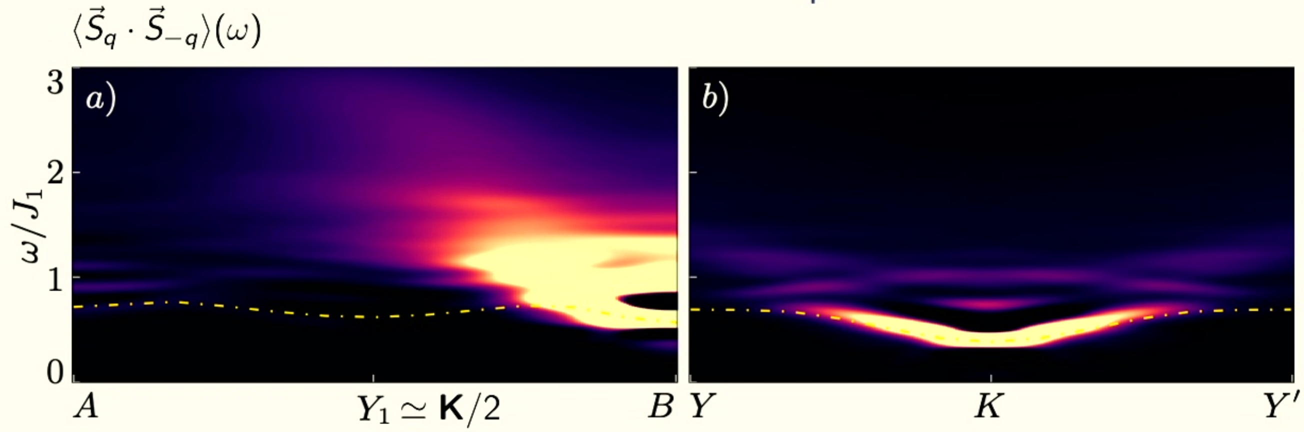


DMRG simulation — J_1-J_2 triangular lattice, no distortion

Drescher, Vanderstraeten, Moessner, Pollmann, arXiv:2209.03344

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$





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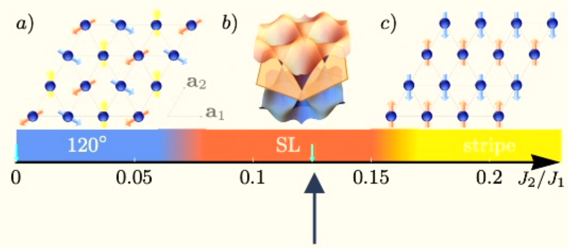
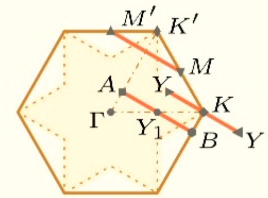
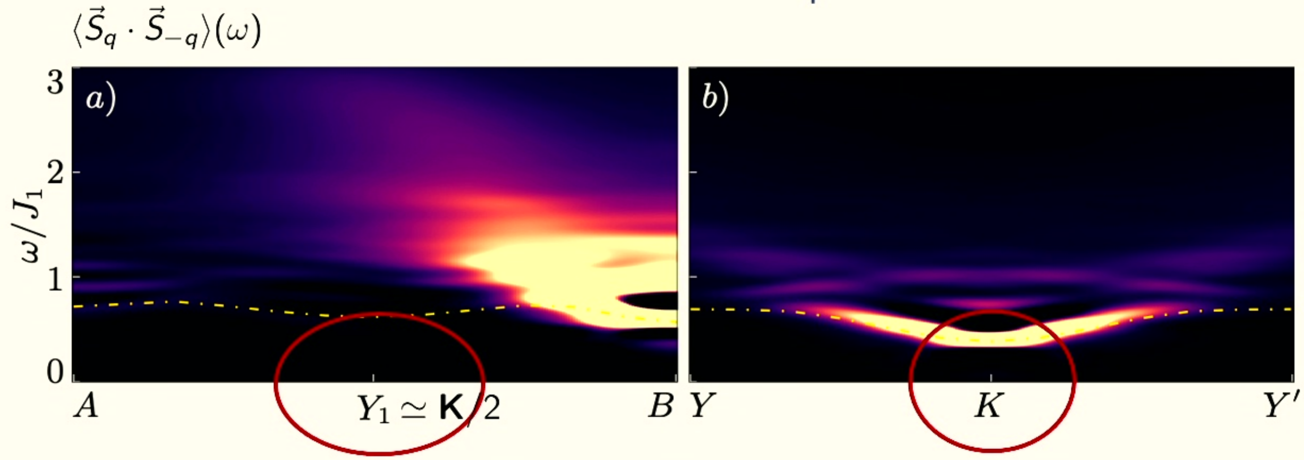






Spin-singlet, valley-triplet monopoles (VBS order parameters): Not visible in spin structure factor!

Spin-triplet, valley-singlet monopoles (120° order parameters)

Signature of relevant coupling in finite-size systems

Use DMRG to study response to **explicit** lattice distortions

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad \text{where} \quad J_{ij} = J^{(0)} \left[1 - \alpha\, \mathbf{u}_{ij} \cdot \mathbf{R}_{ij}/|\mathbf{R}_{ij}|\right]$$

From field theory, expect for distortions with **relevant wavevectors** $\mathbf{k}_a/2$

$$\Delta\mathcal{E}_{\mathrm{DSL}}[\mathbf{u}] = -\lim_{\beta\to\infty} \frac{1}{2\beta V} \langle S_g^2 \rangle_{\mathrm{QED}_3} \simeq -\mathrm{a}_0^{-2} \times g^2\, L^{3-2\Delta_\Phi} |\vec{u}|^2 + \ldots$$

divergent as $L \to \infty$!

All other distortion modes (irrelevant couplings): $\Delta\mathcal{E}_{\mathrm{DSL}}[\mathbf{u}] \sim -|\mathbf{q}|^{2\Delta_\Phi - 1} g^2 |\mathbf{u}_\mathbf{q}|^2$

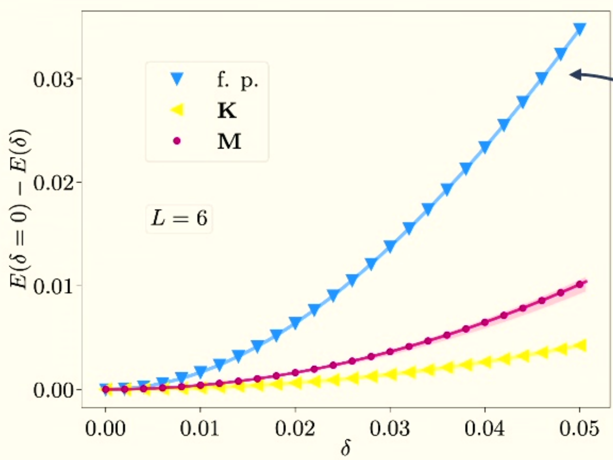
regular as $L \to \infty$!

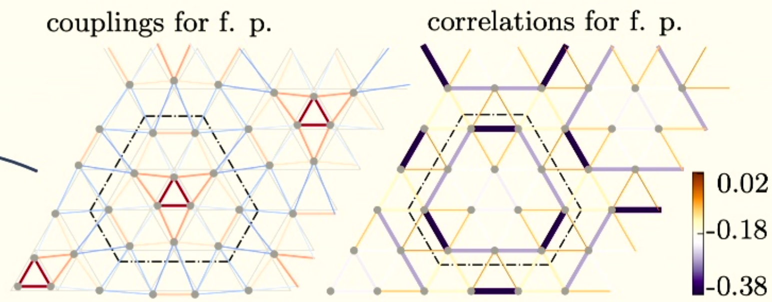
DMRG simulation — Response to distortion fields

Use DMRG to study response to **explicit** lattice distortions

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad \text{where} \quad J_{ij} = J^{(0)} \left[1 - \alpha\, \mathbf{u}_{ij} \cdot \mathbf{R}_{ij} / |\mathbf{R}_{ij}|\right]$$

Ground-state energy
as a function of distortion strength

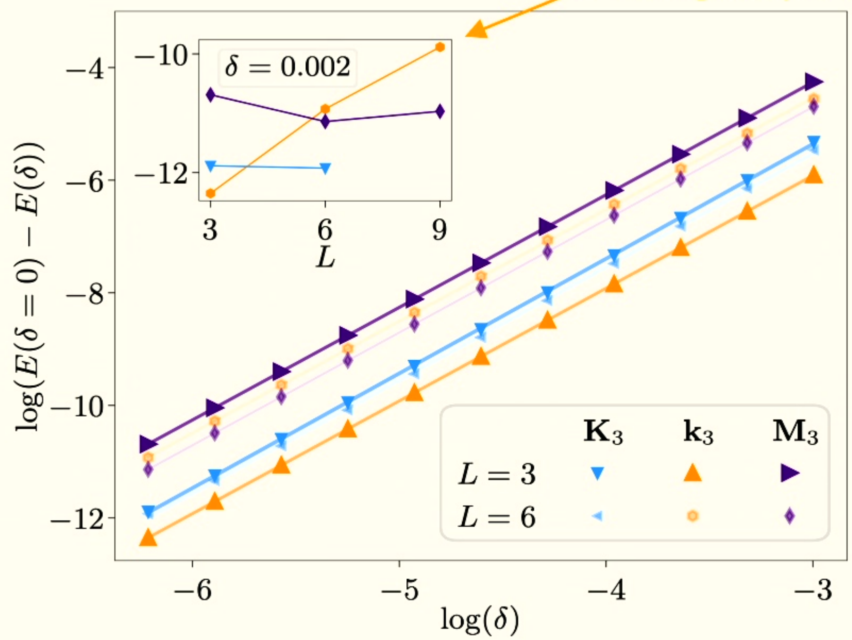




DMRG study — System-size dependence

Extract scaling behaviour $\Delta E(\delta) \simeq -f(L)\delta^2$

strong L-dependence for $\mathbf{k}_3$!



⇒ Signatures of **relevant monopole-lattice** coupling in DMRG study of microscopic spin Hamiltonian!

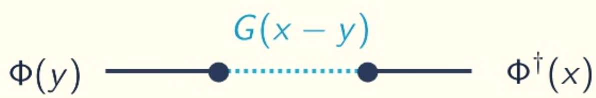
Quantum-mechanical distortion modes (phonons)

Now: phonons as quantum mechanical lattice distortions (again at $\mathbf{k}_a = \mathbf{K}_a/2$)

$$S_{\mathrm{ph}} = \int \mathrm{d}\tau \int \mathrm{d}^2\mathbf{x} \left[\rho|\partial_\tau u_a(\tau,\mathbf{x})|^2 + \kappa|u_a(\tau,\mathbf{x})|^2\right] \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{\kappa}{\rho}}$$

Integrate out phonons: retarded monopole-monopole interaction

$$S = S_{\mathrm{QED}_3} + S_{\Phi\Phi} \quad \text{with} \quad S_{\Phi\Phi} = -g^2 \sum_a |\mathbf{k}|^2 \int \mathrm{d}^3x\, \mathrm{d}^3y\, \Phi_a(x)^\dagger G(x-y) \Phi_a(y)$$



$$G(x-y) = \frac{\delta^{(2)}(\mathbf{x}-\mathbf{y})}{2\rho\omega_0} \mathrm{e}^{-\omega_0|\tau_x - \tau_y|}$$

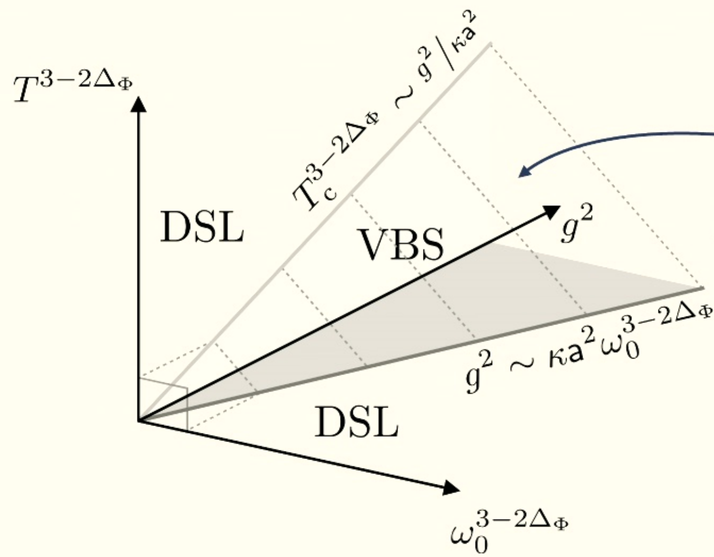
For $|\tau_x - \tau_y| \gg 1/\omega_0$, interaction is suppressed!

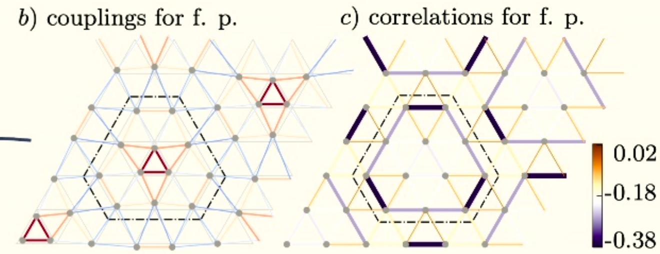
Rescaling coordinates yields dimensionless coupling: $1 \sim g^2 \omega_0^{2\Delta_\Phi - 3}/(\mathrm{a}_0\kappa)$
(also from perturbation theory)

⇒ **Finite phonon gap** yields **non-zero critical coupling** for stability!

Summary — Spin-Peierls instability of U(1) Dirac spin liquid

UFPS, Willsher, Drescher, Pollmann, Knolle, arXiv:2307.12295





- Symmetry-allowed coupling of monopoles to *classical lattice distortion modes* leads to instability with VBS ordering
- Finite phonon frequencies allow for a stable parameter regime

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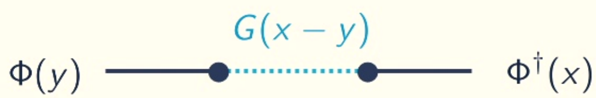

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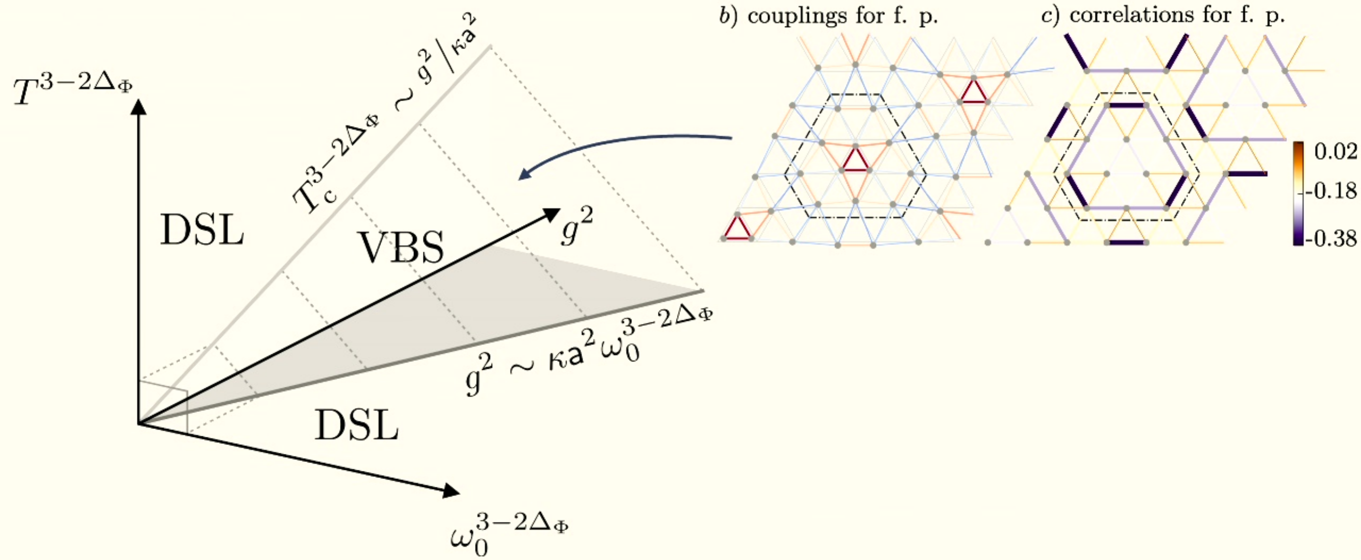

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