Title: Advancing Stochastic Gravitational Wave Background Detection with Spectrogram Correlated Stacking (SpeCs)

Speakers: Ramit Dey

Series: Strong Gravity

Date: September 21, 2023 - 1:00 PM

URL: https://pirsa.org/23090103

Abstract: A stochastic gravitational wave background (SGWB) originates from numerous faint gravitational wave (GW) signals arising from coalescing compact binary objects. Based on the current estimated merger rate, the SGWB signal is expected to originate from non-overlapping GW waveforms where the chirping nature of individual events is expected to be preserved. In this talk, we present a novel technique, Spectrogram Correlated Stacking (or SpeCs), which goes beyond the usual cross-correlation (and to higher frequencies) by exploiting the higher-order statistics in the time-frequency domain. This method would account for the chirping nature of the individual events that comprise SGWB and enable us to extract more information from the signal due to its intrinsic non-gaussianity. We show that SpeCs improve the signal-to-noise for the detection of SGWB by a factor close to 8, compared to standard optimal cross-correlation methods which are tuned to measure only the power spectrum of the signal. SpeCs can probe beyond the power spectrum and its application to the GW data available from the current and next-generation detectors would speed up the SGWB discovery.

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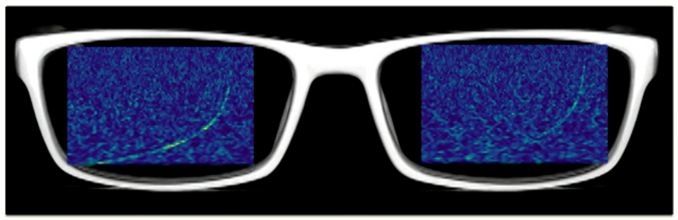
Zoom link: https://pitp.zoom.us/j/91002244803?pwd=a0dnMjZEYTEwSHBCVGRSeHB2Y2pJdz09

Advancing Stochastic Gravitational Wave Background Detection with

SpeCS: Spectrogram Correlated Stacking



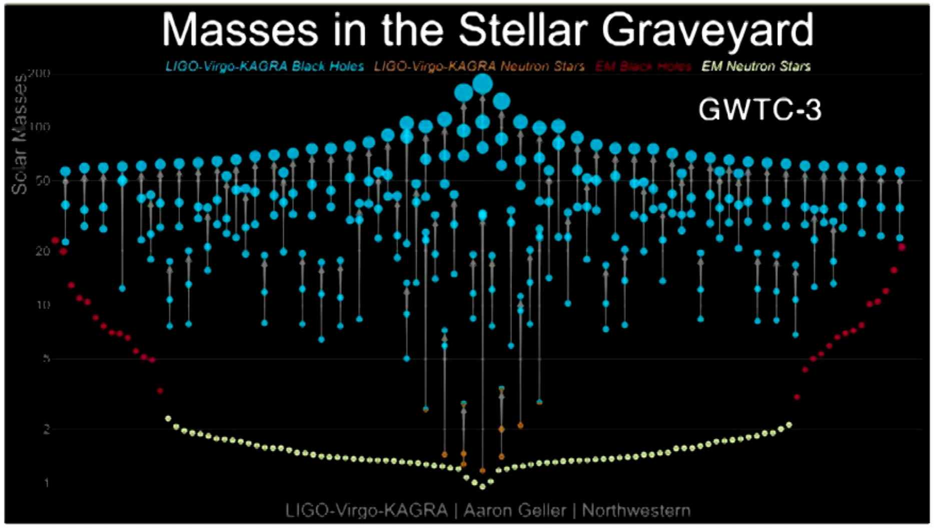
Ramit Dey
Western University

strong gravity seminar
21st Sept 2023

Based on: arXiv:2305.03090

In collaboration with: Luis F Longo, Suvodip Mukherjee, Niayesh Afshordi

Gravitational waves detection



Loud signals detected as individual events by LVK

Sub-threshold events and the stochastic background

Unresolved stellar binary mergers adds up incoherently and produces the stochastic gravitational wave background (SGWB)

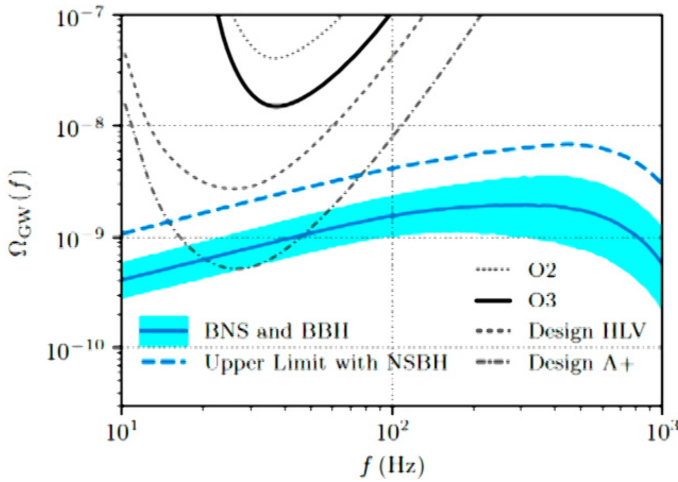
The spectrum is defined as

Astrophysical

GW source

$$\Omega_{GW}(f, \theta_k) = \frac{f}{\rho_c H_0} \int_0^{z_{max}} dz \frac{R_m(z, \theta_k)(dE/df_s)}{(1+z)E(\Omega_M, \Omega_\Omega, z)}$$

Cosmological



Detection of astrophysical SGWB

Detecting the Stochastic Gravitational Wave background:
detector signal cross-correlation (cross-power spectrum)

Detection statistic $S := \int_{-T/2}^{T/2} dt s_1(t) s_2(t)$

Signal-to-Noise-Ratio $\frac{\mu}{\sigma}$

This analysis relies on the stationarity and Gaussianity of the SGWB

Time-delay between detectors (± 10 ms) kills SNR at frequency > 30 Hz

Allen, B. and Romano, J.D., 1999

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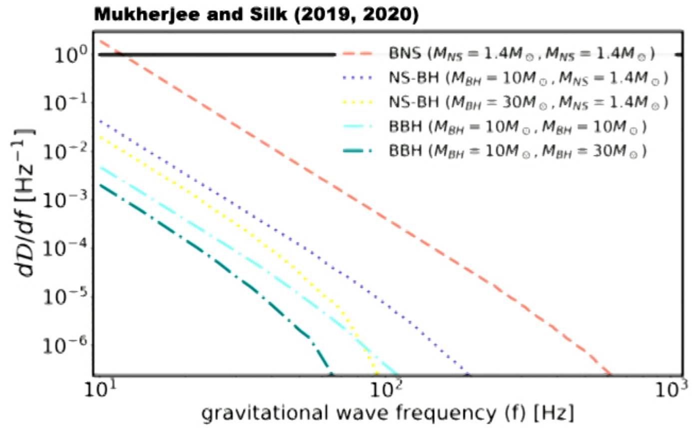
Limitations of current efforts

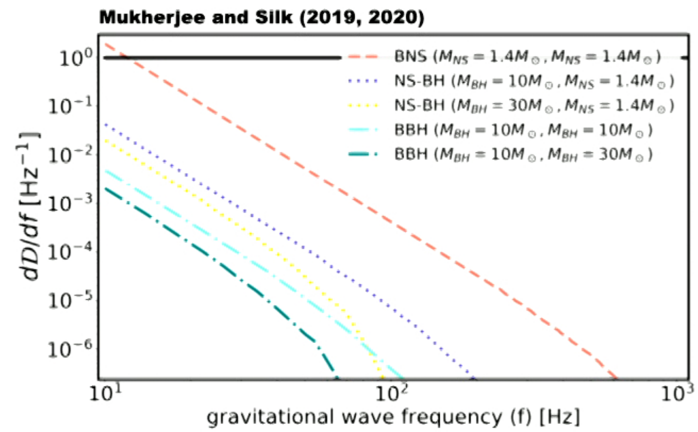
Time-scale over which events are overlapping at a particular frequency (Duty cycle).

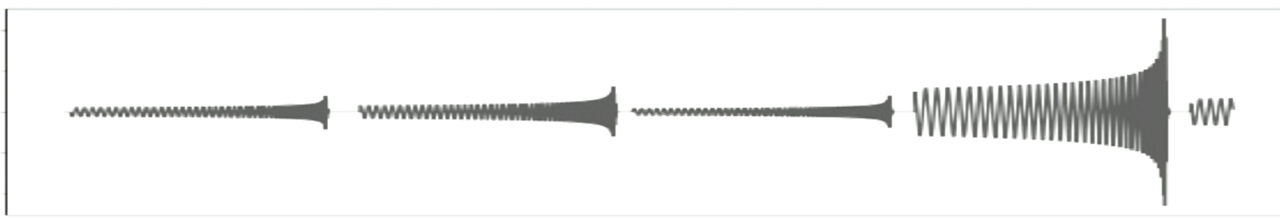


Limitations of current efforts

Time-scale over which events are overlapping at a particular frequency (Duty cycle).

- Does not take into account the time dependent background
- Assumes Gaussianity and the sources are overlapping
- Power spectrum based analysis cannot trace excess information in ‘popcorn’ SGWB

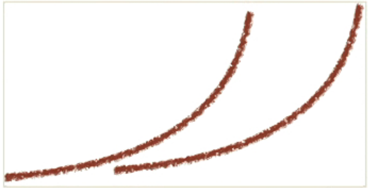




Signal beyond the power spectrum

The temporal dependence of the signal is not captured by the power spectrum

Non-overlapping GW signal will preserve its unique *chirp* behaviour

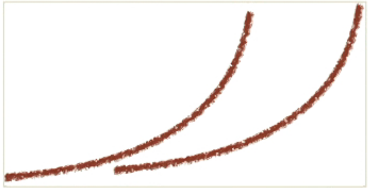


Signal beyond the power spectrum

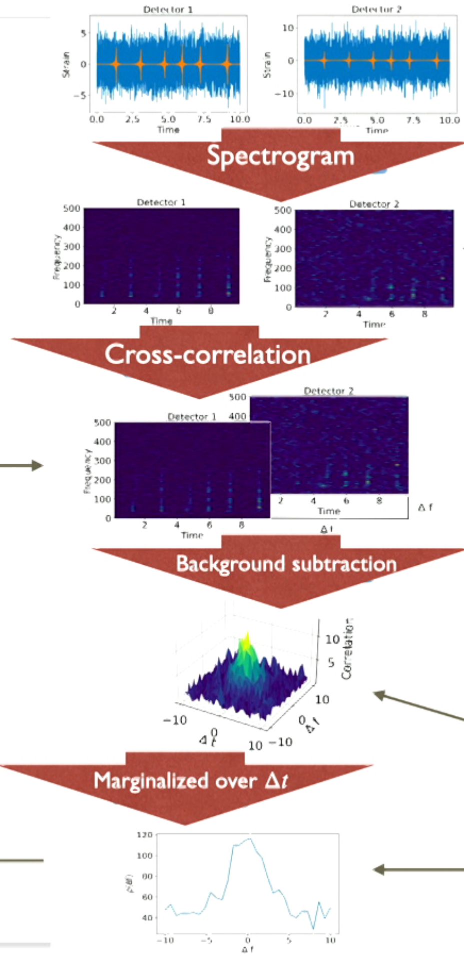
The temporal dependence of the signal is not captured by the power spectrum

Non-overlapping GW signal will preserve its unique *chirp* behaviour

Frequency correlations as a function of time



SpeCs flowchart



$$W_{s_i}(t_m, f) = \sum_{n=-\infty}^{\infty} s_i(t_n) g(t_n - t_m) e^{2\pi i f n}$$

$$S_{s_i}(t, f) = |W_{s_i}(t, f)|^2$$

$$P_{cc}(\Delta t, \Delta f) \equiv \iint_{f_{\min}}^{f_{\max}} S_{s_1}(t, f) S_{s_2}(t - \Delta t, f - \Delta f) \mathrm{d}t \mathrm{d}f$$

Signal extraction

$$P_{cc}^{\text{aperture}}(\Delta t_i \pm \epsilon, \Delta f) \equiv \frac{1}{2\epsilon} \sum_{x=\Delta t_i - \epsilon}^{\Delta t_i + \epsilon} P_{cc}(x, \Delta f),$$

$$\mathbf{P}^{\text{back}}(\Delta f) := \{P_1^{\text{aperture}}(\Delta t_1 \pm \epsilon, \Delta f), P_2^{\text{aperture}}(\Delta t_2 \pm \epsilon, \Delta f), \ldots, P_{N_t}^{\text{aperture}}(\Delta t_{N_t} \pm \epsilon, \Delta f)\}.$$

$$P_{cc}^{\text{clean}}(\Delta t, \Delta f) \equiv P_{cc}(\Delta t, \Delta f) - \bar{P}^{\text{back}}(\Delta f)$$

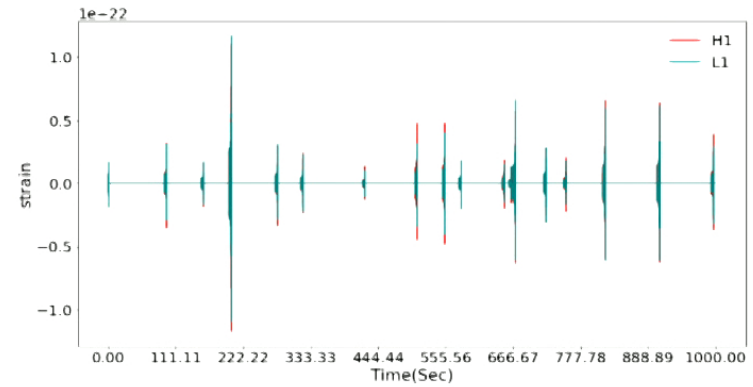
$$P^{\text{sig}} = \frac{1}{2\epsilon} \sum_{\Delta t = 0 - \epsilon}^{0 + \epsilon} P_{cc}^{\text{clean}}$$

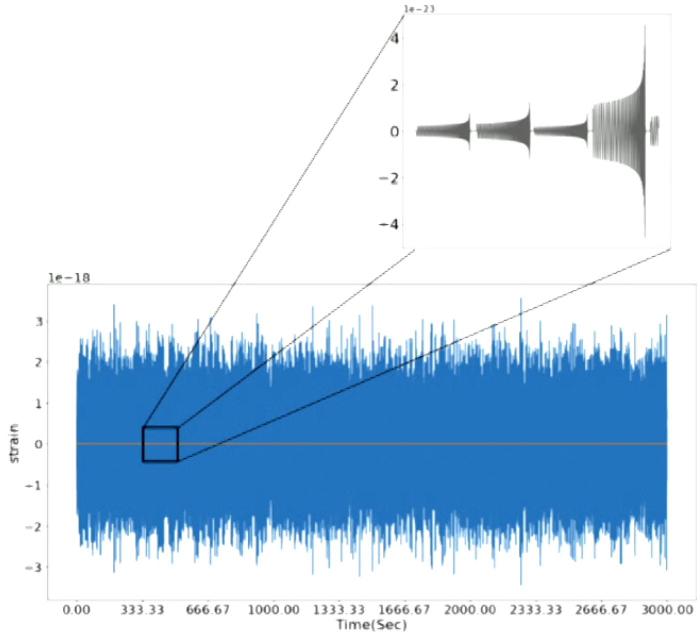
$$\rho_P(\Delta t = 0 \pm \epsilon) = \sqrt{\sum_{\Delta f \Delta f'} \hat{P}_{cc}^{\text{sig}}(\Delta f) C^{-1}(\Delta f, \Delta f') \hat{P}_{cc}^{\text{sig}}(\Delta f')}.$$

Simulated SGWB

- Synthetic noise generated using O3 PSD
- Dataset I: 10^4 waveforms in 3×10^4 sec
- Dataset II: 10^4 waveforms in 5×10^5 sec (~5.7days)
- Masses $10 - 90M_{\odot}$, D 6000-30000 Mpc

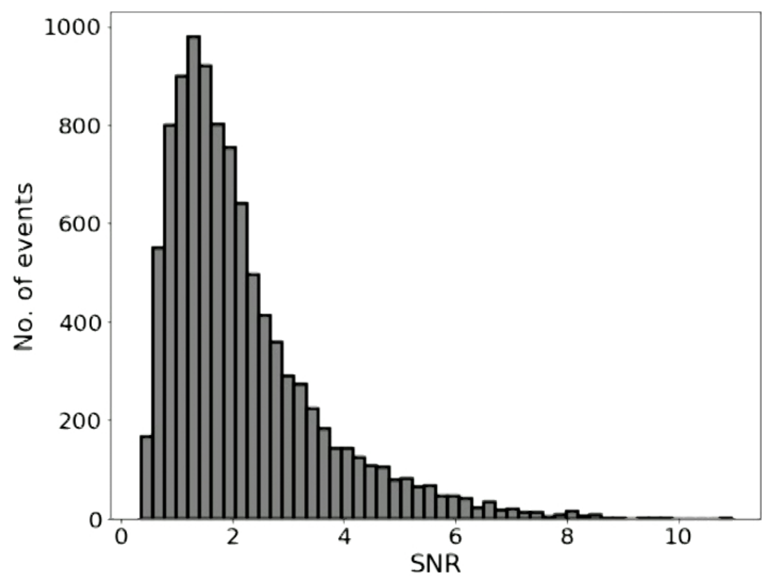
Dataset II (Low merger rate)





Dataset I (High merger rate)

SNR distribution of the injections



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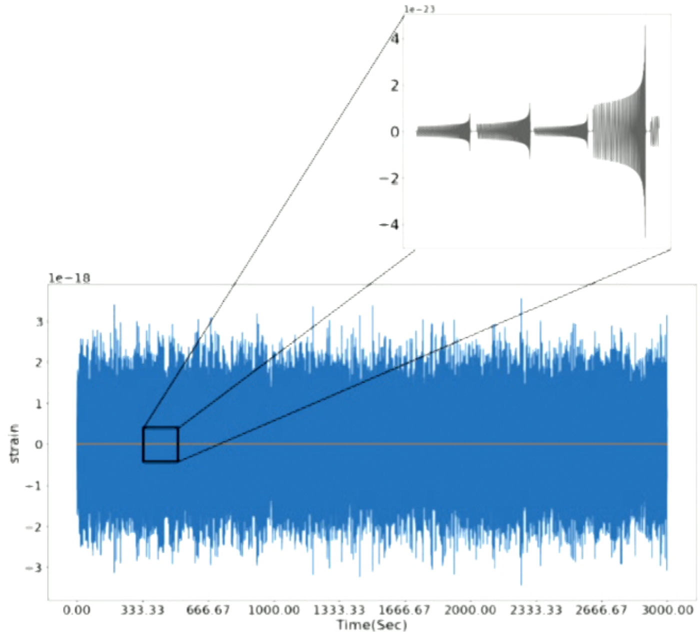

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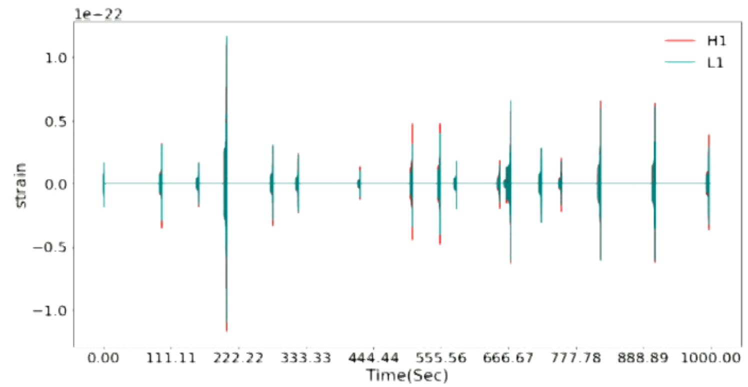

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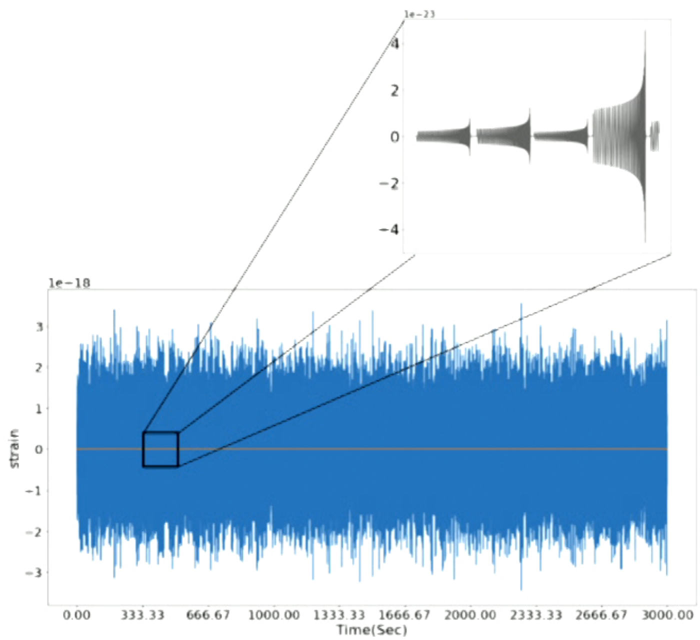

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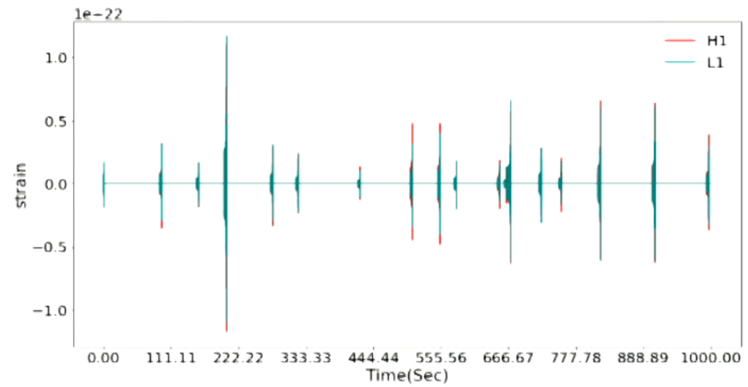
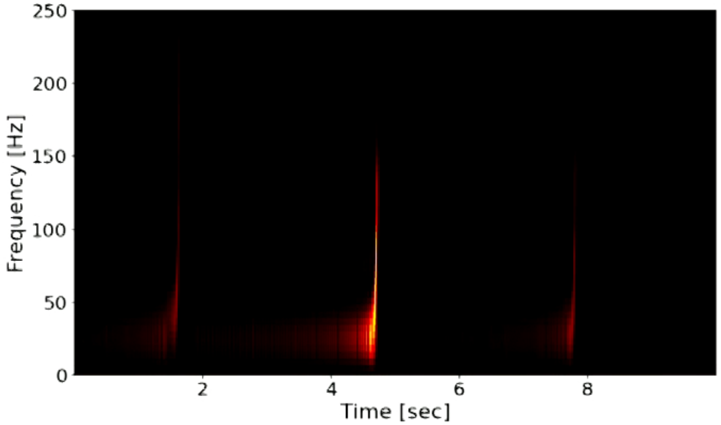
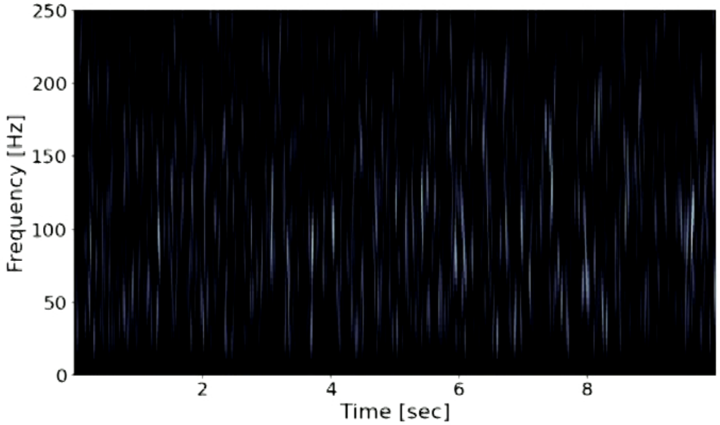


SpeCs analysis

Sample spectrograms generated using Short-time-Fourier-Transform



Signal

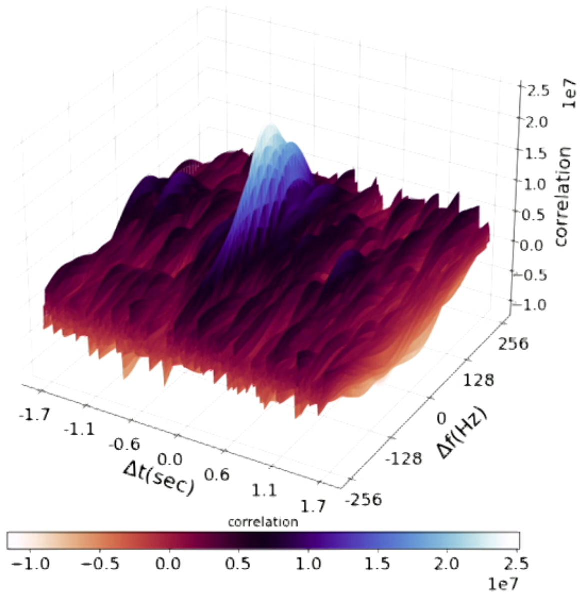


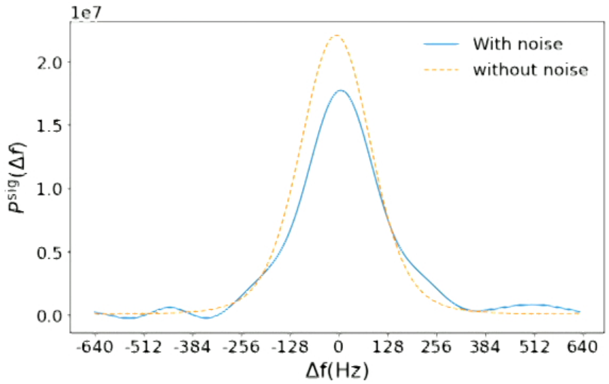
Noise

Segment the data into 500sec slices
Crop the spectrogram beyond $f_{max} = 640Hz$

Time-Frequency domain correlation

Contribution from signal peaks at
$(\Delta f, \Delta t) \sim 0$





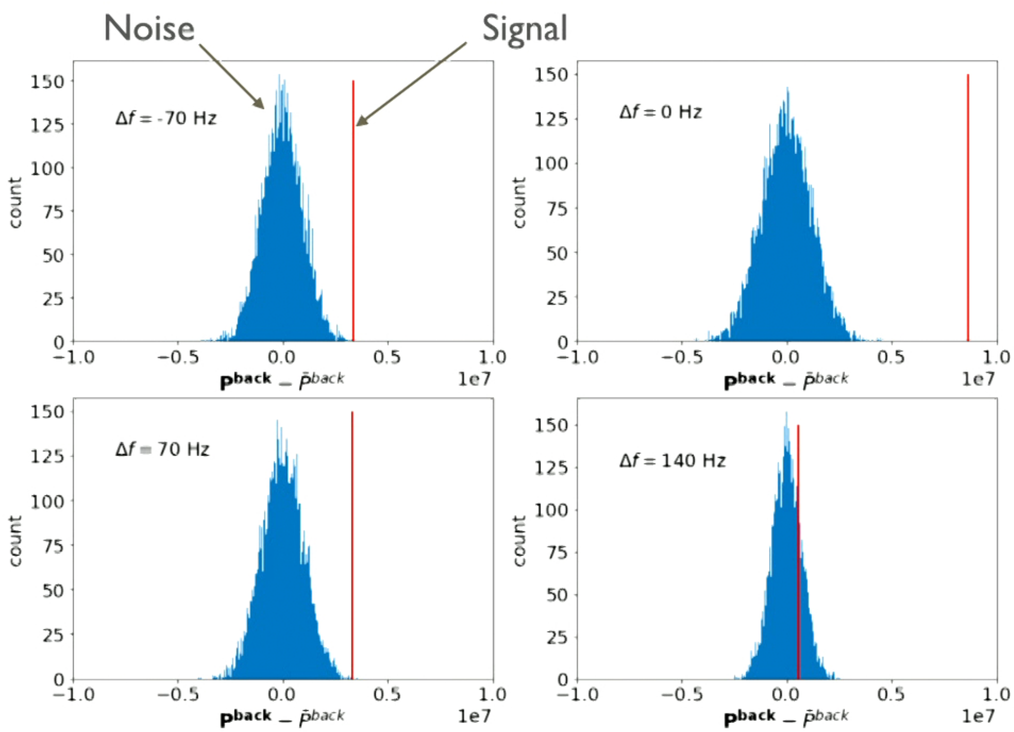
$$P^{\mathrm{sig}} = \frac{1}{2\epsilon} \sum_{\Delta t = 0 - \epsilon}^{0+\epsilon} P_{cc}^{\mathrm{clean}}$$

$$P_{cc}^{\mathrm{clean}}(\Delta t, \Delta f) \equiv P_{cc}(\Delta t, \Delta f) - \bar{P}^{\mathrm{back}}(\Delta f)$$

Distribution of background for a Gaussian noise

$$\mathbf{P}^{\mathbf{back}}(\Delta f) := \{P_1^{\rm aperture}(\Delta t_1 \pm \epsilon, \Delta f), P_2^{\rm aperture}(\Delta t_2 \pm \epsilon, \Delta f), \ldots, P_{N_t}^{\rm aperture}(\Delta t_{N_t} \pm \epsilon, \Delta f)\}.$$

$$\bar{P}^{\rm back}(\Delta f) \equiv \langle \mathbf{P}^{\mathbf{back}}(\mathbf{\Delta f}) \rangle \quad \text{for } |\Delta t_i| > \epsilon.$$



Covariance matrix

Covariance from the background noise distribution

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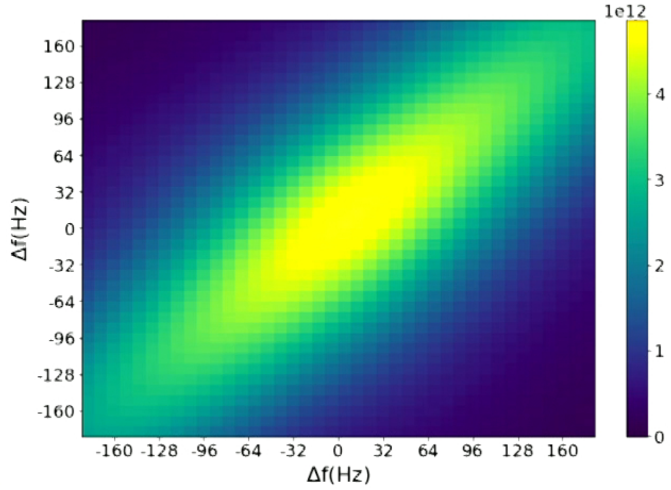
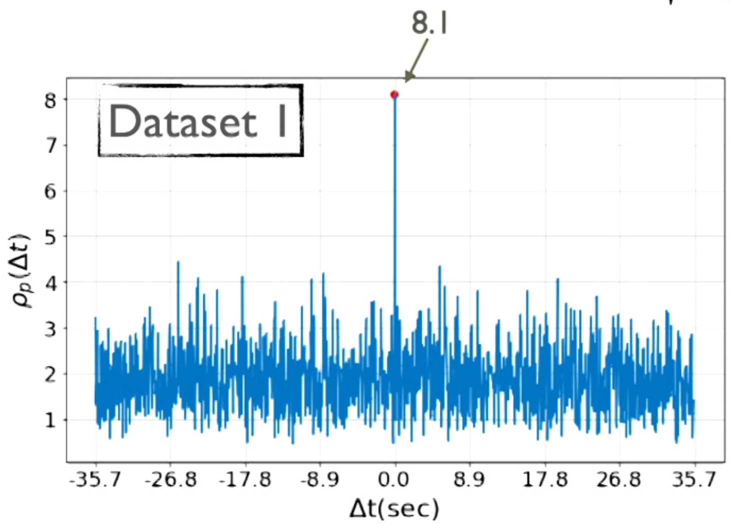
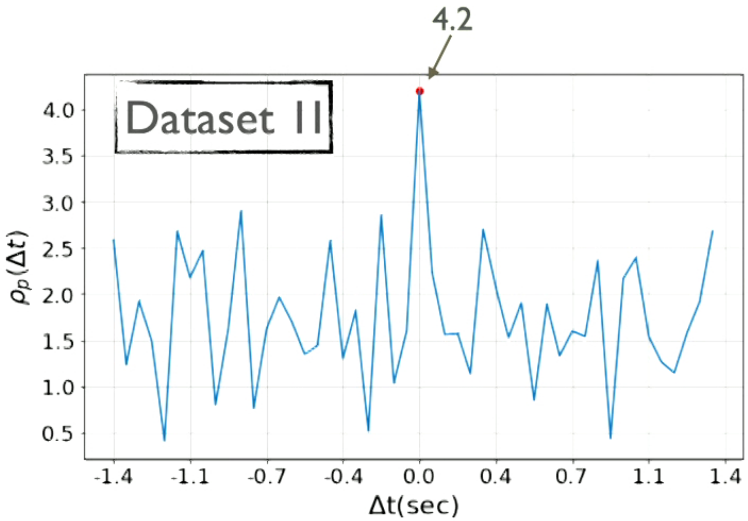
$$C(\Delta f, \Delta f') \equiv \text{Cov}[\mathbf{P}^{\mathbf{back}}(\Delta f), \mathbf{P}^{\mathbf{back}}(\Delta f')]$$



Signal to noise ratio from SpeCs

$$\rho_P(\Delta t = 0 \pm \epsilon) = \sqrt{\sum_{\Delta f \Delta f'} \hat{P}^{\rm sig}_{cc}(\Delta f) C^{-1}(\Delta f, \Delta f') \hat{P}^{\rm sig}_{cc}(\Delta f')}.$$





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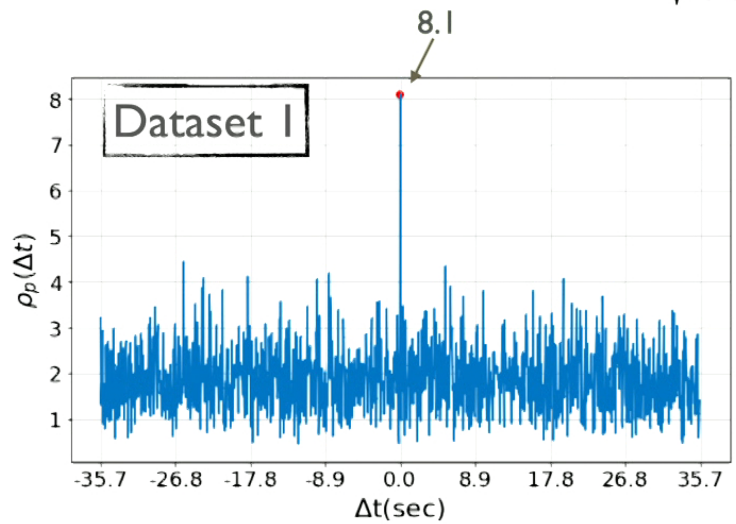
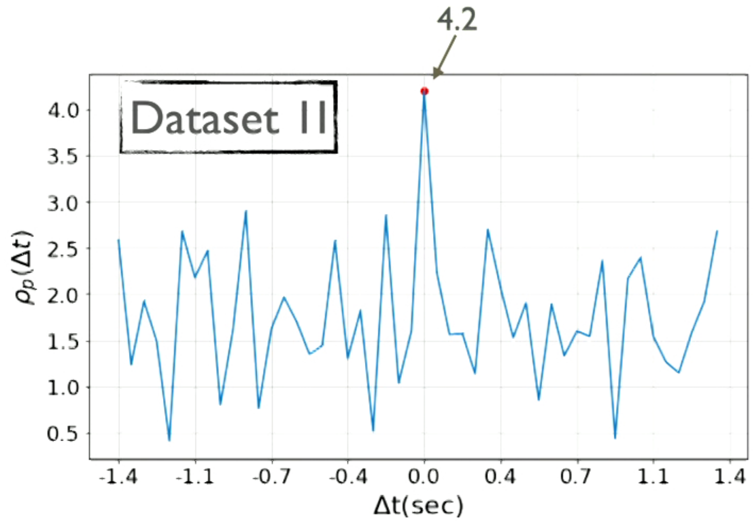
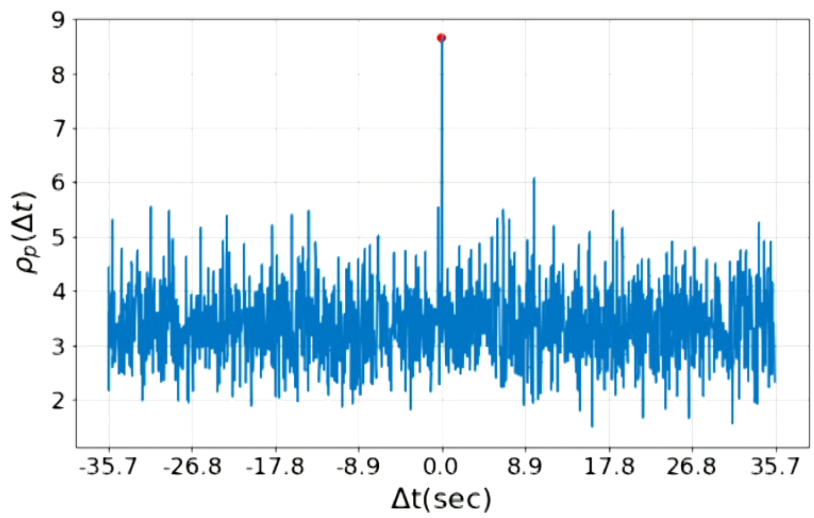




Results combining three LVH detectors

$$\rho^2_{\text{total}} = \sum_{I,J>I} (\rho^2_P)_{IJ}(\Delta t = 0 \pm \epsilon)$$

Peak SNR~8.7 for dataset I



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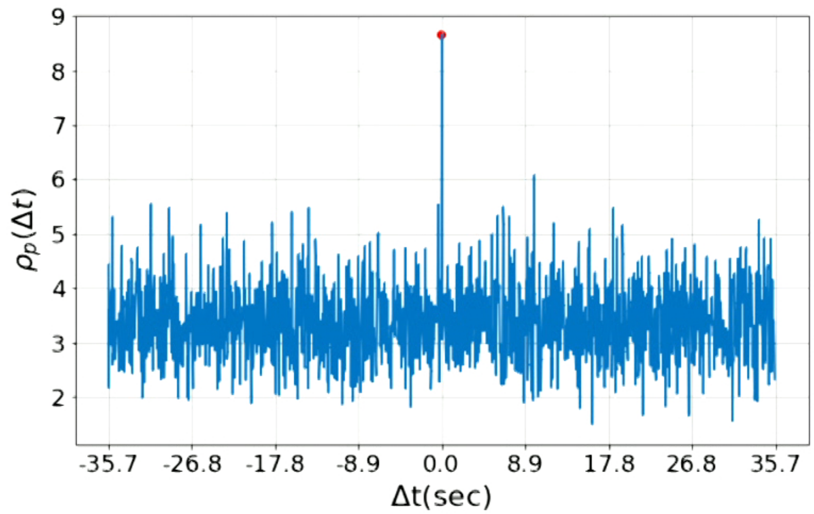

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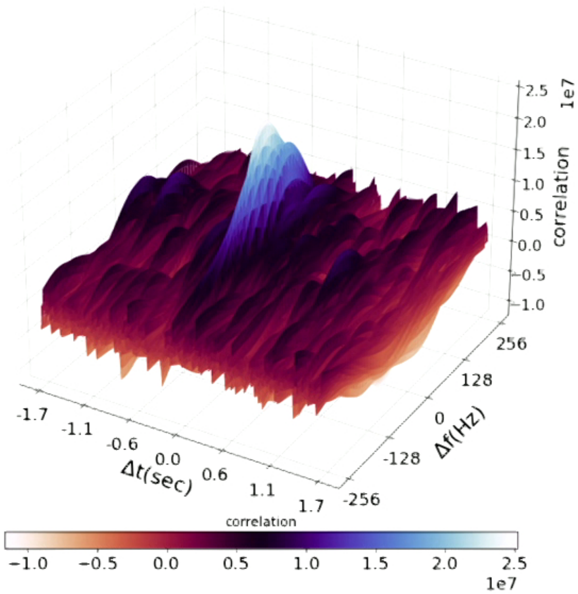

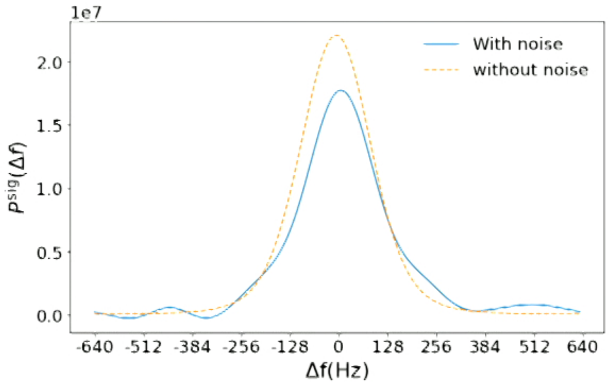

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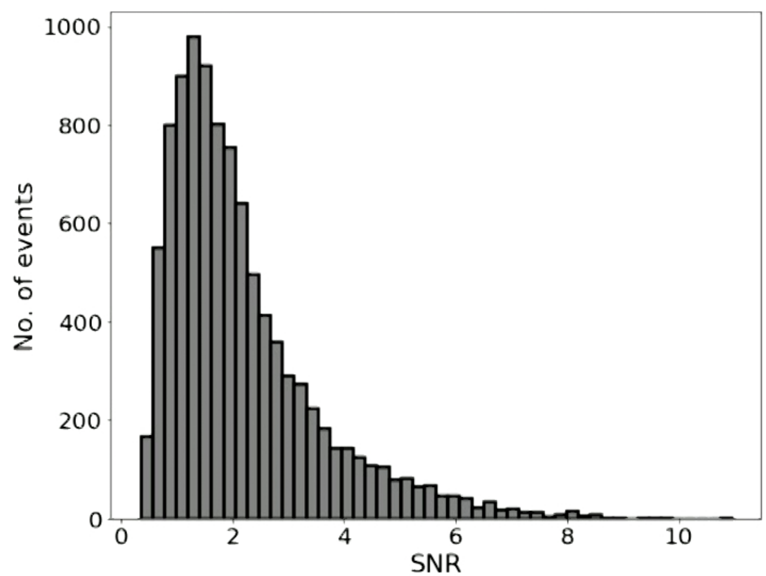
SNR distribution of the injections



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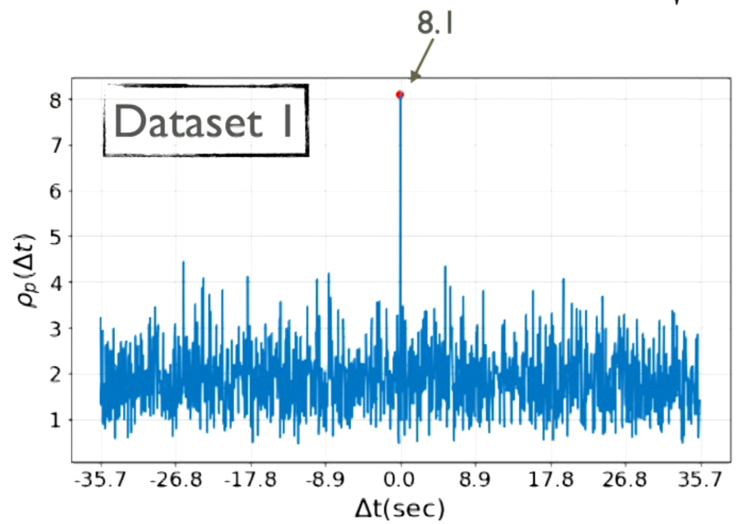
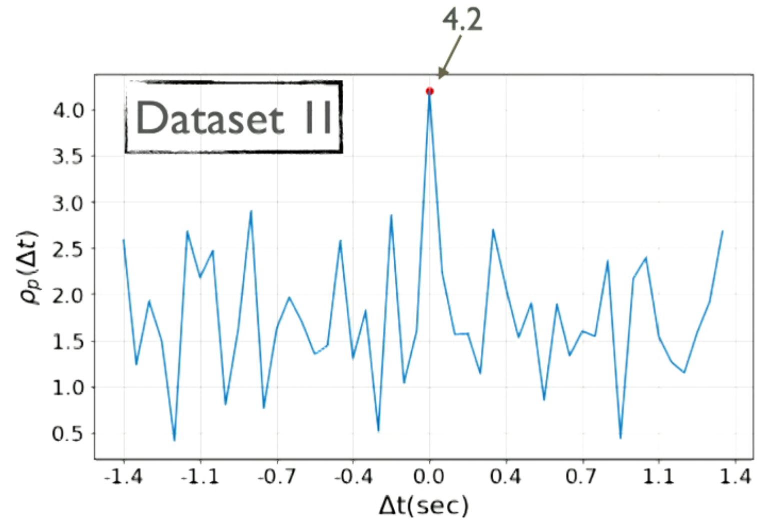
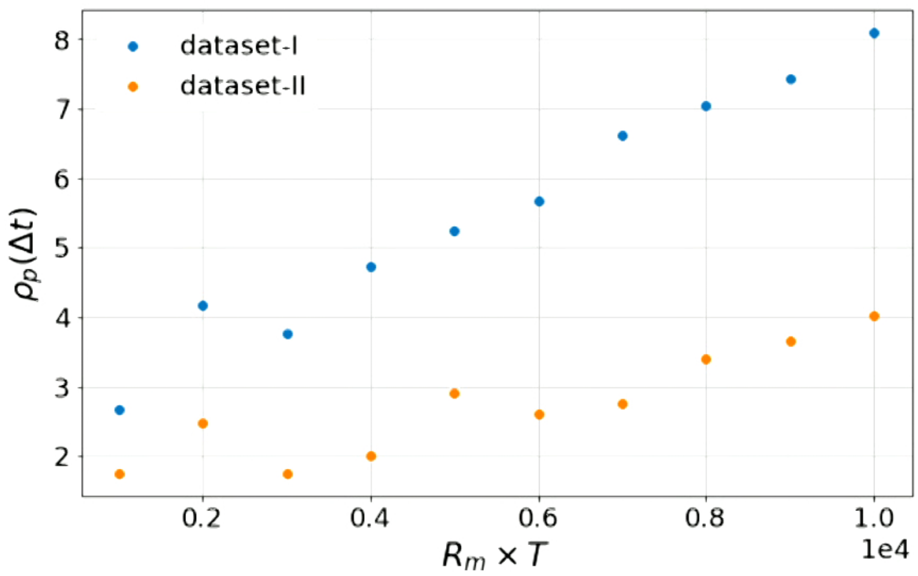
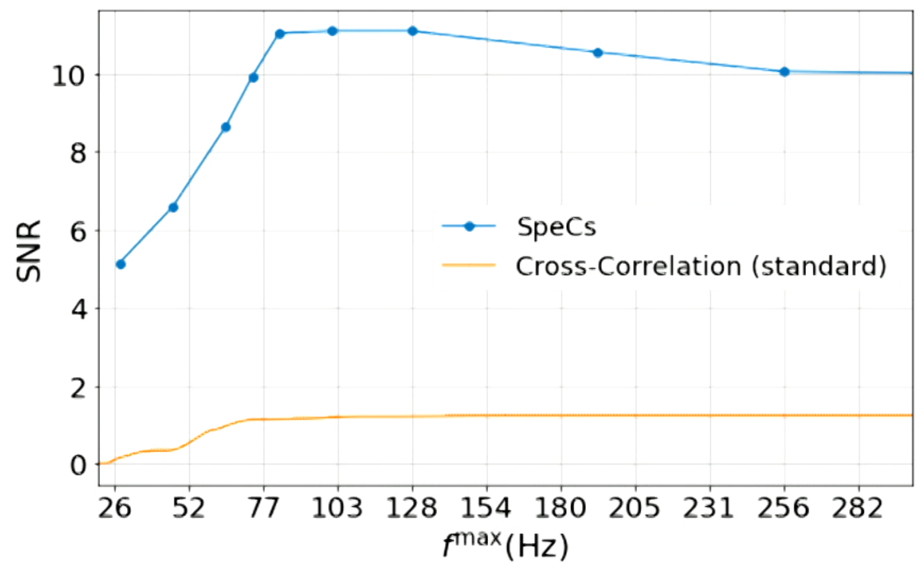




Variation in SNR with number of events

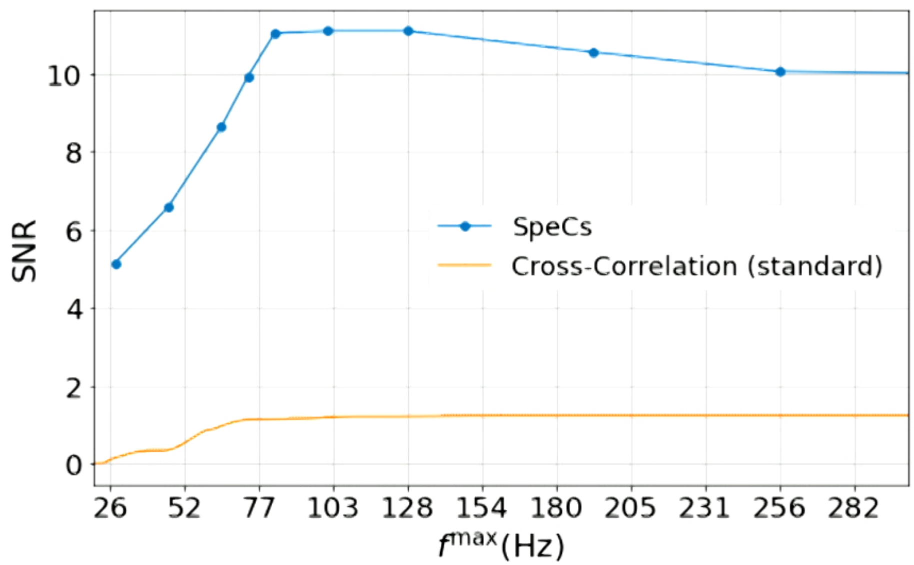


Comparison with cross-correlation technique



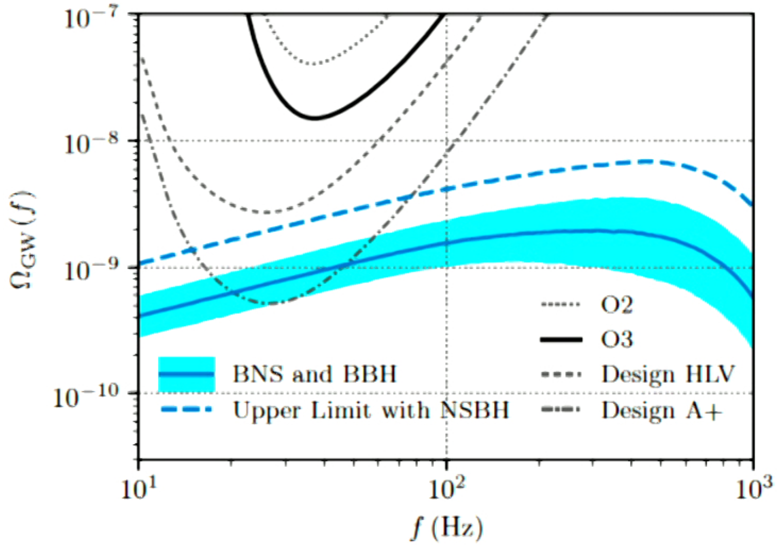
Approx **8 times** gain in SNR

Comparison with cross-correlation technique

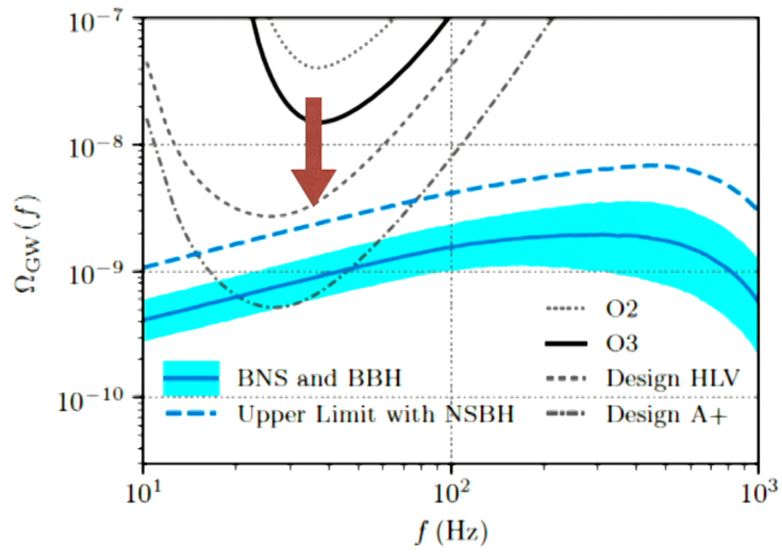


Approx **8 times** gain in SNR

Implication of improved SNR*8



Implication of improved SNR*8



Take home message

- SpeCS: A higher-order statistic to detect time-dependent (low duty cycle) stochastic GW background.
- Difference in the underlying statistical property of the SGWB and noise helps in extracting the signal.
- The method is capable of capturing *more* information than the traditional cross-correlation analysis for a signal with low duty cycle.

Ongoing work!

- Parameter estimation
- Glitches and SpeCs analysis using observed LIGO data

Implication of improved SNR*8