Title: Asymptotic structure and the characterisation of gravitational

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Series: Quantum Gravity

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Abstract: With the main purpose of identifying the existence of gravitational radiation at infinity (scri), a novel approach to the asymptotic structure of spacetime is presented, focusing mainly in cases with non-negative cosmological constant. The basic idea is to consider the strength of tidal forces experienced by scri. To that end I will introduce the asymptotic (radiant) super-momentum, a causal vector defined at scri with remarkable properties that, in particular, provides an innovative characterization of gravitational radiation valid for the general case with ? >= 0 (and which has been proven to be equivalent when ? = 0 to the standard one based on the News tensor). This analysis is also shown to be supported by the initial- (or final-) value Cauchy-type problem defined at scri. The implications are discussed in some detail. The geometric structure of scri, and of its cuts, is clarified. The question of whether or not a News tensor can be defined in the presence of a positive cosmological constant is addressed. Several definitions of asymptotic symmetries are presented. Conserved charges that may detect gravitational radiation are exhibited. Balance laws that might be useful as diagnostic tools to test the accuracy of model waveforms discussed. An interpretation of the Geroch `rho' tensor is found. The whole thing will be complemented with a series of illustrative examples based on exact solutions. In particular we will see that exact solutions with black holes will be radiative if, and only if, they are accelerated.

Zoom link https://pitp.zoom.us/j/96816406686?pwd=eGZINlo2R0d1YkZMRGhackRNMzBVUT09

$\rho_{ab} = \rho_{(ab)}$, $n^a \rho_{ab} = 0$, $\bar{\nabla}_{[c}\rho_{a]b} = 0$ $\left(\Rightarrow h^{ab}\rho_{ab} = \frac{K}{2}\right)$. $\boxed{N_{ab} = \bar{S}_{ab} - \rho_{ab}}$ News tensor

On any cut: if χ is CKV with a fixed point $\Rightarrow \mathcal{L}_\chi \rho_{AB} = - D_A D_B \left(\frac{D_C \chi^C}{2}\right)$ (Adding this, $\exists \rho_{AB}$ for any other topology)

$\hat{\Pi}_{(\mu)} = \omega \Pi_{(\mu)}$

$$D_A D_B \Pi_{(\mu)} - \frac{1}{2} q_{AB} \Delta \Pi_{(\mu)} + \left(\rho_{AB} - \frac{K}{2} q_{AB}\right) \Pi_{(\mu)} = 0$$ ← 4 solutions ($l = 0,1$ spherical harmonics)

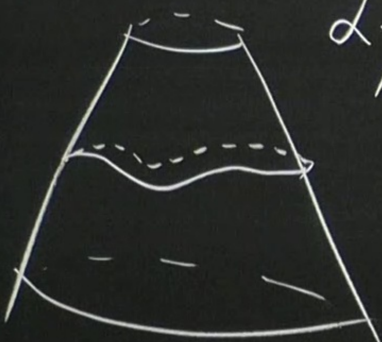
$$2\bar{\nabla}_{[b} S_{c]a} = \boxed{2\bar{\nabla}_{[b} N_{c]a} = - n^\rho d_{\rho\alpha\beta\lambda} e^\alpha_a e^\beta_b e^\lambda_c}$$ $\left(\Rightarrow \mathcal{L}_n N_{ca} = - n^\rho n^\lambda d_{\rho\beta\lambda\mu} e^\beta_a e^\mu\right)$

$$\Pi_\alpha = - n^\beta n^\lambda n^\mu \mathcal{D}_{\alpha\beta\lambda\mu}$$

"asymptotic super-momentum"

$\Lambda = 0$: $Q_\alpha := \Pi_\alpha |_{\mathscr{I}}$ (null future)

$\Lambda > 0$: $P_\alpha := \Pi_\alpha |_{\mathscr{I}}$ (causal future)



$$\mathcal{L}_Y (h_{ab} n^c n^d) = 0 \iff \mathcal{L}_Y h_{ab} = 2\phi h_{ab}, \ \mathcal{L}_Y n^a = -\phi n^a$$

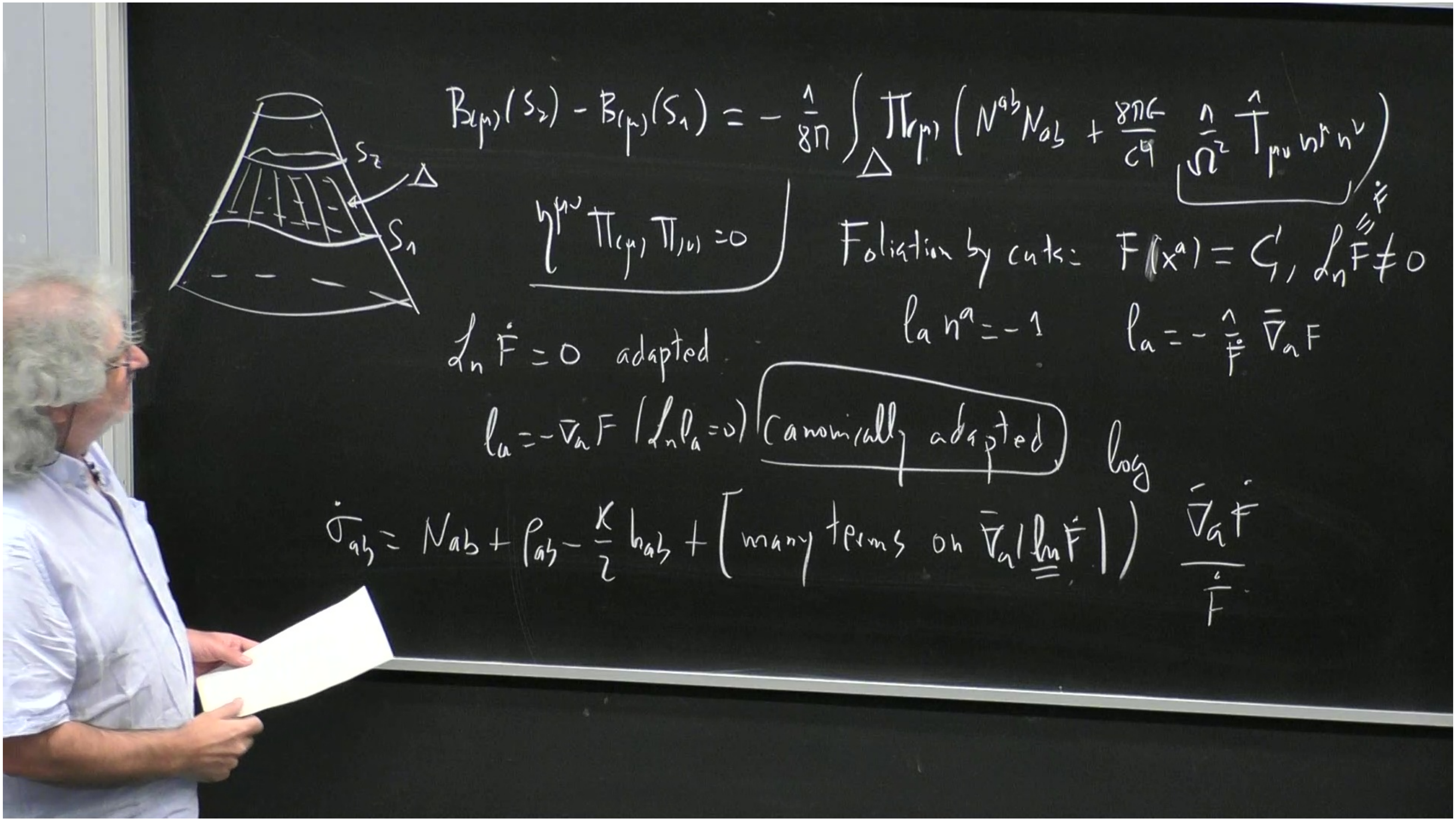
$Y^a = \alpha n^a$ ($\mathcal{L}_n \alpha = 0$, $\alpha \to \alpha \omega$) Super-translations

ξ KV on $(\hat{M}, \hat{g}) \to Y^a_\xi = \alpha_\xi n^a$ $\quad \mathcal{L}_n \bar{S}_{ab} = 0$

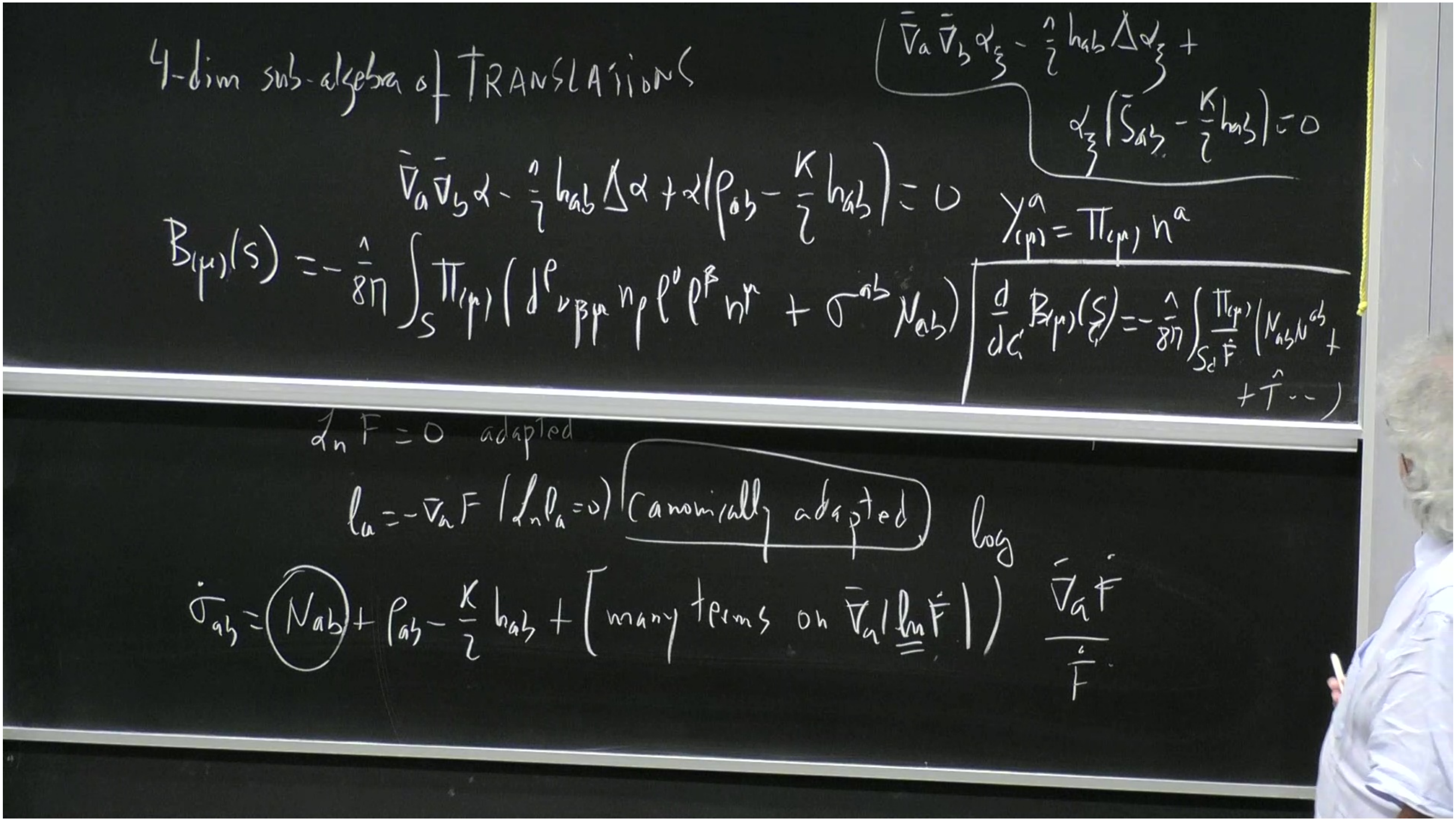
$$\bar{\nabla}_a \bar{\nabla}_b \alpha_\xi - \frac{1}{2} h_{ab} \Delta \alpha_\xi + \alpha_\xi \left(\bar{S}_{ab} - \frac{K}{2} h_{ab}\right) = 0$$

4-dim sub-algebra of TRANSLATIONS

$$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{1}{2} h_{ab} \Delta \alpha + \alpha \left(\rho_{ab} - \frac{K}{2} h_{ab}\right) = 0$$



S2
S1
Δ
B(μ)(S2) - B(μ)(S1) = - 1/8π ∫Δ Π(μ) (N^ab N_ab + 8πG/c⁴ 1/Ω² T̂_μν n^μ n^ν)
η^μν Π(μ) Π(ν) = 0
Foliation by cuts: F(x^a) = C, L_n F ≠ 0
l_a n^a = -1
l_a = - 1/Ḟ ∇̄_a F
L_n Ḟ = 0 adapted
l_a = -∇_a F (L_n l_a = 0)
canonically adapted
log
σ̇_ab = N_ab + ρ_ab - K/2 h_ab + (many terms on ∇̄_a (L_n Ḟ))
∇̄_a Ḟ / Ḟ


4-dim sub-algebra of TRANSLATIONS
$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{\Lambda}{2} h_{ab} \Delta \alpha + \alpha (\rho_{ab} - \frac{K}{2} h_{ab}) = 0$
$\bar{\nabla}_a \bar{\nabla}_b \alpha_\xi - \frac{\Lambda}{2} h_{ab} \Delta \alpha_\xi + \alpha_\xi (\bar{S}_{ab} - \frac{K}{2} h_{ab}) = 0$
$Y^a_{(\mu)} = \Pi_{(\mu)} n^a$
$B_{(\mu)}(S) = -\frac{\Lambda}{8\pi} \int_S \Pi_{(\mu)} (d^\rho{}_{\nu\beta\mu} n_\rho \ell^\nu \ell^\beta n^\mu + \sigma^{ab} N_{ab})$
$\frac{d}{dC} B_{(\mu)}(S_C) = -\frac{\Lambda}{8\pi} \int_{S_C} \frac{\Pi_{(\mu)}}{\dot{F}} (N_{ab} N^{ab} + + \hat{F} \dots)$
$\mathcal{L}_n F = 0$ adapted
$\ell_a = -\bar{\nabla}_a F \; (\mathcal{L}_n \ell_a = 0)$ (canonically adapted)
log
$\dot{\sigma}_{ab} = N_{ab} + \rho_{ab} - \frac{K}{2} h_{ab} + [\text{many terms on } \bar{\nabla}_a (\ln \dot{F})])$
$\frac{\bar{\nabla}_a \dot{F}}{\dot{F}}$

4-dim sub-algebra of TRANSLATIONS

$$\bar{\nabla}_a \bar{\nabla}_b \alpha_\xi - \frac{\hat{}}{2} h_{ab} \Delta \alpha_\xi + \alpha_\xi \left(\bar{S}_{ab} - \frac{K}{2} h_{ab} \right) = 0$$

$$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{\hat{}}{2} h_{ab} \Delta \alpha + \alpha \left(\rho_{ab} - \frac{K}{2} h_{ab} \right) = 0$$

$$Y^a_{(\mu)} = \Pi_{(\mu)} n^a$$

$$B_{(\mu)}(S) = -\frac{\hat{}}{8\pi} \int_S \Pi_{(\mu)} \left(d^\rho{}_{\nu\beta\mu} n_\rho \ell^\nu \ell^\beta n^\mu + \sigma^{ab} N_{ab} \right)$$

$$\frac{d}{dc} B_{(\mu)}(S_c) = -\frac{\hat{}}{8\pi} \int_{S_c} \frac{\Pi_{(\mu)}}{\dot{F}} \left(N_{ab} N^{ab} + \hat{\dot{F}} \cdots \right)$$

$\mathcal{L}_n \dot{F} = 0$ adapted $\qquad \ell_a n^a = -1 \qquad \ell_a = -\frac{\hat{}}{\dot{F}} \bar{\nabla}_a F$

$\ell_a = -\nabla_a F \; (\mathcal{L}_n \ell_a = 0)$ (canonically adapted) $\qquad$ log

$$\dot{\sigma}_{ab} = N_{ab} + \rho_{ab} - \frac{K}{2} h_{ab} + \left[\text{many terms on } \bar{\nabla}_a (\ln \dot{F}) \right) \qquad \frac{\bar{\nabla}_a \dot{F}}{\dot{F}}$$

$$\nabla_a \nabla_b \alpha - \frac{1}{2} h_{ab} \Delta \alpha + \alpha \left(R_{ab} - \frac{R}{2} h_{ab} \right) = 0 \qquad Y^a_{(m)} = \Pi_{(m)} n^a$$

any cut S, $Q^\alpha |_S = W \ell^\alpha + Z n^\alpha + Q^\alpha_T$ $\qquad 2WZ + Q^\alpha_T Q_{T\alpha} = 0$

$W = 2 \dot{N}_{AB} \dot{N}^{AB}$, $Z = 2 D_C N^{CA} D_B N^B{}_A$, $Q_T = \cdots$

$(\mathcal{L}_n N_{ab}) E^a_A E^b_B$

$$Z = 0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow \underline{N^{CA} = 0}$$

$$\nabla_a \nabla_b \alpha - \frac{1}{2} h_{ab} \Delta \alpha + \alpha \left(R_{ab} - \frac{R}{2} h_{ab} \right) = 0 \qquad Y^a_{(n)} = \Pi_{(n)} n^a$$

any cut S, $Q^\alpha|_S = W \ell^\alpha + Z n^\alpha + Q^\alpha_T$ $\qquad 2WZ + Q^\alpha_T Q_{T\alpha} = 0$

$W = 2 \mathring{N}_{AB} \mathring{N}^{AB}$, $Z = 2 D_C N^{CA} D_B N^B{}_A$, Q_T : - - -

$(\mathcal{L}_n N_{ab}) E^a_A E^b_B$ $\qquad Z = 0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow N^{CA} = 0$

Δ open portion on $\mathcal{I}$, $\quad N_{ab}|_\Delta = 0 \Leftrightarrow Q^\alpha|_\Delta = 0$

any cut S, $Q^a|_S = W\ell^a + Z n^a + Q_T^a$, $\quad 2WZ + Q_T^a Q_{Ta} = 0$

$W = 2\dot{N}_{AB}\dot{N}^{AB}$, $Z = 2 D_C N^{CA} D_B N^B{}_A$, Q_T : - - -

$(\mathcal{L}_n N_{ab}) E^a_A E^b_B$ $\quad Z = 0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow N^{CA} = 0$

Δ open portion on $\mathcal{I}$, $\quad N_{ab}|_\Delta = 0 \Longleftrightarrow Q^a|_\Delta = 0$

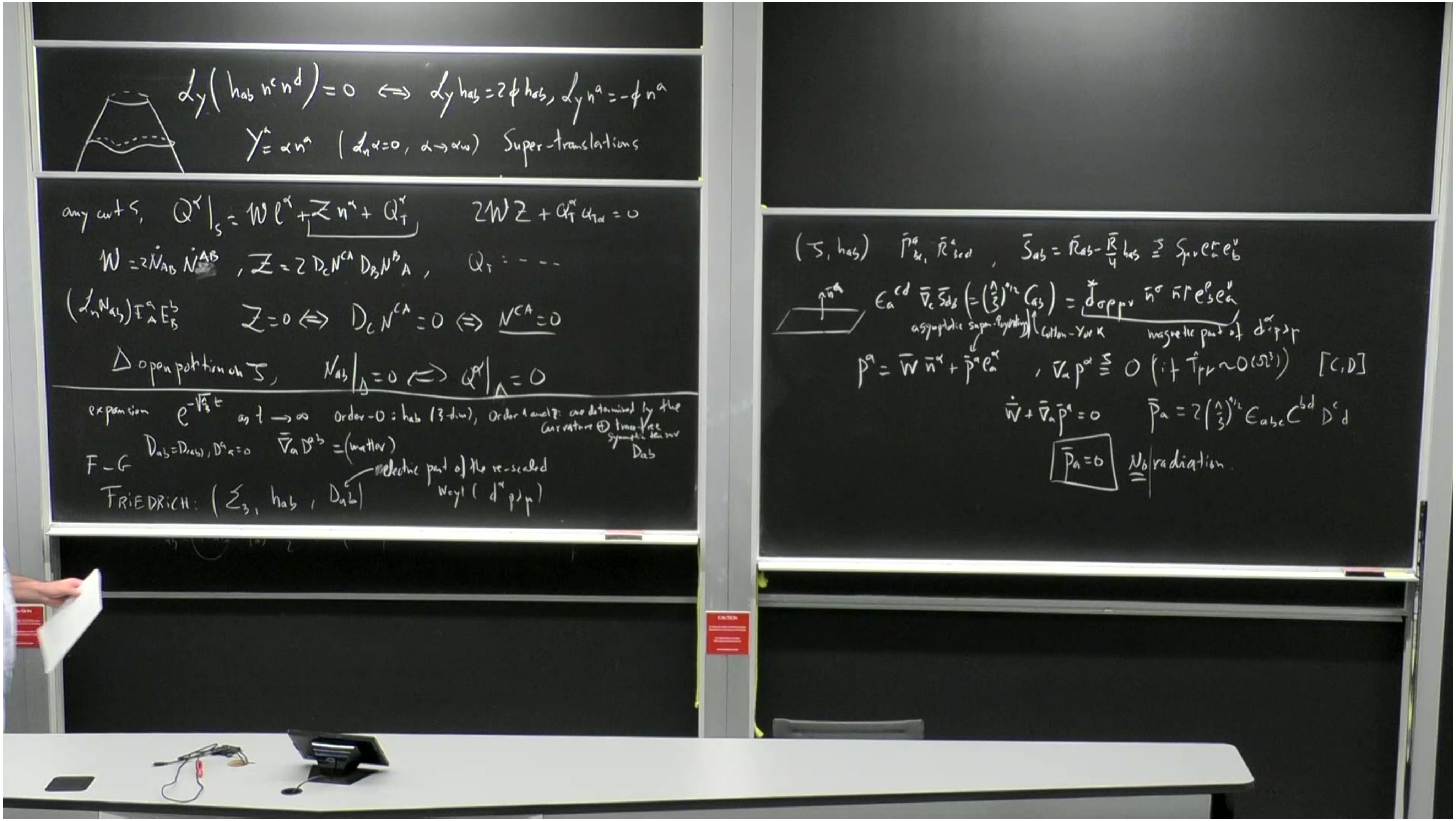
expansion $e^{-\sqrt{\frac{\Lambda}{3}}t}$ as $t \to \infty$ Order-0: h_{ab} (3-dim), Order 1 and 2: are determined by the curvature ⊕ trace-free symmetric tensor D_{ab}

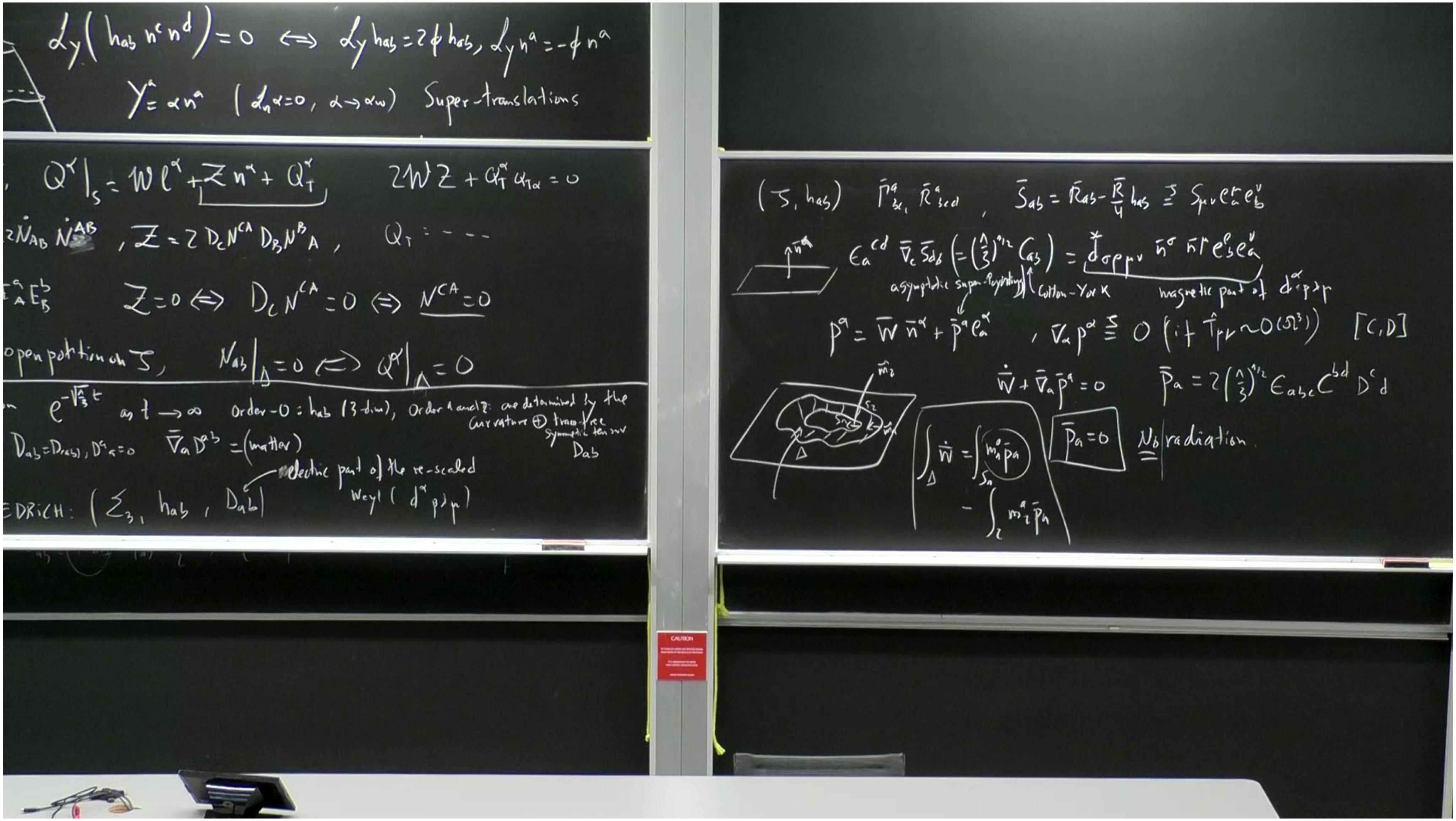
$\bar{\nabla}_a D^{ab} = $ (matter)

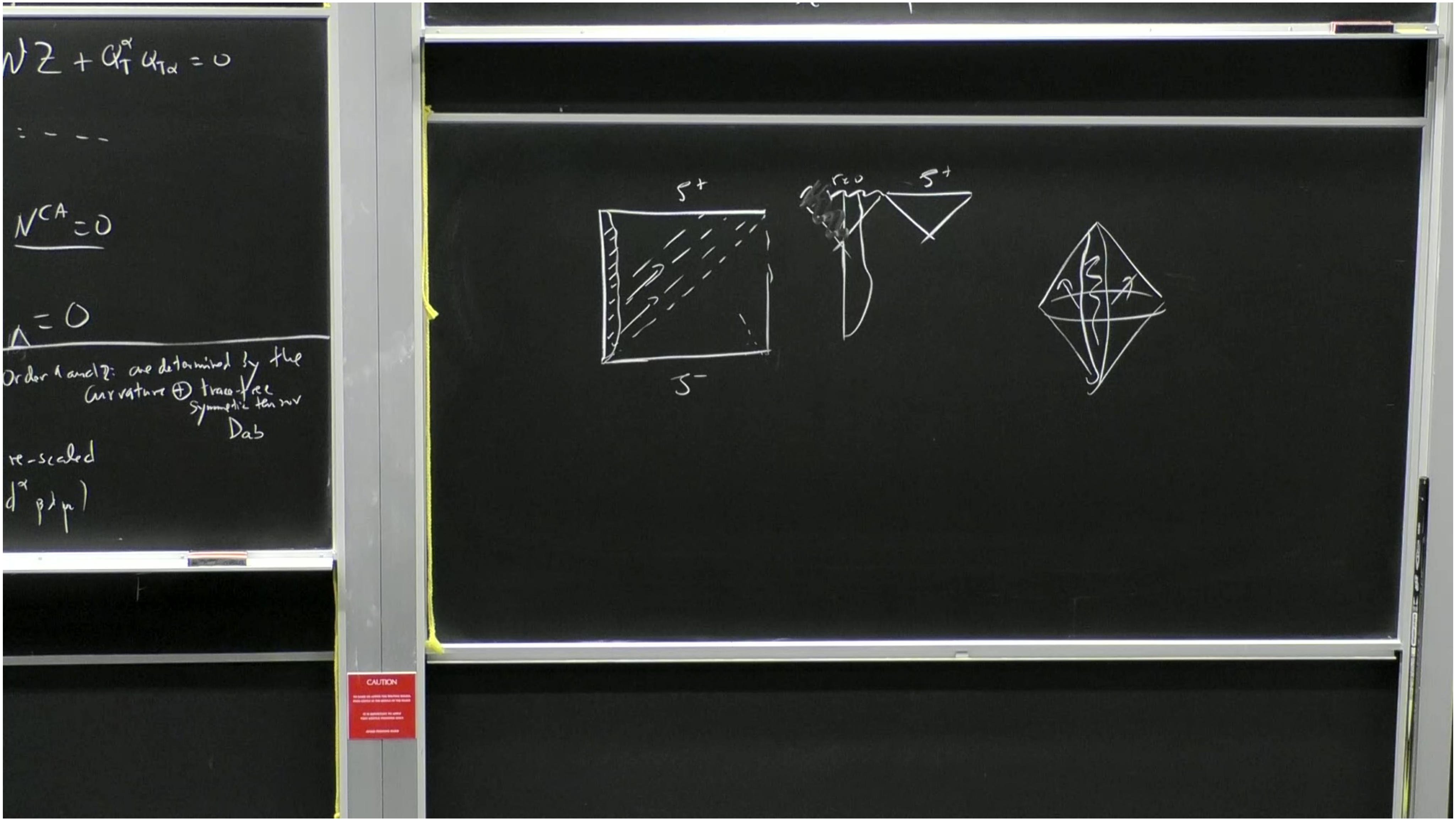
F-G

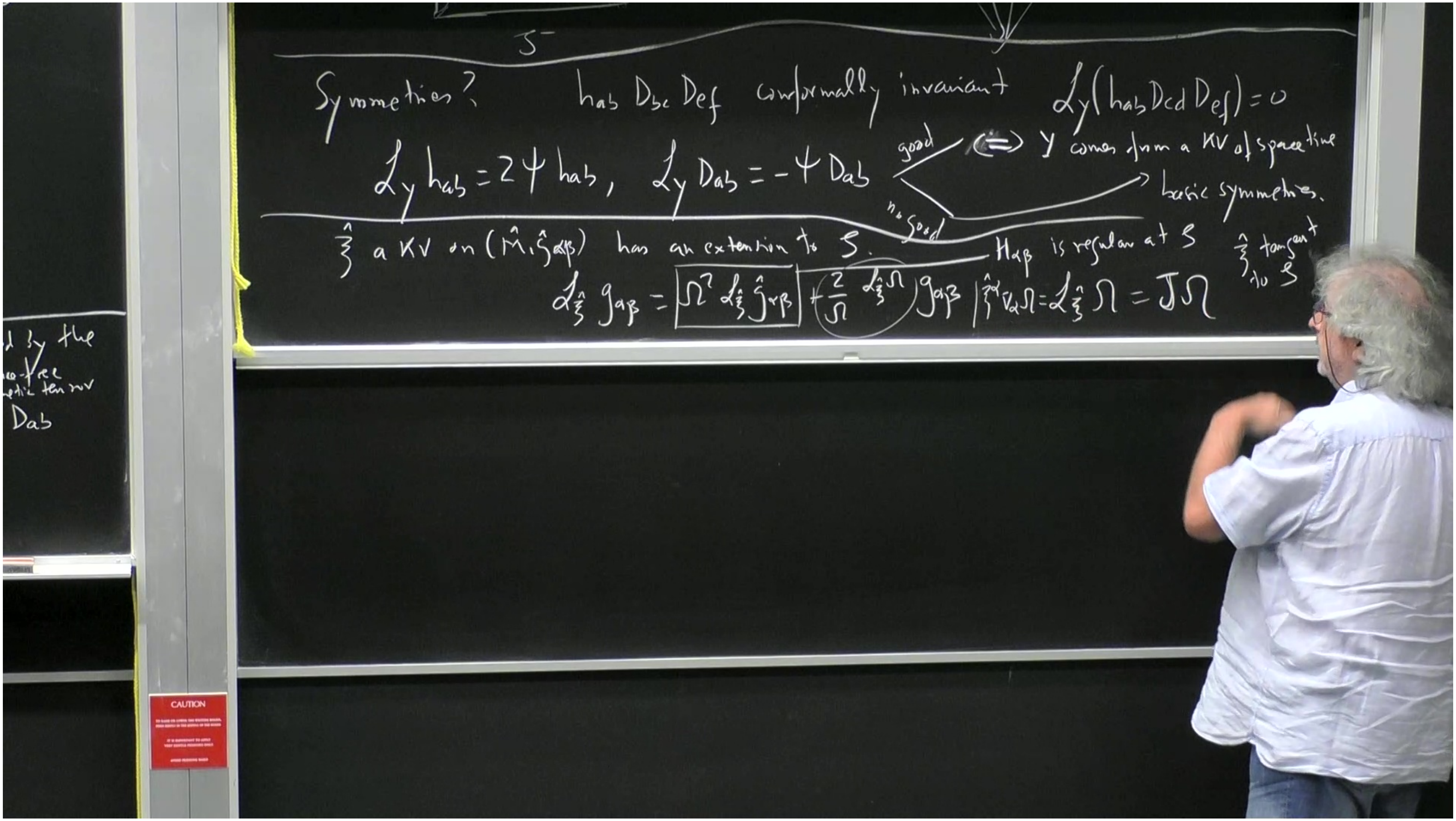
FRIEDRICH: $(\Sigma_3, h_{ab}, D_{ab})$ ← electric part of the re-scaled Weyl ($d^a{}_{p b q}$)

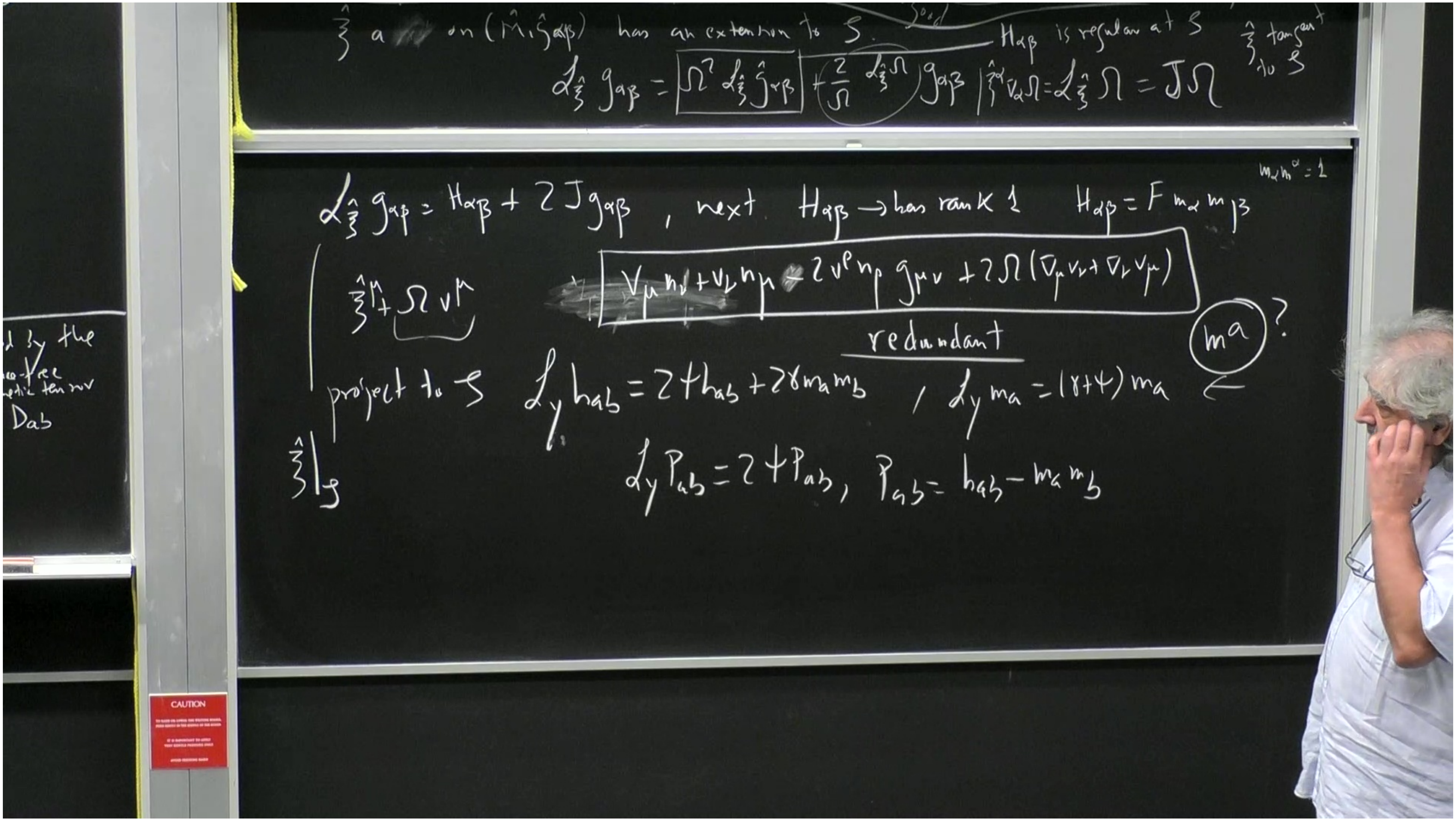
$\dot{\sigma}_{ab} = (N_{ab}) + \rho_{ab} - \frac{K}{2} h_{ab} + $ (many terms on $\nabla_a \ln F$) $\quad \frac{\nabla_a}{\dot{F}}$

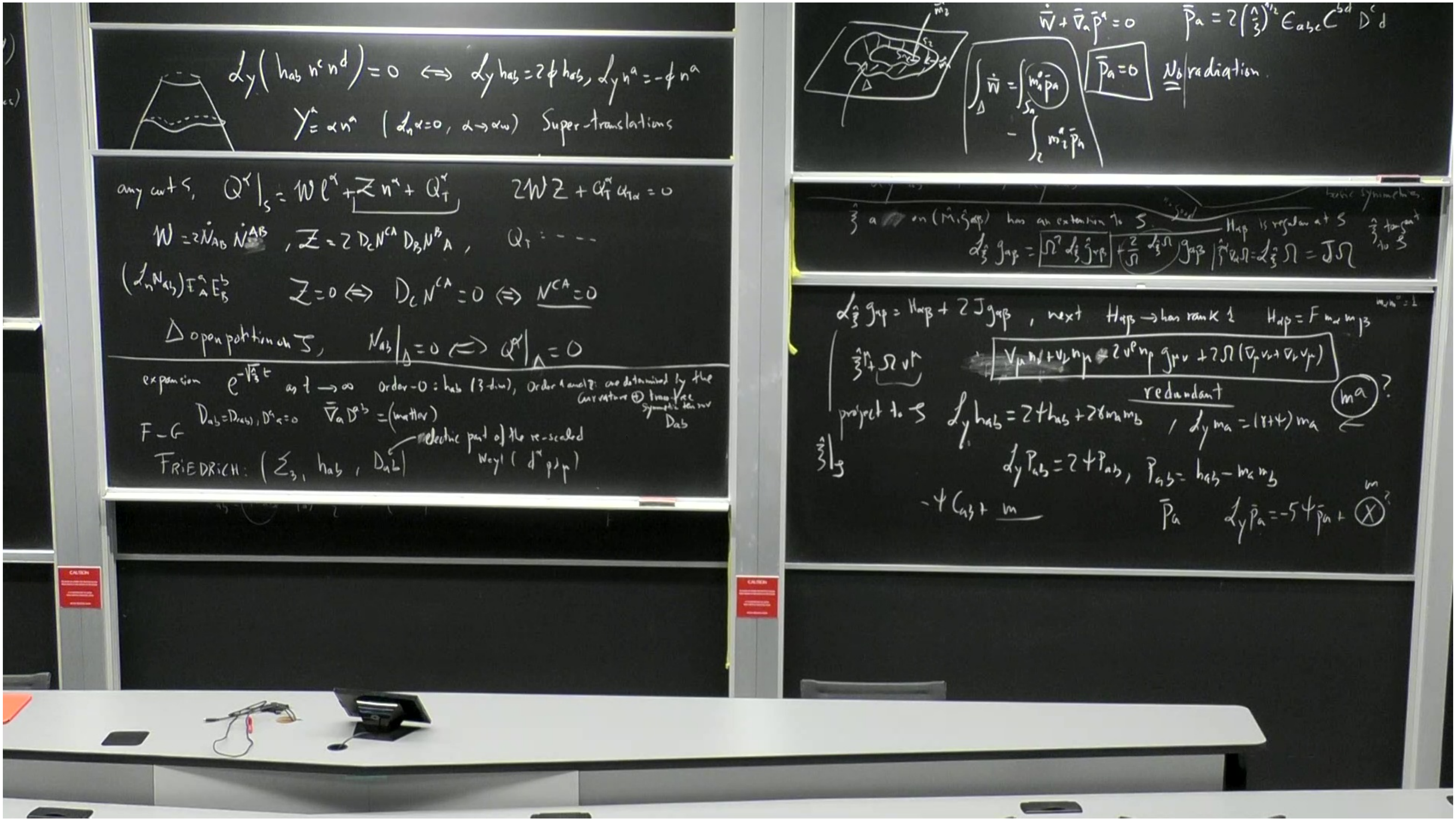

L_Y(h_ab n^c n^d) = 0 ⟺ L_Y h_ab = 2φ h_ab, L_Y n^a = −φ n^a
Y^a = α n^a (L_n α = 0, α → αω) Super-translations
any cut S, Q^α|_S = W ℓ^α + Z n^α + Q_T^α
2WZ + Q_T^α Q_Tα = 0
W = 2 N_AB N^AB, Z = 2 D_C N^CA D_B N^B_A, Q_T : - - -
Z = 0 ⟺ D_C N^CA = 0 ⟺ N^CA = 0
Δ open portion on ℐ, N_ab|_Δ = 0 ⟺ Q^α|_Δ = 0
expansion e^{−√(Λ/3) t} as t → ∞ order-0 : h_ab (3-dim), Order 4 coeff : are determined by the Curvature ⊕ trace-free Symmetric tensor D_ab
F-G
∇̄_a D^ab = (matter)
electric part of the re-scaled Weyl
FRIEDRICH: (Σ_3, h_ab, D_ab)
(ℐ, h_ab)
S̄_ab = R̄_ab − (R̄/4) h_ab
asymptotic super-translation
Cotton-York
magnetic part of
p^a = W̄ n̄^α + p̄^a e_a^α, ∇̄_a p^a ≙ 0 (if T_μν ~ O(Ω^3)) [C,D]
Ẇ + ∇̄_a p̄^a = 0
p̄_a = 2(Λ/3)^{1/2} ε_abc C^bd D^c_d
p̄_a = 0
No radiation.

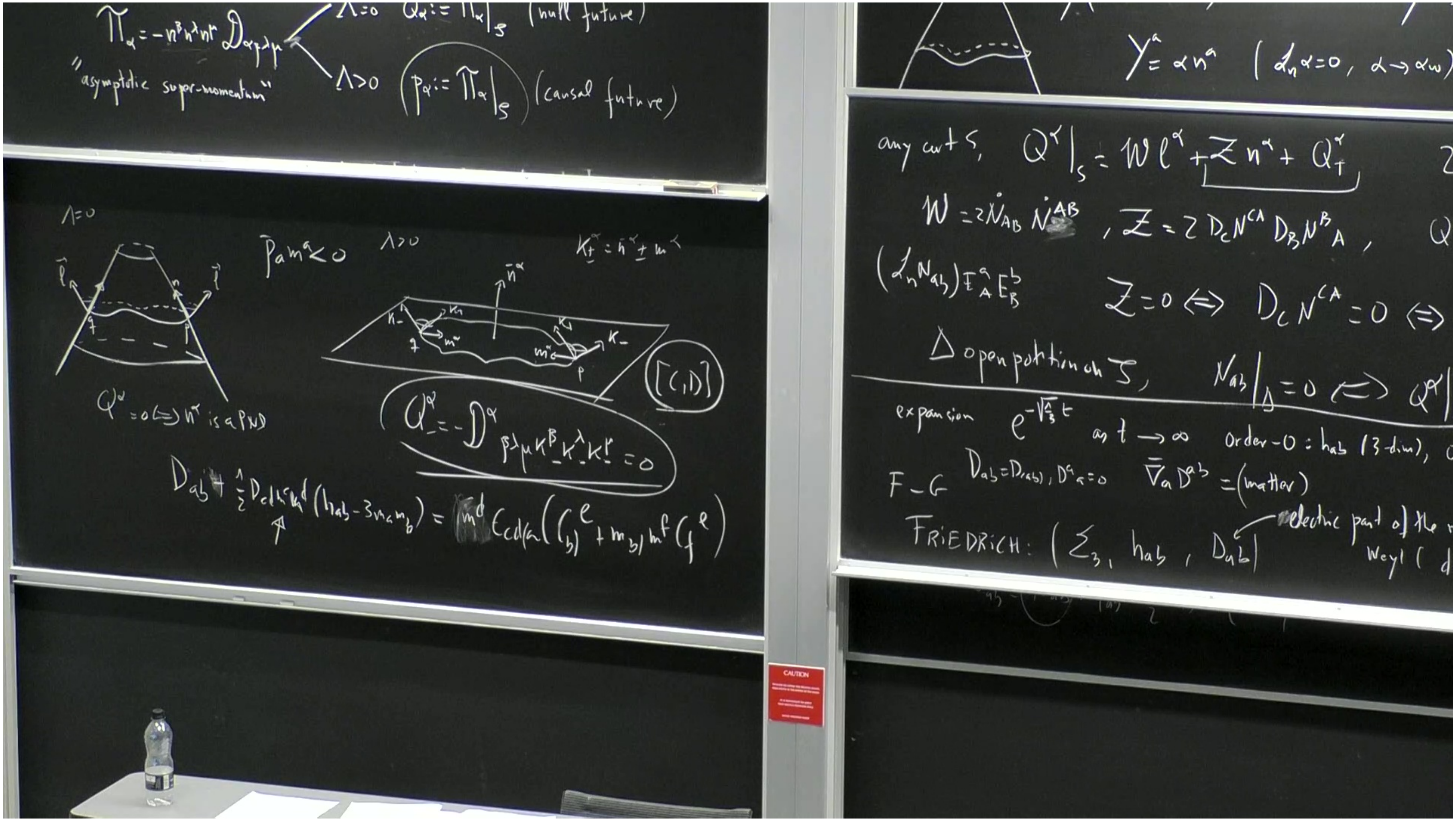

Super-translations
asymptotic super-Poynting
Cotton-York
magnetic part of
No radiation
electric part of the re-scaled Weyl
CAUTION


NZ + Q_T^α Q_Tα = 0
N^CA = 0
Order 1 and 2: are determined by the curvature ⊕ trace-free
symmetric tensor
Dab
re-scaled
J+
J-
r=0


Symmetries?
hab Dbc Def conformally invariant
£Y(hab Dcd Def) = 0
£Y hab = 2ψ hab, £Y Dab = −ψ Dab
good
⟺ Y comes from a KV of spacetime
no good
basic symmetries.
ξ̂ a KV on (M̂, ĝαβ) has an extension to ℐ.
Hαβ is regular at ℐ
ξ̂ tangent to ℐ
£ξ̂ gαβ = Ω² £ξ̂ ĝγβ + 2/Ω £ξ̂ Ω gαβ
ξ̂α ∇α Ω = £ξ̂ Ω = J Ω
Dab
CAUTION


ξ̂ a on (M̂, ĝαβ) has an extension to 𝒮.
Hαβ is regular at 𝒮
ξ̂ tangent to 𝒮
redundant
project to 𝒮
CAUTION

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