Title: Quantum Theory Lecture - 092623

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Dyson, Wick, Propagator

$$\langle f|S|i\rangle \overset{\text{LSZ}}{\longleftrightarrow} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \overset{\text{Dyson}}{\longleftrightarrow} \langle 0|T\varphi_1\cdots\varphi_m|0\rangle$$

Heisenberg

interaction picture

$$\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\,\varphi_1 \cdots\cdots \varphi_m|0\rangle \xleftrightarrow{\text{Wick}} \langle 0|T\,\varphi_1\varphi_2|0\rangle$$

interaction picture

Dyson, Wick, Propagator

$$\langle f|S|i\rangle \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_m|0\rangle$$

Heisenberg

interaction picture

there exists a refence time t_0

$H = S = I$

Wick

$\longleftrightarrow \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle$

$\varphi_0(\vec{x}, t) = e^{iH_0(t-t_0)} \varphi_S(\vec{x}, t_0) e^{-iH_0(t-t_0)}$

interaction picture

Dyson, Wick, Propagator

$$\langle f|S|i\rangle \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_m|0\rangle \xleftarrow{\text{Wick}}$$

Heisenberg

interaction picture

there exists a refence time t_0

$$H = S = I$$

$$H = H_0 + H_1$$

free (KG)

interaction or perturbation

Propagator

SZ

$$\longrightarrow \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_m|0\rangle \xleftrightarrow{\text{Wick}} \langle 0|T\varphi_1\varphi_2|0\rangle$$

Heisenberg

interaction picture

there exists a refence time t_0

$$H = S = I$$

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interaction or perturbation

Wick

$\varphi_m|0\rangle \xleftrightarrow{\text{Wick}} \langle 0|T\varphi_1\varphi_2|0\rangle$

ure

t_0

raction

perturbation

$\varphi_0(\vec{x},t) = e^{iH_0(t-t_0)}\varphi_S(\vec{x},t_0)e^{-iH_0(t-t_0)}$

interaction picture

$= e^{iH_0(t-t_0)}e^{-iH(t-t_0)}\varphi_H(\vec{x},t)e^{iH(t-t_0)} \cdot e^{-iH_0(t-t_0)}$

$= U(t,t_0)\,\varphi_H(\vec{x},t)\,U^\dagger(t,t_0)$

Wick

$\varphi_m|0\rangle \longleftrightarrow \langle 0|T\varphi_1\varphi_2|0\rangle$

ure

t_0

raction

perturbation

$$\varphi_0(\vec{x},t) = e^{iH_0(t-t_0)}\varphi_S(\vec{x},t_0)e^{-iH_0(t-t_0)}$$

interaction picture

aka

"free fields"

$$= e^{iH_0(t-t_0)}e^{-iH(t-t_0)}\varphi_H(\vec{x},t)e^{iH(t-t_0)}\cdot e^{-iH_0(t-t_0)}$$

$$= U(t,t_0)\,\varphi_H(\vec{x},t)\,U^\dagger(t,t_0)$$

$$U(t_2,t_1) = T\exp\left[-i\int_{t_1}^{t_2}dt\,H\right]$$

Schrödinger

interacting theory

$$a_{\vec{p}}(t_0)\,|\Omega(t)\rangle\Big|_{t=\pm\infty} = 0$$

$$a_{\vec{p}}(t_0)\, e^{iH(t-t_0)}\,|\Omega(t_0)\rangle\Big|_{t=\pm\infty} = 0$$

Schrödinger

interacting theory

$$a_{\vec{p}}(t_0)\,|\Omega(t)\rangle\Big|_{t=\pm\infty} = 0$$

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free theory

$$a_{\vec{p}}(t_0)\,|0(t)\rangle\Big|_{t=\pm\infty} = 0$$

$$a_{\vec{p}}(t_0)\, e^{-iH_0(t-t_0)}\,|0(t_0)\rangle\Big|_{t=\pm\infty} = 0$$

Schrödinger

interacting theory

$$a_{\vec{p}}(t_0)\,|\Omega(t)\rangle\Big|_{t=\pm\infty} = 0$$

$$a_{\vec{p}}(t_0)\, e^{-iH(t-t_0)}\,|\Omega(t_0)\rangle\Big|_{t=\pm\infty} = 0$$

free theory

$$a_{\vec{p}}(t_0)\,|0(t)\rangle\Big|_{t=\pm\infty} = 0$$

$$a_{\vec{p}}(t_0)\, e^{-iH_0(t-t_0)}\,|0(t_0)\rangle\Big|_{t=\pm\infty} = 0$$

If vac

$|\Omega$

If vacuum is unique

$$|\Omega\rangle = N_i \lim_{t \to -\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)} |0\rangle$$

$$= N_i \lim_{t \to -\infty} U_{0\,-\infty}$$

ck

$\rightarrow \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle$

$$\varphi_0(\vec{x},t) = e^{iH_0(t-t_0)} \varphi_S(\vec{x},t_0) e^{-iH_0(t-t_0)}$$

interaction picture
aka
"free fields"

$$= e^{iH_0(t-t_0)} e^{-iH(t-t_0)} \varphi_H(\vec{x},t) e^{iH(t-t_0)} \cdot e^{-iH_0(t-t_0)}$$

$$= U(t,t_0)\, \varphi_H(\vec{x},t)\, U^\dagger(t,t_0)$$

$$U(t_2,t_1) = T\exp\left[-i\int_{t_1}^{t_2} dt\, H\right] \equiv U_{21}$$

If vacuum is unique

$$|\Omega\rangle = N_i \lim_{t \to -\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)} |0\rangle$$

$$= N_i \lim_{t \to -\infty} U_{0,-\infty} |0\rangle$$

$$\langle \Omega | = N_f \lim_{t \to +\infty} \langle 0 | U_{\infty,0}$$

$$\varphi_n|\Omega\rangle \overset{t_1>t_2\cdots>t_n}{=} N_i N_f \langle 0| U_{\infty 0} U_{10}\, \varphi_0(x_1) U_{01} \cdots \cdots \varphi_0(x_n) U_{n0} U_{0-\infty}|0\rangle$$

$$= N_i N_f \langle 0| T\, U_{\infty 1}\, \varphi_0(x_1) U_{12}\, \varphi_0(x_2) \cdots\cdots U_{n-\infty}|0\rangle \qquad (U_{32}U_{21} = U_{31})$$

$$= N_i N_f \langle 0| T\, \varphi_0(x_1) \cdots \varphi(x_n)\, U_{\infty-\infty}|0\rangle$$

$t_1 > t_2 \cdots > t_n$

$$\langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle = N_i N_f \langle 0 | U_{\infty 0} U_{01} \varphi_0(x_1) U_{10} \cdots \varphi_0(x_n) U_{n0} U_{0-\infty} | 0 \rangle$$

$$= N_i N_f \langle 0 | T U_{\infty 1} \varphi_0(x_1) U_{12} \varphi_0(x_2) \cdots U_{n-\infty} | 0 \rangle$$

$$= N_i N_f \langle 0 | T \varphi_0(x_1) \cdots \varphi(x_n) U_{\infty -\infty} | 0 \rangle$$

$$U_{01}\,\varphi_0(x_1)\,U_{10}\cdots\cdots\varphi_0(x_n)\,U_{n0}\,U_{0-\infty}|0\rangle$$

$$\infty 1\;\varphi_0(x_1)\,U_{12}\,\varphi_0(x_2)\cdots\cdots U_{n-\infty}|0\rangle \qquad \left(U_{32}U_{21}=U_{31}\right)$$

$$T\;\varphi_0(x_1)\cdots\varphi(x_n)\;U_{\infty-\infty}|0\rangle$$

$$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \overset{t_1>t_2>\cdots>t_n}{=} N_i N_f\langle 0|U_{\infty 0}U_{01}\varphi_0(x_1)U_{10}\cdots\cdots\varphi_0(x_n)U_{n0}U_{0-\infty}|0\rangle$$

$$= N_i N_f\langle 0|T\,U_{\infty 1}\varphi_0(x_1)U_{12}\varphi_0(x_2)\cdots\cdots U_{n-\infty}|0\rangle$$

$$= N_i N_f\langle 0|T\,\varphi_0(x_1)\cdots\varphi(x_n)\,U_{\infty-\infty}|0\rangle$$

Assume $\langle\Omega|\Omega\rangle = 1$

$$N_i N_f\langle 0|U_{\infty 0}U_{0-\infty}|0\rangle = 1$$

$$N_i N_f = \frac{1}{\langle 0|U_{\infty,-\infty}|0\rangle}$$

$$\ldots\varphi_n|\Omega\rangle \overset{t_1>t_2\cdots>t_n}{=} N_i N_f \langle 0| U_{\infty 0} U_{01}\, \varphi_0(x_1) U_{10} \cdots\cdots \varphi_0(x_n) U_{n0} U_{0-\infty}|0\rangle$$

$$= N_i N_f \langle 0| U_{\infty 1}\, \varphi_0(x_1) U_{12}\, \varphi_0(x_2) \cdots\cdots U_{n-\infty}|0\rangle \qquad \left(U_{32}U_{21} = U_{31}\right)$$

$$= N_i N_f \langle 0| T\, \varphi_0(x_1)\cdots\varphi(x_n)\, U_{\infty-\infty}|0\rangle$$

Assume $\langle\Omega|\Omega\rangle = 1$

$$N_i N_f \langle 0| U_{\infty 0} U_{0-\infty}|0\rangle = 1$$

$$N_i N_f = \frac{1}{\langle 0| U_{\infty,-\infty}|0\rangle}$$

$\mathcal{L}_{int}$ is polynomial in fields

$$H_1 = -\int d^3x\, \mathcal{L}_{int}$$

$\langle\Omega$

$|0\rangle$

$\infty|0\rangle \quad (U_{32}U_{21}=U_{31})$

$\rangle$

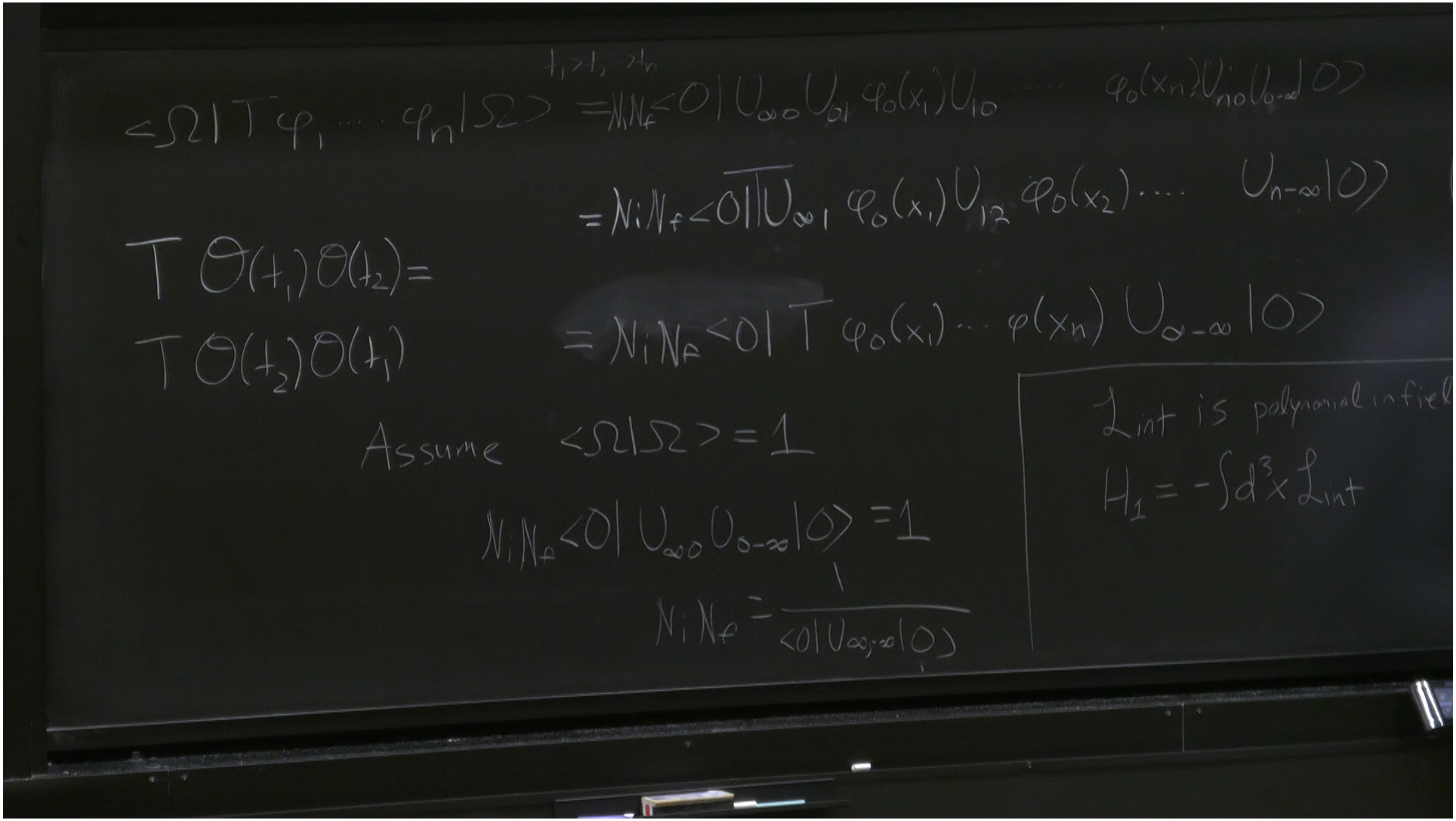
ynomial in fields

$\mathcal{L}_{int}$

$$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

Dyson

$$\exp[i\int d^4x\,\mathcal{L}_{int}] = 1 + i\int d^4x\,\mathcal{L}_{int} + \cdots$$


$t_1>t_2>\cdots>t_n$
$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = N_iN_f\langle 0|U_{\infty 0}U_{01}\varphi_0(x_1)U_{10}\cdots\varphi_0(x_n)U_{n0}U_{0-\infty}|0\rangle$
$= N_iN_f\langle 0|\overline{T}U_{\infty 1}\varphi_0(x_1)U_{12}\varphi_0(x_2)\cdots U_{n-\infty}|0\rangle$
$T\,\theta(t_1)\theta(t_2)=$
$T\,\theta(t_2)\theta(t_1)$
$= N_iN_f\langle 0|T\varphi_0(x_1)\cdots\varphi(x_n)U_{\infty-\infty}|0\rangle$
Assume $\langle\Omega|\Omega\rangle = 1$
$N_iN_f\langle 0|U_{\infty 0}U_{0-\infty}|0\rangle = 1$
$N_iN_f = \frac{1}{\langle 0|U_{\infty,-\infty}|0\rangle}$
$\mathcal{L}_{int}$ is polynomial in fiel
$H_1 = -\int d^3x\,\mathcal{L}_{int}$

$$|0\rangle \qquad (U_{32}U_{21}=U_{31})$$

al in fields

int

$$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

Dyson

$$\exp[i\int d^4x\,\mathcal{L}_{int}] = 1 + i\int d^4x\,\mathcal{L}_{int} + \cdots$$

$$\left(U_{32}U_{21} = U_{31}\right)$$

$$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

Dyson

ields

$$\exp[i\int d^4x\,\mathcal{L}_{int}] = 1 + i\int d^4x\,\mathcal{L}_{int} + \cdots$$

convergence?

still useful approximation

Wick's Theorem

$\langle 0 | T \varphi_1 \cdots \varphi_m | 0 \rangle$

free fields

time-ordered product $\longleftrightarrow$ normal ordered product

$\langle 0 | :\varphi_1 \cdots \varphi_m: | 0 \rangle = 0$ for $m > 0$

$$T \varphi_1 \cdots \varphi_n = \sum :\text{all contractions}:$$

related to commutator

$n > 0$

$n=2 \qquad T\varphi_1\varphi_2$

$$\varphi_{1+} = \int \frac{d^3k}{(2\pi)^3\, 2E_{\vec k}}\, a^{\dagger}_{\vec k}\, e^{ik\cdot x_1}$$

$$\varphi_{1-} = \int \frac{d^3k}{(2\pi)^3\, 2E_{\vec k}}\, a_{\vec k}\, e^{-ik\cdot x_1}$$

$$T\varphi_1\varphi_2 \overset{t_1>t_2}{=} (\varphi_{1+}+\varphi_{1-})(\varphi_{2+}+\varphi_{2-})$$

$$= \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \varphi_{1-}\varphi_{2+} + \varphi_{1-}\varphi_{2-}$$

$n>0$

$n=2 \qquad T\varphi_1\varphi_2$

$$\varphi_{1+} = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\, a^\dagger_{\vec{k}}\, e^{ik\cdot x_1}$$

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$n>0$

$n=2 \qquad T\varphi_1\varphi_2$

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$$= \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \boxed{\varphi_{1-}\varphi_{2+}} + \varphi_{1-}\varphi_{2-}$$

$$= \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$$

$$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + \boxed{\begin{aligned} &+[\varphi_{1-},\varphi_{2+}]\,\Theta(t_1-t_2) \\ &+[\varphi_{2-},\varphi_{1+}]\,\Theta(t_2-t_1)\end{aligned}}$$

contraction $\quad \varphi_1\varphi_2 = \square$

Wick's Theorem

$\langle 0 | T \varphi_1 \cdots \varphi_m | 0 \rangle$ ← free fields

time-ordered product $\leftrightarrow$ normal ordered product

$\langle 0 | : \varphi_1 \cdots \varphi_m : | 0 \rangle = 0$ for $m > 0$

$$T \varphi_1 \cdots \varphi_n = \sum : \text{all contractions} :$$

related to commutator

$$\varphi_1 \varphi_2 = [\varphi_{1-}, \varphi_{2+}]\, \Theta(t_1 - t_2) + [\varphi_{2-}, \varphi_{1+}]\, \Theta(t_2 - t_1)$$

$n=2$ $\quad T\varphi_1\varphi_2$

$$\varphi_{1+} = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} a^+_{\vec{k}} e^{ik\cdot x_1}$$

$$\varphi_{1-} = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} a_{\vec{k}} e^{-ik\cdot x_1}$$

$$T\varphi_1\varphi_2 \overset{t_1>t_2}{=} (\varphi_{1+}+\varphi_{1-})(\varphi_{2+}+\varphi_{2-})$$

$$= \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \boxed{\varphi_{1-}\varphi_{2+}} + \varphi_{1-}\varphi_{2-}$$

$$= \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$$

$$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + \boxed{\begin{aligned} &+[\varphi_{1-},\varphi_{2+}]\,\Theta(t_1-t_2)\\ &+[\varphi_{2-},\varphi_{1+}]\,\Theta(t_2-t_1)\end{aligned}}$$

contraction $\quad \overbrace{\varphi_1\varphi_2} = \square$

$$\langle 0|T\varphi_1\varphi_2|0\rangle = \langle 0|:\varphi_1\varphi_2:|0\rangle + \langle 0|\overbrace{\varphi_1\varphi_2}|0\rangle$$

$$= \overbrace{\varphi_1\varphi_2} = \Delta_F$$

Feynman propagator

$ik\cdot x_1$

e

$-ik\cdot x_1$

e

$1-)(\varphi_{2+}+\varphi_{2-})$

$+\ \varphi_{1+}\varphi_{2-} + \boxed{\varphi_{1-}\varphi_{2+}} + \varphi_{1-}\varphi_{2-}$

$+\ \varphi_{1+}\varphi_{2} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$

$$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + \boxed{\begin{array}{l} +[\varphi_{1-},\varphi_{2+}]\,\Theta(t_1-t_2) \\ +[\varphi_{2-},\varphi_{1+}]\,\Theta(t_2-t_1) \end{array}}$$

contraction $\overbrace{\varphi_1\varphi_2} = \square$

$$\langle 0|T\varphi_1\varphi_2|0\rangle = \langle 0|:\varphi_1\varphi_2:|0\rangle + \langle 0|\overbrace{\varphi_1\varphi_2}|0\rangle = \overbrace{\varphi_1\varphi_2} = \Delta_F$$

Feynman propagator

$$T\varphi_1\varphi_2\varphi_3 = :\varphi_1\varphi_2\varphi_3: + :\overbrace{\varphi_1\varphi_2}\varphi_3: + :\overbrace{\varphi_1\varphi_2\varphi_3}: + :\varphi_1\overbrace{\varphi_2\varphi_3}:$$

Sketch of proof: Induction Base case with $n=2$ proved.

Assume it holds for n fields

$$T\varphi_1\varphi_2\cdots\varphi_{n+1} \overset{t_1>\cdots t_{n+1}}{=} \varphi_1 \sum :\text{contractions involving } \varphi_2\cdots\varphi_m:$$

$$= (\underset{\text{ok}}{\varphi_{1+}} + \underset{\text{move to right}}{\varphi_{1-}}) \sum : \quad :$$

$$\cdots \varphi_{1+}\varphi_{2+} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$$

$$= \varphi_1\varphi_2 = \Delta_F$$

Feynman propagator

one term

$$(\varphi_{1+} + \varphi_{1-}) :\varphi_2 \cdots \varphi_{n+1}: = :\varphi_1 \cdots \varphi_{n+1}: + [\varphi_{1-}, :\varphi_2 \cdots \varphi_{n+1}:]$$

$$= :\varphi_1 \cdots \varphi_{n+1}: + [\varphi_{1-},\varphi_{2+}] :\varphi_3 \cdots \varphi_{n+1}: + \cdots + [\varphi_{1-},\varphi_{n+1+}] :\varphi_2 \cdots \varphi_n:$$

$$a^+ :\sim: = :a^+ \sim:$$

$$+\varphi_{1+}\varphi_{2+} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$$

Feynman propagator $= \varphi_1\varphi_2 = \Delta_F$

one term

$$(\varphi_{1+}+\varphi_{1-}):\varphi_2\cdots\varphi_{n+1}: = :\varphi_1\cdots\varphi_{n+1}: + [\varphi_{1-}, :\varphi_2\cdots\varphi_{n+1}:]$$

$$= :\varphi_1\cdots\varphi_{n+1}: + [\varphi_{1-},\varphi_{2+}]:\varphi_3\cdots\varphi_{n+1}: + \cdots + [\varphi_{1-},\varphi_{n+1+}]:\varphi_2\cdots\varphi_n:$$

$$a^+ :\sim: = :a^+\sim:$$

statement about operators

only completely contracted survive Vacuum expectation value

$$\cdots \varphi_{1+}\varphi_{2+} + \varphi_{1+}\varphi_{2-} + \varphi_{2+}\varphi_{1-} + [\varphi_{1-},\varphi_{2+}] + \varphi_{1-}\varphi_{2-}$$

Feynman propagator $= \varphi_1\varphi_2 = \Delta_F$

one term

$$(\varphi_{1+} + \varphi_{1-}) :\varphi_2 \cdots \varphi_{n+1}: = :\varphi_1 \cdots \varphi_{n+1}: + [\varphi_{1-}, :\varphi_2 \cdots \varphi_{n+1}:]$$

$$= :\varphi_1 \cdots \varphi_{n+1}: + [\varphi_{1-},\varphi_{2+}] :\varphi_3 \cdots \varphi_{n+1}: + \cdots + [\varphi_{1-},\varphi_{n+1+}] :\varphi_2 \cdots \varphi_n:$$

$$a^+ :\sim\sim: = :a^+ \sim\sim:$$

statement about operators

only completely contracted survive Vacuum expectation value

$$\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle = \varphi_1\varphi_2\varphi_3\varphi_4 + \varphi_1\varphi_2\varphi_3\varphi_4 + \varphi_1\varphi_2\varphi_3\varphi_4$$

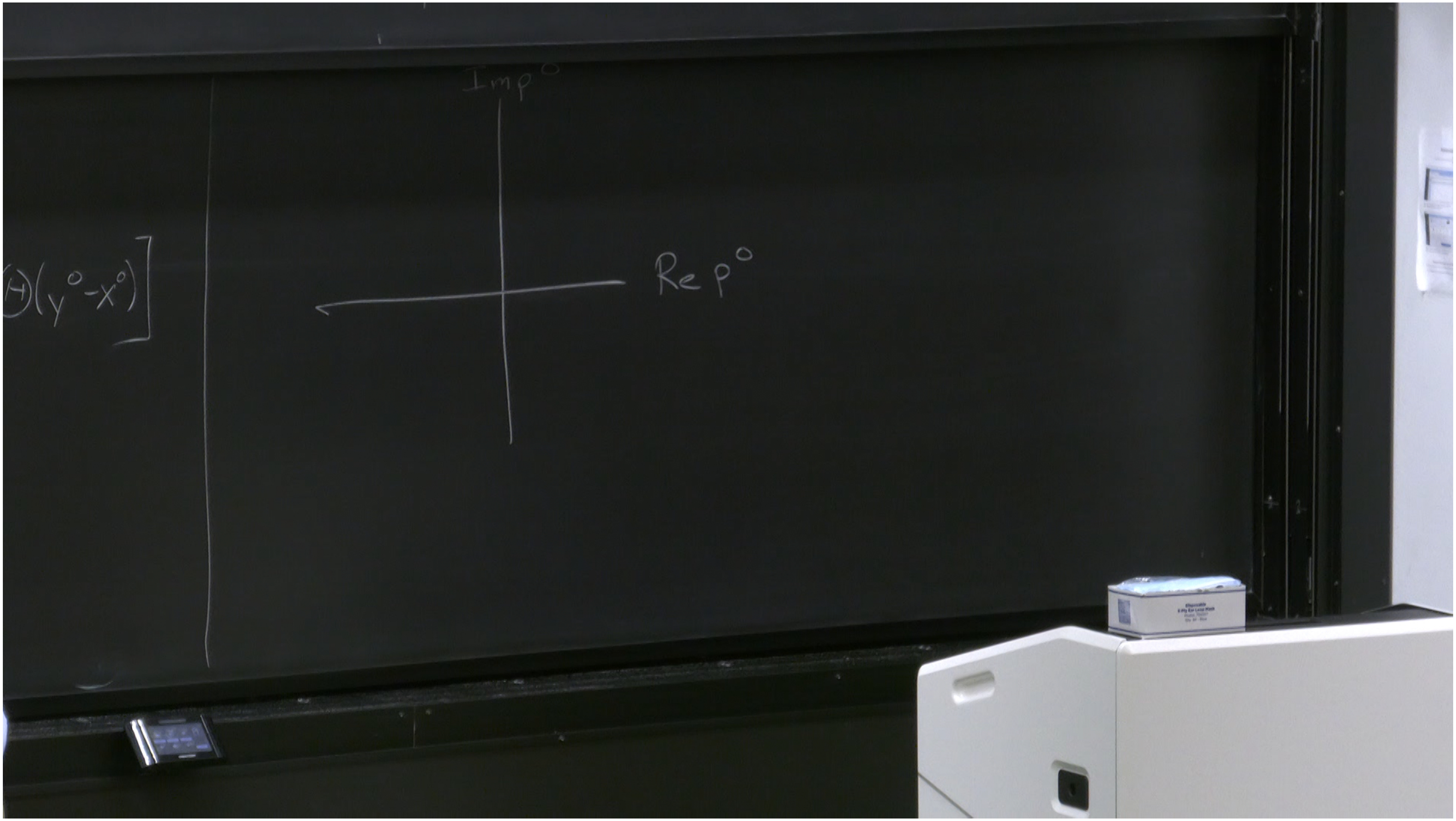
$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip\cdot(x-y)}$$

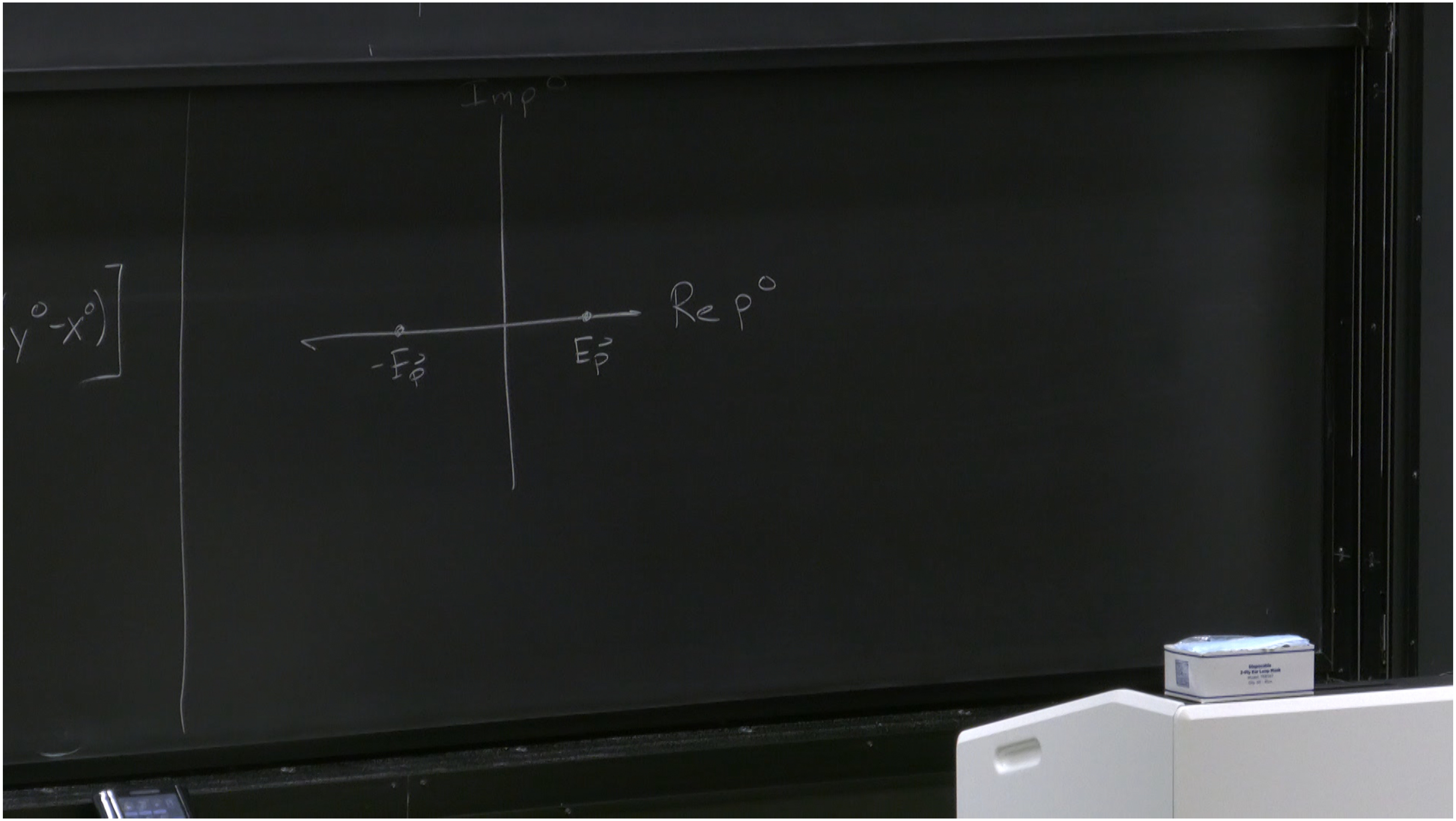
$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left[e^{-ip\cdot(x-y)}\Theta(x^0-y^0) + e^{-ip(y-x)}\Theta(y^0-x^0) \right]$$

$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}}\, e^{-ip\cdot(x-y)}$$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}}\left[e^{-ip\cdot(x-y)}\,\Theta(x^0-y^0) + e^{-ip(y-x)}\,\Theta(y^0-x^0)\right]$$

$$= \int_{C_F} \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2}$$

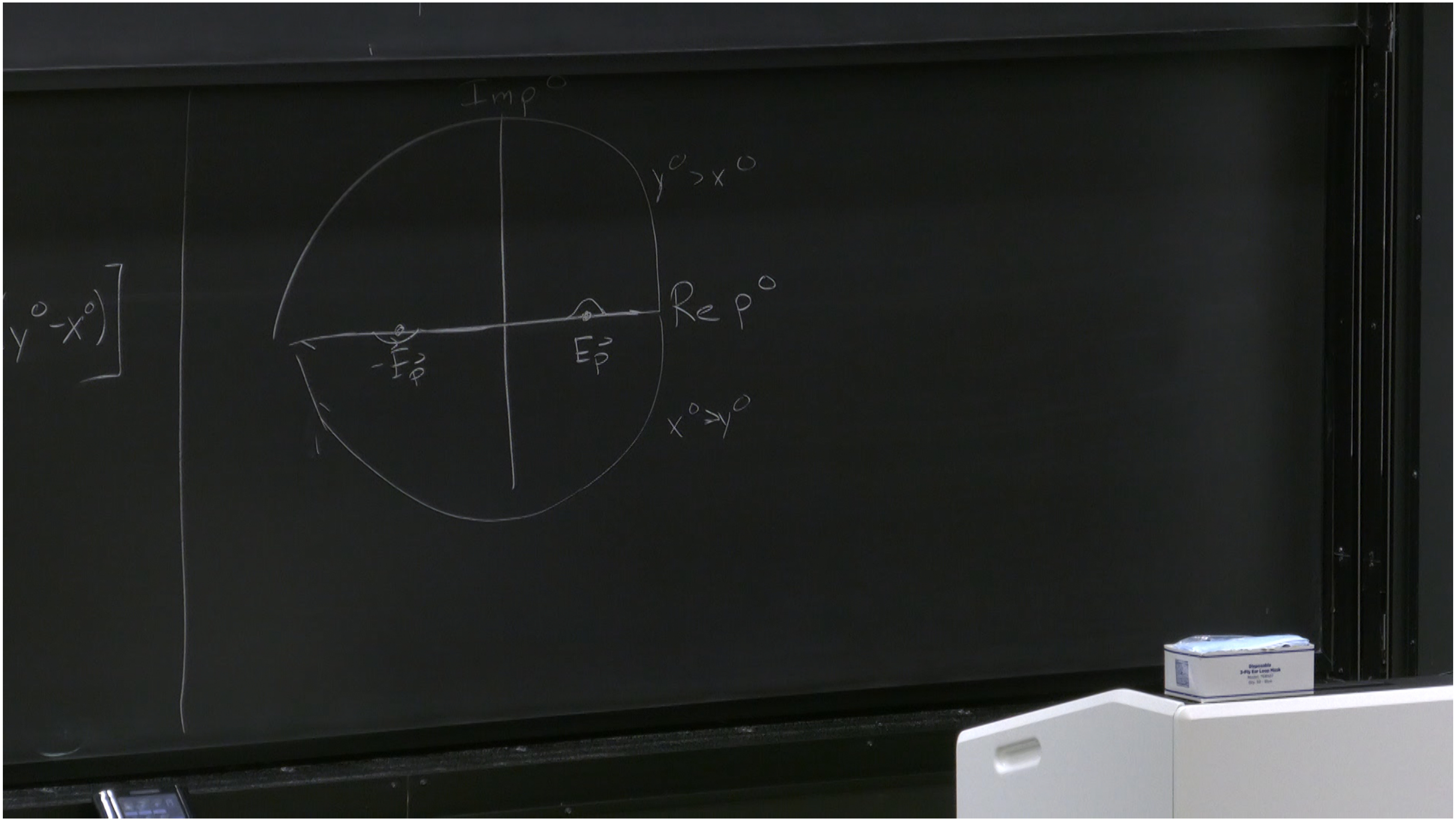

$\Theta(y^0 - x^0)]$
$\mathrm{Im}\,p^0$
$\mathrm{Re}\,p^0$


$y^0-x^0)]$
$\mathrm{Im}\,p^0$
$\mathrm{Re}\,p^0$
$-E_{\vec{p}}$
$E_{\vec{p}}$

$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} e^{-ip\cdot(x-y)}$$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} \left[e^{-ip\cdot(x-y)}\Theta(x^0-y^0) + e^{-ip(y-x)}\Theta(y^0-x^0)\right]$$

$$= \int_{C_F} \frac{d^4p}{(2\pi)^4} \frac{i\, e^{-ip\cdot(x-y)}}{p^2-m^2}$$

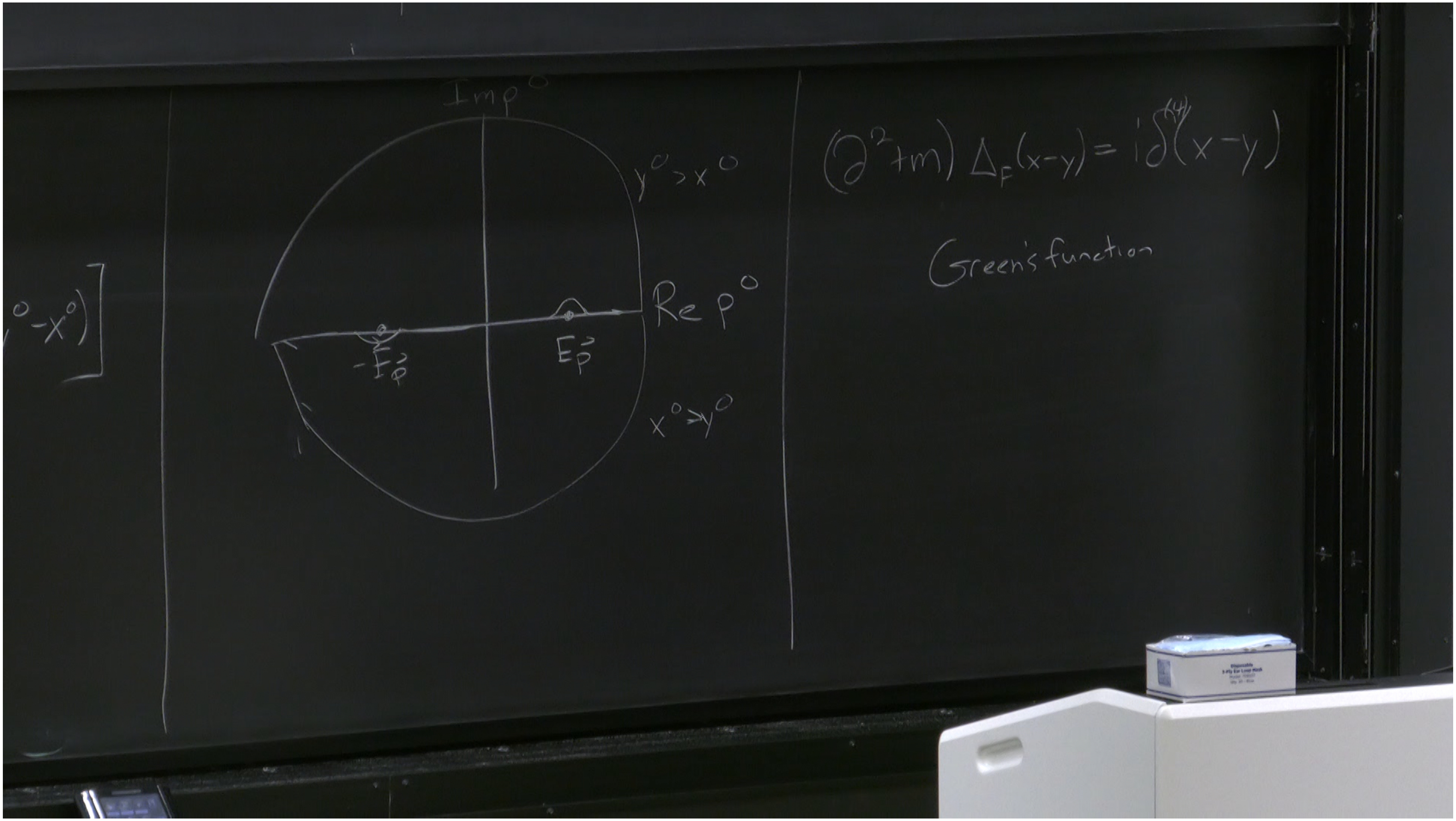


$y^0 - x^0)$
$\mathrm{Im}\, p^0$
$y^0 > x^0$
$\mathrm{Re}\, p^0$
$-E_{\vec{p}}$
$E_{\vec{p}}$
$x^0 > y^0$

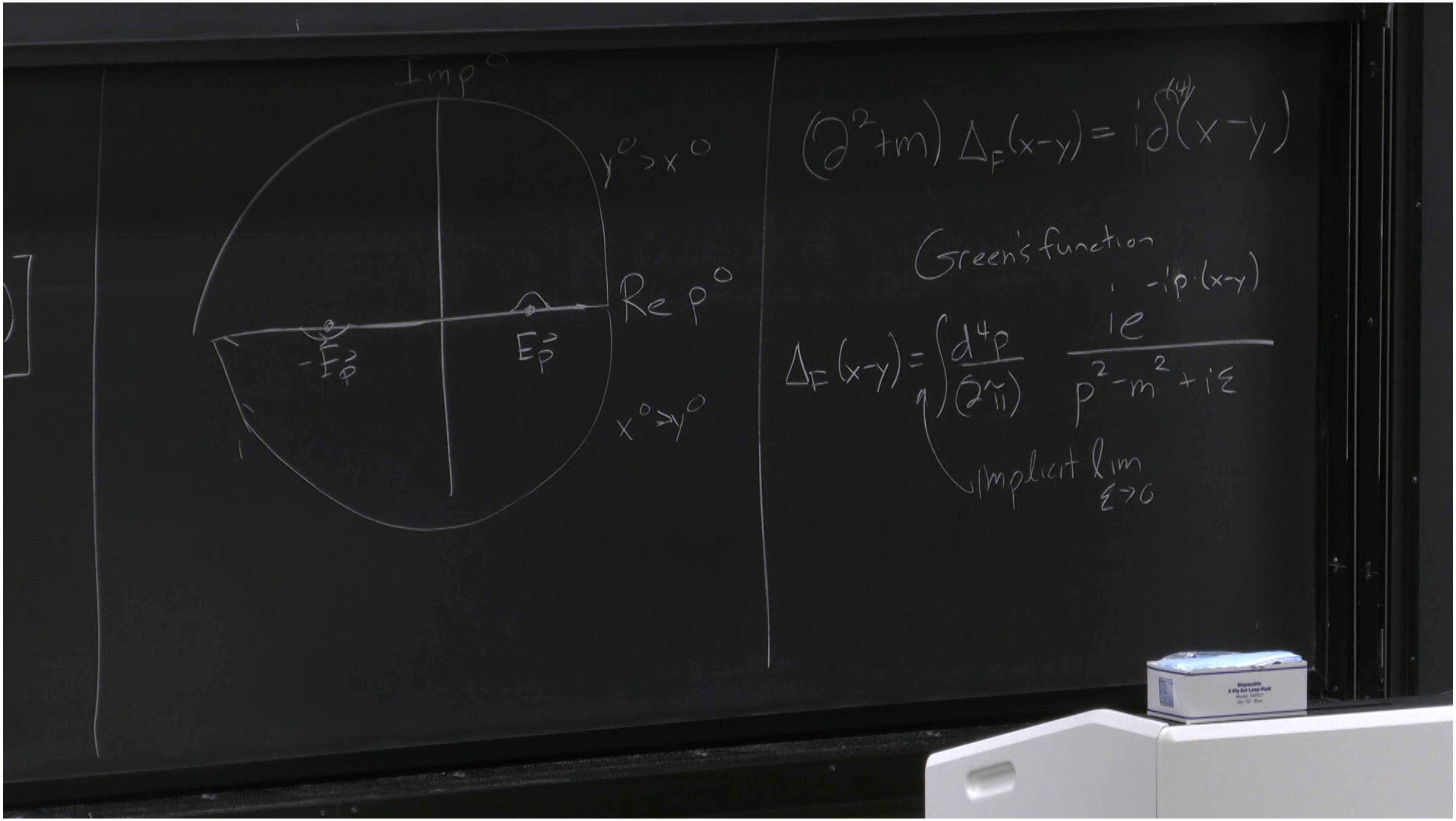
$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip\cdot(x-y)}$$

$$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left[e^{-ip\cdot(x-y)} \Theta(x^0-y^0) + e^{-ip(y-x)} \Theta(y^0-x^0) \right]$$

$$= \int_{C_F} \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip\cdot(x-y)}}{p^2-m^2} = \Delta_F(x-y)$$



$\mathrm{Im}\, p^0$
$y^0 > x^0$
$\mathrm{Re}\, p^0$
$-E_{\vec{p}}$
$E_{\vec{p}}$
$x^0 > y^0$
$(\partial^2 + m)\, \Delta_F(x-y) = i\delta^{(4)}(x-y)$
Green's function


$\mathrm{Im}\, p^0$
$y^0 > x^0$
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$-E_{\vec{p}}$
$E_{\vec{p}}$
$x^0 > y^0$
$(\partial^2 + m)\, \Delta_F(x-y) = i\delta^{(4)}(x-y)$
Green's function
$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)} \frac{ie^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon}$
Implicit $\lim_{\varepsilon \to 0}$

Next: each term in perturbative expansion $\xleftrightarrow[\text{Feynman rules}]{}$ Feynman diagram