Title: Cauchy Characteristic Matching

Speakers: Sizheng Ma

Series: Strong Gravity

Date: September 14, 2023 - 1:00 PM

URL: https://pirsa.org/23090094

Abstract: Two major approaches are used when numerically solving the Einstein field equations. The first one is to use spatial Cauchy slices and treat the system as a standard Cauchy initial value problem. Cauchy-characteristic evolution (CCE) serves as the second approach, which evolves spacetime based on null hypersurfaces. The Cauchy formulation is suitable for the strong field region but is computationally expensive to extend to the wave zone, whereas the Characteristic approach is fast in the wave zone but fails near the binary system where the null surfaces are ill-defined. By combining those two techniques -- simulating the inner region with Cauchy evolution and the outer region with CCE, Cauchy-Characteristic matching (CCM) enables us to take advantage of both methods. In this talk, I present our recent implementation of CCM based on a numerical relativity code SpECTRE. I also discuss how CCM improves the accuracy of Cauchy boundary conditions -- a benefit that allows us to evolve less of the wave zone in the Cauchy code without losing precision.

Zoom link https://pitp.zoom.us/j/98246275227?pwd=QWtmUDNkMlF6bXROLzBoYXVVTGpldz09

Cauchy-Characteristic Matching

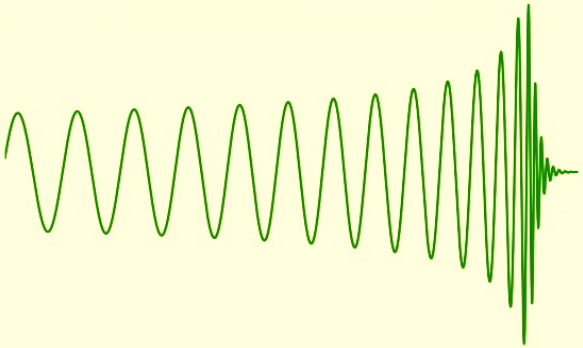
Sizheng Ma

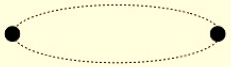
arXiv: 2308.10361

Outline

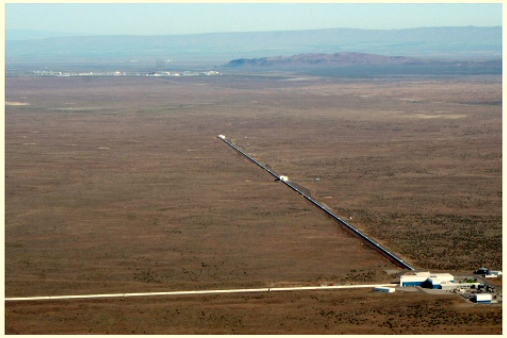
- Introduction
- SpECTRE CCM
- Results
- Current status and future plans

Numerical Relativity and Binary Black-Hole Mergers





Gravitational Waves



Binary Black-Hole

LIGO

Near Zone

Cauchy Evolution

Wave Zone

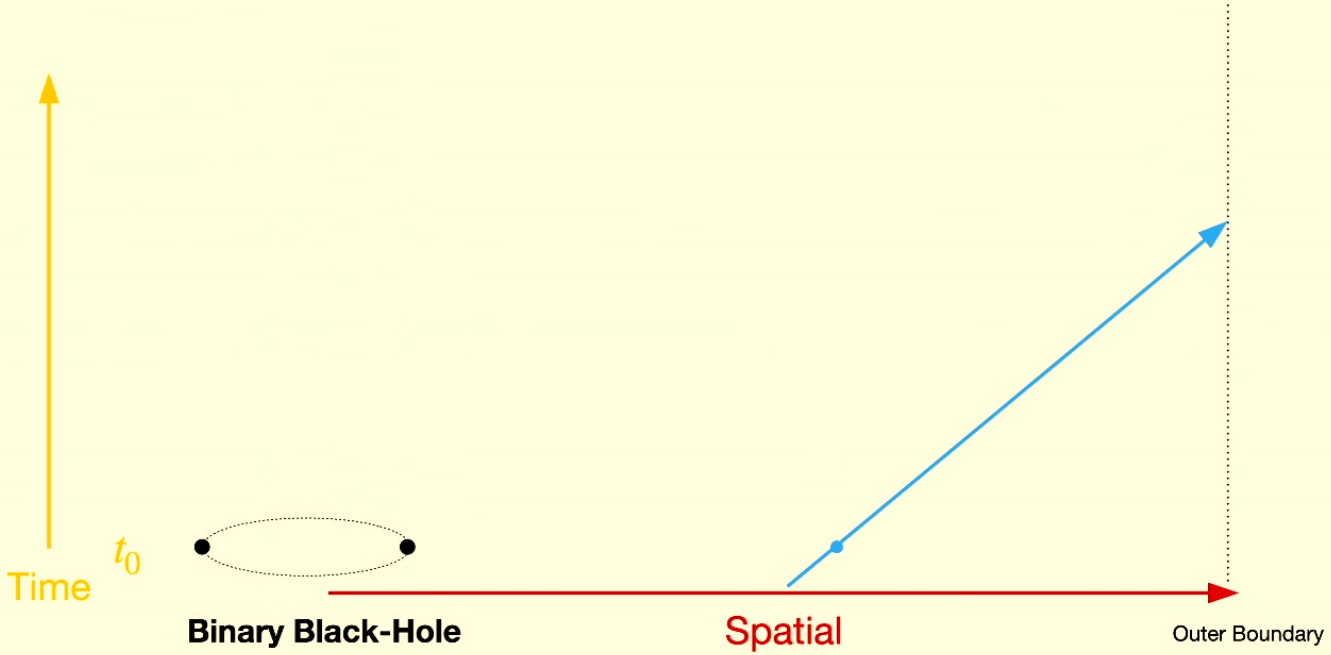
Characteristic Evolution

Three formulations in numerical relativity

- Cauchy evolution (near zone)
- Characteristic evolution (wave zone)
- Hyperboloidal slicing (near+wave zone)

Cauchy evolution and wave extraction

GW detectors



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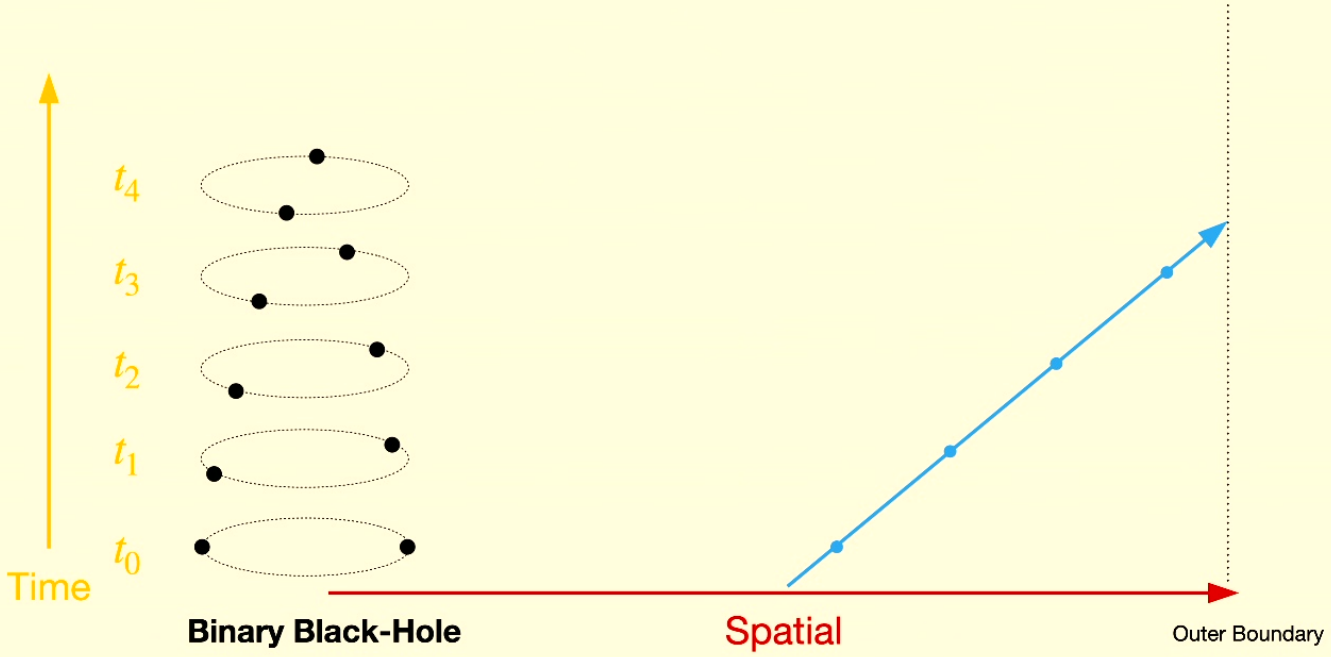

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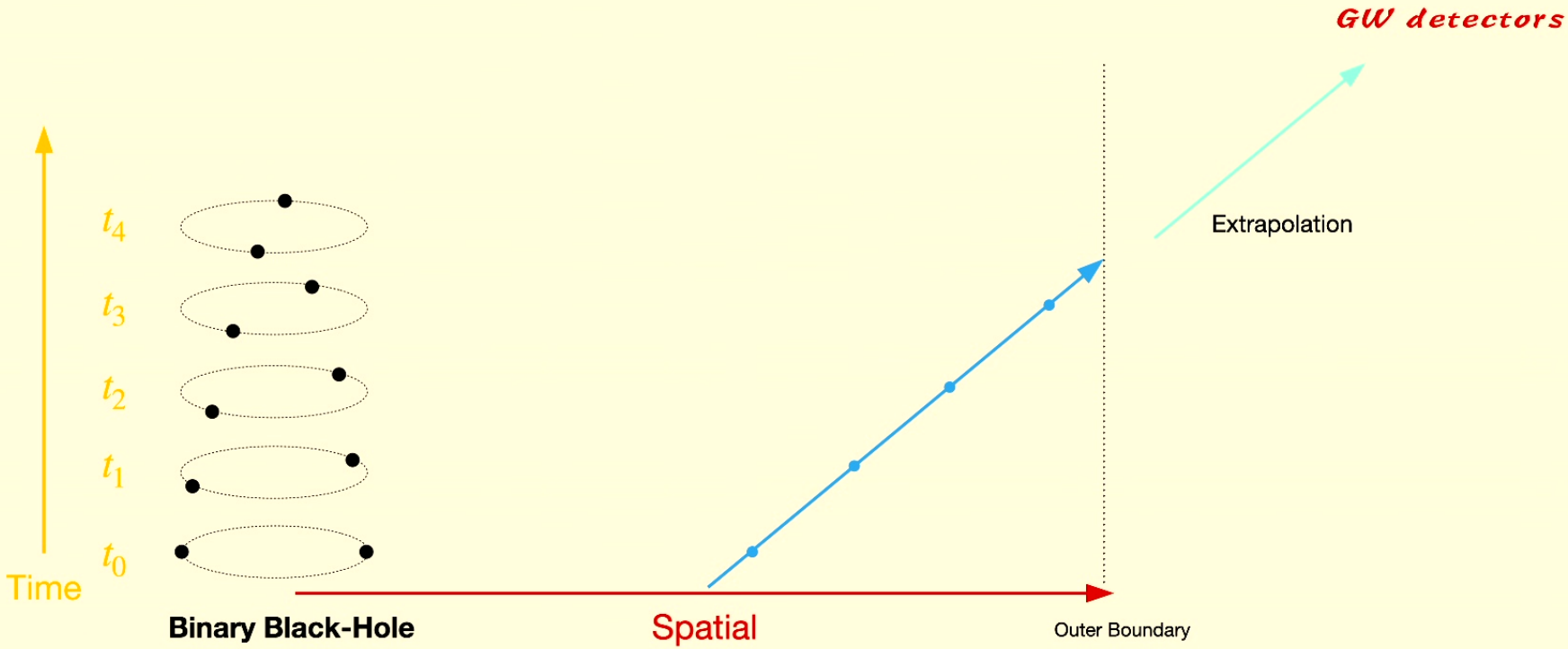
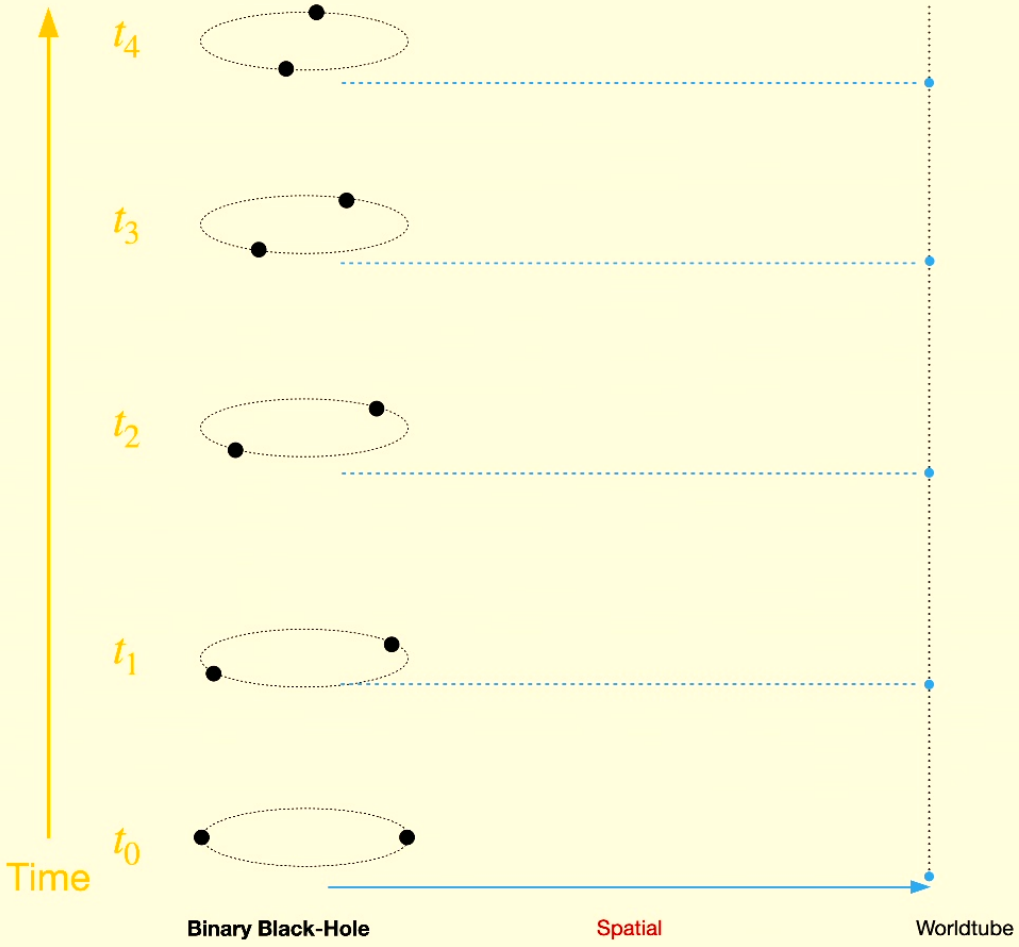


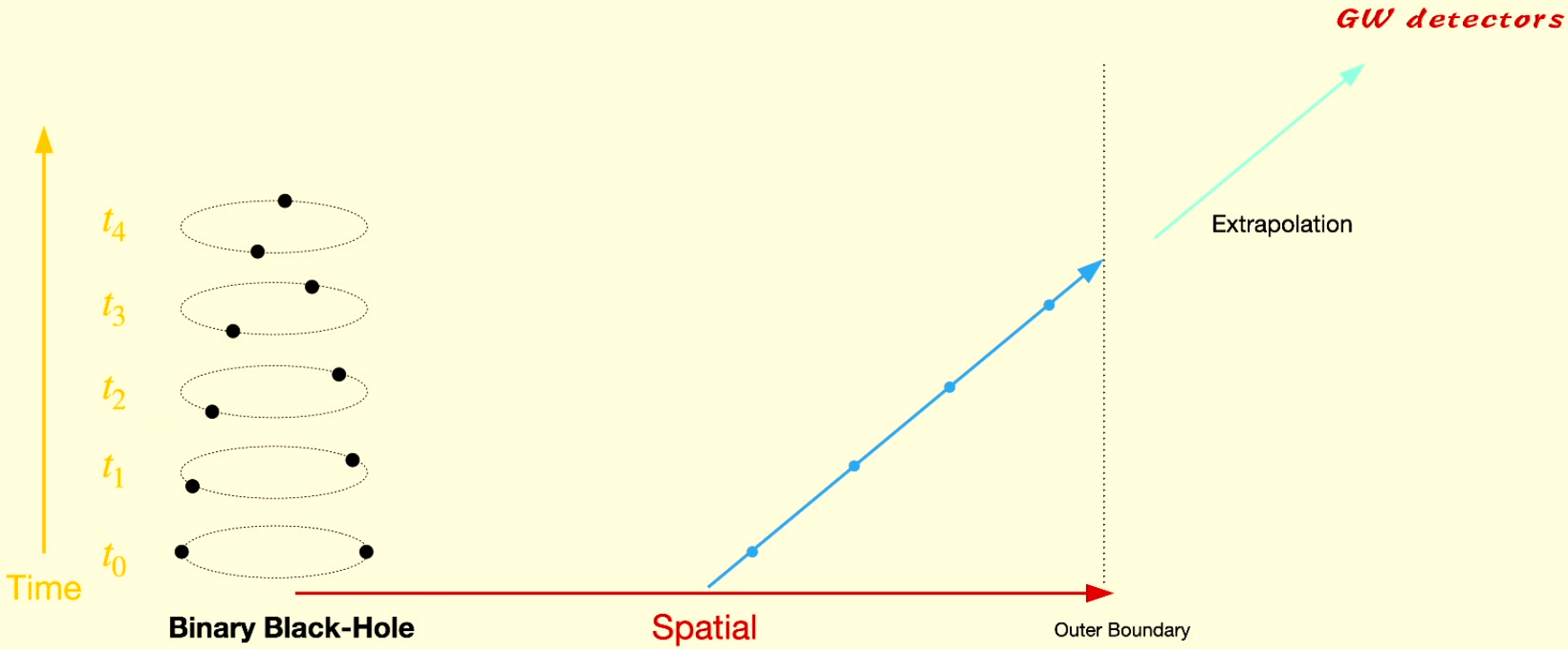
Characteristic evolution



GW detectors

wave zone

Cauchy evolution and wave extraction



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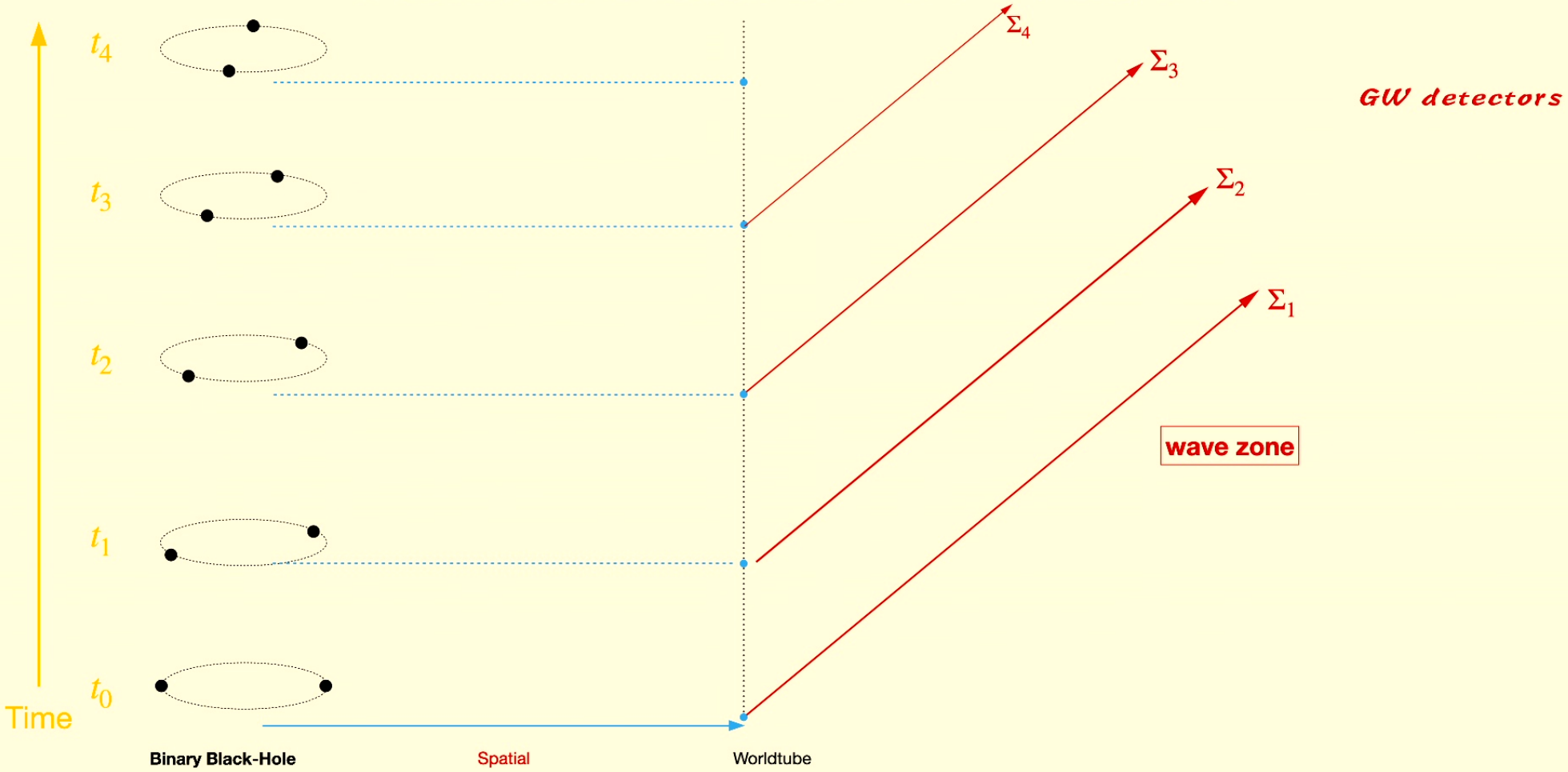


Pros and cons

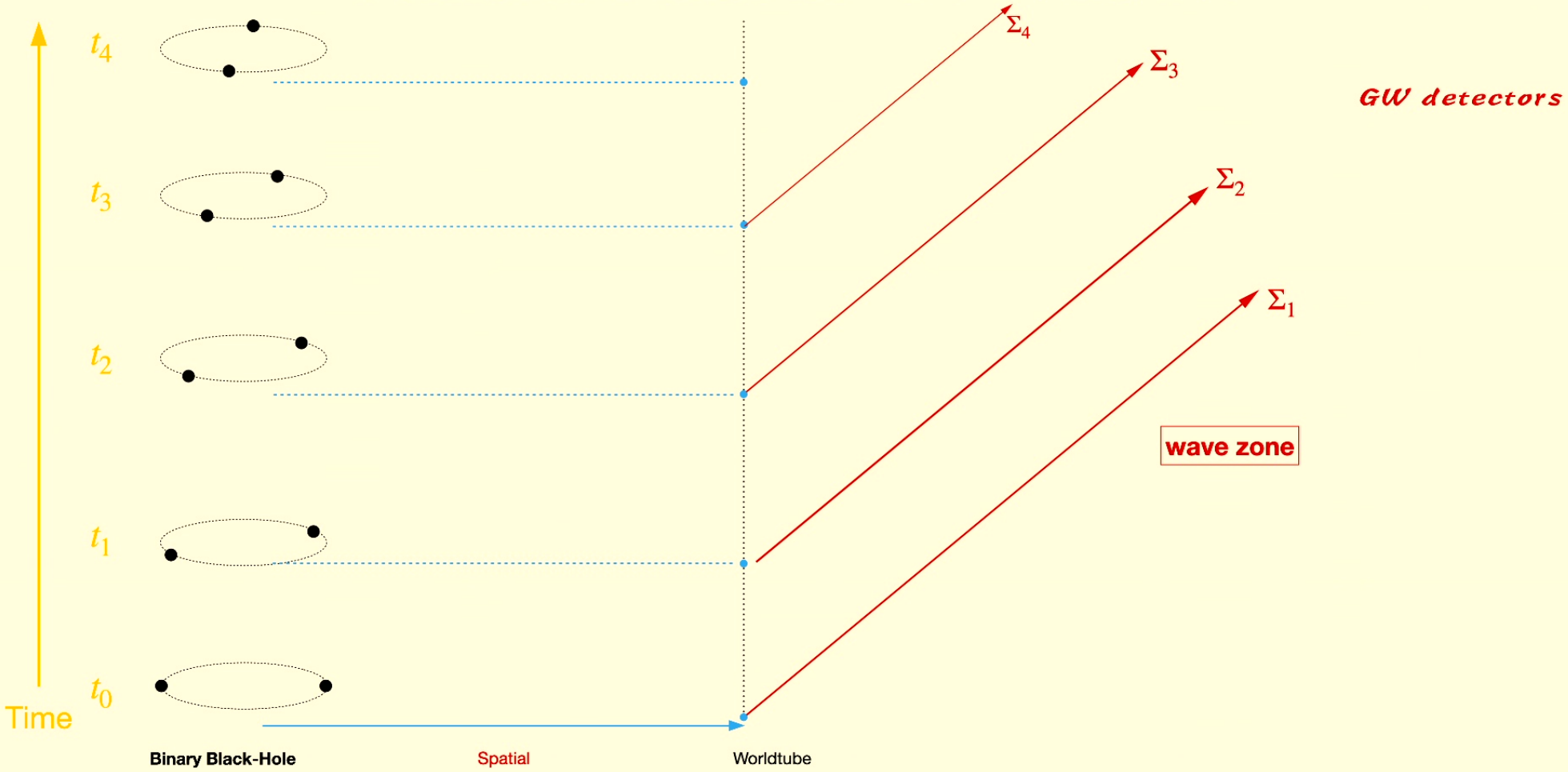
Cauchy evolution

- ✓ Can handle strong-gravity region
- × Computationally expensive (weeks to months)
- × Infinity is not in its computational domain

Characteristic evolution

- ✓ Infinity is in its computational domain
- ✓ Computationally cheap (minutes to hours)
- × Can’t handle strong-gravity region (near zone)

Characteristic evolution



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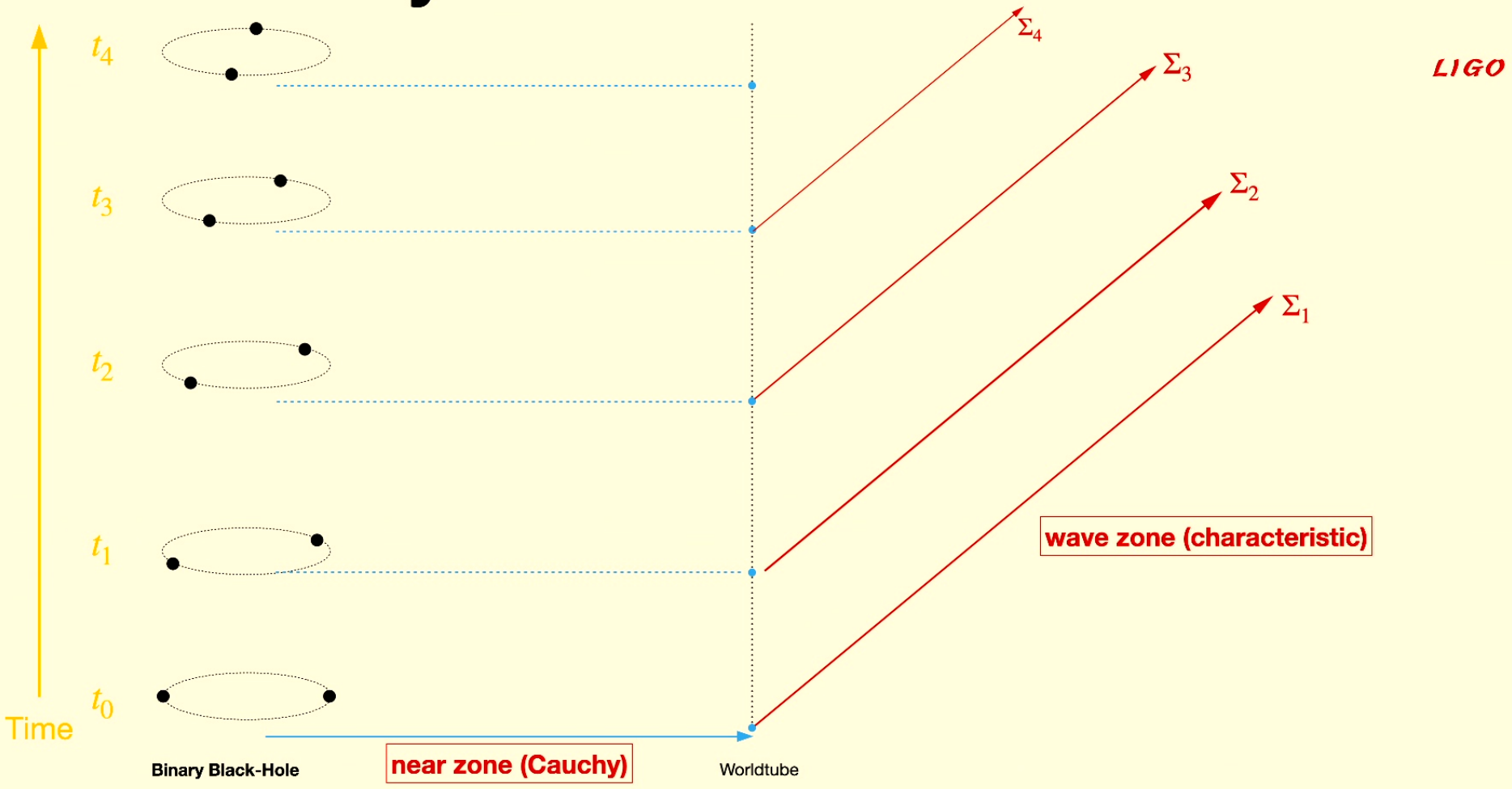

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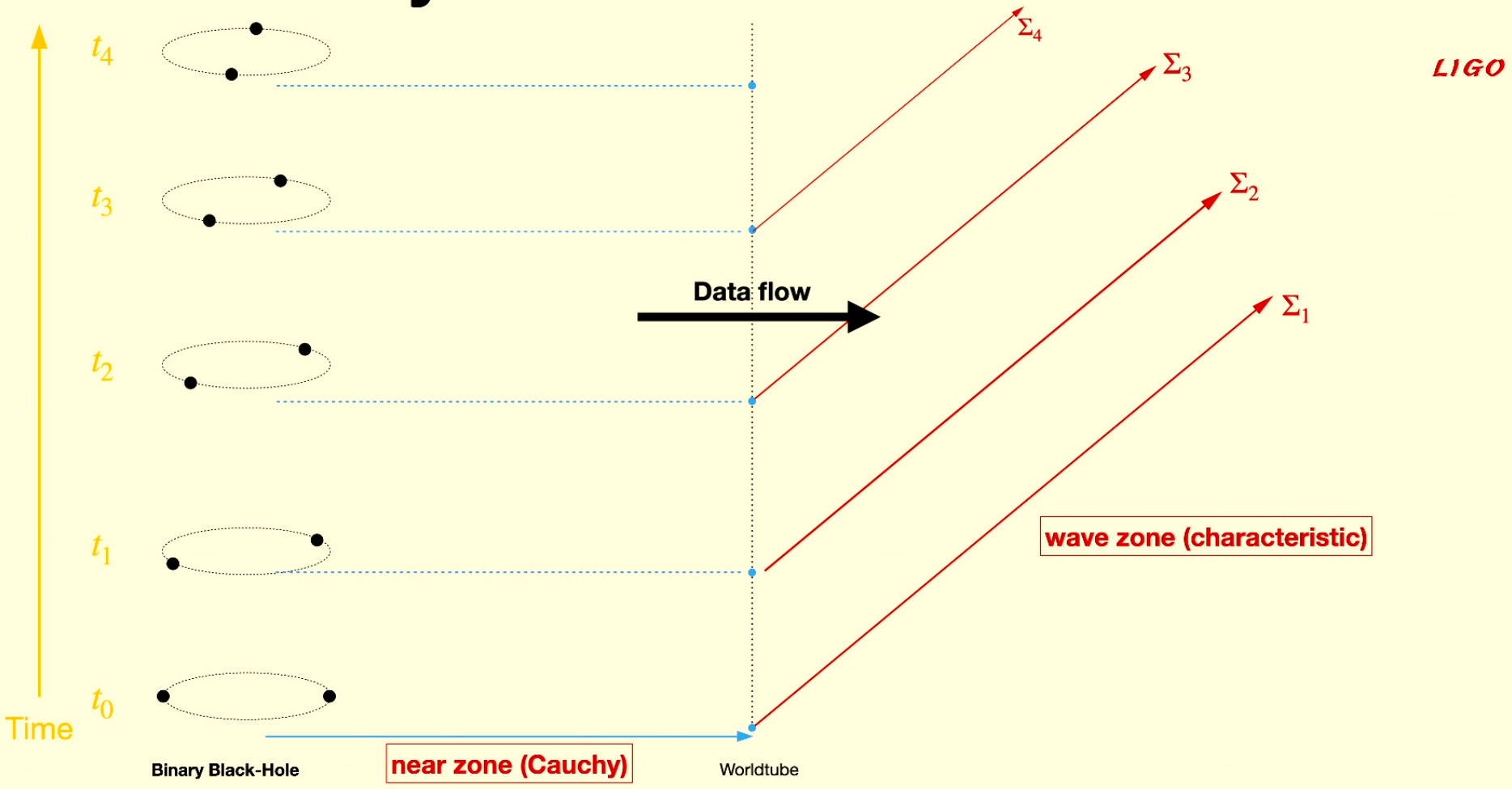
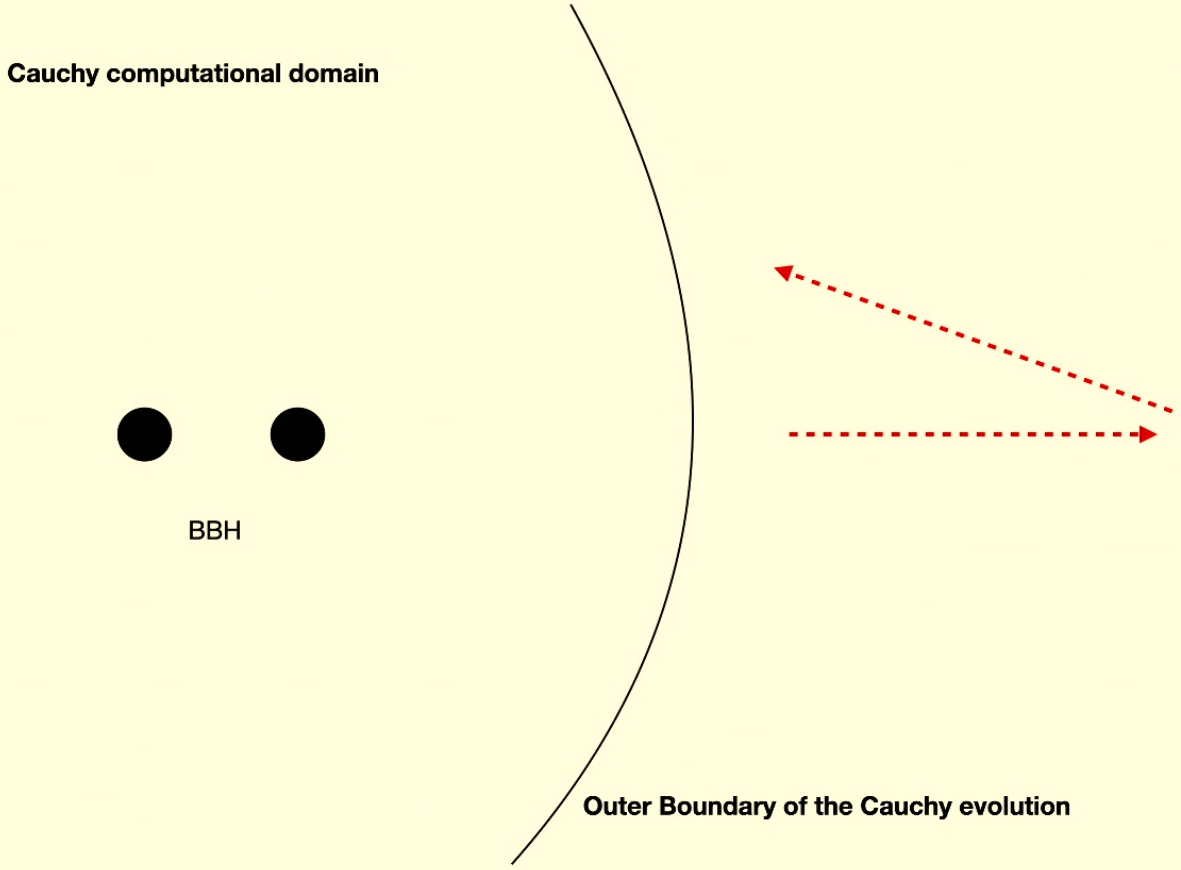


The backscattered gravitational wave



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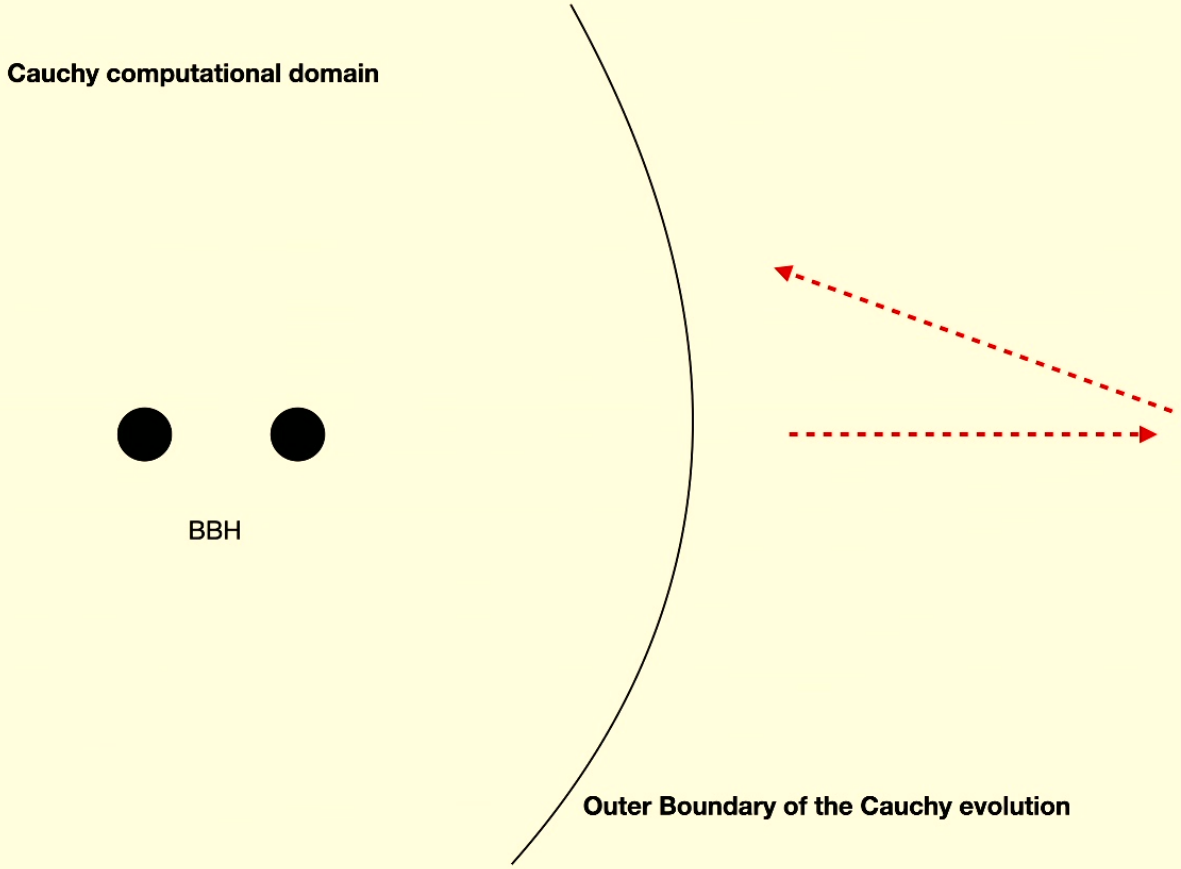

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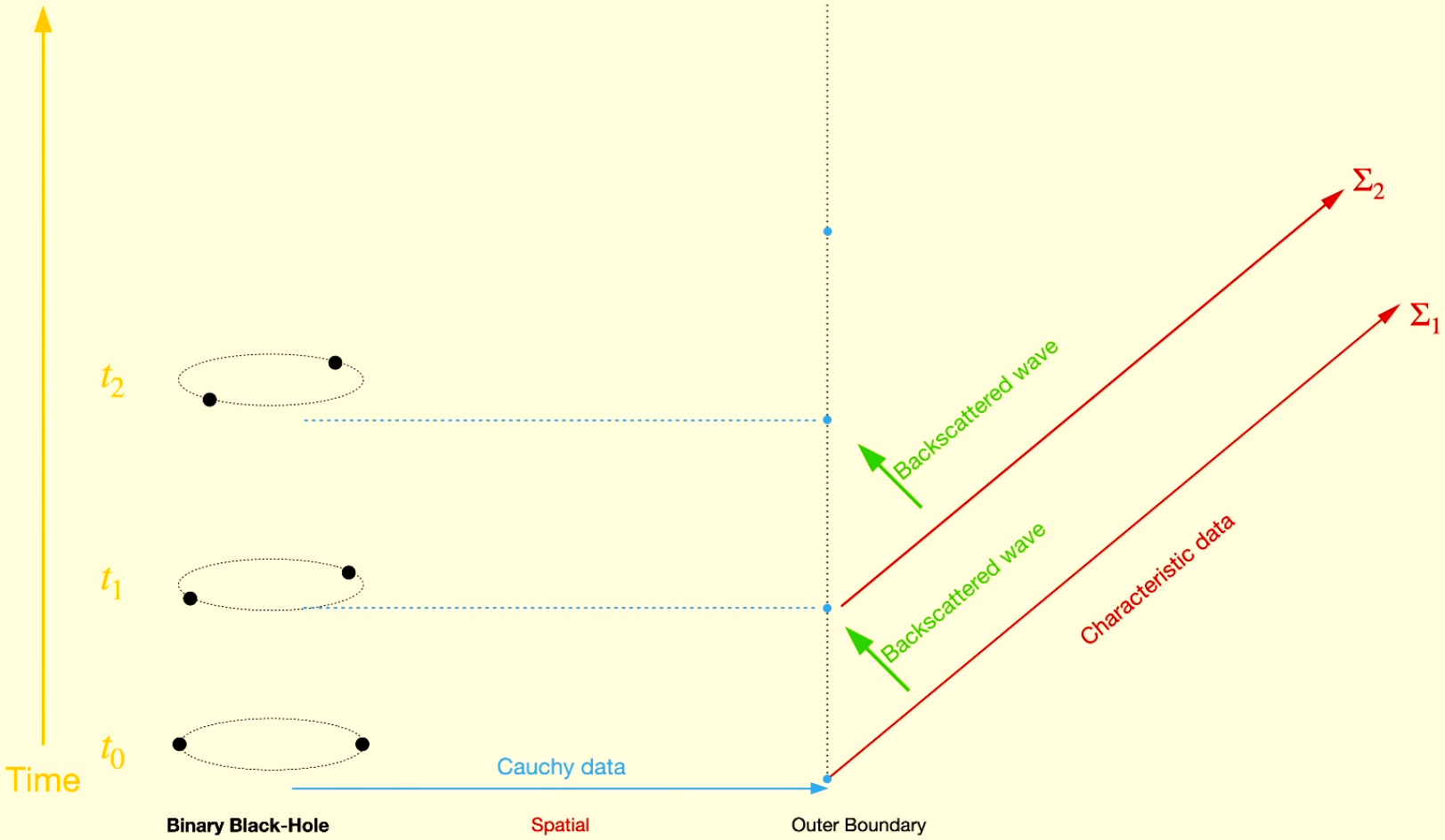

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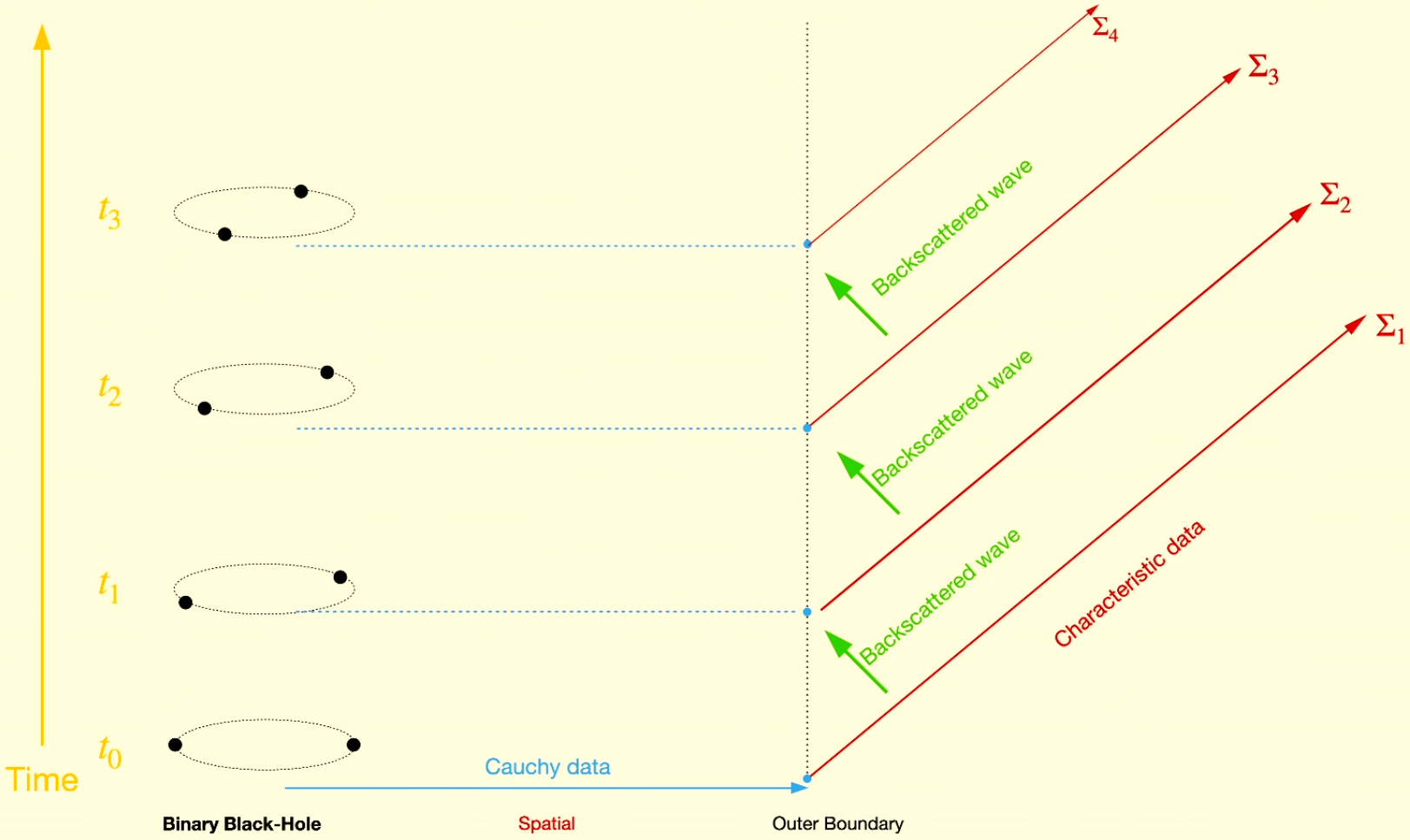

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- SpECTRE CCM arXiv: 2308.10361
- Results
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SpECTRE

- Open source (spectre-code.org, https://github.com/sxs-collaboration/spectre)
- Simulating eXtreme Spacetimes (SXS)
- CCE module (spectral method) Moxon et al., arXiv: 2007.01339, 2110.08635
  - Fast and accurate
  - Streamlined flexibility (standalone vs integrated)

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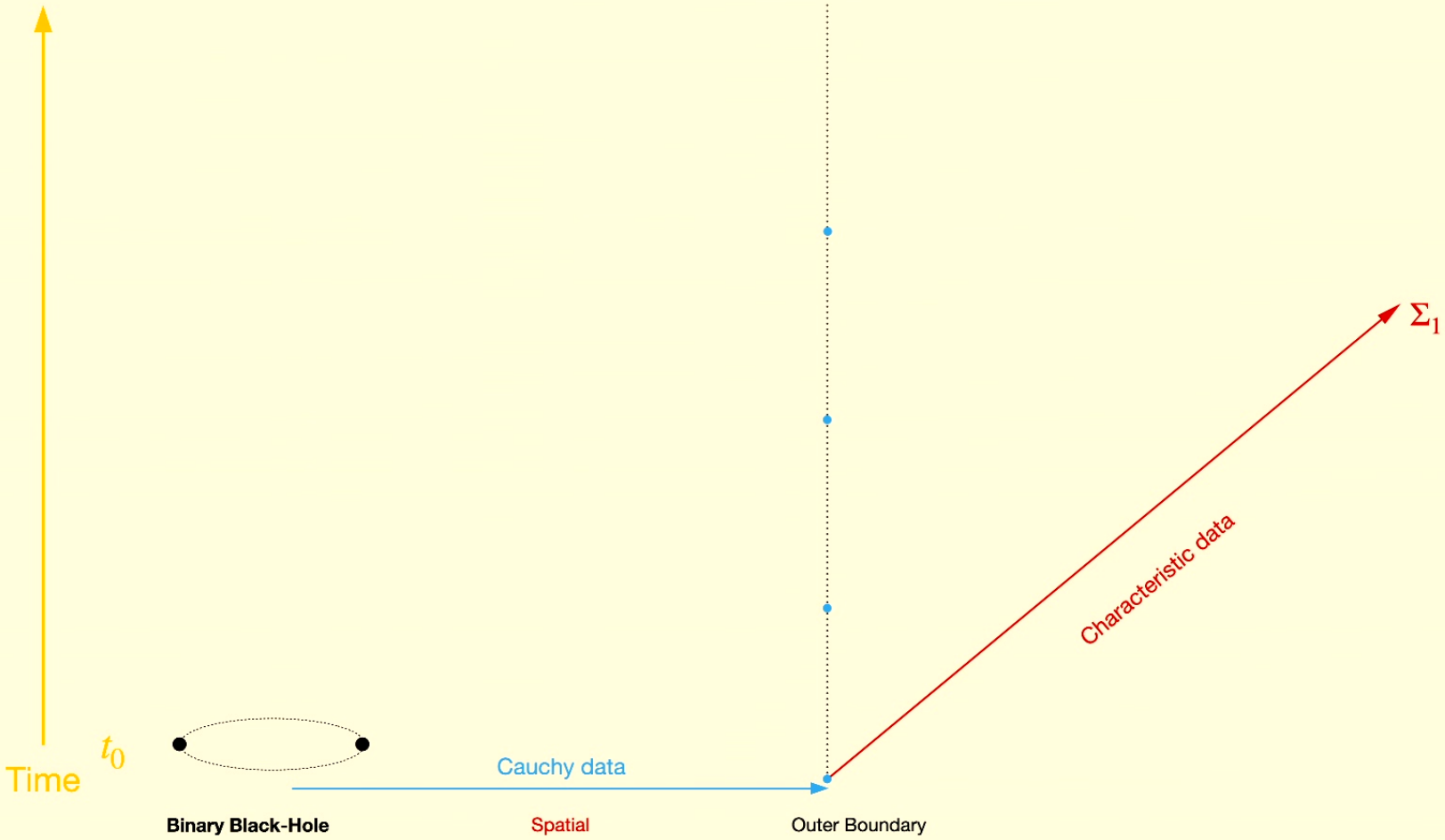

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Cauchy module: First-order generalized harmonic

$$g_{\mu\nu}\Box x^{\mu} = H_{\nu}(x, g)$$

$$\partial_t u^{\alpha} + A^{k\alpha}_{\beta}\partial_k u^{\beta} = F^{\alpha}$$

$$u^{\alpha} \in [g_{\mu\nu},\ \partial_t g_{\mu\nu},\ \partial_i g_{\mu\nu}]$$

Lindblom et al., arXiv: 0512093

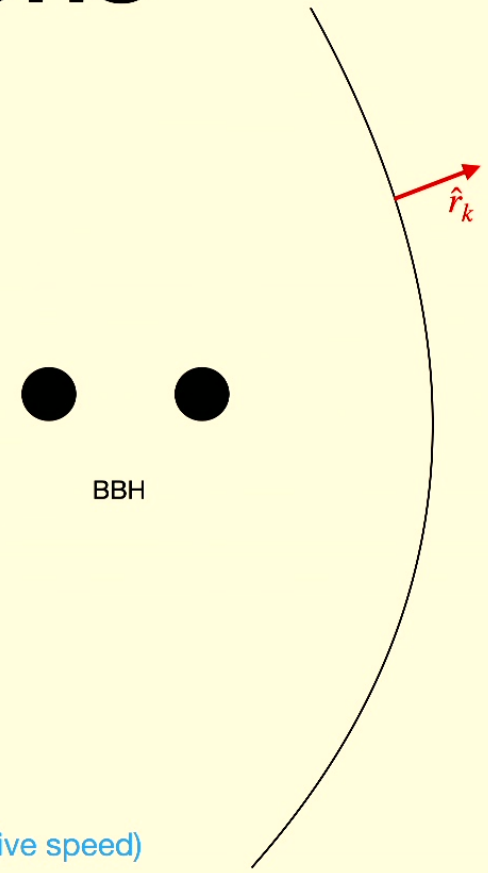
Boundary conditions

$$\partial_t u^\alpha + A^{k\alpha}_\beta \partial_k u^\beta = F^\alpha$$

$\hat{r}_k A^{k\alpha}_\beta$

Eigenvalues: characteristic speeds

Eigenvectors: characteristic fields



Impose boundary conditions on the characteristic fields that enter the domain (negative speed)

Boundary conditions

Incoming characteristic field (10 degrees of freedom)

Lindblom et al., arXiv: 0512093

4 degrees of freedom: Constraint-preserving

- **Freeze constraint-violation fluxes**
- **Has nothing to do with CCM**

Boundary conditions

Incoming characteristic field (10 degrees of freedom)

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4 degrees of freedom: Constraint-preserving (has nothing to do with CCM)

2 degrees of freedom : Two polarization states of backscattered GWs (CCM)

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- 4 degrees of freedom: Constraint-preserving (has nothing to do with CCM)
- 2 degrees of freedom : Two polarization states of backscattered GWs (CCM)
- 4 degrees of freedom: Gauge part (CCM? future work)

Boundary conditions

$$\partial_t u^\alpha + A^{k\alpha}_\beta \partial_k u^\beta = F^\alpha$$

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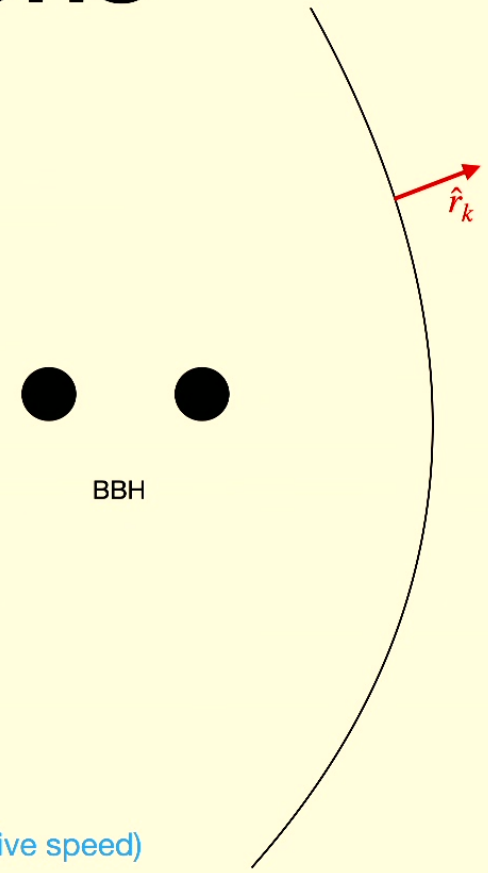
Eigenvalues: characteristic speeds

Eigenvectors: characteristic fields



Impose boundary conditions on the characteristic fields that enter the domain (negative speed)

Matching physical subset

Cauchy Evolution
(Generalized Harmonic gauge)

Characteristic Evolution
(Bondi-Sachs gauge)

Backscattered GWs $= \mathrm{Re}\left(\Psi_0^{\mathrm{Cauchy}} \bar{m}_a^{\mathrm{Cauchy}} \bar{m}_b^{\mathrm{Cauchy}}\right)$

(Well-defined, no ambiguity)

Lorentz Transformations

Blah

arXiv: 2308.10361

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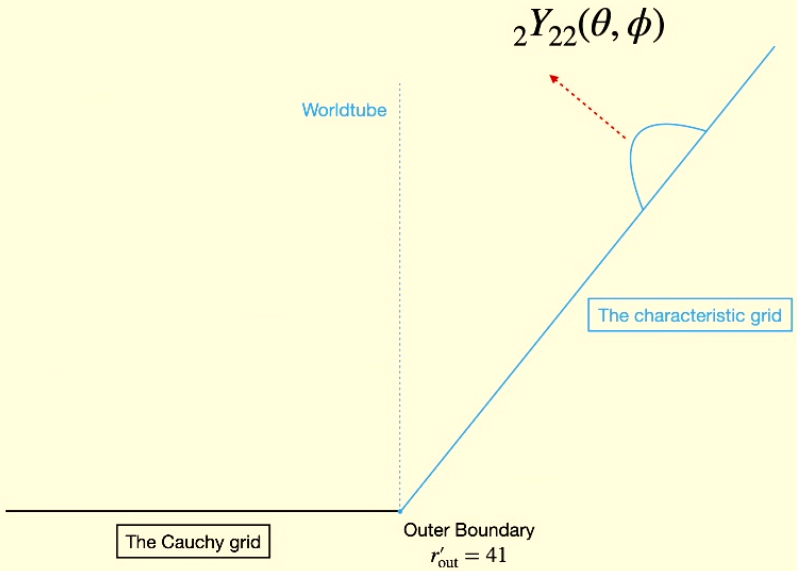
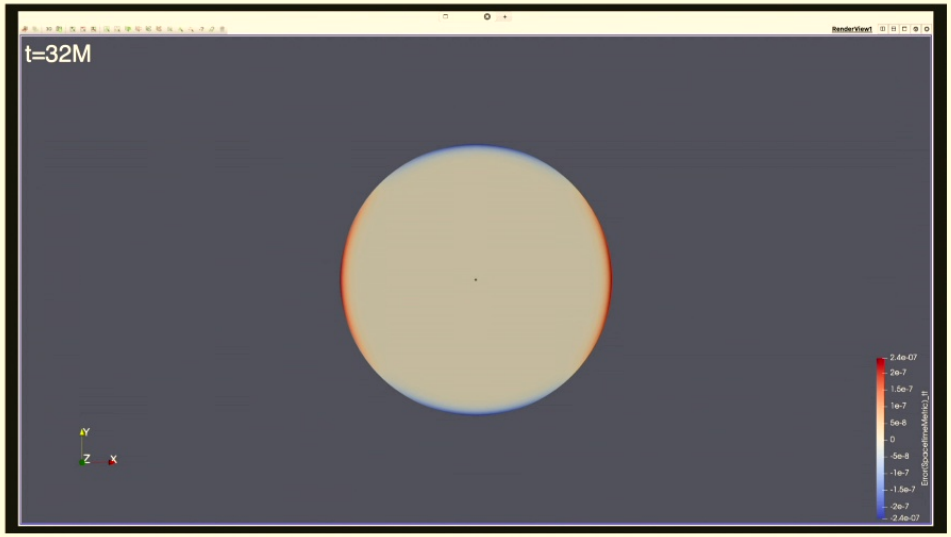
Test I: a pulse on the Characteristic grid



$$g_{tt}^{\rm num}-g_{tt}^{\rm stationary}$$



Cauchy grid

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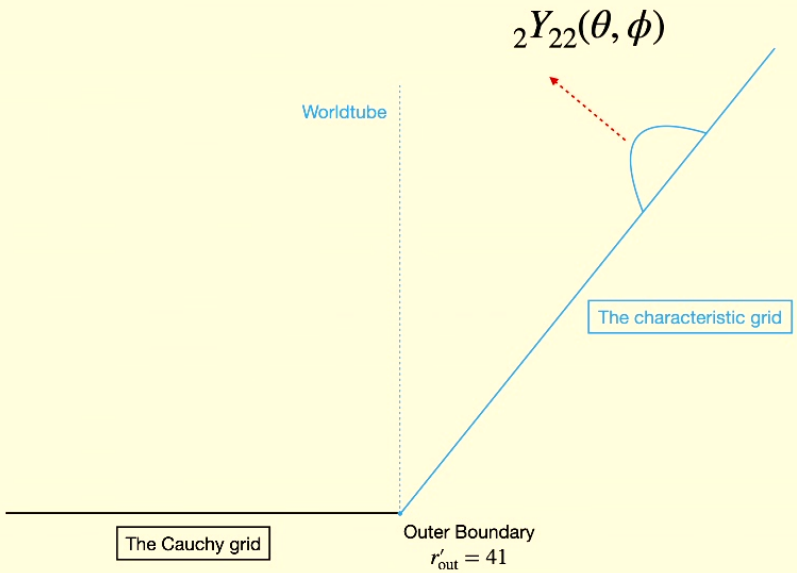

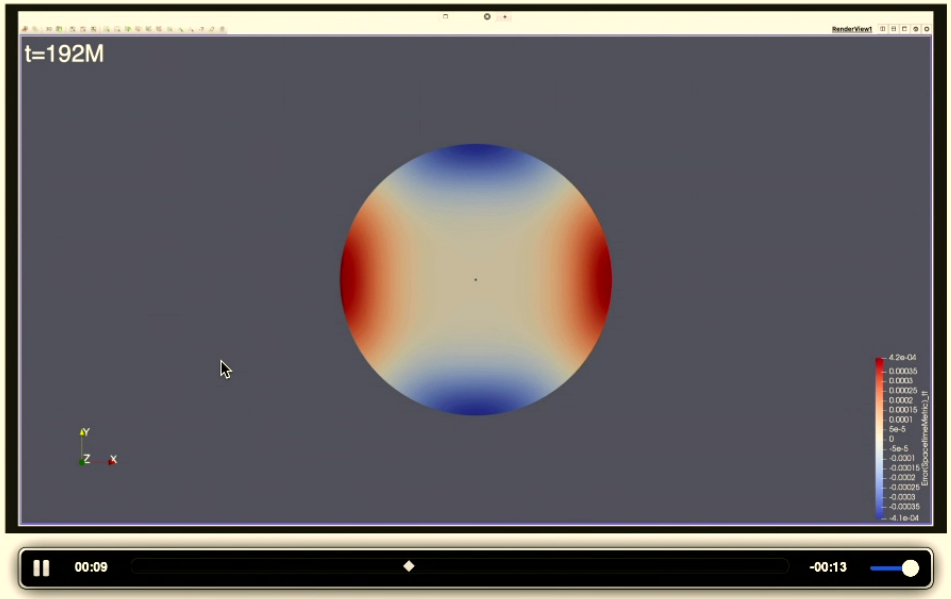

Cauchy grid

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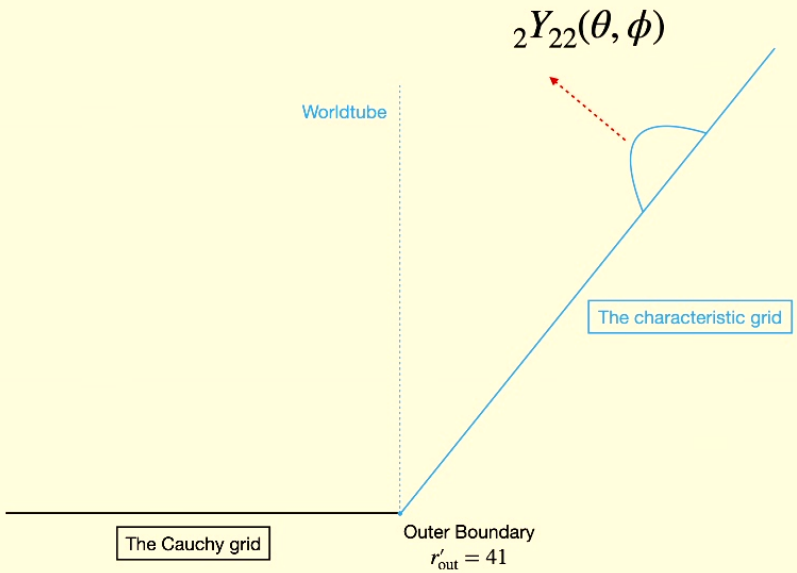
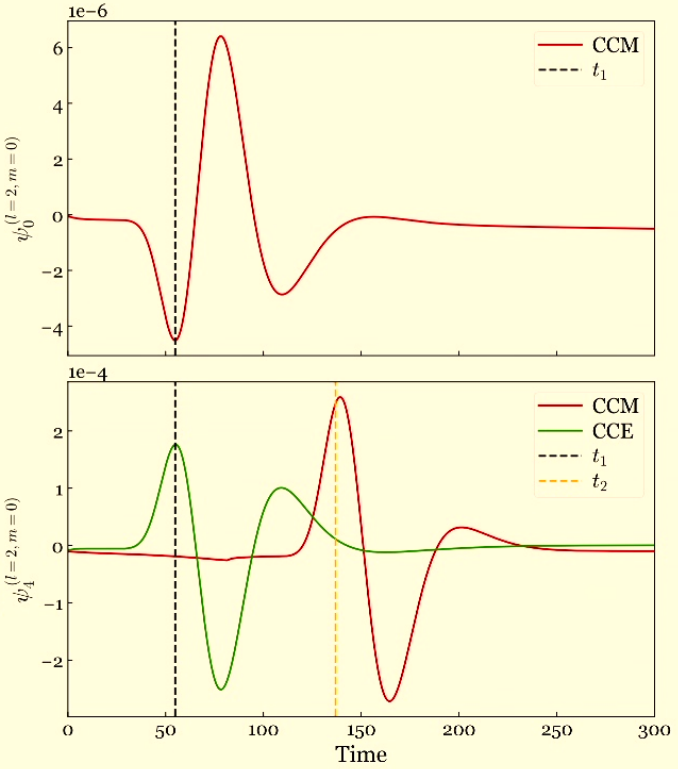


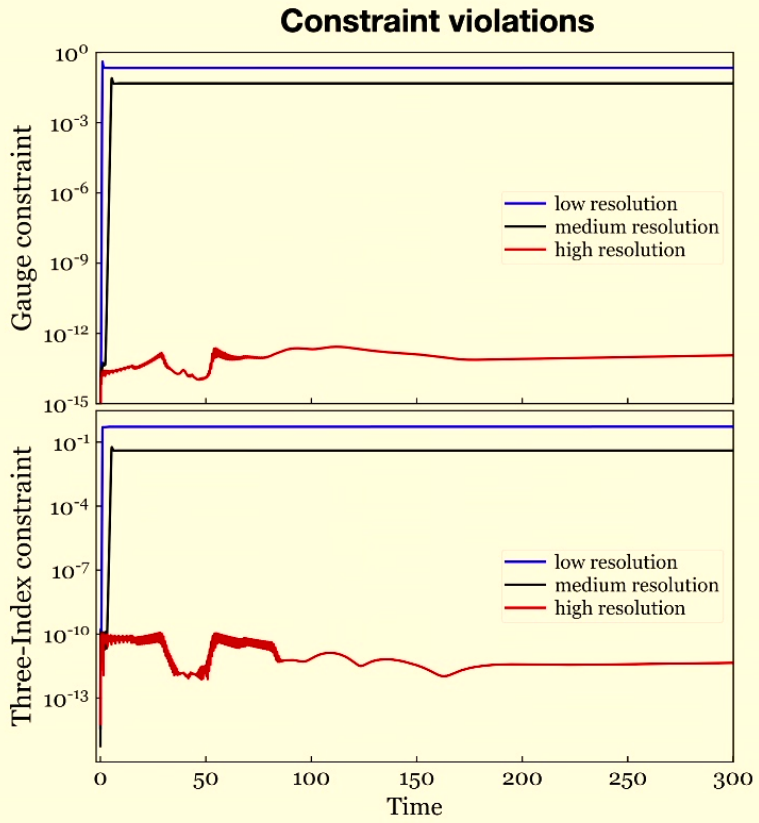
ψ_0 at outer boundary

ψ_4 at future null infinity



Test I: a pulse on the Characteristic grid

No numerical instability



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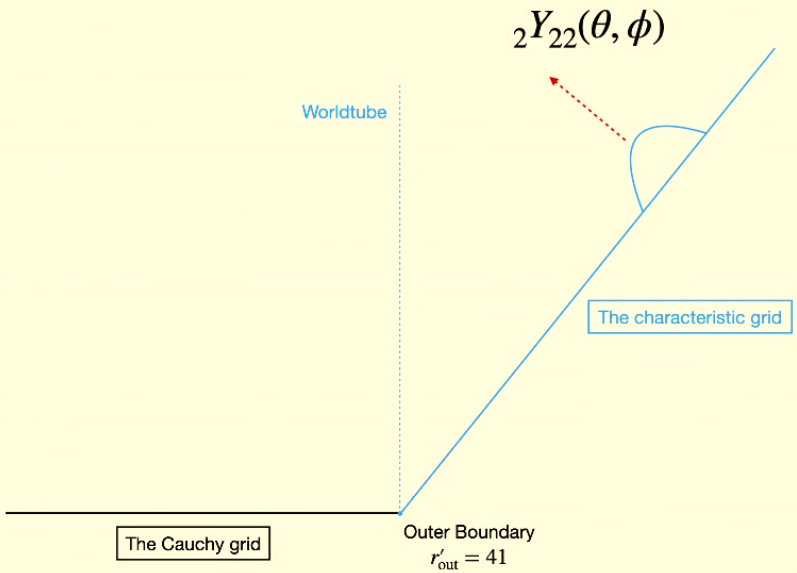

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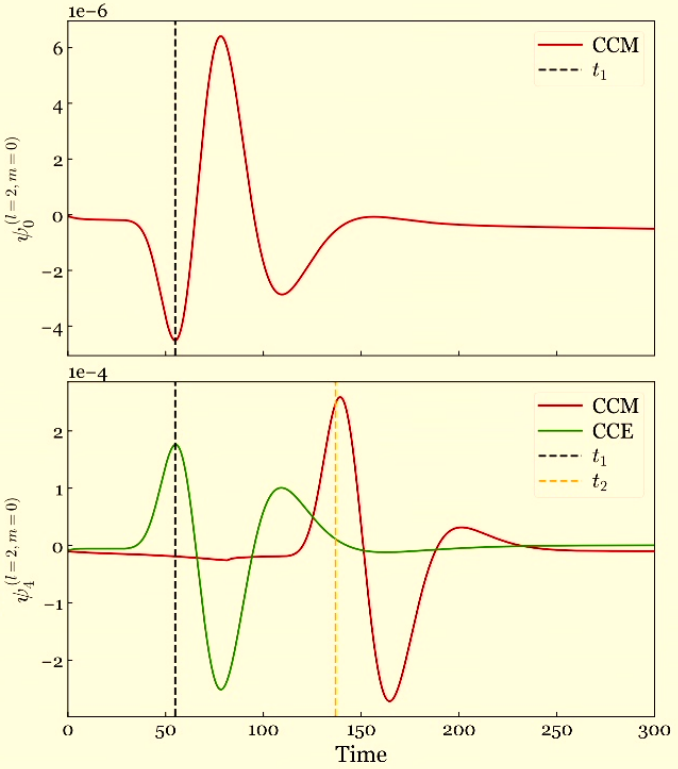

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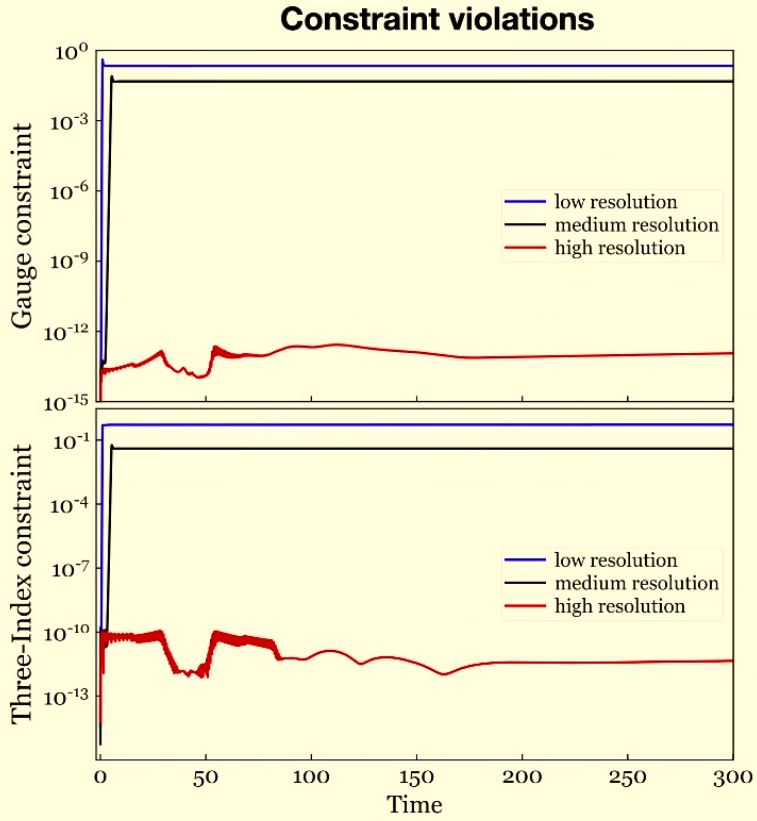
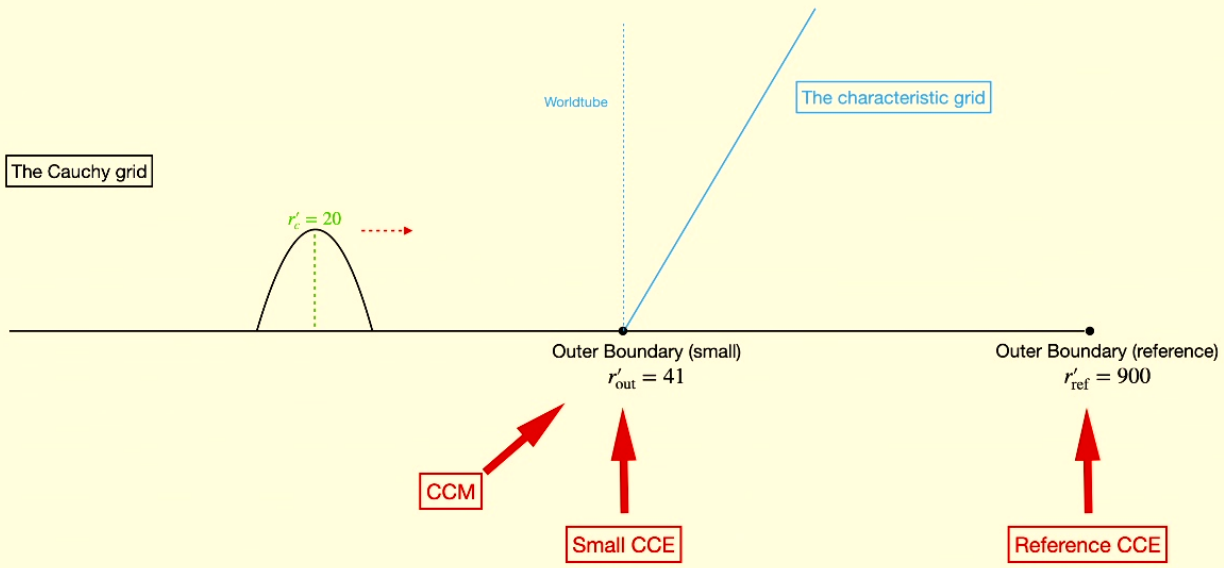
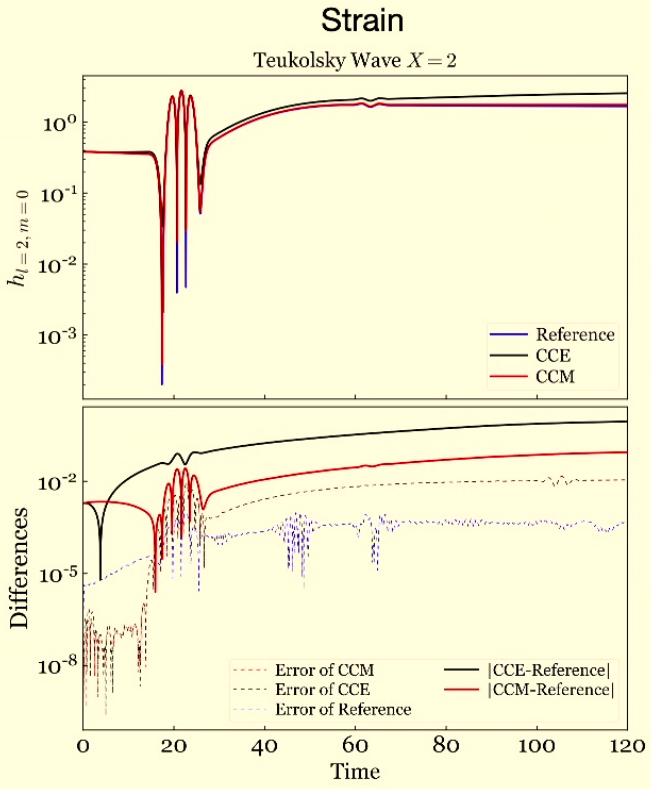


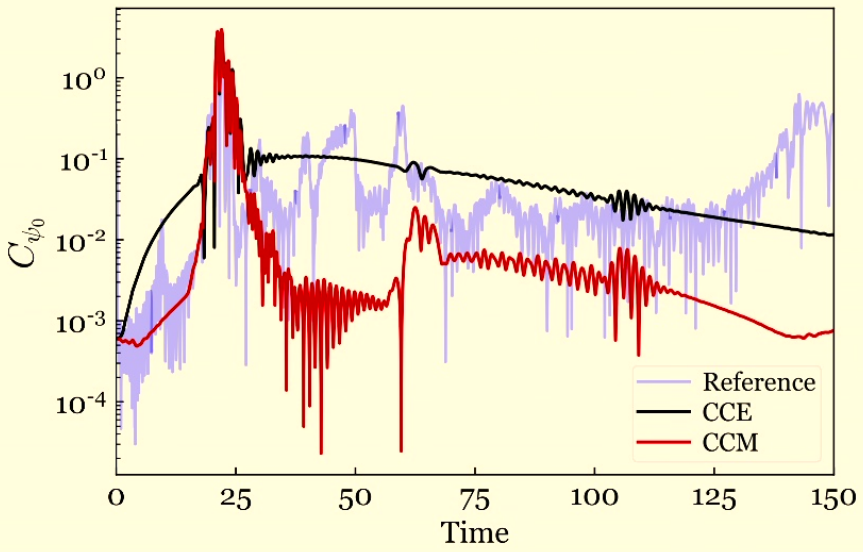
Test II: a pulse on the Cauchy grid



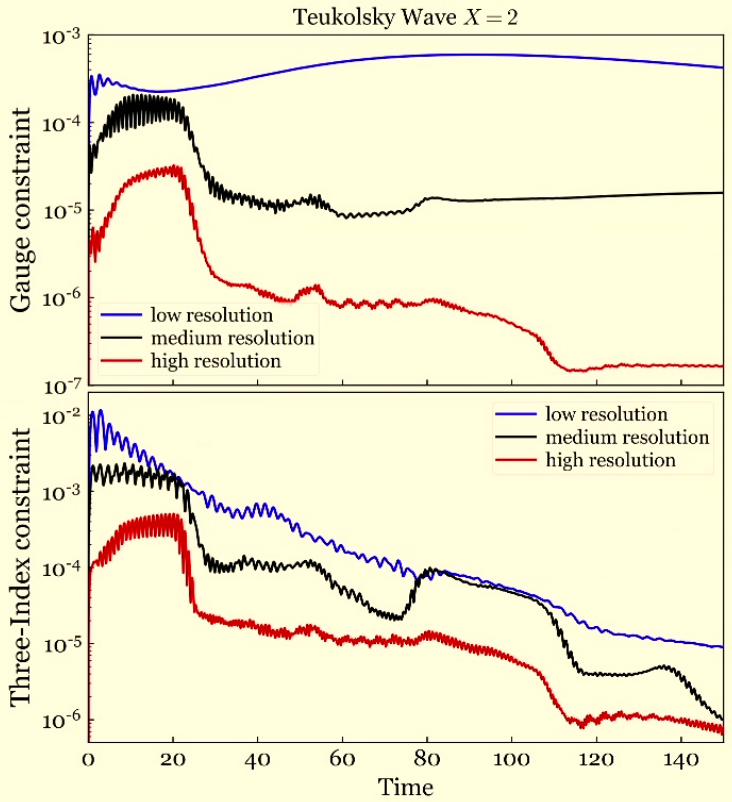


Test II: a pulse on the Cauchy grid

Violations of Bianchi identities
(In terms of Weyl scalars)



Constraint violations



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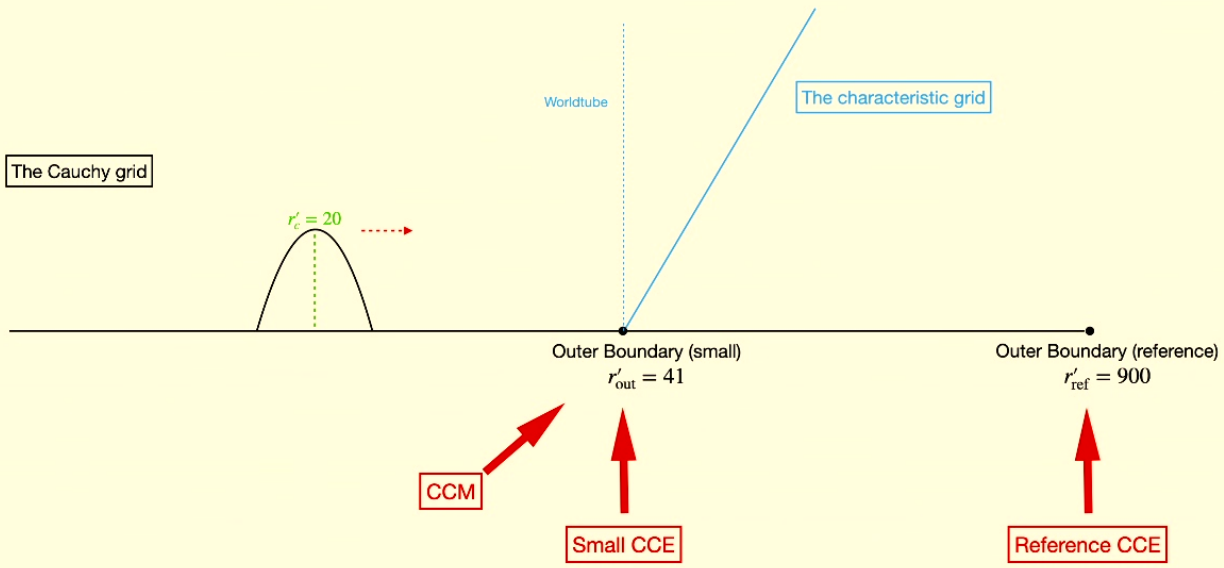

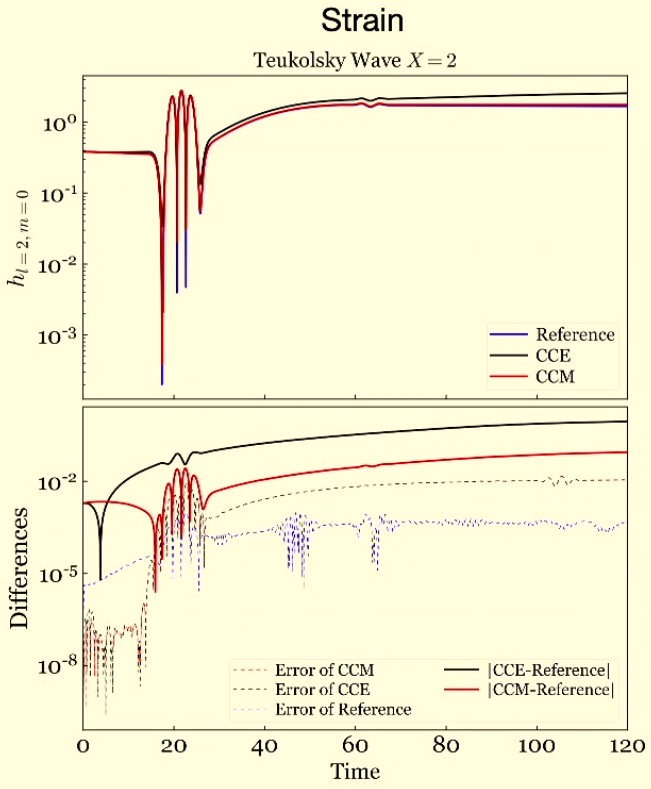

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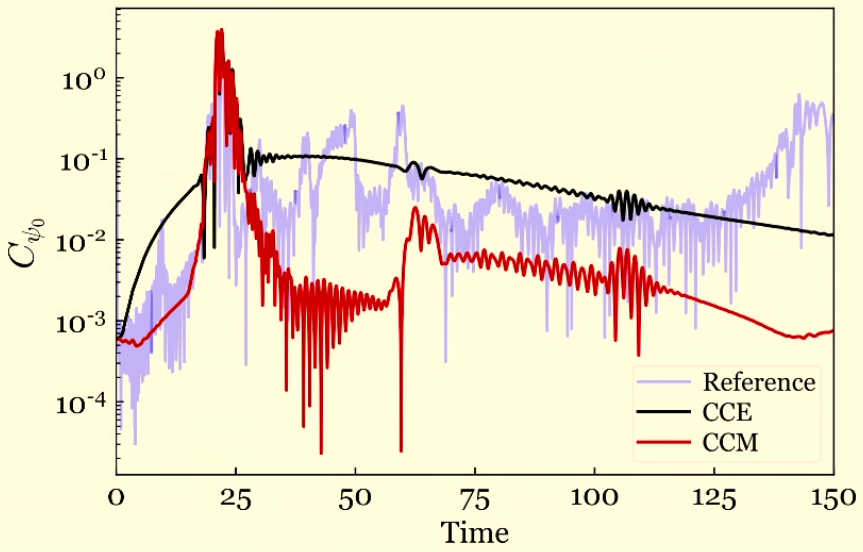

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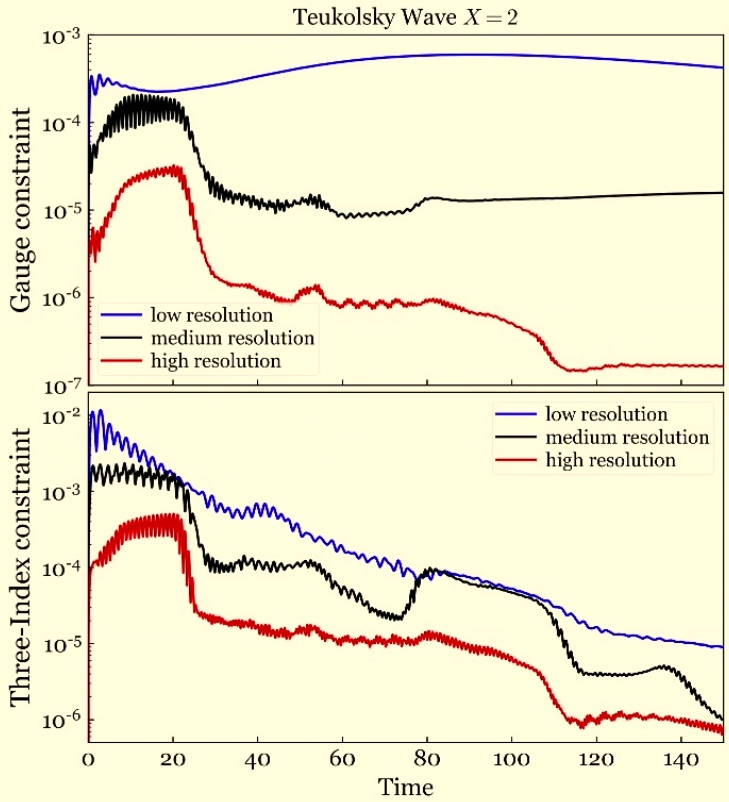

- The third test involves nonlinear perturbation of a Kerr BH, no numerical instability was found.

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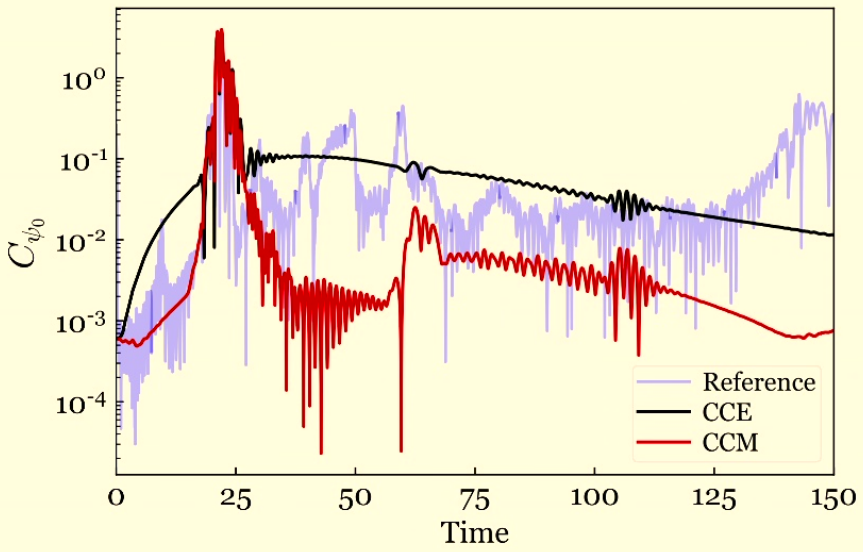

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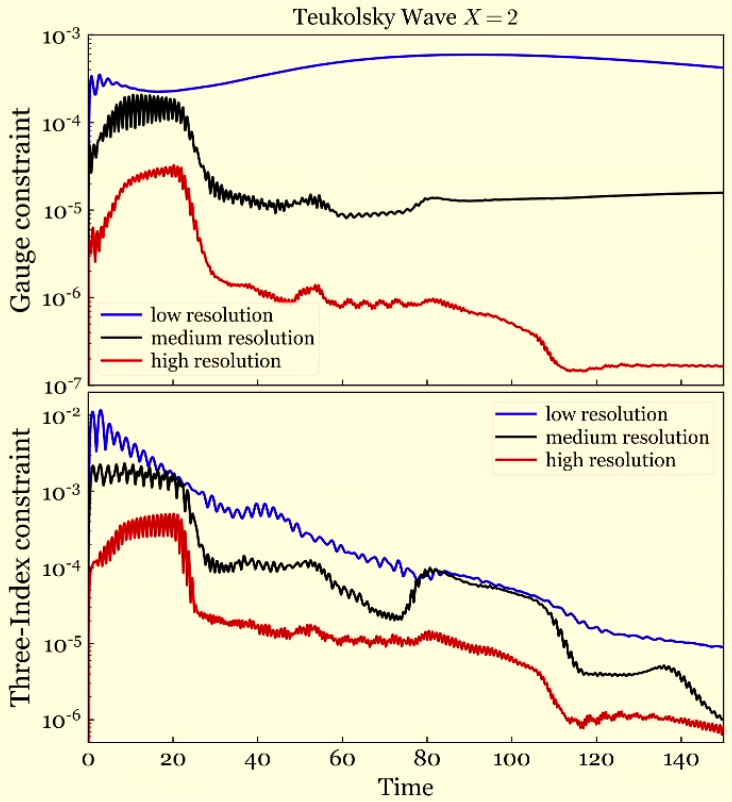

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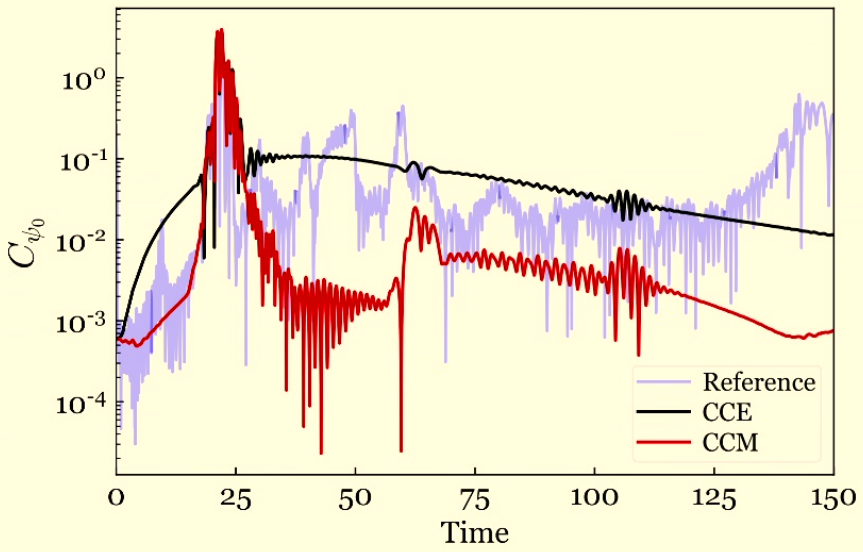

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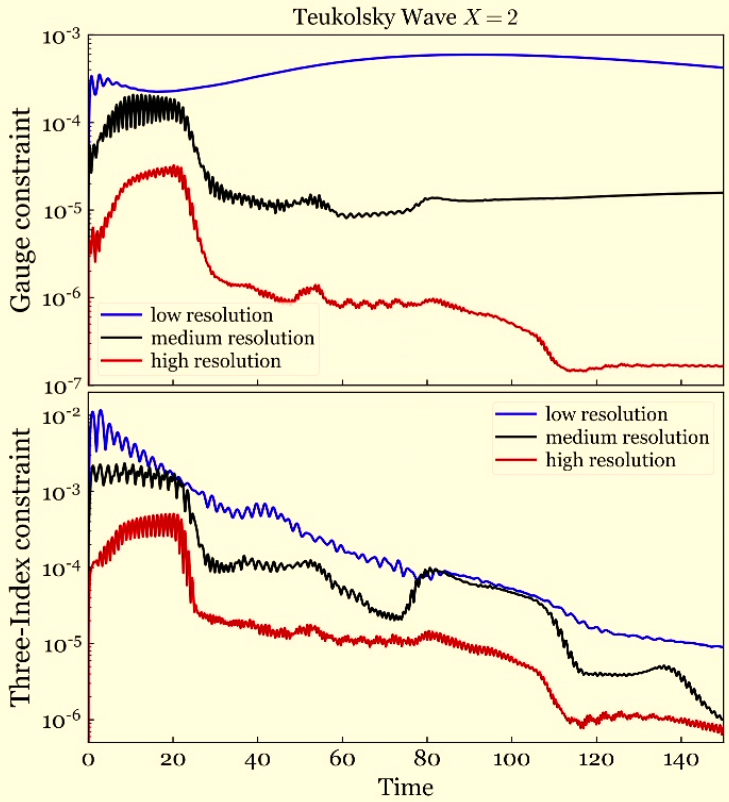

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SpECTRE-SpEC CCM

- Binary black-hole mergers are not ready in SpECTRE yet.
  - Needs months to years to make efficient production Cauchy runs, which is the major limitation of the current SpECTRE CCM system
- SpECTRE-SpEC CCM
  - Link SpECTRE characteristic system as a library to SpEC executable
  - Run SpECTRE on the fly while SpEC BBH proceeds
  - The hybrid SpECTRE-SpEC CCM system is about to be done.

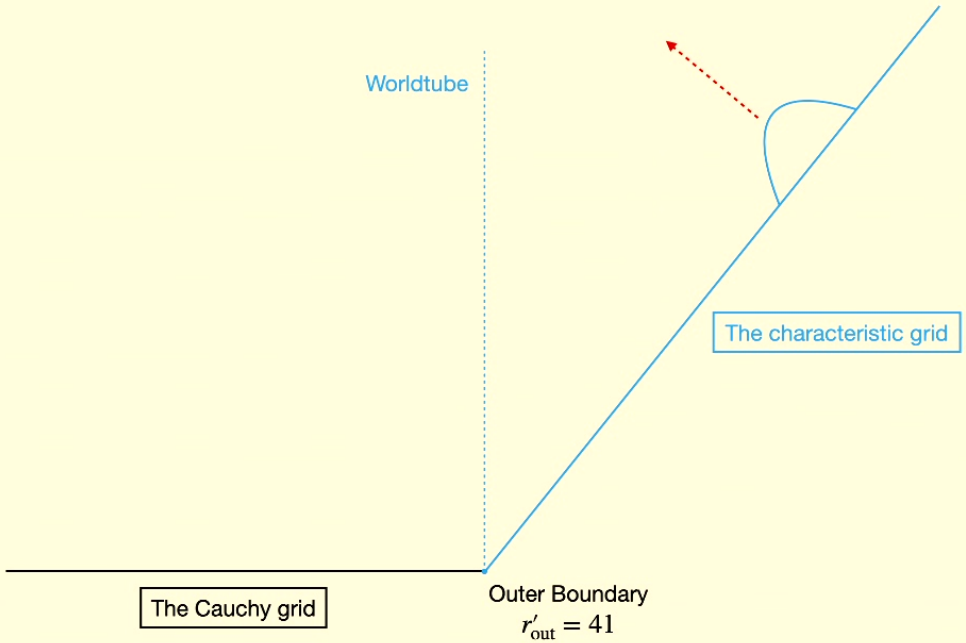
Gauge boundary conditions

Incoming characteristic fields (10 degrees of freedom)

- 4 degrees of freedom: Constraint-preserving (has nothing to do with CCM)
- 2 degrees of freedom : Two polarization states of backscattered GWs (CCM)
- 4 degrees of freedom: Gauge part (I have no intuition at all!)

- **What will happen when we send gauge flux into the Cauchy domain?**
- **How does it interact with the damped harmonic gauge?**

Initial data and junk radiation



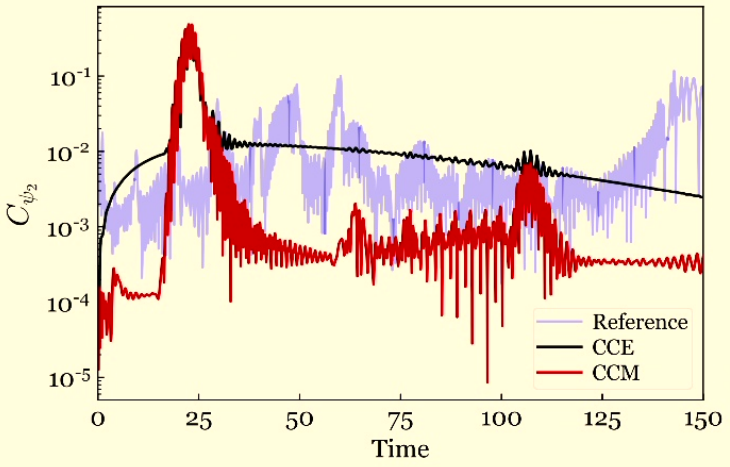
Scalar CCE/CCM

- Black hole-neutron star mergers in scalar-tensor gravity (Ma et al., arXiv: 2304.11836)
- Extract waveforms with CCE (memory effects)
  - The scalar+metric CCE system in SpECTRE has been well tested (Ma et al, in progress)
- Match the scalar part

High-quality waveforms

- Fix BMS freedom with higher accuracy
- High-quality waveforms for 3G detectors
- High-quality waveforms for long simulations

Violation of Bianchi identities
(In terms of ψ_2)



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- 2 degrees of freedom : Two polarization states of backscattered GWs (CCM)
- 4 degrees of freedom: Gauge part (I have no intuition at all!)

- **What will happen when we send gauge flux into the Cauchy domain?**
- **How does it interact with the damped harmonic gauge?**