

Title: Dissipative State Preparation and the Dissipative Quantum Eigensolver

Speakers: Toby Cubitt

Series: Perimeter Institute Quantum Discussions

Date: September 13, 2023 - 11:00 AM

URL: <https://pirsa.org/23090093>

Abstract: Finding ground states of quantum many-body systems is one of the most important---and one of the most notoriously difficult---problems in physics, both in scientific research and in practical applications. Indeed, we know from a complexity-theoretic perspective that all methods (quantum or classical) must necessarily fail to find the ground state efficiently in general. The ground state energy problem is already NP-hard even for classical, frustration-free, local Hamiltonians with constant spectral gap. For general quantum Hamiltonians, the problem becomes QMA-hard.

Nonetheless, as ground state problems are of such importance, and classical algorithms are often successful despite the theoretical exponential worst-case complexity, a number of quantum algorithms for the ground state problem have been proposed and studied. From quantum phase estimation-based methods, to adiabatic state preparation, to dissipative state engineering, to the variational quantum eigensolver (VQE), to quantum/probabilistic imaginary-time evolution (QITE/PITE).

Dissipative state engineering was first introduced in 2009 by Verstraete, Cirac and Wolf and by Kraus et al. However, it only works for the restricted class of frustration-free Hamiltonians.

In this talk, I will show how to construct a dissipative state preparation dynamics that provably produces the correct ground state for arbitrary Hamiltonians, including frustrated ones. This leads to a new quantum algorithm for preparing ground states: the Dissipative Quantum Eigensolver (DQE). DQE has a number of interesting advantages over previous ground state preparation algorithms:

- o The entire algorithm consists simply of iterating the same set of simple local measurements repeatedly.
- o The expected overlap with the ground state increases monotonically with the length of time this process is allowed to run.
- o It converges to the ground state subspace unconditionally, without any assumptions on or prior information about the Hamiltonian (such as spectral gap or ground state energy bound).
- o The algorithm does not require any variational optimisation over parameters.
- o It is often able to find the ground state in low circuit depth in practice.
- o It has a simple implementation on certain types of quantum hardware, in particular photonic quantum computers.
- o It is immune to errors in the initial state.
- o It is inherently fault-resilient, without incurring any fault-tolerance overhead. I.e. not only is it resilient to errors on the quantum state, but also to faulty implementations of the algorithm itself; the overlap of the output with the ground state subspace degrades smoothly with the error rate, independent of the total run-time.

I give a mathematically rigorous analysis of the DQE algorithm and proofs of all the above properties, using non-commutative generalisations of methods from classical probability theory.

Zoom link <https://pitp.zoom.us/j/96022753460?pwd=SWlUVkVta1RyY3dsWUJWckRqOHdNdz09>



PHASECRAFT

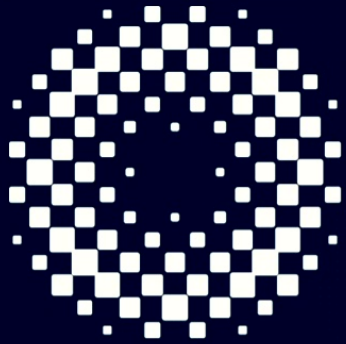
Dissipative Ground State Preparation & the Dissipative Quantum Eigensolver

Toby Cubitt

[arXiv: 2303.11962]

THE QUANTUM
ALGORITHMS
COMPANY

 UCL



PHASECRAFT



Ashley
Montanaro



John
Morton



Toby
Cubitt

- Intro & Background
 - AGSPs
 - Conditionally stopped process
 - Epsilon schedules
 - Fault-resilience
 - Resampling strategies
 - Optimal stopping
 - Fixed-point process
 - Conclusions & Outlook
-
- main
- bonus

Ground State Problem

Input:

(Description of) k -local Hamiltonian

$$H = \sum_{i=1}^m h_i \quad h_i \text{ } k\text{-local}$$

Output:

$$H |\psi_0\rangle = \lambda_{\min} |\psi_0\rangle$$

$$|\tilde{\psi}\rangle \text{ s.t. } \|\ |\tilde{\psi}\rangle - |\psi_0\rangle \|\leq \epsilon$$

Ground State Problem

Input:

(Description of) k -local Hamiltonian

$$H = \sum_{i=1}^m h_i \quad h_i \text{ } k\text{-local}$$

Output:

$$H \Pi_0 = \lambda_{\min} \Pi_0$$

$$\tilde{\rho} \quad \text{s.t.} \quad \text{tr}(\Pi_0 \tilde{\rho}) \geq 1 - \epsilon$$

Ground State Problem

QMA-hard \rightarrow exponential time even on quantum computer

Physical assumptions on H do not make ground state problem easy:

- Frustration-free
- Constant spectral gap
- Commuting local terms
- Polynomial density of states

\rightarrow still NP-hard.

| Algorithm | Adiabatic | QPE | i -time | VQE |
|---------------------------------------|-----------|-----|-----------|-----|
| No conditions on H | | | | ✓ |
| Provably succeeds | | ✓ | ✓ | |
| Low-depth in practice | | | | ✓ |
| No parameter optimisation | ✓ | ✓ | ✓ | |
| Efficient to generate multiple copies | | | | ✓ |
| Convenient implementation | | | | ✓ |
| Insensitive to initial state | | | | |
| Noise- and fault-resilient | | | | |

| Algorithm | Adiabatic | QPE | i -time | VQE | Dissipative |
|---------------------------------------|-----------|-----|-----------|-----|----------------|
| No conditions on H | | | | ✓ | |
| Provably succeeds | | ✓ | ✓ | | ✓ |
| Low-depth in practice | | | | ✓ | |
| No parameter optimisation | ✓ | ✓ | ✓ | | ✓ |
| Efficient to generate multiple copies | | | | ✓ | |
| Convenient implementation | | | | ✓ | ✓ |
| Insensitive to initial state | | | | | ✓ |
| Noise- and fault-resilient | | | | | ✓ ¹ |

Dissipative state engineering & quantum computation

Dissipative...

- ground state engineering
- MPS / PEPS / stabilizer state engineering
- Quantum computation

[VWC '09]

Dissipative state engineering & quantum computation

Dissipative...

- ground state engineering
→ exp-time

[VWC '09]

Dissipative state engineering

Def. Frustration-free

$$H = \sum_i h_i \quad H |\psi_0\rangle = \lambda_{\min}(H) |\psi_0\rangle$$

$$h_i |\psi_0\rangle = \lambda_{\min}(h_i) |\psi_0\rangle$$

For H frustration-free:

$$H = \sum_i h_i \quad \longrightarrow \quad H = \sum_i \pi_i \quad \text{w.l.o.g.}$$

Dissipative state engineering

$$H = \sum_{i=1}^m \pi_i \quad n \text{ qubits, } \dim = 2^n =: D$$

$$\mathcal{E}(\rho) = \frac{1}{m} \sum_{i=1}^m \left(\pi_i \rho \pi_i + (1 - \text{tr}(\pi_i \rho)) \frac{\mathbb{1}}{D} \right)$$

1. Pick random $i \in \{1, \dots, m\}$
2. Measure $\{\pi_i, \pi_i^\perp\}$
3. On π_i^\perp outcome, replace state with maximally-mixed state

Dissipative state engineering

$$H = \sum_{i=1}^m \pi_i \quad n \text{ qubits, } \dim = 2^n =: D$$

$$\mathcal{E}(\rho) = \frac{1}{m} \sum_{i=1}^m \left(\pi_i \rho \pi_i + (1 - \text{tr}(\pi_i \rho)) \frac{\mathbb{1}}{D} \right)$$

1. Pick random $i \in \{1, \dots, m\}$

2. Measure $\{\pi_i, \pi_i^\perp\} \longrightarrow \text{tr}(\mathcal{E}^\infty(\rho) \pi_0) = 1$

3. On π_i^\perp outcome, replace state with maximally-mixed state

Dissipative state engineering

Advantages:

- Simple, local procedure
- Succeeds independent of initial state
- Succeeds even if state hit by errors during computation (but only proven for error rate = 0)

Dissipative state engineering

Advantages:

- Simple, local procedure
- Succeeds independent of initial state
- Succeeds even if state hit by errors during computation (but only proven for error rate = 0)

Disadvantages:

- Only works for frustration-free H
- (Exp-time)

| Algorithm | Adiabatic | QPE | i -time | VQE | Dissipative | DQE |
|---------------------------------------|-----------|-----|-----------|-----|----------------|-----|
| No conditions on H | | | | ✓ | | ✓ |
| Provably succeeds | | ✓ | ✓ | | ✓ | ✓ |
| Low-depth in practice | | | | ✓ | | ✓ |
| No parameter optimisation | ✓ | ✓ | ✓ | | ✓ | ✓ |
| Efficient to generate multiple copies | | | | ✓ | | |
| Convenient implementation | | | | ✓ | ✓ | ✓ |
| Insensitive to initial state | | | | | ✓ | ✓ |
| Noise- and fault-resilient | | | | | ✓ ¹ | ✓ |

Dissipative Quantum Eigensolver

Advantages:

- Simple, local procedure
- Works for any H
- Does not require knowledge of properties of H
- Fault-resilient (without any overhead)

Disadvantages:

- (Exp-time)

Dissipative Quantum Eigensolver

Two new ingredients:

1. Use weak measurements rather than projective.
2. Make use of information from measurement outcomes.

- Intro & Background

- AGSPs

- Conditionally stopped process

- Epsilon schedules

- Fault-resilience

- Resampling strategies

- Optimal stopping

- Fixed-point process

- Conclusions & Outlook

main

bonus

Approximate Ground State Projectors

Def. AGSP

K Hermitian is $(\Delta, \Gamma, \varepsilon)$ -AGSP for π_0 if

$$(i) [K, \pi] = 0$$

$$(ii) K \pi \geq \sqrt{\Gamma} \pi$$

$$(iii) K \pi^\perp \leq \sqrt{\Delta} \pi^\perp$$

$$(iv) \|\pi - \pi_0\| \leq \varepsilon$$

(cf. [AKLV '13])

Approximate Ground State Projectors

$$H = \sum_{i=1}^m h_i, \quad H \Pi_0 = \lambda_{\min} \Pi_0$$

Lem.

$$K = \prod_{i=1}^m \left((1-\varepsilon) \mathbb{1} + \varepsilon k_i k_i \right) \prod_{i=m}^1 \left((1-\varepsilon) \mathbb{1} + \varepsilon k_i k_i \right)$$

is a $(\Gamma, \Delta, O(\varepsilon^2))$ -AGSP for Π_0 , where

$$k_i = \frac{1}{2}(\mathbb{1} - h_i / \|h_i\|), \quad \kappa = \sum_i \|h_i\|, \quad \kappa_i = \|h_i\| / \kappa$$

$$\Gamma = (1-\varepsilon)^{2m-1} \left(1 - \frac{\varepsilon \lambda_1}{\kappa} \right) + O(\varepsilon^2)$$

$$\Delta = (1-\varepsilon)^{2m-1} \left(1 - \frac{\varepsilon \lambda_0}{\kappa} \right) - O(\varepsilon^2)$$

Approximate Ground State Projectors

AGSP K can be implemented by local measurements.

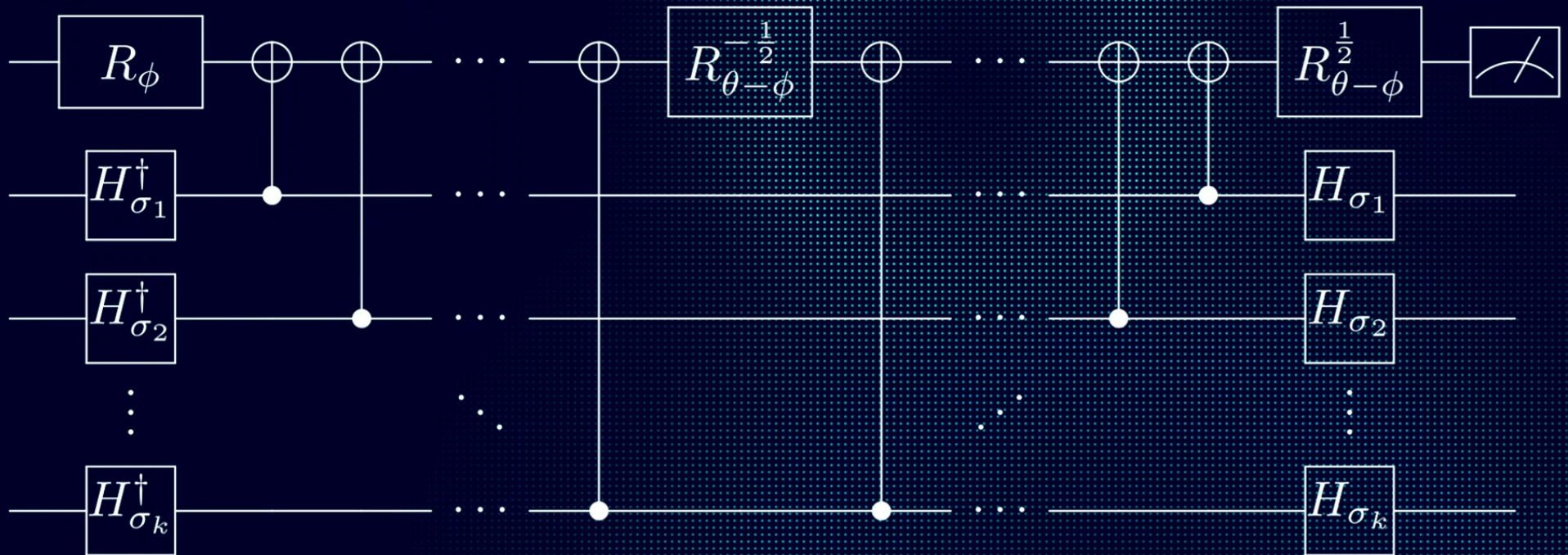
$$K = \prod_{i=1}^m \left((1-\varepsilon) \mathbb{1} + \varepsilon k_i \right) \prod_{i=m}^1 \left((1-\varepsilon) \mathbb{1} + \varepsilon k_i \right)$$
$$= \prod_{i=1}^m k_i \prod_{i=m}^1 k_i$$

Weak-measure each (suitably normalised)

local term in Hamiltonian in turn

→ K if obtain all " $(1-\varepsilon) \mathbb{1} + \varepsilon k_i$ " outcomes.

Approximate Ground State Projectors



Weak-measurement of $h_i = \sigma_1 \otimes \sigma_2 \otimes \dots \otimes \sigma_k$
 $\theta = \cos^{-1}(1-\epsilon)$, $\phi = \cos^{-1}(1-\epsilon(1-k_i))$

- Intro & Background

- AGSPs

- Conditionally stopped process

- Epsilon schedules

- Fault-resilience

- Resampling strategies

- Optimal stopping

- Fixed-point process

- Conclusions & Outlook

main

bonus

Conditionally stopped process

Quantum instrument $\{\Sigma_0, \Sigma_1\}$

$$\Sigma_0(\rho) = K\rho K^\dagger, \quad \Sigma_1(\rho) = (1 - \text{tr}(K\rho K^\dagger)) \frac{\mathbb{1}}{D}$$

Iterate instrument starting from $\rho = \frac{\mathbb{1}}{D}$
until obtain n "0"s $\rightarrow \rho_n$.

Thm.

$$\text{tr}(\Pi_0 \rho_n) \geq 1 - \varepsilon - \frac{D}{N} \left(\frac{\Delta}{\Gamma}\right)^n$$

$D := \text{total dim.}$, $N := \text{tr } \Pi_0 = \text{g.s. degeneracy}$

Conditionally stopped process

Thm.

$$\text{tr}(\Pi_0 \rho_n) \geq 1 - \varepsilon - \frac{D}{N} \left(\frac{\Delta}{\Gamma}\right)^n$$

Pf.

Recall $[k, \Pi] = 0$, $\|\Pi - \Pi_0\| \leq \varepsilon$ (Def. AGSP)

$$\text{tr}(\Pi k^n \rho k^n) = \text{tr}((k\Pi)^n \rho (k\Pi)^n) \geq \Gamma^n \text{tr} \Pi \rho.$$

Similarly, $\text{tr}(\Pi^\perp k^n \rho k^n) \leq \Delta^n (1 - \text{tr} \Pi \rho).$

Conditionally stopped process

$$\rho_n = \frac{\xi_0^n(\rho)}{\text{tr} \xi_0^n(\rho)} = \frac{k^n \rho k^n}{\text{tr}(k^n \rho k^n)}$$

$$\text{tr}(\pi \rho_n) = \frac{\text{tr}(\pi k^n \rho k^n)}{\text{tr}(k^n \rho k^n)}$$

$$= 1 - \frac{\text{tr}(\pi^\perp k^n \rho k^n)}{\text{tr}(\pi k^n \rho k^n) + \text{tr}(\pi^\perp k^n \rho k^n)}$$

$$\geq 1 - \frac{\Delta^n (1 - \text{tr} \pi \rho)}{\Gamma^n \text{tr} \pi \rho}$$

by ineqs.
shown before

Conditionally stopped process

$$\text{tr } \pi \rho_0 = \text{tr } \pi \frac{\mathbb{1}}{D} = \frac{N}{D}$$

$$\text{tr } \pi \rho_n \geq 1 - \frac{1 - \text{tr } \pi \rho_0}{\text{tr } \pi \rho_0} \left(\frac{\Delta}{\Gamma} \right)^n \geq 1 - \frac{D}{N} \left(\frac{\Delta}{\Gamma} \right)^n$$

$$\text{tr } \pi_0 \rho_n \geq \text{tr } \pi \rho - \varepsilon$$

$$= 1 - \varepsilon - \frac{D}{N} \left(\frac{\Delta}{\Gamma} \right)^n \quad \square$$

$\|\pi - \pi_0\| \leq \varepsilon$
Def. AGSP

Conditionally stopped process

$$\Sigma_0(\rho) = K\rho K^T, \quad \Sigma_1(\rho) = (1 - \text{tr}(K\rho K^T)) \frac{\mathbb{1}}{D}$$

$\rho_0 = \frac{\mathbb{1}}{D}$, iterate until n "0"s.

Thm.

$\tau_n :=$ stopping time = # iterations until run of n zeros

Expected stopping time:

$$\mathbb{E}(\tau_n) = \frac{1}{\text{tr}(K^{2n})} \text{tr} \left(\frac{1 - K^{2n}}{1 - K^2} \right) \leq \frac{1}{\Gamma^n} \left(n + \frac{1 - \Delta^n}{1 - \Delta} \left(\frac{D}{N} - 1 \right) \right)$$

Conditionally stopped process

Pf.

Imagine betting on sequence of 0/1 outcomes according to following strategy:

- At time step t , bet all winnings so far + additional ϵ_t on "0", at fair odds.
- Outcome "0" \rightarrow get back stake \times odds.
- Outcome "1" \rightarrow lose entire stake.

Conditionally stopped process

Random vars $X_t := t$ 'th 0/1 outcome

$M_t :=$ net winnings at time t

$$M_0 = 0$$

$$M_t = \begin{cases} (M_{t-1} + t) \times \frac{1}{P_r(X_t=0 | X_{t-1}, X_{t-2}, \dots, X_1)} & X_t = 0 \\ -t & X_t = 1 \end{cases}$$

Conditionally stopped process

Random vars $X_t := t$ 'th 0/1 outcome

$M_t :=$ net winnings at time t

$$M_0 = 0$$

$$M_t = \begin{cases} (M_{t-1} + t) \times \frac{1}{P_r(X_t=0 | X_{t-1}, X_{t-2}, \dots, X_1)} & X_t = 0 \\ -t & X_t = 1 \end{cases}$$

fair odds

Conditionally stopped process

$$\begin{aligned} \mathbb{E}(M_t | X_{t-1} \dots X_1) &= \Pr(0 | X_{t-1} \dots X_1) \cdot \left(\frac{M_{t-1} + t}{\Pr(0 | X_{t-1} \dots X_1)} - t \right) \\ &\quad - \Pr(1 | X_{t-1} \dots X_1) \cdot t \quad (\text{Def. } M_t) \\ &= M_{t-1} \end{aligned}$$

$\Rightarrow M_t$ is a Martingale wrt X_t . (fair odds)

$\Rightarrow \mathbb{E}(M_{\tau_n})$ is also a Martingale (Doob's optional stopping Thm.)

Conditionally stopped process

$$\begin{aligned} \mathbb{E}(M_t | X_{t-1} \dots X_1) &= \Pr(0 | X_{t-1} \dots X_1) \cdot \left(\frac{M_{t-1} + t}{\Pr(0 | X_{t-1} \dots X_1)} - t \right) \\ &\quad - \Pr(1 | X_{t-1} \dots X_1) \cdot t \quad (\text{Def. } M_t) \\ &= M_{t-1} \end{aligned}$$

$\Rightarrow M_t$ is a Martingale wrt X_t . (fair odds)

$\Rightarrow \mathbb{E}(M_{\tau_n})$ is also a Martingale (Doob's optional stopping Thm.)

$\Rightarrow \mathbb{E}(M_{\tau_n}) = \mathbb{E}(M_0)$

Conditionally stopped process

Reset state to $\frac{1}{D}$ after a "1"

\Rightarrow state at run of k "0"s is

$$P_k = \frac{K^k \frac{1}{D} K^k}{\text{tr}(\text{---})} = \frac{K^{2k}}{\text{tr}(K^{2k})}$$

Conditionally stopped process

Reset state to $\frac{1}{D}$ after a "1"

\Rightarrow state at run of k "0"s is

$$P_k = \frac{K^k \frac{1}{D} K^k}{\text{tr}(\text{---})} = \frac{K^{2k}}{\text{tr}(K^{2k})}$$

$$P_{\sim}(010^k1\dots) = \frac{\text{tr}(K P_k K)}{\text{tr}(K^{2k})}$$

Conditionally stopped process

Just had run of n "0"s since last "1".

$$M_{\tau_n - n} = -\tau_n + n$$

$$M_{\tau_n - k} = \frac{1}{P_r(010^{n-1}1\dots)} (M_{\tau_n - k - 1} + \tau_n - k) - \tau_n + k$$

(Def. M_t)

$$P_r(010^k1\dots) = \frac{\text{tr}(k^{2k+2})}{\text{tr}(k^{2k})} \quad (\text{from previously})$$

Solve recurrence:

$$\Rightarrow M_{\tau_n} = \frac{1}{\text{tr}(k^{2n})} \text{tr}\left(\frac{1 - k^{2n}}{1 - k^2}\right) - \tau_n$$

Conditionally stopped process

$$\mathbb{E}(M_{\tau_n}) = \frac{1}{\text{tr}(k^{2n})} \text{tr} \left(\frac{1 - k^{2n}}{1 - k^2} \right) - \mathbb{E}(\tau_n)$$

$\stackrel{=}{=} 0$ from previously

Conditionally stopped process

$$0 = \frac{1}{\text{tr}(k^{2n})} \text{tr} \left(\frac{1 - k^{2n}}{1 - k^2} \right) - \mathbb{E}(\mathcal{T}_n)$$

Rearranging:

$$\mathbb{E}(\mathcal{T}_n) = \frac{1}{\text{tr}(k^{2n})} \text{tr} \left(\frac{1 - k^{2n}}{1 - k^2} \right)$$

Conditionally stopped process

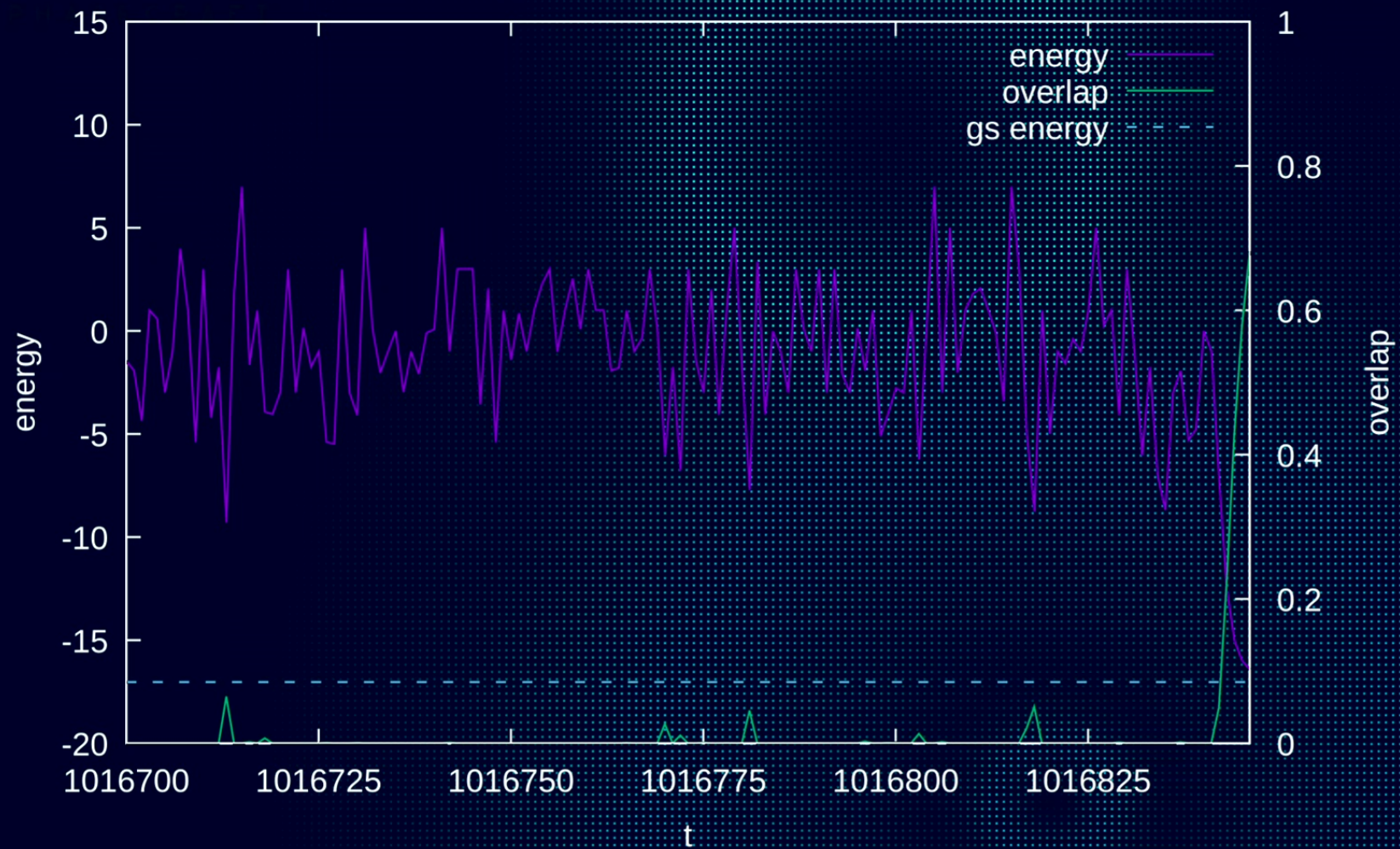
$$0 = \frac{1}{\text{tr}(k^{2n})} \text{tr} \left(\frac{1 - k^{2n}}{1 - k^2} \right) - \mathbb{E}(\mathcal{T}_n)$$

Rearranging:

$$\mathbb{E}(\mathcal{T}_n) = \frac{1}{\text{tr}(k^{2n})} \text{tr} \left(\frac{1 - k^{2n}}{1 - k^2} \right)$$

$$\approx \frac{1}{\text{tr}(k^{2n})} \left(n + \frac{1 - \Delta^n}{1 - \Delta} \left(\frac{D}{N} - 1 \right) \right) \quad \square$$

1D Heisenberg chain Length 10, $n=4$



- Intro & Background
 - AGSPs
 - Conditionally stopped process
 - Epsilon schedules
 - Fault-resilience
 - Resampling strategies
 - Optimal stopping
 - Fixed-point process
 - Conclusions & Outlook
-
- main
- bonus

Epsilon Schedules

Thm.

$$\text{tr}(\Pi_0 \rho_n) \geq 1 - \epsilon - \frac{D}{N} \left(\frac{\Delta}{\Gamma}\right)^n$$

$$\Gamma = (1 - \epsilon)^{2m-1} \left(1 - \epsilon \lambda_1 / \kappa\right) + O(\epsilon^2)$$

$$\Delta = (1 - \epsilon)^{2m-1} \left(1 - \epsilon \lambda_0 / \kappa\right) - O(\epsilon^2)$$

Epsilon Schedules

Solution: choose ϵ_t decreasing with t .

Thm.

$\{\epsilon_0, \epsilon_1\}$, stop after n "0"s

$$\epsilon_t = \frac{\epsilon_0}{t - t_1} \text{ where } t_1 = \text{time of last "1"}$$

ρ_n = stopping state

$$\lim_{n \rightarrow \infty} \text{tr}(\Pi_0 \rho_n) = 1.$$

Epsilon Schedules

Thm. $\lim_{n \rightarrow \infty} \text{tr}(\Pi_0 \rho_n) = 1.$

harmonic series
is divergent
 \Rightarrow product convergent

Pf. (intuition)

• Once $\epsilon_t \ll \lambda_1 - \lambda_0 \rightarrow$ convergence $\sim \prod_t (1 - \frac{\epsilon}{t}) \rightarrow 0$

• Error accumulated up to that point

$$\sim \sum_t \frac{1}{t^2} \prod_{s=1}^t (1 - \frac{\epsilon}{s})^{-1} \leq e^{2\epsilon} \zeta(2(1-\epsilon)) = o(1).$$

Epsilon Schedules

Thm. $\lim_{n \rightarrow \infty} \text{tr}(\Pi_0 \rho_n) = 1.$

harmonic series
is divergent
 \Rightarrow product convergent

Pf. (intuition)

- Once $\epsilon_t \ll \lambda_1 - \lambda_0 \rightarrow$ convergence $\sim \prod_t (1 - \frac{\epsilon}{t}) \rightarrow 0$

($r, \Delta, o(\epsilon^2)$) - AGSP

- Error accumulated up to that point

$$\sim \sum_t \frac{1}{t^2} \prod_{s=1}^t (1 - \frac{\epsilon}{s})^{-1} \leq e^{2\epsilon} \zeta(2(1-\epsilon)) = o(1).$$

- Intro & Background
 - AGSPs
 - Conditionally stopped process
 - Epsilon schedules
 - **Fault-resilience**
 - Resampling strategies
 - Optimal stopping
 - Fixed-point process
 - Conclusions & Outlook
-
- main
- bonus

Fault-Resilience

- What if errors occur during the process?
- What if the process itself is implemented using faulty operations?

Both possibilities encompassed by:

$$\{\Sigma_0, \Sigma_1\} \longrightarrow \{\Sigma'_0, \Sigma'_1\}$$

$$\|\Sigma'_0 - \Sigma_0\|_1, \|\Sigma'_1 - \Sigma_1\|_1 \leq \delta$$



Fault - Resilience

Thm.

$$\{\xi'_0, \xi'_1\} \text{ s.t. } \|\xi'_0 - \xi_0\| \leq \delta$$

Iterate $\{\xi'_0, \xi'_1\}$ until run of n "0"s.

$\rho_n =$ stopping state.

$$\lim_{n \rightarrow \infty} \text{tr}(\Pi_0 \rho_n) = 1 - O(\epsilon) + O(\delta)$$

Fault-Resilience

Thm. $\lim_{n \rightarrow \infty} \text{tr}(\Pi_0 \rho_n) = 1 - O(\epsilon) + O(\delta)$

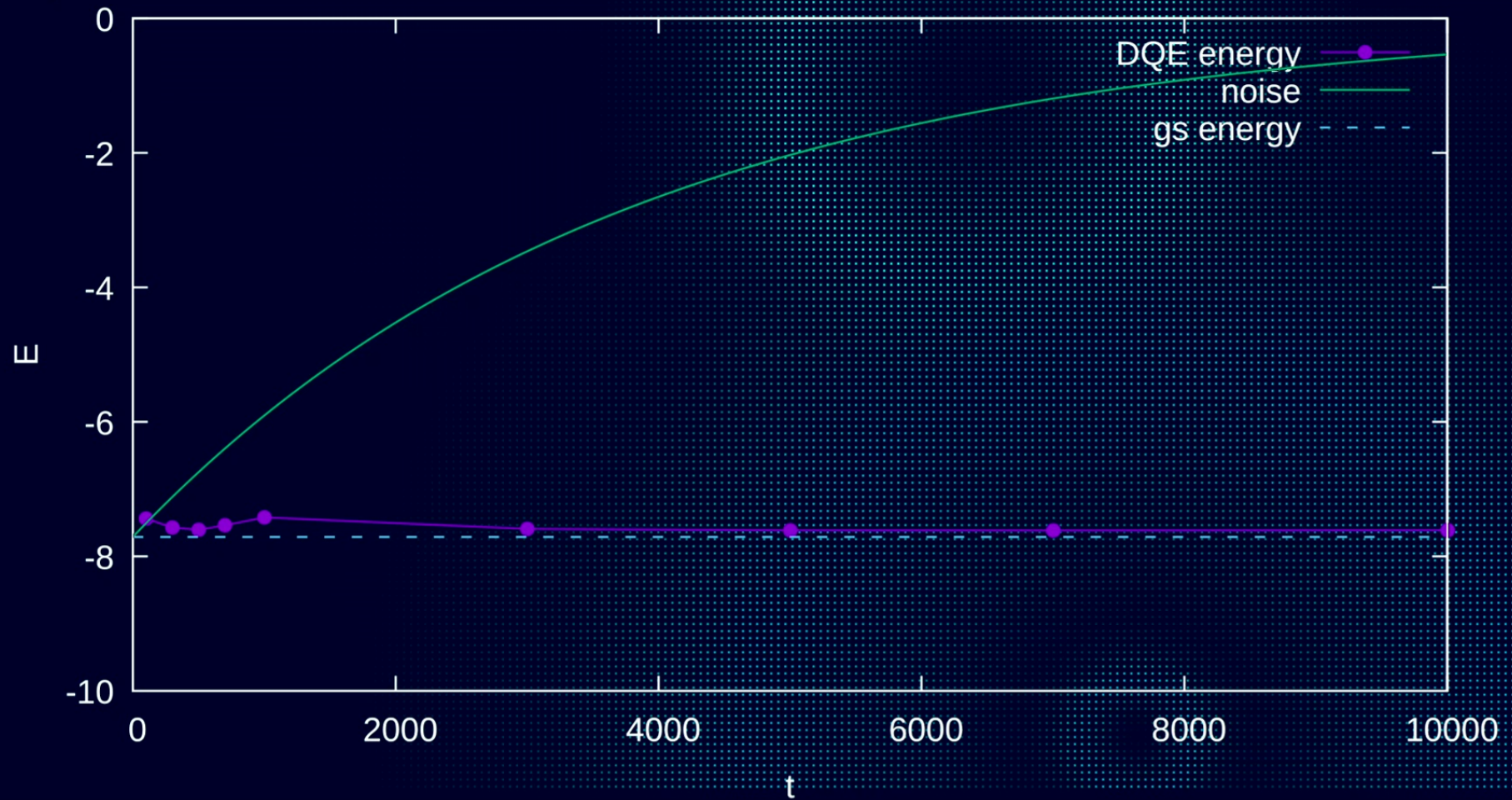
Pf. (hint)

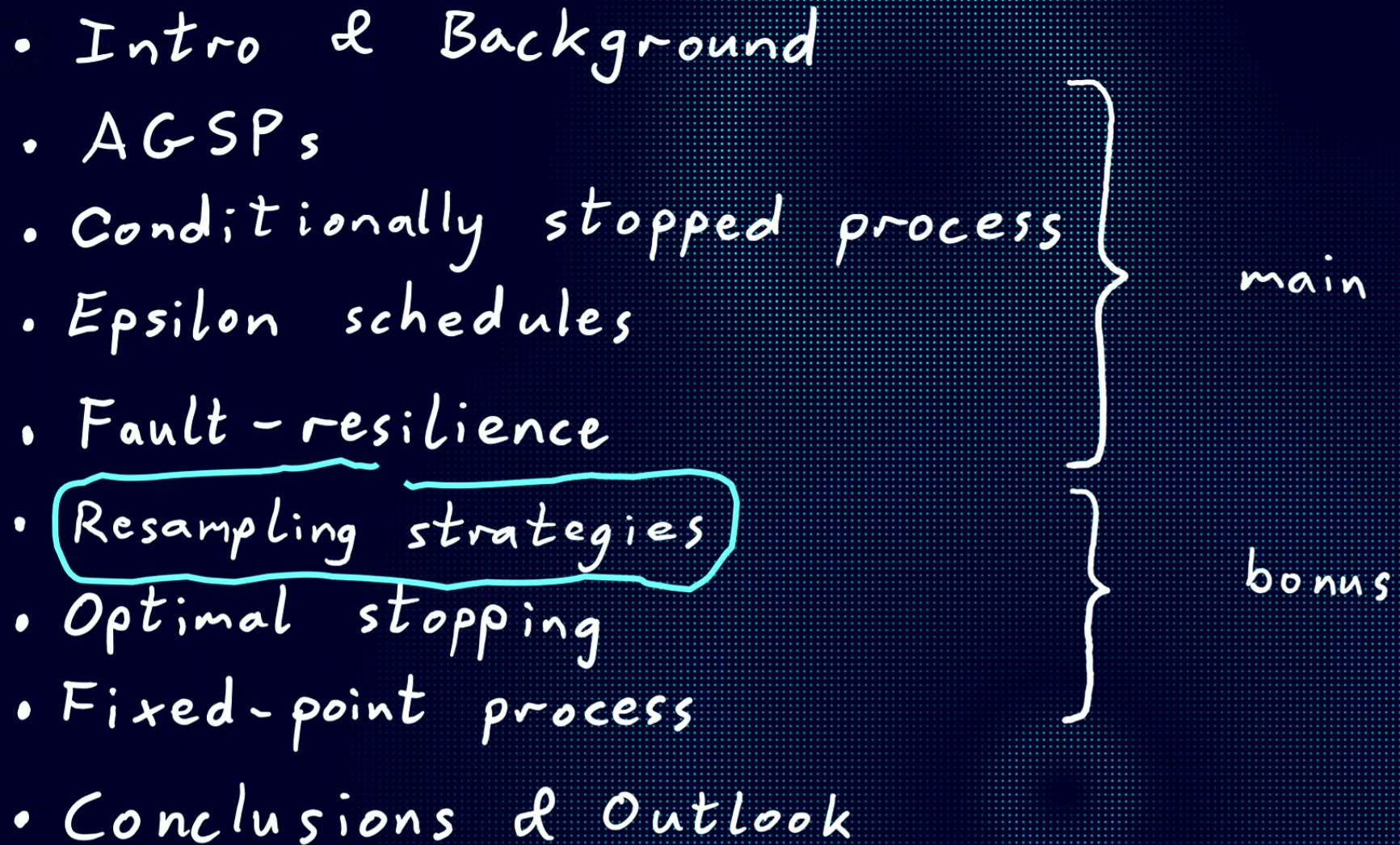
$$\mathcal{E}_0^n(\rho) \equiv E_0^n | \rho \rangle \rangle$$

Non-commutative Perron-Frobenius theory
+ Schur structure of transfer matrix
+ eigenspace perturbation theory

Fault - Resilience

1D Heisenberg $n=5$, noise= 10^{-4}

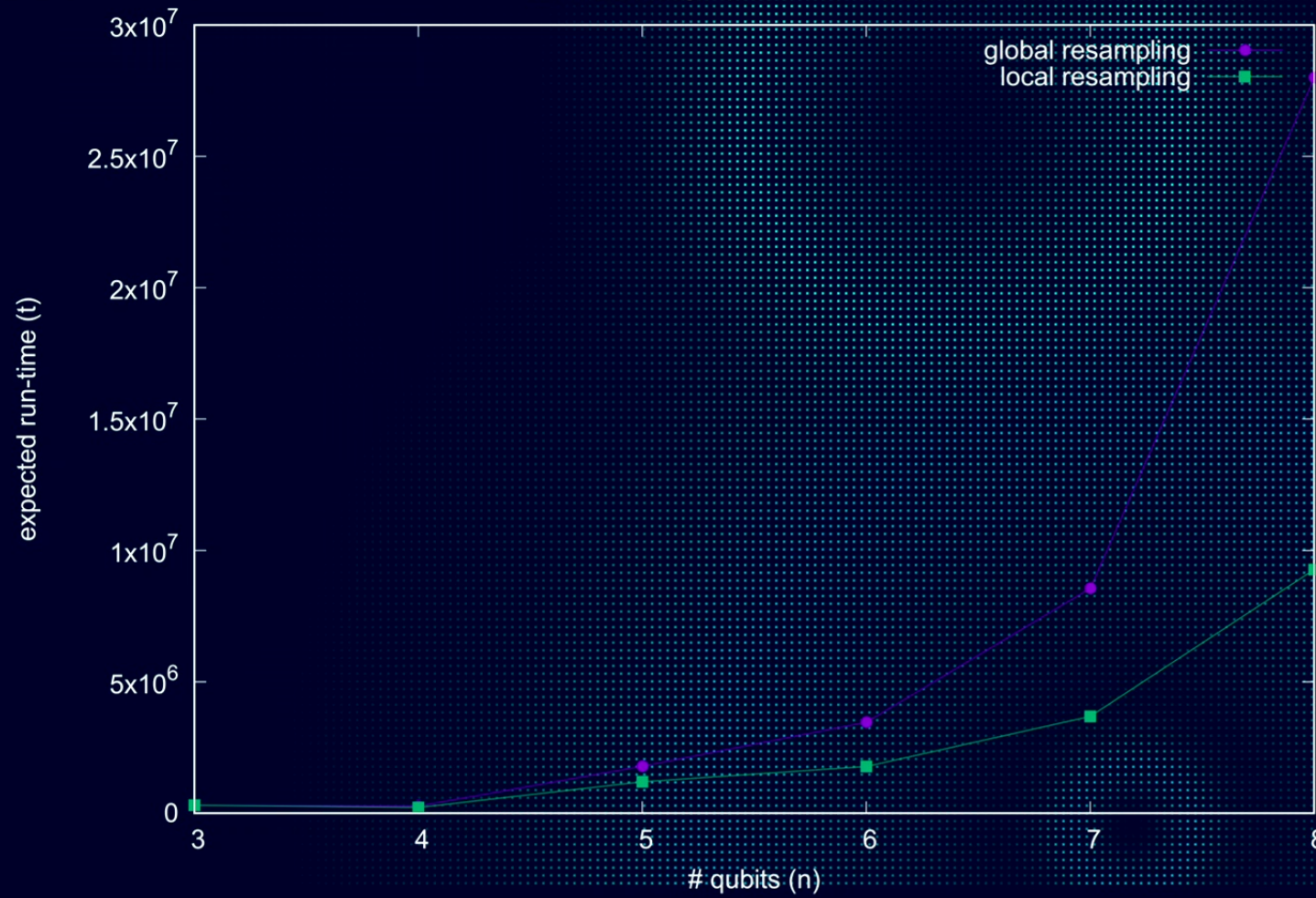


- Intro & Background
 - AGSPs
 - Conditionally stopped process
 - Epsilon schedules
 - Fault-resilience
 - Resampling strategies
 - Optimal stopping
 - Fixed-point process
 - Conclusions & Outlook
- main
- bonus
- 



Resampling Strategies

1D Heisenberg, epsilon=0.1, zeros=256

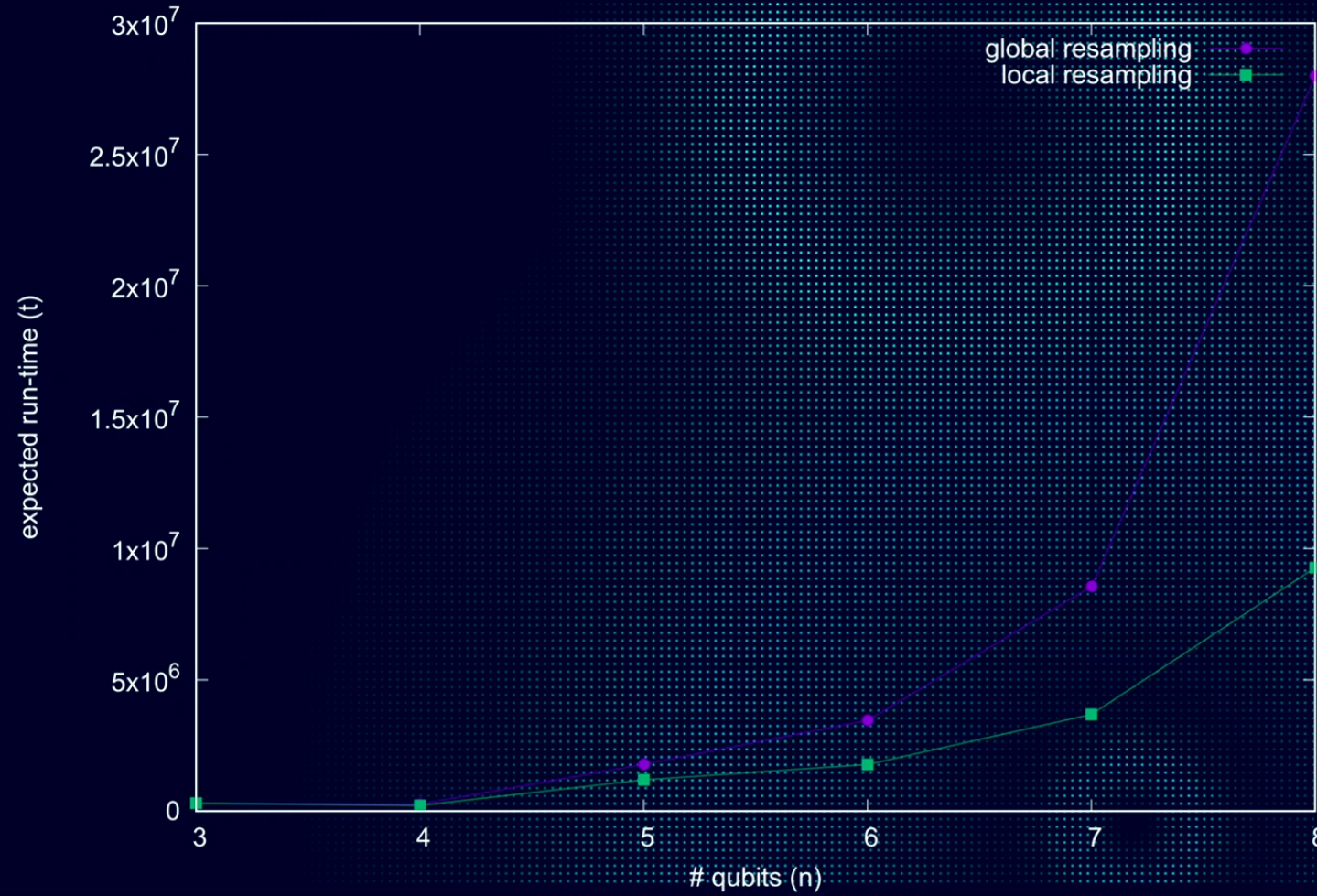


- Intro & Background
 - AGSPs
 - Conditionally stopped process
 - Epsilon schedules
 - Fault-resilience
 - Resampling strategies
 - **Optimal stopping**
 - Fixed-point process
 - Conclusions & Outlook
- } main
- } bonus



Resampling Strategies

1D Heisenberg, epsilon=0.1, zeros=256



Dissipative Gibbs Sampling

- Similar benefits to DQE : simple local procedure, fault-resilience, no complex q. subroutines required (no H . simulation, phase estimation, qubitization, etc.)
- Exact analytic expressions for run-time, no hard-to-analyse quantities (e.g. mixing times).
- Optimal(?) run-time scaling with precision.