Title: Quantization via SQFT

Speakers: Davide Gaiotto

Series: Quantum Fields and Strings

Date: September 26, 2023 - 2:00 PM

URL: https://pirsa.org/23090111

Abstract: I will review how protected correlation functions in certain SQFTs can be used to ``quantize" phase spaces built from their spaces of vacua.

Zoom link https://pitp.zoom.us/j/96543843231?pwd=U0d5d3EvVXJGT0VzR2JYUzBQVGJlUT09

QUANTIZATION OF COULOMB (OR HIGGS) BRANCHES

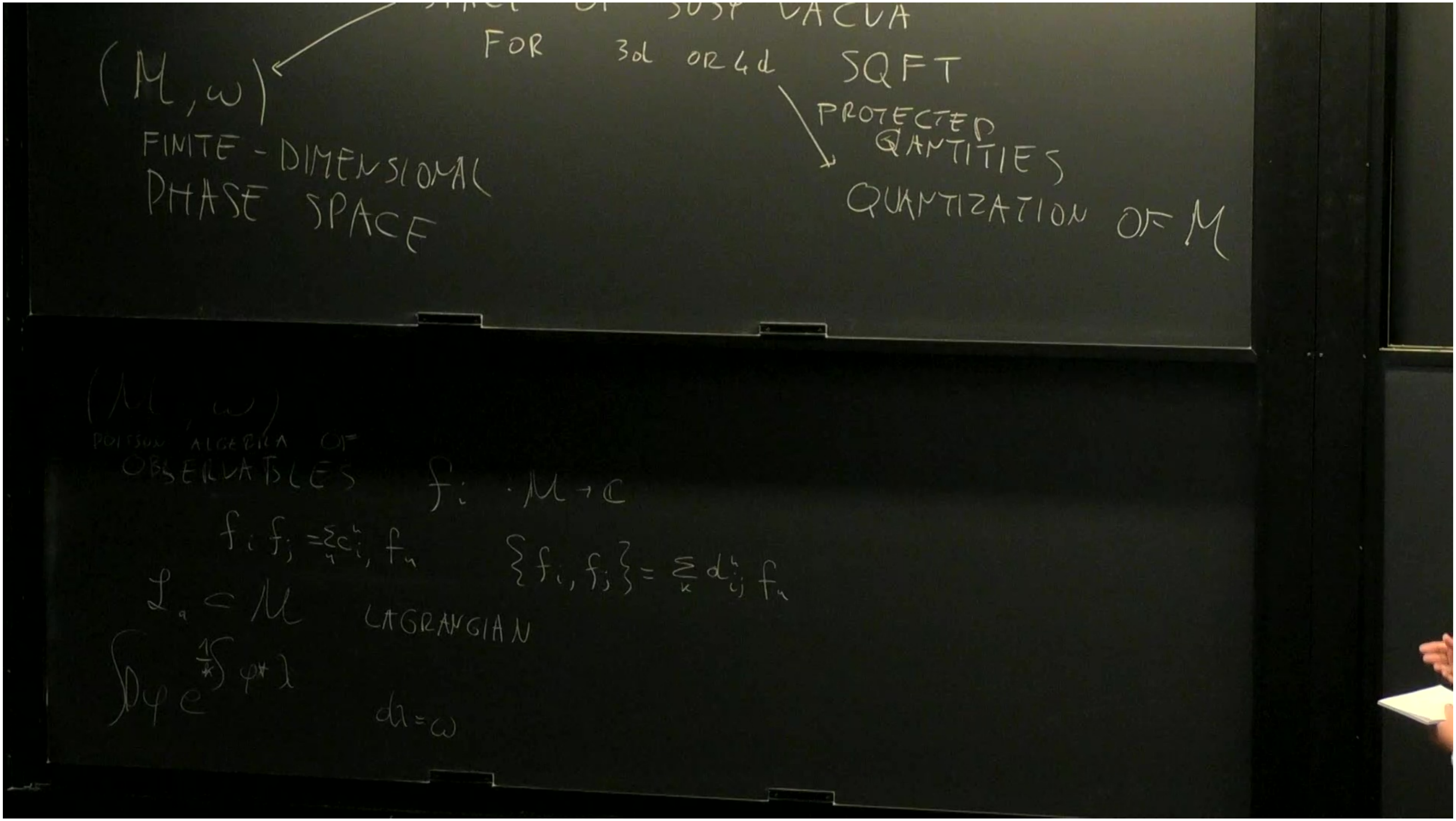
SPACE OF SUSY VACUA
FOR 3d OR 4d SQFT

$(\mathcal{M}, \omega)$

FINITE - DIMENSIONAL
PHASE SPACE

PROTECTED
QUANTITIES

QUANTIZATION OF $\mathcal{M}$


SUSY VACUA
FOR 3d OR 4d SQFT
(M, ω)
FINITE-DIMENSIONAL PHASE SPACE
PROTECTED QUANTITIES
QUANTIZATION OF M
(M, ω)
POISSON ALGEBRA OF OBSERVABLES $f_i : M \to \mathbb{C}$
$f_i f_j = \sum_k c^k_{ij} f_k$
$\{f_i, f_j\} = \sum_k d^k_{ij} f_k$
$\mathcal{L}_a \subset M$ LAGRANGIAN
$\int D\varphi \, e^{\frac{1}{\hbar}\int \varphi^* \lambda}$
$d\lambda = \omega$

SY VACUA

R 4d SQFT

PROTECTED QANTITIES

QUANTIZATION OF $\mathcal{M}$

$\mathcal{H}$ HILBERT SPACE

$\hat{}$ALGEBRA$\hat{}$ $A_\hbar$

$\hat{f}_i$

$\hat{f}_i \hat{f}_j = \sum_k \hat{c}^k_{ij}(\hbar) \hat{f}_k$

$\hat{c}^k_{ij} = c^k_{ij} + i\hbar d^k_{ij} + \dots$

$d^k_{ij} f_k$

$|\mathcal{L}_a\rangle$

$\mathcal{M} = T^*N \qquad (p,q) \in M$

$f_i = f_i^{(0)}(q) + p \cdot f^{(1)}(q) + p^2 \cdot f^{(2)}(q) \cdot$

$\hat{f}_i = \hat{f}^{(0)} + \hat{f}^{(1)}(-i\hbar\partial) + \dots$

$H = L^2(N, \sqrt{\mathrm{vol}}) \ni \psi(q)\sqrt{\wedge dq^i}$

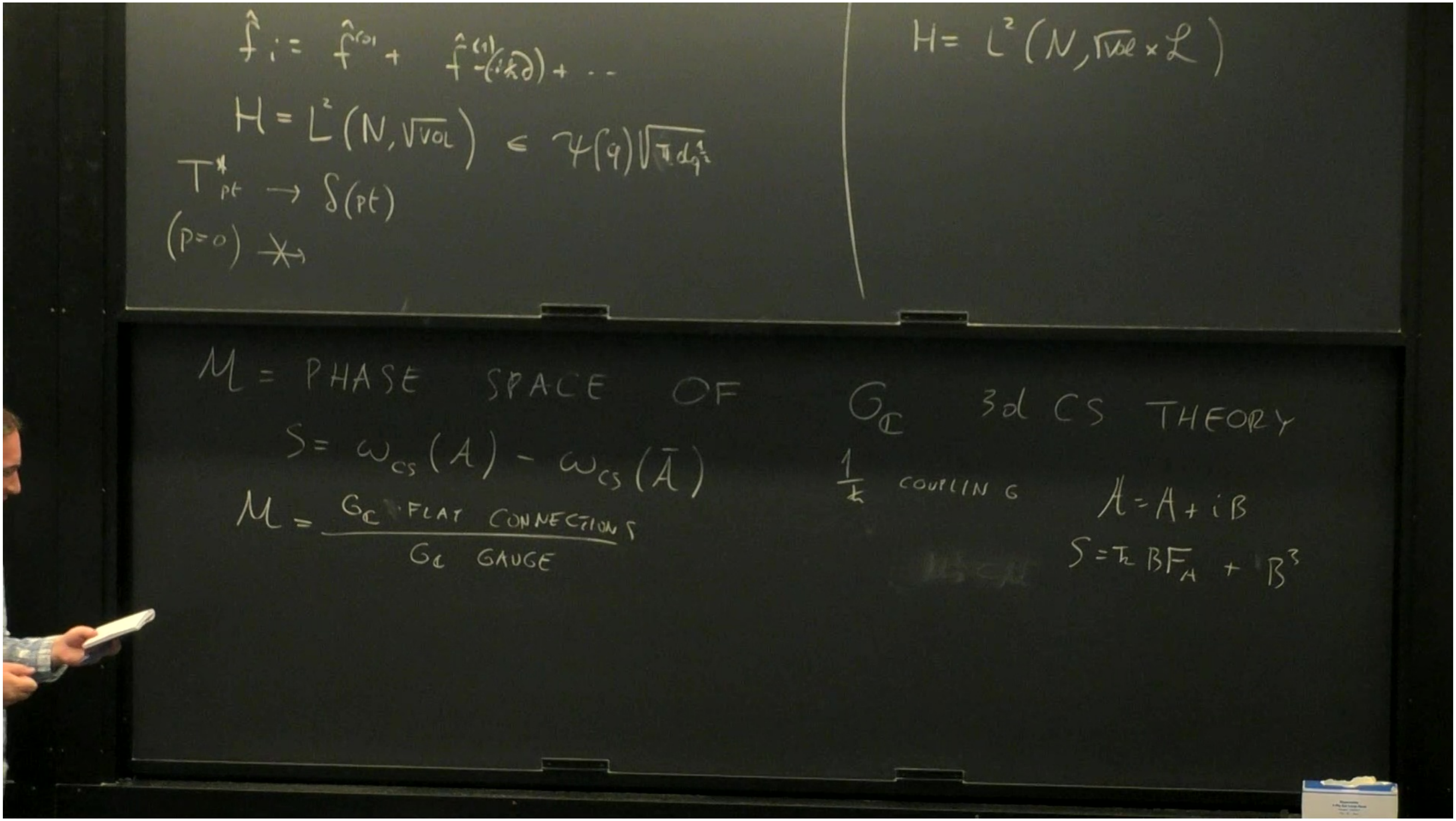
$T^*_{pt} \to \delta(pt)$

$(p=0) \not\mapsto$

TWISTED COTANGENT

$p' = \frac{\partial q}{\partial q'} p + c(q)$

$H = L^2(N, \sqrt{\mathrm{vol}} \times \mathcal{L})$


$\hat{f}_i = \hat{f}^{(0)} + \hat{f}^{(1)} (-i\hbar\partial) + \dots$
$H = L^2(N, \sqrt{\mathrm{VOL}}) \ni \psi(q)\sqrt{\Pi\, dq^i}$
$T^*_{pt} \to \delta(pt)$
$(p=0) \not\to$
$H = L^2(N, \sqrt{\mathrm{vol}} \times \mathcal{L})$
$\mathcal{M}$ = PHASE SPACE OF $G_{\mathbb{C}}$ 3d CS THEORY
$S = \omega_{CS}(\mathcal{A}) - \omega_{CS}(\bar{\mathcal{A}})$
$\frac{1}{k}$ COUPLING
$\mathcal{M} = \frac{G_{\mathbb{C}} \text{ FLAT CONNECTIONS}}{G_{\mathbb{C}} \text{ GAUGE}}$
$\mathcal{A} = A + iB$
$S = \mathrm{Tr}\, BF_A + B^3$

$\hat{f}_i := \hat{f}^{(0)} + \hat{f}^{(1)}(-i\hbar\partial) + \dots$

$H = L^2(N, \sqrt{\mathrm{VOL}}) \in \Psi(q)\sqrt{\pi\, dq}$

$T^*_{pt} \to \delta(pt)$

$(p=0) \not\to$

$H = L^2(N, \sqrt{\mathrm{vol}} \times \mathcal{L})$

$S = \omega_{CS}(\mathcal{A}) - \omega_{CS}(\bar{\mathcal{A}})$

$\frac{1}{k}$ COUPLING

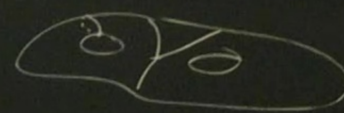
$\mathcal{A} = A + iB$

$\mathcal{M} = \dfrac{G_{\mathbb{C}} \text{ FLAT CONNECTIONS ON 2d SURFACE } C}{G_{\mathbb{C}} \text{ GAUGE}}$

$S = \mathrm{Tr}\, B F_A + B^3$

$W_{\ell,R} = \mathrm{Tr}_R\, P\exp \int_\ell \mathcal{A}$ $\quad \widetilde{W}_{\ell,R} = \mathrm{Tr}_R\, P\exp \int_\ell \bar{\mathcal{A}}$

$\widetilde{W}_{\ell,R} = W^{\dagger}_{\ell^{-1},R}$ $\quad W_a$ $\quad \widetilde{W}_a = W^{\dagger}_{\rho(a)}$



$\omega = \mathrm{Im}\, \Omega$

$\Omega = \int \delta\mathcal{A} \wedge \delta\mathcal{A}$

$\hat{f} := \hat{f}^{(0)} + \hat{f}^{(1)}(i\hbar\partial) + \dots$

$H = L^2(N, \sqrt{\mathrm{VOL}}) \ni \psi(q)\sqrt{\Pi\, dq^i}$

$T^*_{pt} \to \delta(pt)$

$(p=0) \not\to$

$H = L^2(N, \sqrt{\mathrm{vol}} \times \mathcal{L})$

$S = \omega_{CS}(\mathcal{A}) - \omega_{CS}(\bar{\mathcal{A}})$ $\quad \frac{1}{k}$ COUPLING $\quad \mathcal{A} = A + iB$

$$\mathcal{M} = \frac{G_{\mathbb{C}} \text{ FLAT CONNECTIONS ON 2d SURFACE } C}{G_{\mathbb{C}} \text{ GAUGE}}$$

$S = \mathrm{Tr}\, BF_A + B^3$

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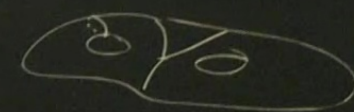
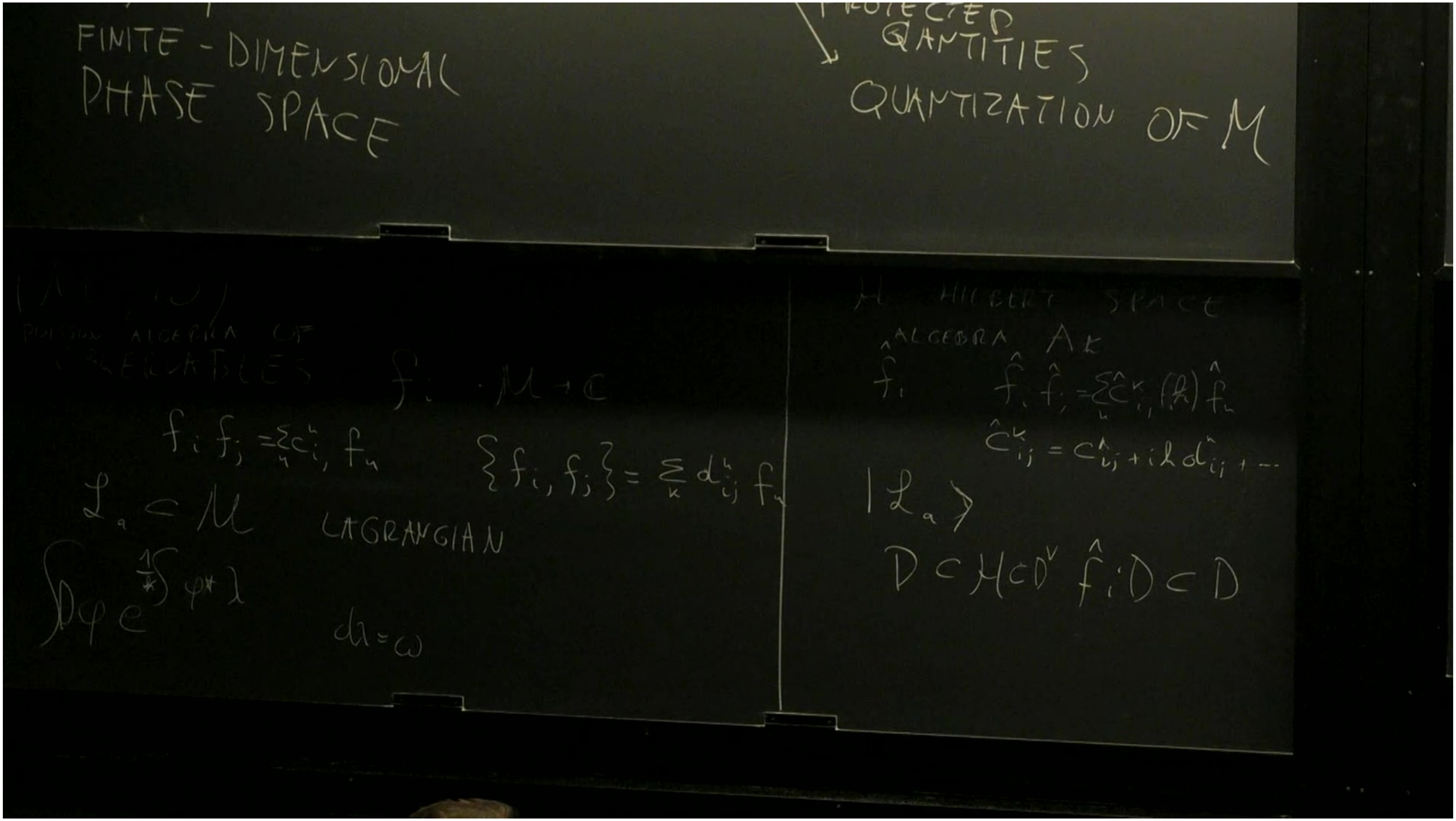


$\tilde{W}_{\ell,R} = W^\dagger_{\ell',R}$ $\quad W_a$ $\quad \tilde{W}_a = W^\dagger_{\ell(a)}$

$\omega = \mathrm{Im}\,\Omega$ $\quad \Omega = \int \delta\mathcal{A} \wedge \delta\mathcal{A}$

$M_3 \mid \partial M_3 = C$ $\quad \mathcal{L}(M_3) \subset \mathcal{M}$


PROTECTED QUANTITIES
FINITE-DIMENSIONAL PHASE SPACE
QUANTIZATION OF M
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ALGEBRA $A_\hbar$
$\hat{f}_i \quad \hat{f}_i \hat{f}_j = \sum_k \hat{c}^k_{ij}(\hbar) \hat{f}_k$
$\hat{c}^k_{ij} = c^k_{ij} + i\hbar d^k_{ij} + \dots$
$|\mathcal{L}_a\rangle$
$D \subset \mathcal{H} \subset D^\vee \quad \hat{f}_i : D \subset D$

FINITE - DIMENSIONAL PHASE SPACE

PROTECTED QUANTITIES

QUANTIZATION OF $\mathcal{M}$

EQUIP C W/ COMPLEX STRUCTURE τ

$$A = A_{(1,0)} + A_{(0,1)} \qquad \Psi[A_{(0,1)}]$$

$\mathcal{M} \approx$ TWISTED COTANGENT TO $\mathrm{BUN}(G,C) = \{ A_{(0,1)}$ MOD GAUGE $\}$

$$\mathcal{H} = L^2\left(\mathrm{BUN}, |K|^{1+\frac{2i}{\hbar}}\right)$$

$|M_3\rangle$

FINITE-DIMENSIONAL PHASE SPACE

QUANTITIES

QUANTIZATION OF $\mathcal{M}$

EQUIP C W/ COMPLEX STRUCTURE τ

$A = A_{(1,0)} + A_{(0,1)}$ $\qquad \Psi[A_{(0,1)}]$

$\mathcal{M} \approx$ TWISTED COTANGENT TO $\{ A_{(0,1)}$ MOD GAUGE $\}$ $=$ BUN(G,C)

$\mathcal{H} = L^2\left(\mathrm{BUN}, |K|^{1+\frac{2i}{\hbar}}\right)$

$|M_3\rangle$

$M = L(N, \sqrt{VOL}) \in \mathcal{F}(q)\sqrt{\prod dq^{\frac{1}{2}}}$

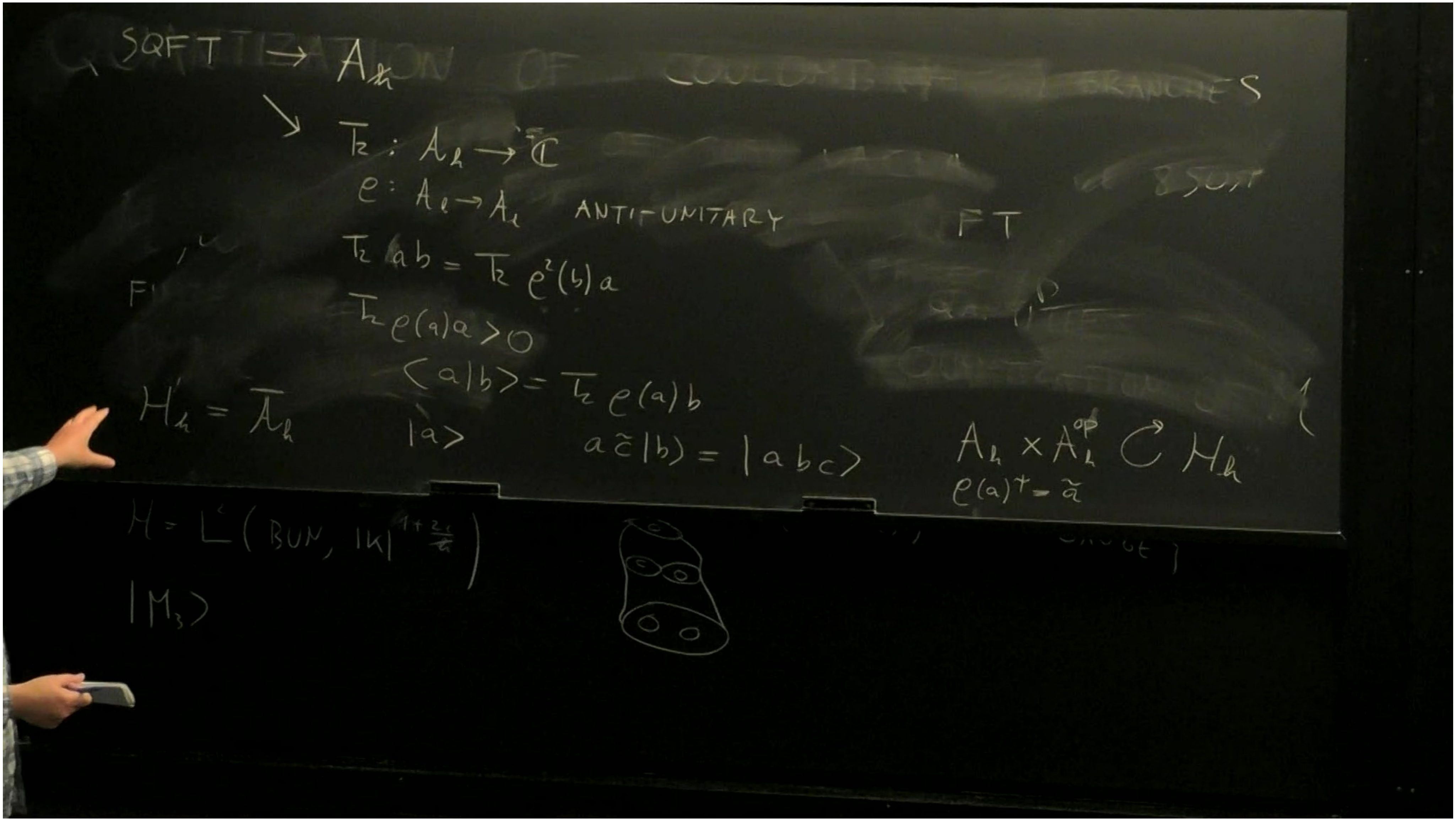
$T^*_{pt} \to \delta(pt)$

$(p=0) \not\to \qquad D = C^\infty(N)$

$$\mathcal{M}^{ADE}(G,C) = \text{“3d” (“ON } S^1 \times R^3\text{”) COULOMB BRANCH OF “K-THEORETIC” 4d } N=2 \text{ SQFT OF CLASS } S$$

$\downarrow$ SCHUR INDICES

QUANTIZATION OF $\mathcal{M}(G,C)$



SQFT $\to A_\hbar$
$\mathrm{Tr} : A_\hbar \to \mathbb{C}$
$\rho : A_\hbar \to A_\hbar$ ANTI-UNITARY
$\mathrm{Tr}\, ab = \mathrm{Tr}\, \rho^2(b)\, a$
$\mathrm{Tr}\, \rho(a) a > 0$
$\langle a|b\rangle = \mathrm{Tr}\, \rho(a)\, b$
$H_\hbar = \overline{A_\hbar}$
$|a\rangle$
$a\tilde{c}|b\rangle = |abc\rangle$
$A_\hbar \times A_\hbar^{op} \circlearrowright H_\hbar$
$\rho(a)^\dagger = \tilde{a}$
$H = L^2(\mathrm{BUN}, |K|^{1+\frac{2i}{\hbar}})$
$|M_3\rangle$

$\mathcal{H} = L(N, \sqrt{\mathrm{vol}}) \in \mathcal{V}(q) \sqrt{\prod dq^{\frac{1}{2}}}$

$T^*_{pt} \to \delta(pt)$

$(p=0) \not\to \qquad D = C^{\infty}(N)$

$\mathcal{M}^{ADE}(G, C) =$ "ON $S^1 \times R^3$" "3d" COULOMB BRANCH OF "K-THEORETIC" 4d N=2 SQFT OF CLASS S

$\downarrow$ SCHUR INDICES

QUANTIZATION OF $\mathcal{M}(G,C)$

4d : 6d ON C

4d ON S^1 : 6d ON $C \times S^1$

= 5d SYM ON C

SQFT T $\to A_\hbar$ ... COULOMB ... BRANCHES

$\searrow \quad \mathrm{Tr}: A_\hbar \to \mathbb{C}$

$\rho: A_\hbar \to A_\hbar$ ANTI-UNITARY

$\mathrm{Tr}\, ab = \mathrm{Tr}\, \rho^2(b)\, a$

$\mathrm{Tr}\, \rho(a)\, a > 0$

$\langle a|b\rangle = \mathrm{Tr}\, \rho(a)\, b$

$\mathcal{H}'_\hbar = \bar{A}_\hbar \qquad |a\rangle \qquad a\tilde{c}|b\rangle = |abc\rangle$

BOUNDARY B FOR T

$\langle B|a\rangle$

$\langle B| \in A_\hbar^\vee$

$A_\hbar \times A_\hbar^{op} \circlearrowright \mathcal{H}_\hbar$

$\rho(a)^\dagger = \tilde{a}$

$\mathcal{H} = L^2(\mathrm{BUN}, |K|^{1+\frac{2i}{\hbar}})$

$|M_3\rangle$

$H'_\hbar = \bar{A}_\hbar$ $\quad \langle a|b\rangle = \mathrm{Tr}\, \rho(a) b$ $\quad |a\rangle$ $\quad a\tilde{c}|b) = |abc\rangle$

$A_\hbar \times A_\hbar^{op} \circlearrowright$

$\rho(a)^\dagger = \tilde{a}$

4d $N=2$ SQFT SHUR INDEX

$I(q) = \mathrm{Tr}_{Ops} (-1)^F q^{J_3 - I_3}$

$I_{a_1 \ldots a_n}(q)$

$I_a(q) \equiv \mathrm{Tr}\, a$

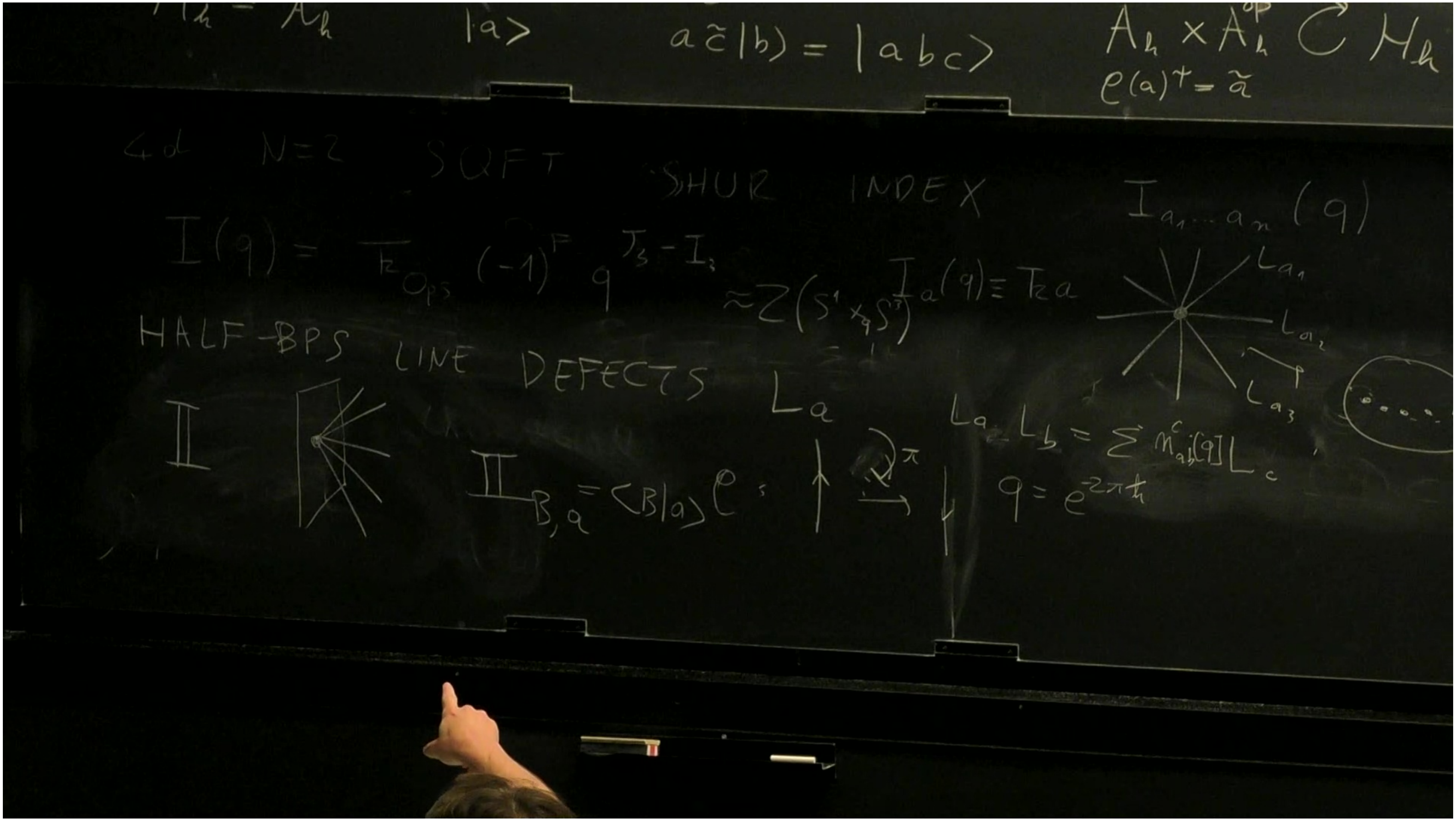
HALF-BPS LINE DEFECTS L_a

I

$\mathrm{II}_{B,a} = \langle B|a\rangle \rho$

$L_a L_b = \sum n_{ab}^c[q] L_c$

$q = e^{-2\pi\hbar}$



$|a\rangle$
$a\tilde{c}|b) = |abc\rangle$
$A_\hbar \times A_\hbar^{op} \circlearrowright \mathcal{H}_\hbar$
$\rho(a)^\dagger = \tilde{a}$
4d N=2 SQFT SHUR INDEX
$I(q) = \mathrm{Tr}_{Ops}(-1)^F q^{J_3 - I_3} \approx Z(S^1 \times_q S^3)$
$I_a(q) \equiv \mathrm{Tr}\, a$
$I_{a_1 \dots a_n}(q)$
L_{a_1}
L_{a_2}
L_{a_3}
HALF-BPS LINE DEFECTS
L_a
I
II
$\mathrm{II}_{B,a} = \langle B|a)\,\rho$
$L_a L_b = \sum n_{ab}^c[q] L_c$
$q = e^{-2\pi\hbar}$