

Title: Asymptotic structure and the characterisation of gravitational

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Series: Quantum Gravity

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Abstract: With the main purpose of identifying the existence of gravitational radiation at infinity (\mathcal{scri}), a novel approach to the asymptotic structure of spacetime is presented, focusing mainly in cases with non-negative cosmological constant. The basic idea is to consider the strength of tidal forces experienced by \mathcal{scri} . To that end I will introduce the asymptotic (radiant) super-momentum, a causal vector defined at \mathcal{scri} with remarkable properties that, in particular, provides an innovative characterization of gravitational radiation valid for the general case with $\Lambda \geq 0$ (and which has been proven to be equivalent when $\Lambda = 0$ to the standard one based on the News tensor). This analysis is also shown to be supported by the initial- (or final-) value Cauchy-type problem defined at \mathcal{scri} . The implications are discussed in some detail. The geometric structure of \mathcal{scri} , and of its cuts, is clarified. The question of whether or not a News tensor can be defined in the presence of a positive cosmological constant is addressed. Several definitions of asymptotic symmetries are presented. Conserved charges that may detect gravitational radiation are exhibited. Balance laws that might be useful as diagnostic tools to test the accuracy of model waveforms discussed. An interpretation of the Geroch ρ tensor is found. The whole thing will be complemented with a series of illustrative examples based on exact solutions. In particular we will see that exact solutions with black holes will be radiative if, and only if, they are accelerated.

Zoom link <https://pitp.zoom.us/j/96816406686?pwd=eGZINlo2R0d1YkZMRGhackRNMzBVUT09>

$$p_{ab} = p_{(ab)}, \quad n^a p_{ab} = 0, \quad \bar{\nabla}_{[c} p_{a]b} = 0 \quad (\Rightarrow h^{ab} p_{ab} = \frac{\kappa}{2}). \quad \boxed{N_{ab} = \bar{S}_{ab} - p_{ab}} \text{ News tensor}$$

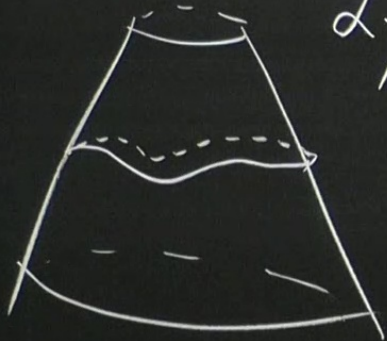
On any cut: if χ is KV with a fixed point $\Rightarrow \mathcal{L}_\chi p_{AB} = -D_A D_B \left(\frac{D_C \chi^C}{2} \right)$ (Adding this, $\int p_{AB}$ for any other topology)

$$D_A D_B \Pi_{(p)} - \frac{1}{2} q_{AB} \Delta \Pi_{(p)} + \left(p_{AB} - \frac{\kappa}{2} q_{AB} \right) \Pi_{(p)} = 0 \quad \leftarrow 4 \text{ solutions } (l=0,1 \text{ spherical harmonics})$$

$$\int \bar{\nabla}_{[b} \delta_{c]a} = \int \bar{\nabla}_{[b} N_{c]a} = -n^p d\rho_{[p]a} e_a^\nu e_b^\beta e_c^\lambda \quad (\Rightarrow \mathcal{L}_n N_{ca} = -n^p n^\lambda d\rho_{[p]a} e_a^\nu e_b^\beta e_c^\lambda)$$

$$\Pi_\alpha = -n^p n^\lambda n^\mu D_{\alpha p \lambda \mu} \begin{cases} \Lambda = 0 & Q_\alpha := \Pi_\alpha|_S \text{ (null future)} \\ \Lambda > 0 & P_\alpha := \Pi_\alpha|_S \text{ (causal future)} \end{cases}$$

"asymptotic super-momentum"



$$\mathcal{L}_Y(h_{ab} n^c n^d) = 0 \iff \mathcal{L}_Y h_{ab} = 2\phi h_{ab}, \mathcal{L}_Y n^a = -\phi n^a$$

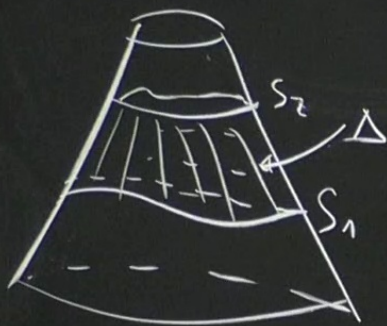
$$Y^a = \alpha n^a \quad (d_n \alpha = 0, \alpha \rightarrow \alpha_w) \quad \text{Super-translations}$$

$$\left\{ \text{KV on } (\hat{M}, \hat{g}) \right\} \rightarrow Y_{\xi}^a = \alpha_{\xi} n^a \quad d_n \bar{S}_{ab} = 0$$

4-dim sub-algebra of TRANSLATIONS

$$\left(\bar{\nabla}_a \bar{\nabla}_b \alpha_{\xi} - \frac{1}{2} h_{ab} \Delta \alpha_{\xi} + \alpha_{\xi} \left(\bar{S}_{ab} - \frac{K}{2} h_{ab} \right) = 0 \right)$$

$$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{1}{2} h_{ab} \Delta \alpha + \alpha \left(\rho_{ab} - \frac{K}{2} h_{ab} \right) = 0$$



$$B_{(p)}(S_2) - B_{(p)}(S_1) = - \frac{1}{8\pi} \int_{\Delta} \Pi_{(p)} \left(N^{ab} N_{ab} + \frac{8\pi G}{c^4} \left[\frac{1}{\sqrt{\Sigma^2}} \hat{T}_{\mu\nu} n^\mu n^\nu \right] \right)$$

$$\int \Pi_{(p)} \Pi_{(q)} = 0$$

Foliation by cuts: $F(x^a) = C$, $\underline{d}_n F \neq 0$

$$\underline{d}_n \dot{F} = 0 \text{ adapted}$$

$$l_a n^a = -1$$

$$l_a = - \frac{1}{\dot{F}} \bar{\nabla}_a F$$

$$l_a = - \bar{\nabla}_a F \quad (\underline{d}_n l_a = 0) \quad \text{canonically adapted}$$

log

$$\dot{\sigma}_{ab} = N_{ab} + P_{ab} - \frac{\kappa}{2} h_{ab} + \left(\text{many terms on } \bar{\nabla}_a (\underline{d}_n \dot{F}) \right) \frac{\bar{\nabla}_a \dot{F}}{\dot{F}}$$

4-dim sub-algebra of TRANSLATIONS

$$\left(\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{\kappa}{2} h_{ab} \Delta \alpha + \alpha \left(\bar{S}_{ab} - \frac{\kappa}{2} h_{ab} \right) \right) = 0$$

$$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{\kappa}{2} h_{ab} \Delta \alpha + \alpha \left(\bar{P}_{ab} - \frac{\kappa}{2} h_{ab} \right) = 0$$

$$y_{(cp)}^a = \pi_{(cp)} n^a$$

$$B_{(p)}(s) = -\frac{1}{8\pi} \int_S \pi_{(p)} \left(d^\rho \nu_{\beta\mu} n_\rho l^\nu l^\beta n^\mu + \sigma^{ab} N_{ab} \right)$$

$$\left. \frac{d}{ds} B_{(p)}(s) = -\frac{1}{8\pi} \int_S \frac{\pi_{(p)}}{S \dot{F}} \left(N_{ab} N^{ab} + \hat{T} \dots \right) \right\}$$

$\hat{d}_n F = 0$ adapted

$$l_a = -\bar{\nabla}_a F \quad (\hat{d}_n l_a = 0) \quad \text{canonically adapted} \quad \text{log}$$

$$\hat{\sigma}_{ab} = \left(N_{ab} + \bar{P}_{ab} - \frac{\kappa}{2} h_{ab} + \left[\text{many terms on } \bar{\nabla}_a \underline{\underline{l_n F}} \right] \right) \frac{\bar{\nabla}_a \dot{F}}{\dot{F}}$$

4-dim sub-algebra of TRANSLATIONS

$$\left\{ \begin{aligned} \bar{\nabla}_a \bar{\nabla}_b \alpha &= \frac{\kappa}{2} h_{ab} \Delta \alpha + \\ \alpha \left(\bar{S}_{ab} - \frac{\kappa}{2} h_{ab} \right) &= 0 \end{aligned} \right.$$

$$\bar{\nabla}_a \bar{\nabla}_b \alpha - \frac{\kappa}{2} h_{ab} \Delta \alpha + \alpha \left(\bar{P}_{ab} - \frac{\kappa}{2} h_{ab} \right) = 0$$

$$y_{(p)}^a = \Pi_{(p)} n^a$$

$$B_{(p)}(s) = -\frac{\hat{1}}{8\pi} \int_S \Pi_{(p)} \left(d^\rho \nu_{\beta\rho} n_\rho l^\nu p^\beta n^\nu + \sigma^{ab} N_{ab} \right)$$

$$\left. \frac{d}{ds} B_{(p)}(s) = -\frac{\hat{1}}{8\pi} \int_{S \cap \dot{F}} \left(N_{ab} N^{ab} + \hat{T} \dots \right) \right.$$

$l_n \dot{F} = 0$ adapted

$l_a n^a = -1$

$l_a = -\frac{\hat{1}}{\dot{F}} \bar{\nabla}_a F$

$l_a = -\bar{\nabla}_a F$ ($l_n l_a = 0$) canonically adapted

log

$\hat{J}_{ab} = \underbrace{(N_{ab})}_{\text{circled}} + \bar{P}_{ab} - \frac{\kappa}{2} h_{ab} + \left[\text{many terms on } \bar{\nabla}_a \underline{\underline{l_n F}} \right] \frac{\bar{\nabla}_a \dot{F}}{\dot{F}}$

$$\nabla_a \nabla_b \alpha - \frac{1}{\gamma} h_{ab} \Delta \alpha + \alpha (\rho_{ab} - \frac{1}{\gamma} h_{ab}) = 0 \quad \gamma_{(m)}^a = \Pi_{(m)} n^a$$

any cut S , $Q^x|_S = W l^x + \underbrace{Z n^x}_{\text{}} + Q_T^x$ $2WZ + Q_T^x Q_{T\alpha}^x = 0$

$W = 2\dot{N}_{AB} \dot{N}^{AB}$, $Z = 2D_C N^{CA} D_B N^B_A$, $Q_T = \dots$

$(\dot{N}_{ab}) E_A^a E_B^b$

$Z=0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow \underline{N^{CA} = 0}$

$$\nabla_a \nabla_b \alpha - \frac{1}{7} h_{ab} \Delta \alpha + \alpha (\rho_{ab} - \frac{1}{7} h_{ab}) = 0 \quad \gamma_{(m)}^a = \Pi_{(m)} n^a$$

any cut \mathcal{S} , $Q^\alpha|_{\mathcal{S}} = W l^\alpha + \underbrace{Z n^\alpha}_{\text{---}} + Q_T^\alpha$, $Z W Z + Q_T^\alpha Q_{T\alpha} = 0$

$W = 2 \dot{N}_{AB} \dot{N}^{AB}$, $Z = 2 D_C N^{CA} D_B N^B_A$, $Q_T = \dots$

$(\dot{N}_{AB}) E_A^a E_B^b$

$Z = 0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow \underline{N^{CA} = 0}$

Δ open portion of \mathcal{S} , $N_{ab}|_{\Delta} = 0 \Leftrightarrow Q^\alpha|_{\Delta} = 0$

any cut S , $Q^\alpha|_S = W l^\alpha + \underbrace{Z n^\alpha + Q_T^\alpha}_{\text{---}}$, $2WZ + Q_T^\alpha Q_{T\alpha} = 0$

$W = 2\dot{N}_{AB} N^{AB}$, $Z = 2D_C N^{CA} D_B N^B{}_A$, $Q_T = \dots$

$(\dot{N}_{AB}) E_A^\alpha E_B^\beta$ $Z=0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow \underline{N^{CA} = 0}$


Δ open portion of S , $N_{ab}|_\Delta = 0 \Leftrightarrow Q^\alpha|_\Delta = 0$

expansion $e^{-\sqrt{3}t}$ as $t \rightarrow \infty$ order -0: h_{ab} (3-dim), order 1 and 2: are determined by the curvature \oplus trace-free symmetric tensor D_{ab}

F-G

FRIEDRICH: $(\Sigma_3, h_{ab}, D_{ab})$ electric part of the re-scaled Weyl $(d^\alpha p)_\mu$

$\dot{\sigma}_{ab} = \underbrace{N_{ab}}_{\text{---}} + P_{ab} - \frac{K}{2} h_{ab} + (\text{many terms on } \nabla_a \ln F)$ $\frac{\dot{\sigma}_{ab}}{F}$



$$\mathcal{L}_Y(h_{ab} n^c n^d) = 0 \Leftrightarrow \mathcal{L}_Y h_{ab} = 2\phi h_{ab}, \mathcal{L}_Y n^a = -\phi n^a$$

$$Y^a = \alpha n^a \quad (\mathcal{L}_n \alpha = 0, \alpha \rightarrow \alpha \omega) \quad \text{Super-translations}$$

any cut \mathcal{S} , $Q^r|_{\mathcal{S}} = W l^r + \underline{Z} n^r + Q_T^r$, $2WZ + Q_T^r u_{ra} = 0$

$W = 2N_{AB} N^{AB}$, $\underline{Z} = 2D_C N^{CA} D_B N^B{}_A$, $Q_T = \dots$


$(\mathcal{L}_n N_{AB}) E_A^r E_B^s$ $Z=0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow N^{CA} = 0$

Δ open portion on \mathcal{S} , $N_{ab}|_{\Delta} = 0 \Leftrightarrow Q^r|_{\Delta} = 0$

expansion $e^{-\int \frac{1}{3} t}$ as $t \rightarrow \infty$ order-0: h_{ab} (3 dim), order-1 and 2: are determined by the curvature \oplus mass \oplus spin \oplus linear momentum D_{ab}

F-G $D_{ab} = D_{(a} \delta_{b)}$, $D^a{}_c = 0$ $\bar{D}_a D^b = (\text{matter})$
 FRIEDRICH: $(\Sigma_3, h_{ab}, D_{ab})$ selective part of the re-scaled Weyl ($d^r p_r$)

(\mathcal{S}, h_{ab}) $\bar{P}^a{}_r, \bar{R}^a{}_{bcd}$, $\bar{S}_{ab} = \bar{R}_{ab} - \frac{\bar{R}}{4} h_{ab} \cong S_{\nu\sigma} e^{\nu} e^{\sigma}$

 $E_a{}^{cd} \bar{D}_c \bar{S}_{db} = (\frac{2}{3})^{1/2} C_{ab} = \underbrace{d^r p_r}_{\text{asymptotic sup. symmetry}} \underbrace{\bar{n}^r \bar{n}^s e^r e^s}_{\text{Cotton-York}} \underbrace{e^a}_{\text{magnetic part of } d^r p_r}$

$p^a = \bar{W} \bar{n}^a + \bar{p}^a e^a$, $\bar{D}_a p^a \cong 0$ (if $\hat{T}_{\mu\nu} \sim O(\Omega^2)$) [C,D]

$\dot{\bar{W}} + \bar{D}_a \bar{p}^a = 0$ $\bar{p}_a = 2(\frac{2}{3})^{1/2} \epsilon_{abc} C^{bd} D^c d$

$\bar{p}_a = 0 \quad \underline{=} \quad \text{No radiation.}$

$$d_Y(\text{has } n^c n^d) = 0 \iff d_Y \text{has} = 2\phi \text{has}, d_Y n^a = -\phi n^a$$

$$Y = \alpha n^a \quad (d_n \alpha = 0, \alpha \rightarrow \alpha \omega) \text{ Super-translations}$$

$$Q^Y|_S = W l^a + \underbrace{Z n^a}_{\substack{2N_{AB} N^{AB} \\ Z = 2D_C N^{CA} D_B N^B A}} + Q_T^Y \quad 2WZ + Q_T^a Q_{T\alpha} = 0$$

$$Z=0 \iff D_C N^{CA} = 0 \iff N^{CA} = 0$$

open portion of S , $N_{ab}|_\Delta = 0 \iff Q^Y|_\Delta = 0$

as $t \rightarrow \infty$ order -0: has (3-dim), order 1 and 2: are determined by the curvature \oplus trans free symplectic form D_{ab}

$D_{ab} = D_{(ab)}, D^a_n = 0 \quad \bar{\nabla}_a D^{ab} = (\text{matter})$

electric part of the re-scaled Weyl $(d^a \rho)_n$

EDRICH: $(\sum_{3,1} \text{has}, D_{ab})$

$$(\Sigma, \text{has}) \quad \bar{P}^a_{bc}, \bar{R}^a_{bcd}, \quad \bar{S}_{ab} = \bar{R}_{ab} - \frac{\bar{R}}{4} \text{has} \cong \text{Sym}^2 e^a_e$$

$$E_a^{cd} \bar{\nabla}_c \bar{S}_{db} = \left(\frac{1}{3}\right)^{1/2} C_{ab} = \underbrace{d^*_{opp} n^\sigma n^\tau e^a_\sigma e^b_\tau}_{\substack{\text{asymptotic super-Ryckhoff} \\ \text{Cotton-York} \\ \text{magnetic part of } d^*_{opp}}}$$

$$p^a = \bar{W} n^a + \bar{p}^a e^a_n, \quad \bar{\nabla}_a p^a \cong 0 \quad (\text{if } \hat{r}_\nu \sim O(\Omega^2)) \quad [C,D]$$

$$\bar{W} + \bar{\nabla}_a \bar{p}^a = 0 \quad \bar{p}_a = 2 \left(\frac{1}{3}\right)^{1/2} \epsilon_{abc} C^{bd} D^c d$$

$$\int_\Delta \dot{\bar{W}} = \int_{S_n} m^a_n \bar{p}_n - \int_c m^a_c \bar{p}_n \quad \boxed{\bar{p}_a = 0} = \text{No radiation}$$

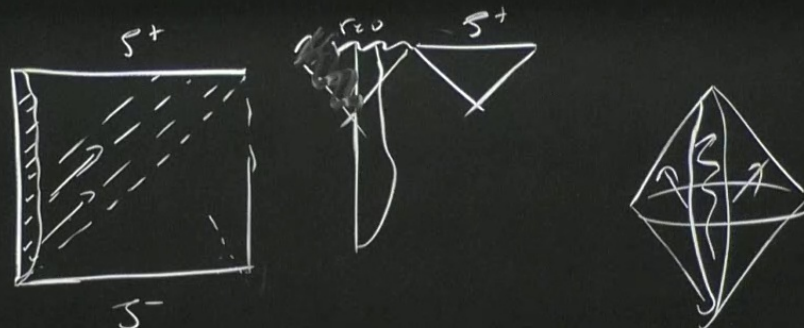
$$N^2 + G_T^{\alpha} G_{Td} = 0$$

$$N^{\alpha} = 0$$

$$\Lambda = 0$$

Order 1 and 2: are determined by the
curvature \oplus trace-free
symmetric tensor
 D_{ab}

re-scaled
 $d^{\alpha} p_{\alpha}$



CAUTION
DO NOT TOUCH THE BOARD SURFACE
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Symmetries?

S^-

has Dsc Def conformally invariant

$$\mathcal{L}_Y(\text{has Dcd Def}) = 0$$

$$\mathcal{L}_Y \text{has} = 2\psi \text{has}, \quad \mathcal{L}_Y \text{Dab} = -\psi \text{Dab}$$

good

\Leftrightarrow

Y comes from a KV of spacetime

no good

basic symmetries.

$\hat{\Sigma}$ a KV on $(\hat{M}, \hat{g}_{\alpha\beta})$ has an extension to S .

$H_{\alpha\beta}$ is regular at S

$\hat{\Sigma}$ tangent to S

$$d_{\hat{\Sigma}}^{\alpha} g_{\alpha\beta} = \left[\Omega^2 d_{\hat{\Sigma}}^{\alpha} \hat{g}_{\alpha\beta} \right] + \left(\frac{2}{\Omega} d_{\hat{\Sigma}}^{\alpha} \Omega \right) g_{\alpha\beta} \quad \hat{g}_{\alpha\beta} \text{ on } \hat{\Sigma} = d_{\hat{\Sigma}}^{\alpha} \Omega = \mathcal{J} \Omega$$

d by the
no free
metric tensor
Dab



$\hat{\Sigma}$ a \mathbb{Z}_2 on $(\hat{M}, \hat{g}_{\alpha\beta})$ has an extension to S . $H_{\alpha\beta}$ is regular at S $\hat{\Sigma}$ tangent to S

$$d_{\hat{\Sigma}}^{\perp} g_{\alpha\beta} = \boxed{\Omega^2 d_{\hat{\Sigma}}^{\perp} \hat{g}_{\alpha\beta}} + \frac{2}{\Omega} d_{\hat{\Sigma}}^{\perp} \Omega g_{\alpha\beta} \Big|_{\hat{\Sigma}} \hat{v}_{\alpha} \Omega = d_{\hat{\Sigma}}^{\perp} \Omega = J\Omega$$

$d_{\hat{\Sigma}}^{\perp} g_{\alpha\beta} = H_{\alpha\beta} + 2Jg_{\alpha\beta}$, next $H_{\alpha\beta} \rightarrow$ has rank 1 $H_{\alpha\beta} = F_{m_a m_b}$ $m_a m_b = 1$

$\hat{\Sigma}^{\perp} \Omega v^{\mu}$

$$\boxed{v_{\mu} n_{\nu} + v_{\nu} n_{\mu} = 2v^{\rho} n_{\rho} g_{\mu\nu} + 2\Omega (\nabla_{\mu} v_{\nu} + \nabla_{\nu} v_{\mu})}$$

redundant

project to S $d_{\hat{\Sigma}}^{\perp} h_{ab} = 2\delta h_{ab} + 2\delta m_a m_b$ / $d_{\hat{\Sigma}}^{\perp} m_a = (1+4)m_a$ m_a ?

$\hat{\Sigma}|_S$ $d_{\hat{\Sigma}}^{\perp} P_{ab} = 2\delta P_{ab}$, $P_{ab} = h_{ab} - m_a m_b$

d by the
no free
metric tensor
Dab





$$d_Y(h_{ab} n^c n^d) = 0 \iff d_Y h_{ab} = 2\phi h_{ab}, d_Y n^a = -\phi n^a$$

$$Y^a = \alpha n^a \quad (d_Y \alpha = 0, \alpha \rightarrow \alpha \omega) \text{ Super-translations}$$

any cut \mathcal{S} , $Q^X|_{\mathcal{S}} = W \ell^X + Z n^X + Q_T^X$, $2WZ + Q_T^X \ell_{TX} = 0$

$$W = 2N_{AB} N^{AB}, Z = 2D_C N^{CA} D_B N^B{}_\Lambda, Q_T = \dots$$

$$(D_C N_{AB}) F^A{}_\Lambda F^B{}_\Sigma \quad Z=0 \iff D_C N^{CA} = 0 \iff N^{CA} = 0$$

Δ open portion on \mathcal{S} , $N_{ab}|_{\Delta} = 0 \iff Q^X|_{\Delta} = 0$

expansion $e^{-\frac{1}{3}\epsilon t}$ as $t \rightarrow \infty$ Order 0: has 13-dim, Order 1 and 2: are determined by the curvature & linear & quadratic terms in D_{ab}

F-G $D_{ab} = D_{(a} \nu_{b)}$, $D^a \nu_a = 0$, $\bar{\nabla}_a \nu^b = (\text{matter})$

FRIEDRICH: (\sum_3, h_{ab}, D_{ab}) reductive part of the re-scaled Weyl ($d^* \rho_{\mathcal{P}}$)

$$\bar{W} + \bar{\nabla}_a \bar{p}^a = 0 \quad \bar{p}_a = 2 \left(\frac{\Lambda}{3}\right)^{1/2} \epsilon_{abc} C^{bd} D^c d$$

$$\int_{\Delta} \bar{W} = \int_{S_{\Delta}} m_i \bar{p}^i - \int_{\mathcal{L}} m_i \bar{p}^i$$

$$\bar{p}_a = 0 \quad \underline{N_b} \text{ radiation}$$

$\frac{1}{3} a$ on (M, g_{ab}) has an extension to S near infinity

$d_{\mathcal{S}}^2 g_{ab} = \int_{\mathcal{S}} d_{\mathcal{S}}^2 j_{ab} + \frac{2}{\Omega} d_{\mathcal{S}}^2 \Omega g_{ab}$ H_{ab} is regular at S

$$\int_{\mathcal{S}} d_{\mathcal{S}}^2 \Omega = \int_{\mathcal{S}} d_{\mathcal{S}}^2 \Omega = \int_{\mathcal{S}} d_{\mathcal{S}}^2 \Omega$$

$d_{\mathcal{S}}^2 g_{ab} = H_{ab} + 2J_{ab}$, next $H_{ab} \rightarrow$ has rank 2 $H_{ab} = F_{m_a m_b}$ $m_a = 1$

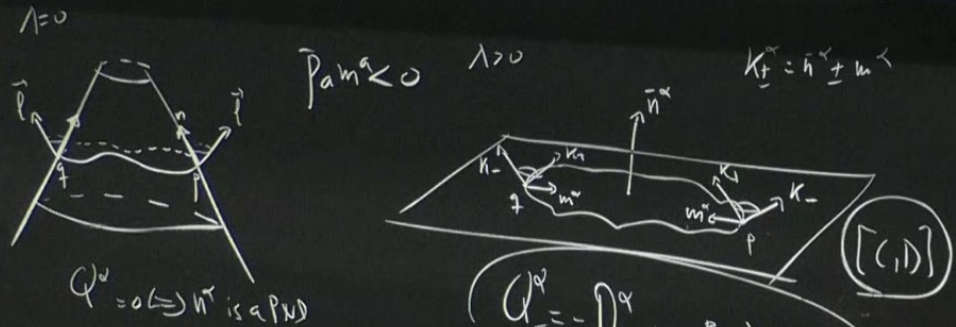
$$\int_{\mathcal{S}} d_{\mathcal{S}}^2 \Omega \nu^a \nu^b \quad \nu_{\mu} n^{\mu} + \nu_{\nu} m^{\nu} = 2\nu^{\mu} n^{\nu} g_{\mu\nu} + 2\Omega(\nu_{\mu} \nu^{\mu} + \nu_{\nu} \nu^{\nu})$$

project to \mathcal{S} $d_Y h_{ab} = 2\phi h_{ab} + 2\delta m_a m_b$ redundant / $d_Y m_a = (1+\psi)m_a$ (ma)?

$$d_Y p_{ab} = 2\psi p_{ab}, p_{ab} = h_{ab} - m_a m_b$$

$$-\psi C_{ab} + m \quad \bar{p}_a \quad d_Y \bar{p}_a = -5\psi \bar{p}_a + \text{X?}$$

$\Pi_\alpha = -n^\beta n^\gamma n^\delta D_{\alpha\beta\gamma\mu}$
 "asymptotic super-momentum"
 $\Lambda = 0 \quad Q_\alpha := \Pi_\alpha|_S \quad (\text{null future})$
 $\Lambda > 0 \quad (P_\alpha := \Pi_\alpha|_S) \quad (\text{causal future})$



$P_{\alpha m} \leq 0 \quad \Lambda > 0$

$K_+ = \bar{n}^\alpha + m^\alpha$

$Q_\alpha = -D^\beta P_{\alpha\beta} = 0$

$Q^\alpha = 0 \Leftrightarrow n^\alpha$ is a PND

$D_{ab} + \frac{1}{2} D_c h^c_d (h_{ab} - 3m_a m_b) = \int_{\mathcal{M}} \epsilon_{cd} (f_a^e + m_b) m^c f^e$

$Y^\alpha = \alpha n^\alpha \quad (d_n \alpha = 0, \alpha \rightarrow \alpha \omega)$

any cut S , $Q^\alpha|_S = W l^\alpha + Z n^\alpha + Q_T^\alpha$

$W = 2N_{AB} N^{AB}, \quad Z = 2D_C N^{CA} D_B N^B_A$

$(\int_n N_{ab}) E_A^a E_B^b$
 $Z = 0 \Leftrightarrow D_C N^{CA} = 0 \Leftrightarrow$

Δ open portion of S , $N_{ab}|_\Delta = 0 \Leftrightarrow Q^\alpha|_\Delta = 0$

expansion $e^{-\sqrt{3}t}$ as $t \rightarrow \infty$ order -0: has (3-dim), 0

$F-G \quad D_{ab} = D_{ab}, D^a_a = 0 \quad \bar{\nabla}_a D^a_b = (\text{matter})$

FRIEDRICH: $(\Sigma_3, h_{ab}, D_{ab})$ electric part of the Weyl (d)