Title: Phase transitions out of quantum Hall states in moire bilayers

Speakers: Senthil Todadri

Series: Quantum Matter

Date: September 19, 2023 - 11:00 AM

URL: https://pirsa.org/23090107

Abstract: Quantum Hall phases are the most exotic experimentally established quantum phases of matter.Recently they have been discovered at zero external magnetic field in two dimensional moire materials. I will describe recent work (with Xue-Yang Song and Ya-Hui Zhang) on their proximate phases and associated phase transitions that is motivated by the high tunability of thede moire systems. These phase transitions (and some of the proximate phases) are exotic as well, and realize novel 'beyond Landau' criticality that have been explored theoretically for many years. I will show that these moiré platforms provide a great experimental opportunity to study these unconventional phase transitions and related unconventional phases, thereby opening a new direction for research in quantum matter.

Zoom link : https://pitp.zoom.us/j/97483204701?pwd=S2x4ck9tNHFjM0RiTDNWNFhaMk9SUT09

Phase transitions out of quantum Hall phases in moire bilayers

T. Senthil (MIT)

Simons Foundation

Collaborators

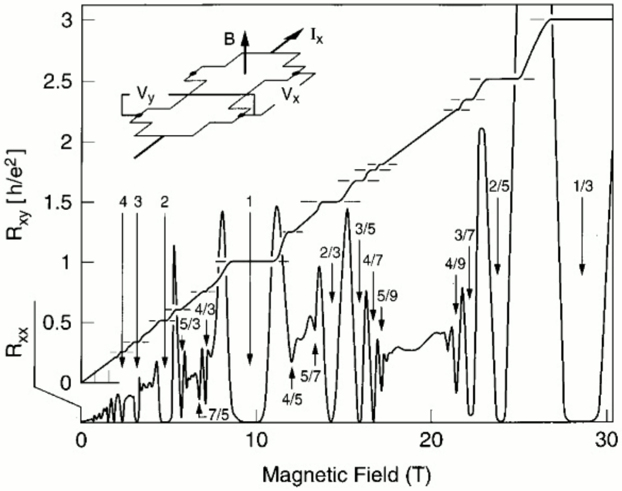
Xue-Yang Song, MIT postdoc

Ya-Hui Zhang, Johns-Hopkins

Classic (fractional) quantum Hall effect

Electrons in 2d in a strong B-field

Landau level physics + Coulomb interactions (+ disorder)



Integer quantum Hall effect without B-field

Lattice systems: fill an integer number of bands with net non-zero Chern number

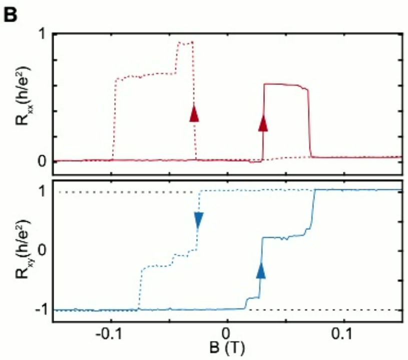
Thouless et al 1983;
Haldane 1988
(2016 Nobel)

``Chern insulator"

Special case: Integer quantum anomalous Hall effect

Chern insulator is obtained by spontaneous breakdown of time reversal symmetry

First observation: magnetically doped topological insulator films



More modern example from a moire graphene material
Serlin, …..Young, Science 2020

Fractional quantum Hall effect without B-field

Fractional filling of a Chern band to realize a fractional quantum hall effect

Theoretical possibility and demonstration in toy models:
Many theoretical papers from ~ 2011

K. Sun, Z. Gu, H. Katsura, and S. D. Sarma, PRL (2011).
D. Sheng, Z.-C. Gu, K. Sun, and L. Sheng, Nature communications (2011).
T. Neupert, L. Santos, C. Chamon, and C. Mudry, PRL (2011).
Y.-F. Wang, Z.-C. Gu, C.-D. Gong, and D. Sheng, PRL (2011).
E. Tang, J.-W. Mei, and X.-G. Wen, PRL (2011). N. Regnault and B. A. Bernevig, PRX (2011).

…

Suitable experimental platforms?? Why interesting??

Rapid progress last few years

Theory (2018 - now) : look for it in moire materials!

Experiment:
2021: Twisted bilayer graphene
FQH states at low field B $\approx 5\ T$ (Flux through unit cell $\ll$ flux quantum)

2023: Twisted transition metal dichalcogenides

More to come?

The era of correlated moire materials

Two monolayers with slightly different (orientation, lattice constant,…..) lattices form a long period moire pattern.

Most famous example: twisted bilayer graphene

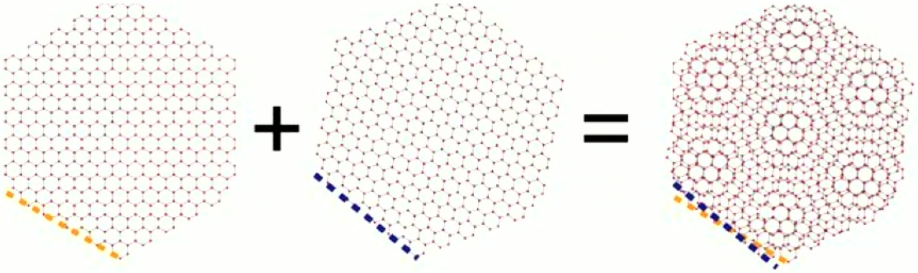
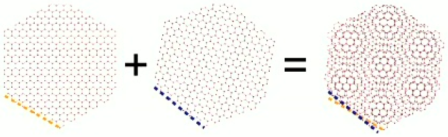


Fig Credit: Kim et al, PNAS 2017

Some other routes to moire graphene materials

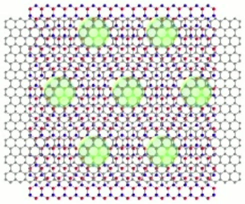
1. Twist n-layer graphene to a magic angle relative to another m-layer graphene

``Classic'': n = m = 1 (twisted bilayer); other experiments: n = m = 2 (twisted double bilayer); n = 1, m = 2 (twisted monolayer-bilayer)



2. n-layer graphene aligned with a hexagonal Boron Nitride (h-BN) substrate

Slight lattice mismatch between graphene and h-BN leads to a large period moire pattern.



Band topology in many moire graphene systems*: Nearly flat Chern bands

Zhang, Mao, Cao, Jarillo-Herrero, TS, 2019 (arXiv,1805: 08232)

Flat bands of two valleys have equal and opposite Chern number C.

ABC-trilayer graphene/h-BN: C = ±2 or ±3 (for one sign of perpendicular displacement field)

Twisted bilayer aligned with h-BN: C = ±1 *(also Bultinck, Chatterjee, Zalatel 19; Zhang, Mao, TS, 19)*

Twisted double bilayer: C = ±2 *(also Lee,…., Vishwanath 19)*

*Systems without C_2T symmetry

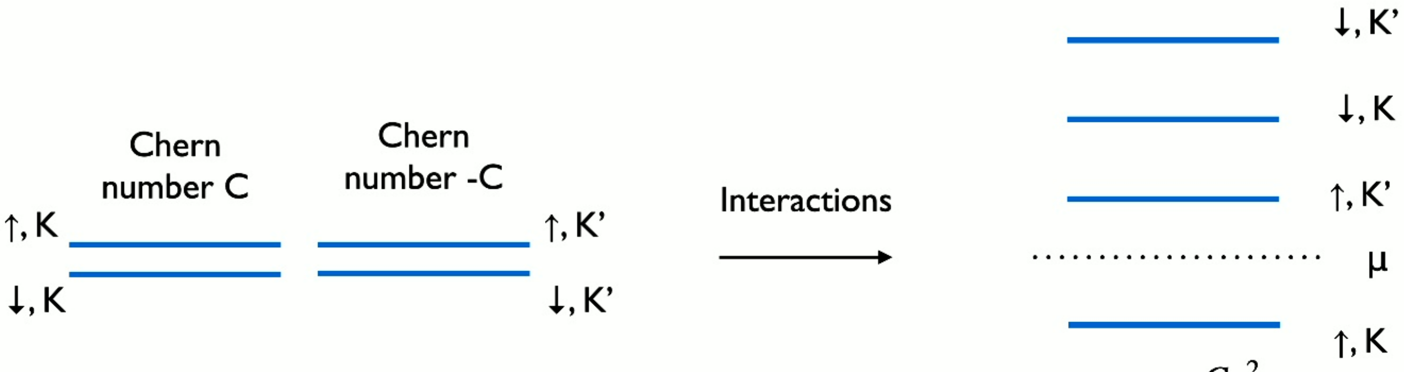
Strong correlations in moire bands

Zhang, Mao, Cao, Jarillo-Herrro, TS 2019; also Zhang, Mao, TS, 2019; Bultinck, Chatterjee, Zaletel, 2020, Liu, Dai 2021.

Focus on total band fillings $\nu_T = 1, 3$ (including spin/valley)

Hartree-Fock theory in flat band limit:

Spin and valley ferromagnetic order to fully fill a band (``Flat band ferromagnetism") and form a Chern insulator



Chern number C

Chern number -C

↑, K

↓, K

↑, K'

↓, K'

Interactions

↓, K'

↓, K

↑, K'

μ

↑, K

$$\sigma_{xy} = \frac{Ce^2}{h}$$

Fractional Quantum Anomalous Hall (FQAH) effect in moire graphene

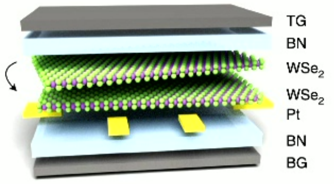
TBLG aligned with hBN: Valley polarization to break time reversal and populate one Chern band

Theory: Numerical calculations/analytic arguments show possibility of FQAH phase with a reasonable gap.

(Repellin, TS, 2020; Ledwith,….,Vishwanath 2020, Bergholtz et al 2020)

Experiments (so far): Xie,……Yacoby 2021
FQH states at small B-fields

Moire Transition Metal Dichalcogenides (TMD)



TMDs: WSe_2, WS_2, $MoTe_2$,……

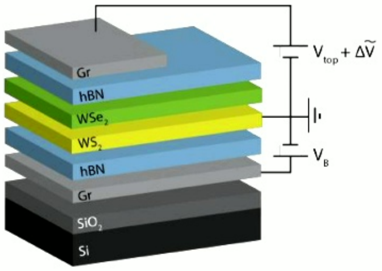
Moire structure: Stack two TMD monolayers with some small difference (Eg, lattice constant mismatch, or twist angle)

Strongly spin-orbit coupled: Spin is locked to valley at a high energy scale

Strong correlated electrons in a single band in each of two valleys

In some cases the bands of the two valleys have equal and opposite Chern number

d



Can tune ratio of interactions/bandwidth electrically using a perpendicular displacement field

Fractional Quantum Anomalous Hall (FQAH) effect in moire graphene

TBLG aligned with hBN: Valley polarization to break time reversal and populate one Chern band

Theory: Numerical calculations/analytic arguments show possibility of FQAH phase with a reasonable gap.

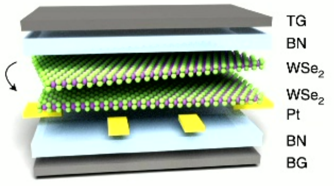
(Repellin, TS, 2020; Ledwith,….,Vishwanath 2020, Bergholtz et al 2020)

Experiments (so far): Xie,……Yacoby 2021
FQH states at small B-fields

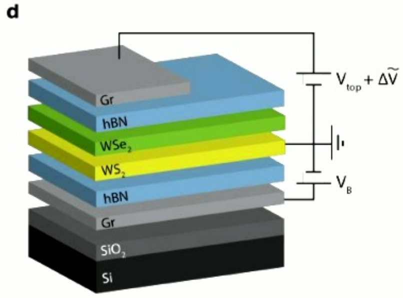
Moire Transition Metal Dichalcogenides (TMD)

TMDs: WSe_2, WS_2, $MoTe_2$,……

Moire structure: Stack two TMD monolayers with some small difference (Eg, lattice constant mismatch, or twist angle)



Strongly spin-orbit coupled: Spin is locked to valley at a high energy scale



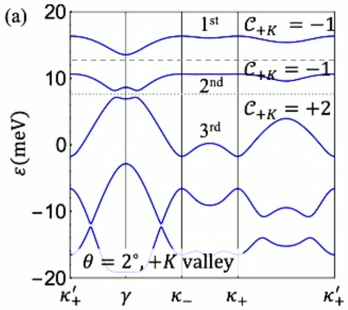
Strong correlated electrons in a single band in each of two valleys

In some cases the bands of the two valleys have equal and opposite Chern number

Can tune ratio of interactions/bandwidth electrically using a perpendicular displacement field

Nearly flat moire Chern bands in TMD homobilayers

Early (pre-experiment) theory: Top most valence bands have nearly flat bands with valley Chern number ± 1 in a range of twist angles near 1.5 deg (Wu,…., MacDonald 2019; Yu, Chen, Yao, 2020; Devakul, …., Fu 2021)

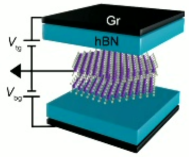


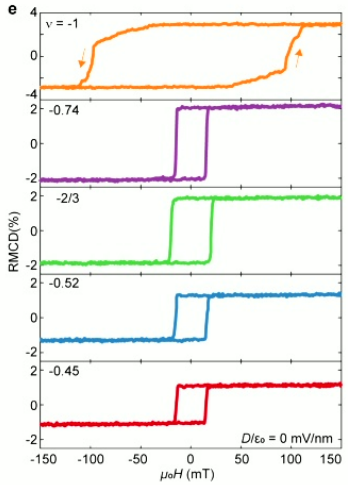
Wu,…., MacDonald 2019

Many body calculations (Li,….Lin 2021; Crepel, Fu, 2023)

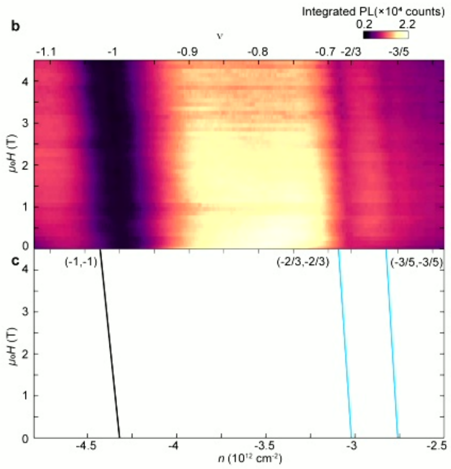
1. Ferromagnetism through valley polarization
2. Evidence for FQAH states

Fractional Quantum Anomalous Hall effect in twisted MoTe2: First signature





Evidence for ferromagnetism
(Valley polarization)



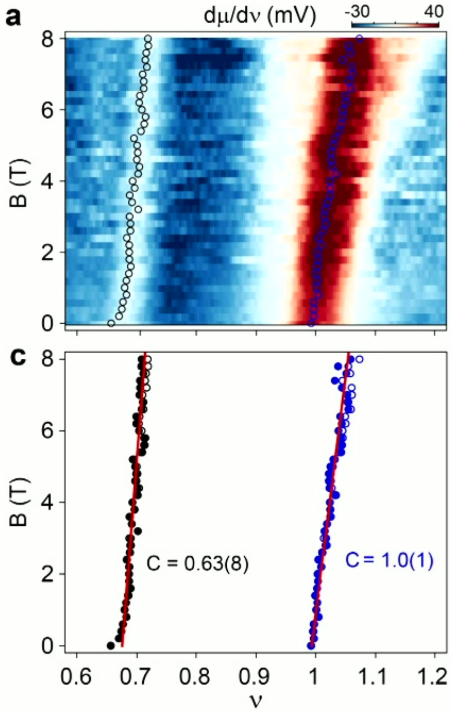
Location of many body gaps (inferred through an optical technique) in n-B plane

Streda formula $\sigma_{xy} = \dfrac{dn}{dB}$

Found at twist angle near 3.5 deg (different from original band theory suggestion)

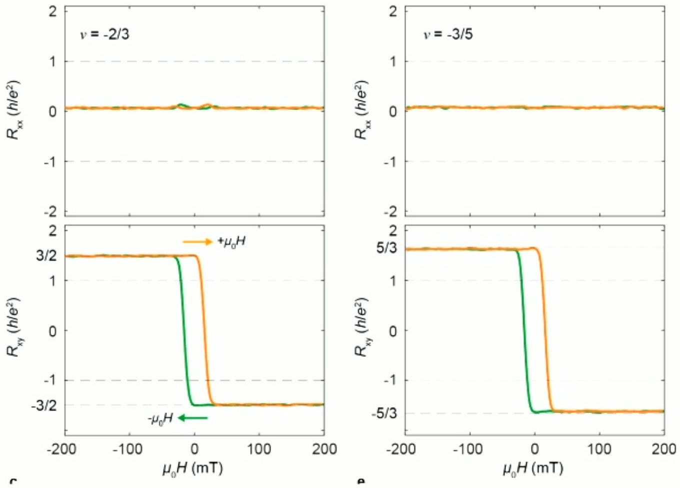
Cai,….., Xiaodong Xu, 2023

Fractional Quantum Anomalous Hall effect in twisted MoTe2: Evidence from compressibility



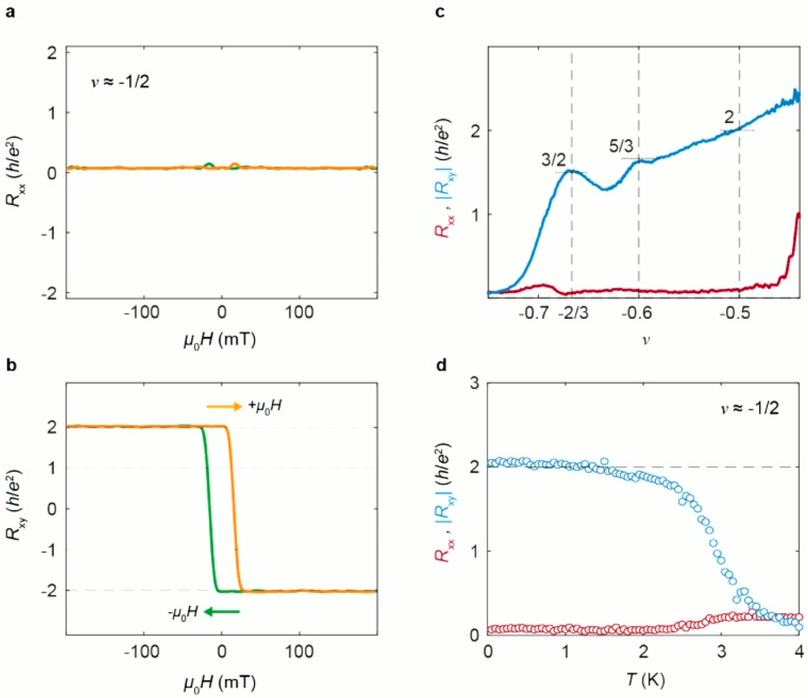
Zeng,…….Mak, Shan 2023

Fractional Quantum Anomalous Hall effect in twisted MoTe2: Transport



Park,……, Xiaodong Xu, 2023

Evidence for B = 0 composite fermi liquid at half-filling



Park,……, Xiaodong Xu, 2023

Some new questions/opportunities

Most striking feature of moire materials - tunable

`Material' parameters can be tuned electrically

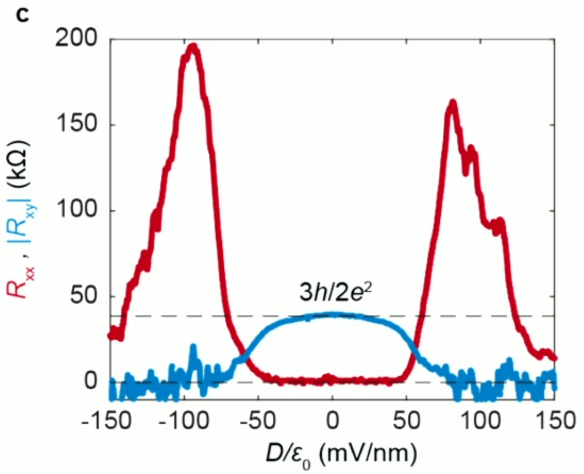
- bandwidth W and hence correlation strength
- electron density in moire unit cell

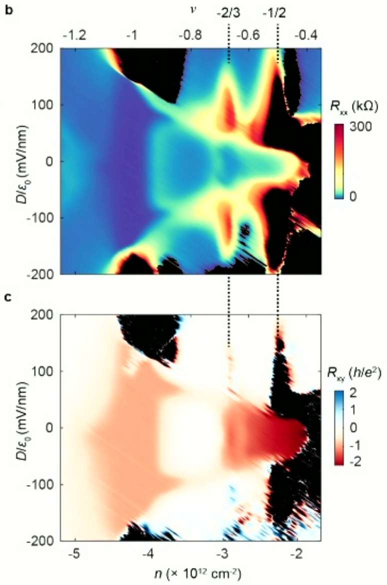
Hope of

- `moire engineering' of phases proximate to quantum Hall
- exploration of quantum phase transitions

These are very likely to be exotic with emergent gapless gauge fields and associated fractionalized excitations (deconfined critical points/phases).

Tuning out of quantum Hall phases in experiments





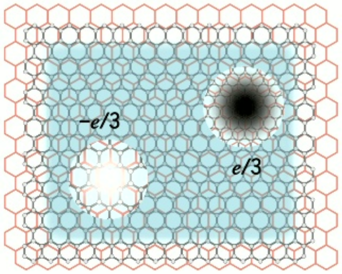
Tune displacement field to get out of quantum Hall into competing phases

Some obvious competitors to FQAH phases

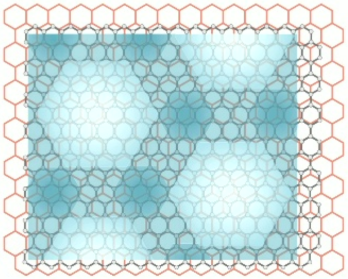
Ferromagnetism survives beyond FQAH in experiments, so assume competition happens when only one valley is populated.

1. At commensurate lattice filling, CDW insulators

2. (With increasing bandwidth) Landau Fermi liquid

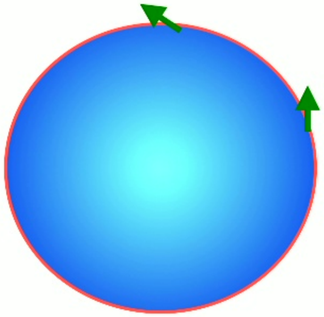


FQAH



CDW insulator

Fig credit: Repellin and Regnualt, 2018



Fermi liquid

Outline

1. Competition of FQAH and CDW ordering

- describe by a `quantum Ginzburg-Landau' theory that can be used to describe the phase transition.
- emergence of proximate CDW* phases with neutral topological order

2. Evolution from composite Fermi liquid- Landau Fermi liquid

- theory and some experimental predictions

General parton framework

Write electron operator as $c = \Phi f$

Φ: bosonic parton that carries charge of electron
f: neutral fermionic parton

Gauge redundancy: $\Phi \rightarrow \Phi e^{i\alpha}, \quad f \rightarrow f e^{-i\alpha}$

Φ and f both have same density (lattice filling) as the electron

Can reformulate the system in terms of Φ and f (instead of the original c-operators) but must include a dynamical U(1) gauge field a.

Parton framework

Schematically we can work with a Lagrangian

$$\mathscr{L} = \mathscr{L}[\Phi, A + a] + \mathscr{L}[f, -a]$$

and discuss the phase diagram/transitions of the electronic system.

(A = external probe U(1) gauge field)

Exploring the phase diagram

$$\mathscr{L} = \mathscr{L}[\Phi, A + a] + \mathscr{L}[f, -a]$$

(A = external probe U(1) gauge field)

Suppose we are interested in exploring the vicinity of some `base' phase P.

Put the parton f itself in the phase P.

If Φ forms a superfluid, $\langle \Phi \rangle \neq 0$, then a ≈ -A i.e, dynamical gauge field is `Higgsed'.

Effective Lagrangian $\mathscr{L} = \mathscr{L}[f, A]$ just describes the electronic system in the phase P.

We can move out of phase P to phases P' by letting the bosons Φ move out of a superfluid.

Example: Fermi liquids and Mott insulators

Put f in a state with a Fermi surface

Fermi liquid: superfluid of Φ
Physical electron c $\sim \langle \Phi \rangle f$, i.e, f just becomes the electron.

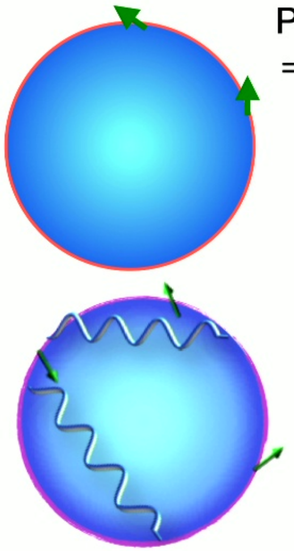
Moving out of the Fermi liquid:
Lattice filling of 1 (spinful) electron per site => Φ is at lattice filling 1.

Put Φ in a (bosonic) Mott insulator => at low energies get an electronic Mott insulator but where the spins form a quantum spin liquid with a neutral fermi surface

Continuous electronic Mott transition

$$\mathscr{L} = \mathscr{L}[\Phi, A + a] + \mathscr{L}[f, -a]$$

(A = external probe U(1) gauge field)



Phase P = Fermi liquid

Phase P' = Mott with spinon fermi surface

Superfluid-insulator transition of Φ in the presence of gauge fields and Fermi surface of f.

Problem is tractable (gauge fields and f-fermions essentially decouple from critical Φ-sector).

Many concrete predictions for experiment, eg, universal jump of residual resistivity

TS, 2008

Plan of attack

Here we are interested in understanding proximate phases and associated phase transitions to FQH phases at rational fractional filling ν_e of a lattice.

Both partons are also at the same filling $\nu_e = \nu_\Phi = \nu_f$

Put f in FQH phase

Φ-superfluid: Electron in FQH phase

Φ in a Mott insulator: Destroy the QH response

Fractional lattice filling: Cannot form a trivial Mott insulator

Simplest option: Φ forms a CDW Mott insulator

FQH-CDW competition gets related to superfluid-CDW competition of Φ

Interlude: competition between superfluid and CDW order for bosons

Rational lattice filling p/q provides important constraints on low energy theory

Eg: LSM restriction - cannot have trivial gapped phase

Naive Landau-Ginzburg theory for superfluid order parameter does not work
(Not faithful to filling constraint; admits a trivial gapped phase)

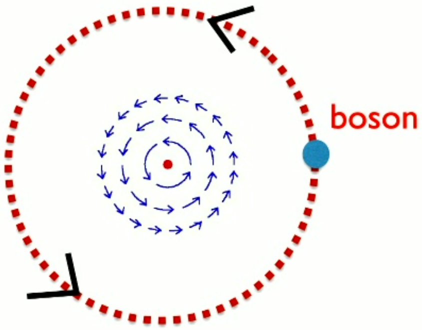
\`Quantum' Landau-Ginzburg theory:

Obtain using duality to a vortex description

Charge-vortex duality for bosons

Alternate description of phases/phase transitions in terms of vortices of superfluid order parameter

In 2d: vortices are point defects.



2π phase

Charge-vortex duality for bosons

A vortex sees

(i) particle density as effective magnetic field $b = 2\pi\rho$

(ii) particle current as effective electric field $\vec{e} = 2\pi(\hat{z} \times \vec{j})$

Fluctuating particle density/current => vortices see fluctuating effective ``electromagnetic field" (represent in terms of effective scalar/vector potentials).

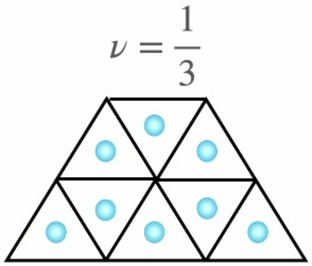
Dual vortex description: Vortices + U(1) gauge field.

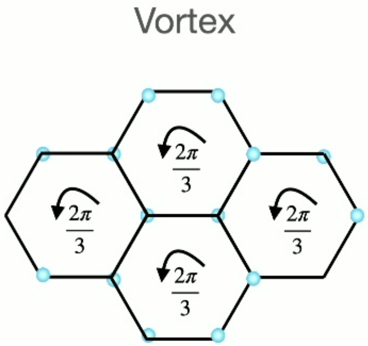
Peskin; Dasgupta, Halperin; Fisher, Lee

Charge-vortex duality for bosons at fractional lattice filling

Lannert, TS, Fisher, 2001, Balents, Bartosch,
Burkov, Sachdev, Sengupta 2005, Burgos, Balents, 2007

Vortices see flux $2\pi/q$ through each lattice plaquette.





Projective action of translation => q degenerate species of vortices Φ_{vI} in dual theory

Superfluid-CDW competition of bosons at fractional lattice filling

Quantum Landau-Ginzburg theory

$$\mathscr{L} = \sum_{I=1}^{q} \mathscr{L}[\Phi_{vI}, b] + \cdots + \frac{1}{2\pi} A db$$

(b = dual gauge field; A = background gauge field)

Φ_{vI} gapped: get superfluid

Φ_{vI} condensed: get Mott insulator but projective translation action gives CDW order.

Theory is

- consistent with filling constraints
- captures both superfluid and CDW orders
- provides a `mean field' starting point for possible Landau-forbidden second order superfluid-CDW transition.

Application to FQAH/CDW competition at fractional filling

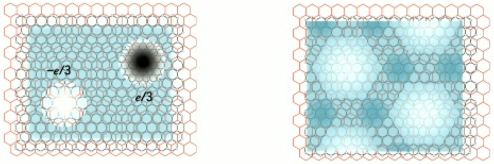


Fig credit: Repellin and Regnualt, 2018

$$\mathscr{L} = \mathscr{L}[\Phi, A + a] + \mathscr{L}[f, -a]$$

(A = external probe U(1) gauge field)

Eg: $\nu = 1/3$: Put f in 1/3-Laughlin state, and let Φ undergo superfluid-CDW insulator transition

$$\mathscr{L}_f = -\frac{3}{4\pi}\alpha d\alpha + \frac{1}{2\pi}\alpha da$$

$$\mathscr{L}_\Phi = \sum_{I=1}^{q} \mathscr{L}[\Phi_{\nu I}, b] + \cdots + \frac{1}{2\pi}(a + A)db$$

Combine to get final theory

$$\mathscr{L} = \sum_{I=1}^{q} \mathscr{L}[\Phi_{\nu I}, b] + \cdots - \frac{3}{4\pi}bdb + \frac{1}{2\pi}Adb$$

Can use to discuss phase transition and other universal aspects of FQAH/CDW competition

Other lattice fillings
Example: $\nu = 2/3$

Particle/hole conjugating theory at 1/3 gives transition from 2/3 FQAH to CDW +IQAH.

Transition to CDW without any quantum Hall response?

Repeat procedure of previous slides with K-matrix of 2/3 state.

Natural CDW state has no quantum Hall response but has a coexisting neutral $(U(1))_2$ topological order (a CDW* phase)

$$\mathscr{L}_f = -\frac{2}{4\pi}\alpha d\alpha$$

Neutral topological order is `dark' - not visible in electrical transport.

Perhaps this state can be stabilized in moire materials??

$\nu = 1/2$:Composite Fermi liquid- Fermi liquid evolution

Evidence for CFL in numerics: Goldman et al, 2023; Dong et al 2023

Experiment: evolve from CFL to FL by tuning displacement field

Theory: use same parton framework (Barkeshli, McGreevy 2014)

$c = \Phi\, f$, and put f in a Fermi surface state

Φ in superfluid: Landau fermi liquid

Φ in $\nu = 1/2$ Laughlin state: get Composite Fermi Liquid

CFL-FL transition: Laughlin-superfluid transition of bosons at $\nu = 1/2$ + coupling to f-fermi surface and gauge field.

Critical theory

2 massless Dirac fermions + U(1) gauge fields

$$\mathscr{L} = \sum_{i=1,2} \bar{\psi}_i(\gamma_\mu(-i\partial_\mu\sigma_0 - \hat{a}_\mu\sigma_0))\psi_i + m_i\bar{\psi}_i\psi_i + \frac{1}{4\pi}d(\hat{a} + A_b)(\hat{a} + A_b) - \frac{1}{4\pi}\hat{a}d\hat{a}$$

CFT in a large-N expansion (possibly true at N = 2 as well).

More basic questions:

1. Bosons are at lattice filling 1/2 => so are the fermions $f_{1,2}$.
How then can the fermions form band insulators?

2. What protects a direct transition where Chern number jumps by 2?

What protects the direct transition?

1. Bosons are at lattice filling 1/2 => so are the fermions $f_{1,2}$.
How then can the fermions form band insulators?

$f_{1,2}$ must see π-flux through each lattice plaquette.
Doubles the unit cell and enables band insulators

2. What protects a direct transition where Chern number jumps by 2?

π-flux => projective action of translation.
Two degenerate fermions at low energy; Dirac band touchings must occur in pairs
Jump of Chern number by 2 is what is natural!

Direct transition protected by lattice translation symmetry alone.
Critical point has power law CDW fluctuations (though no CDW in either phase).

Transport across the transition (in ideal clean system)

Ioffe-Larkin rule for transport in parton theories:

Φ and f resistivity tensors add in series.

Total resistivity $\rho_{ab} = \rho^{\Phi}_{ab} + \rho^{f}_{ab}$

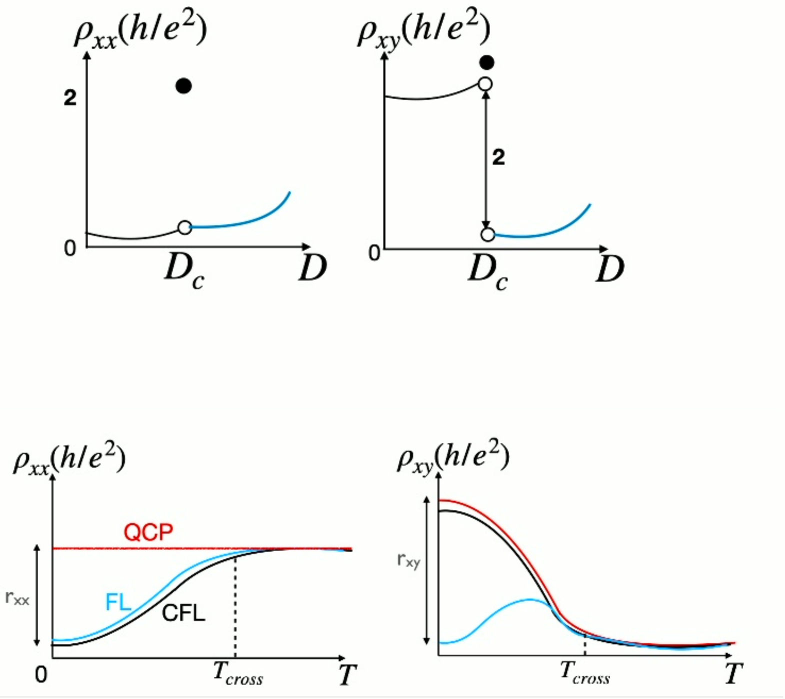
ρ^f smooth across the transition but ρ^{Φ} changes dramatically.

CFL: $\rho^{\Phi} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \frac{h}{e^2}$; FL: $\rho^{\Phi} = 0$

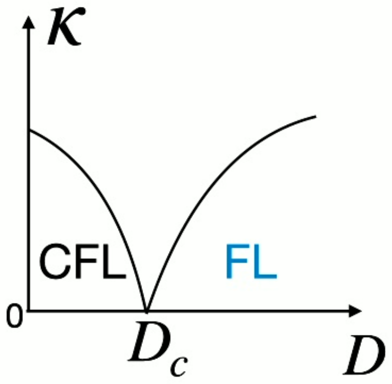
Critical point $\rho^{\Phi}_{ab} = \frac{h}{e^2}\mathscr{R}_{ab}$ with $\mathscr{R}$ a universal tensor.

=> Universal jump of resistivity tensor across CFL-FL transition.

Experimental predictions (clean system): transport



Experimental predictions (clean system): compressibility



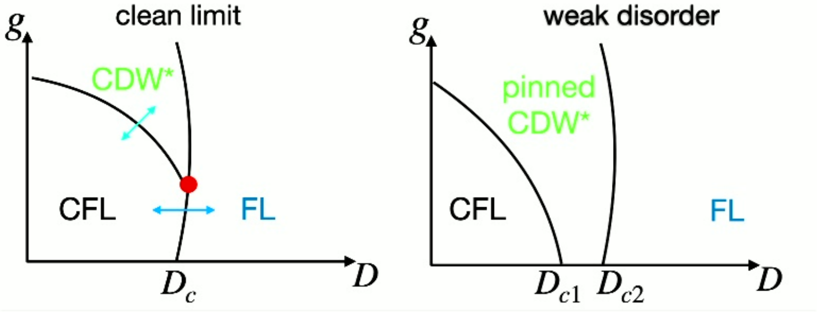
Though both metallic phases are compressible, the critical point has vanishing compressibility (dominated by Φ sector).

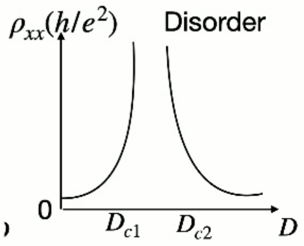
Effect of disorder

Disorder will pin critical CDW fluctuations and introduce an intermediate insulating Phase of the bosons,

However the f-fermions will survive as diffusive neutral excitations.

``Pinned CDW*” phase: insulating electrical transport, local CDW order, but metallic thermal transport more or less unchanged across phase diagram.





Summary

FQH: most exotic experimentally established phases

Recent discovery of FQAH in tunable moire platforms:

Opportunity to probe proximate phases/phase transitions

Theory: inherit some of exotic physics of FQH without experimental signal of (quantized) electrical transport.

Great experimental platform to realize deconfined critical points/phases!