Title: Commuting operations factorise

Speakers: Renato Renner

Series: Quantum Foundations

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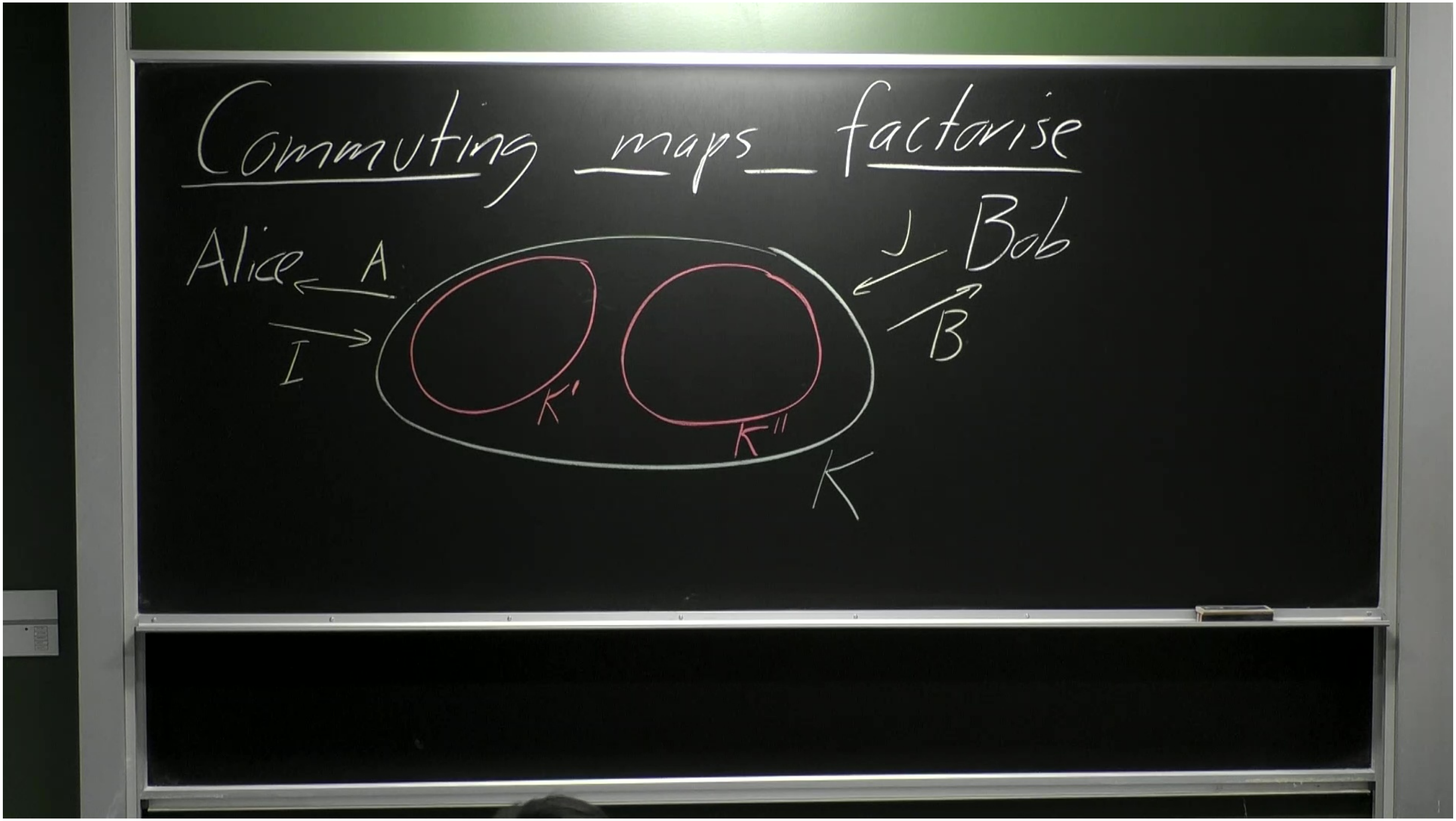
URL: https://pirsa.org/23090057

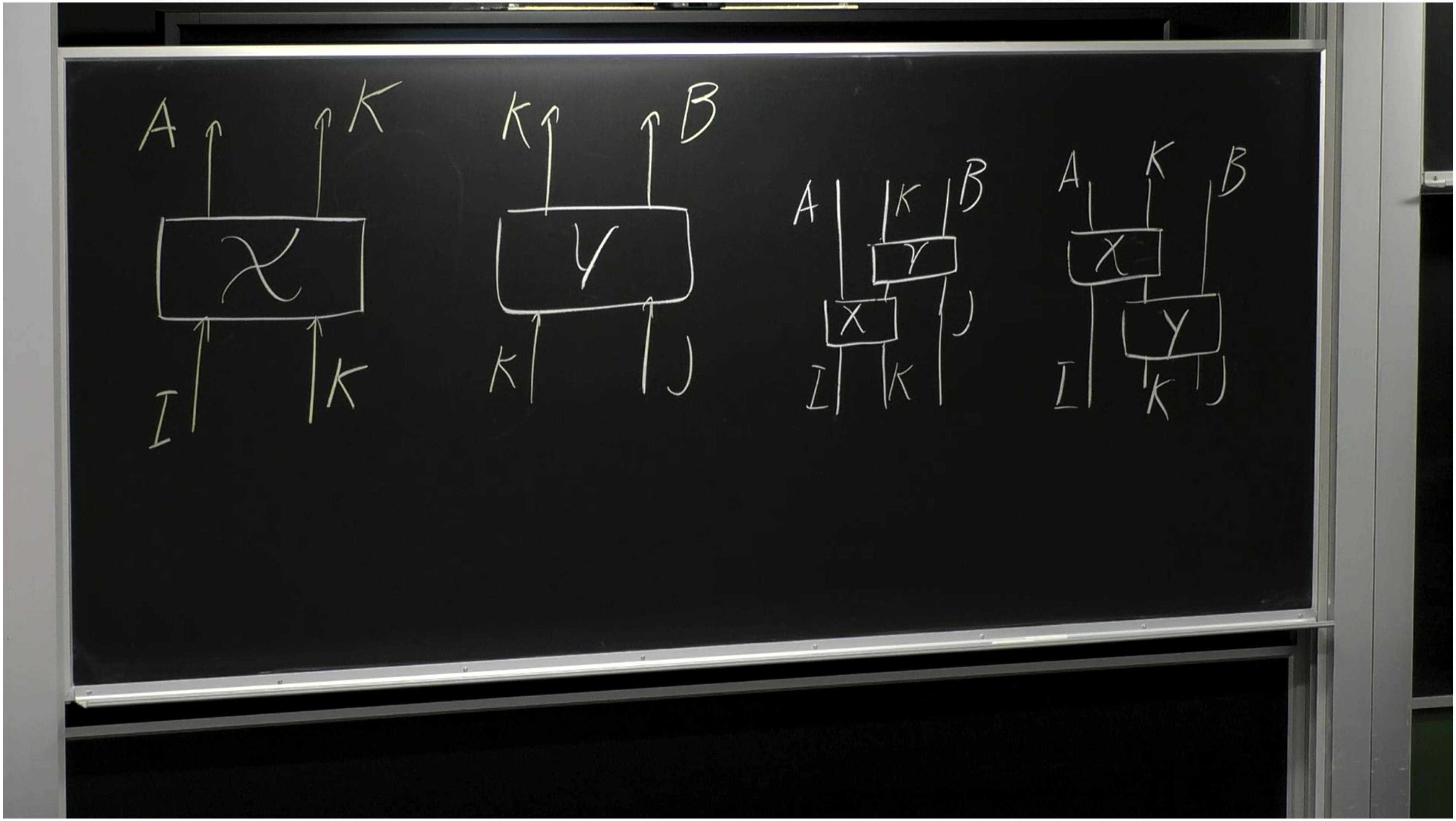
Abstract:

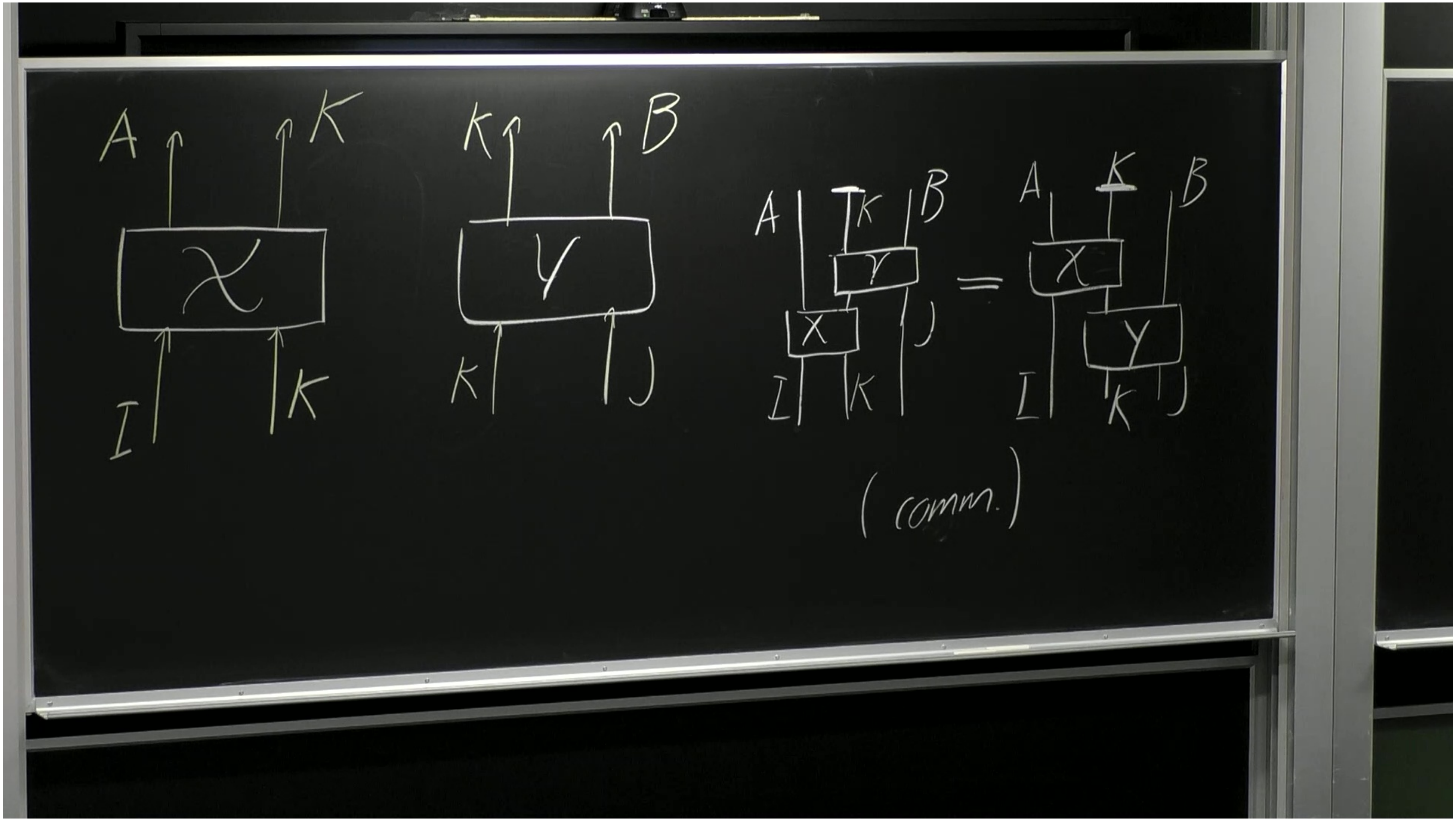
Tsirelson's problem involves two agents, Alice and Bob, who apply measurements on the same quantum system, K. It asks whether commutation, i.e., independence of whether Alice or Bob measures first, is sufficient to conclude that Alice and Bob's measurements can be factorised so that they act non-trivially only on distinct subsystems of K. In this talk, I will present a "fully quantum generalisation" of this problem, where Alice and Bob's measurements are replaced by operations on K that may depend on additional quantum inputs and produce quantum outputs. As for Tsirelson's original problem, it turns out that commutation indeed implies factorisation, provided that all relevant systems are finite-dimensional.

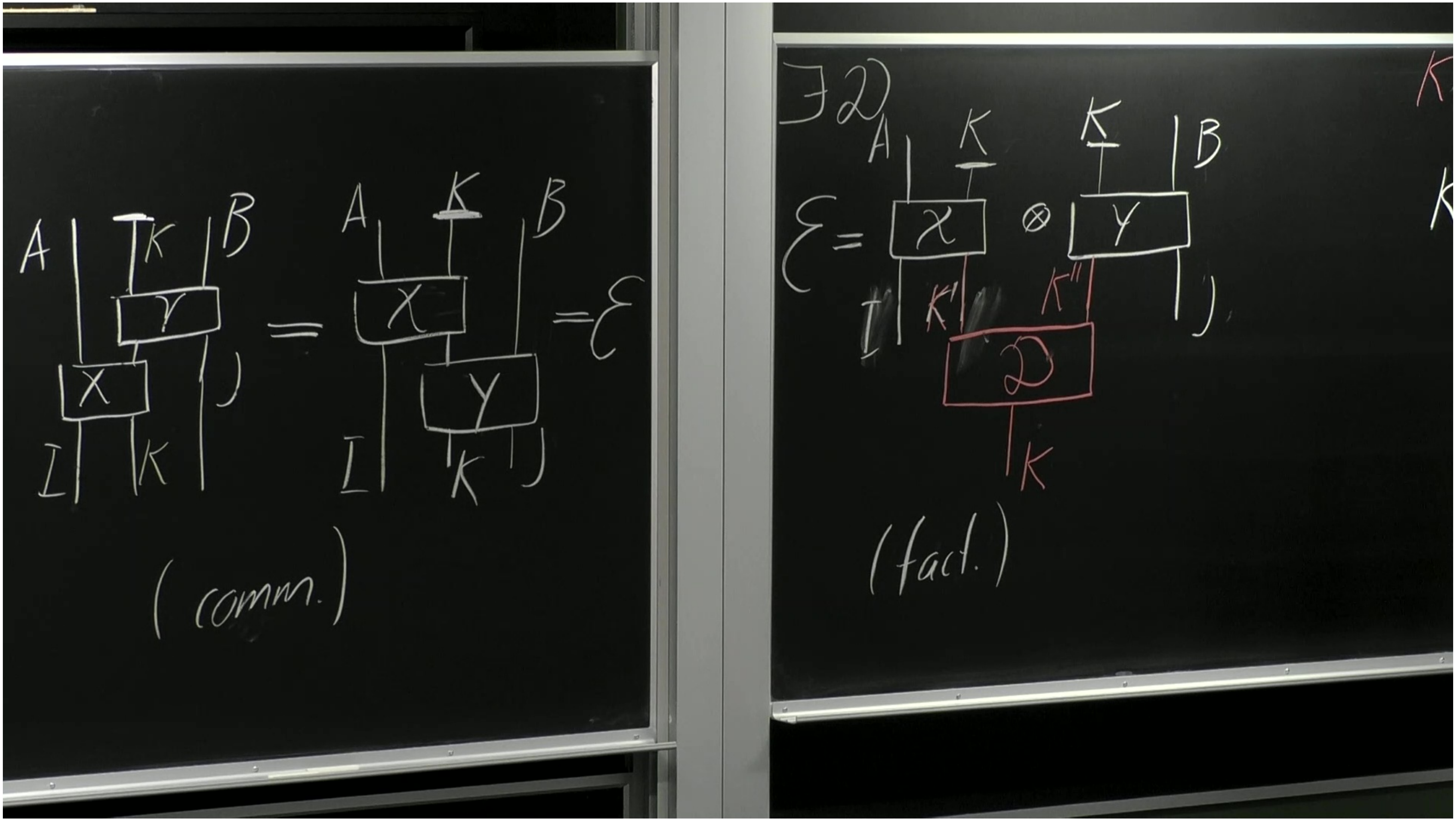
This is joint work with Ramona Wolf; preprint available at arXiv:2308.05792.

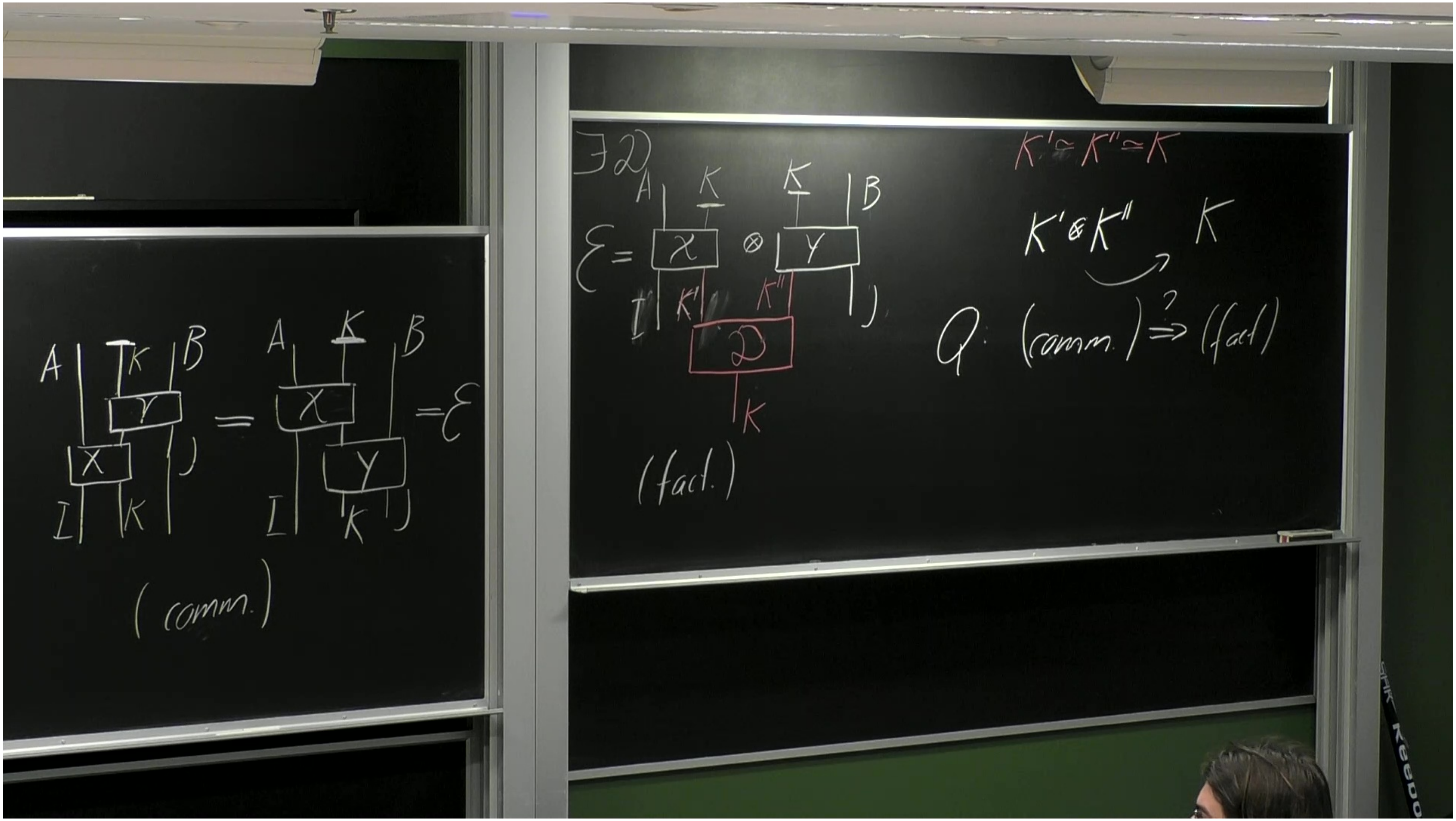
Zoom link https://pitp.zoom.us/j/99031410183?pwd=MzVoQXpPSll6bFp1b1g3U2J4U21rZz09

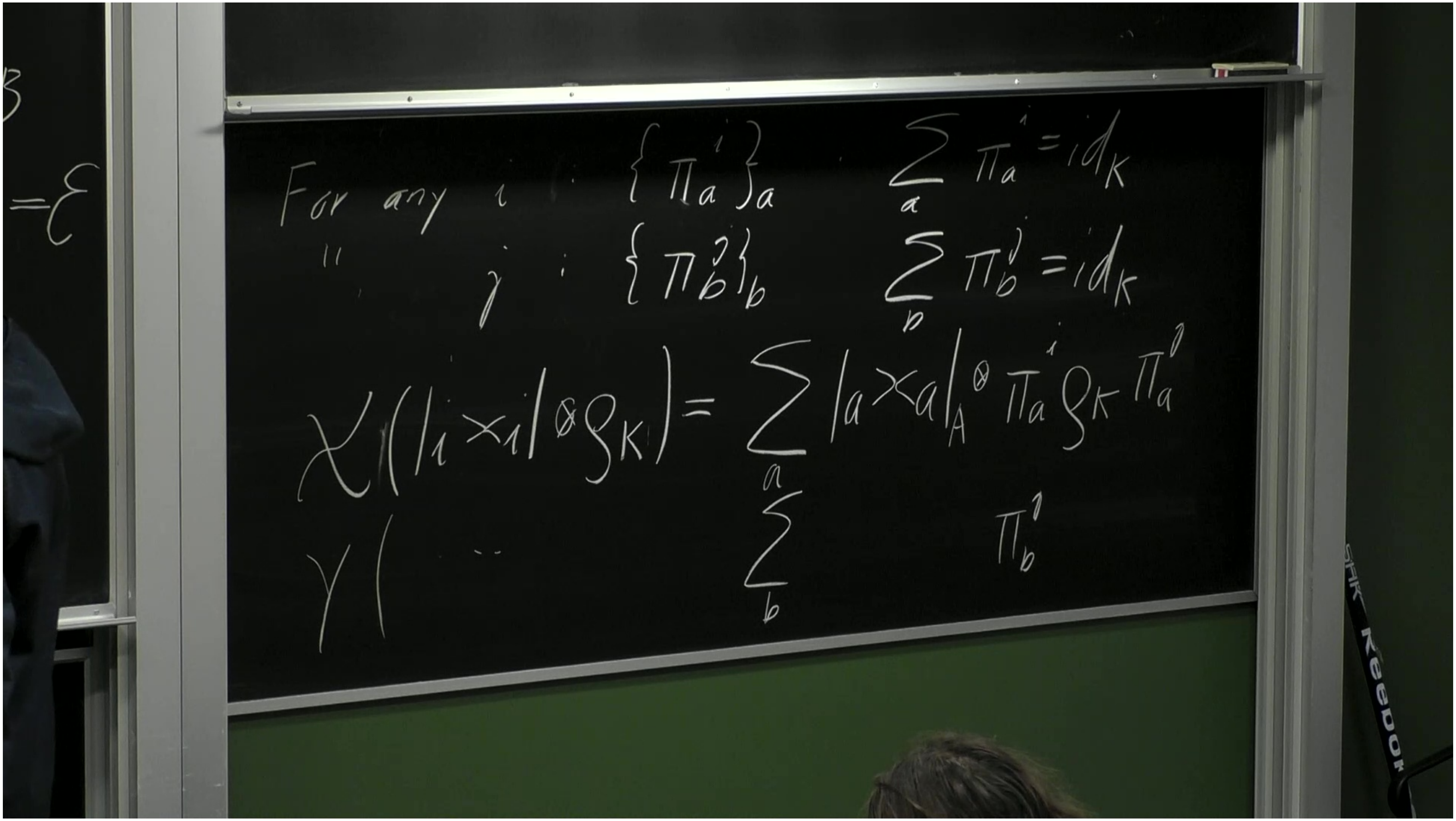

Commuting maps factorise
Alice
A
I
J
Bob
B
K'
K''
K

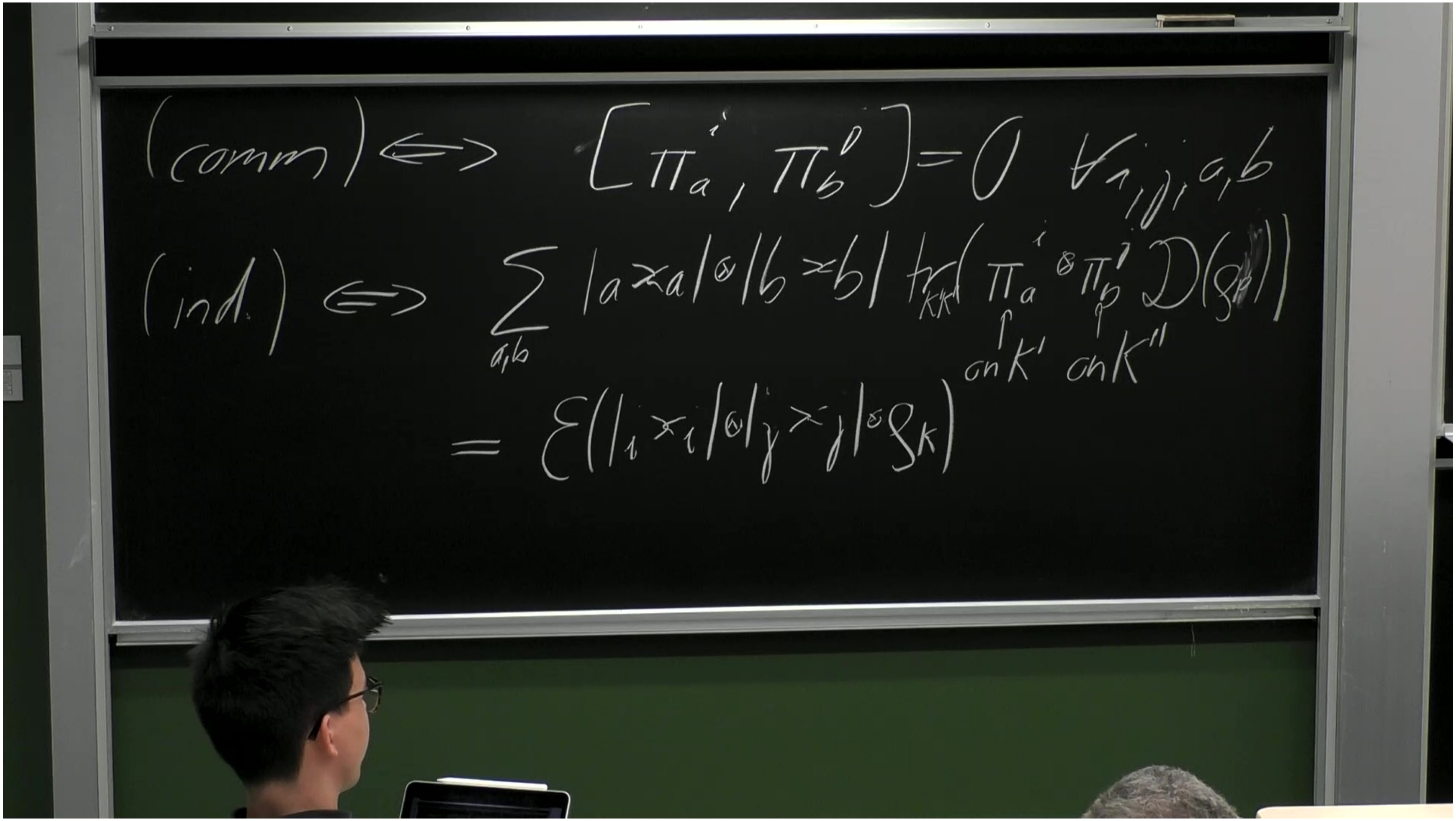

A
K
I
K
X
K
B
K
J
Y
A
K
B
X
Y
I
K
J

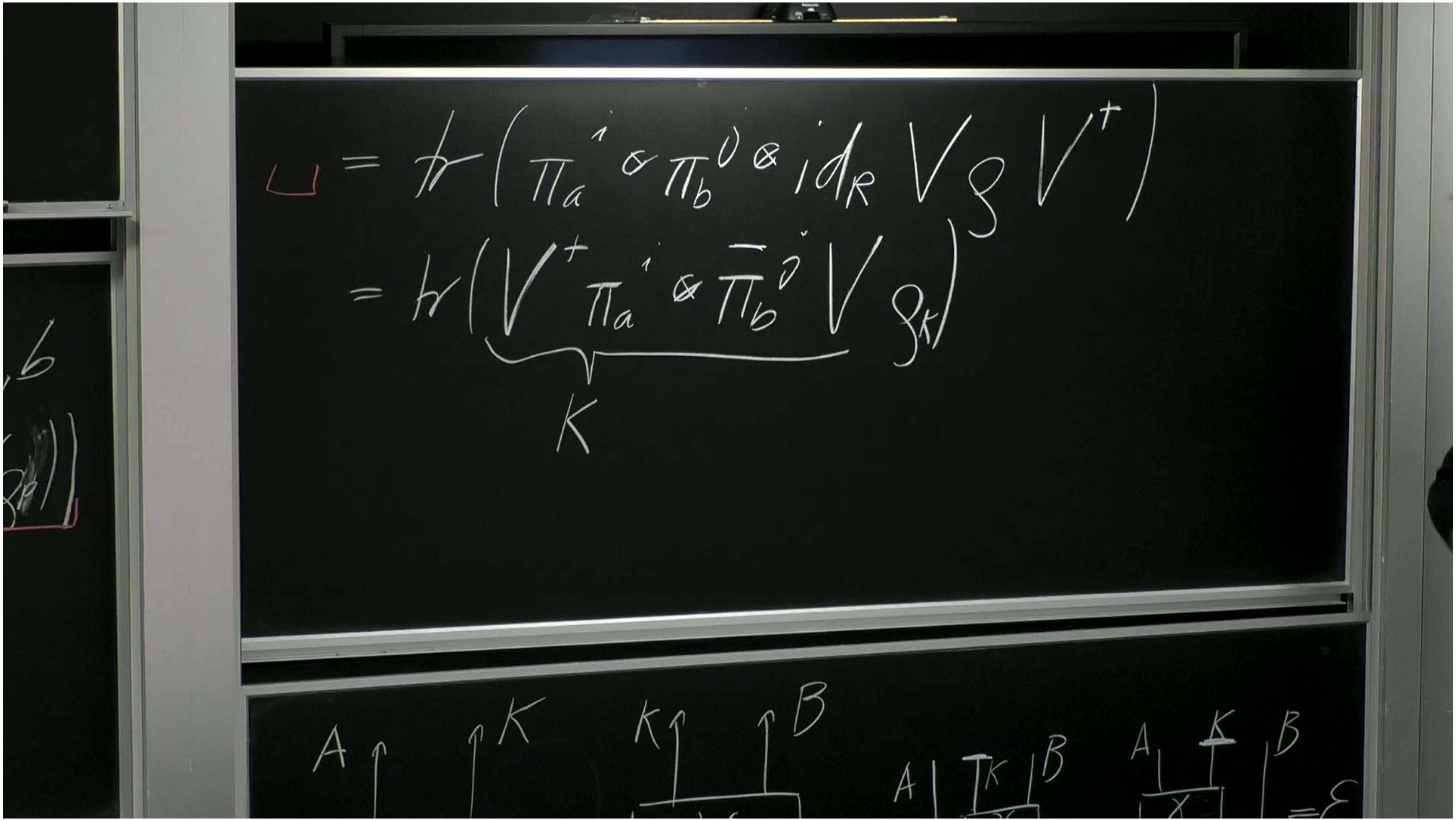

A
K
I
K
χ
K
B
K
J
Y
A
K
B
Y
X
J
I
K
=
A
K
B
X
Y
I
K
J
(comm.)

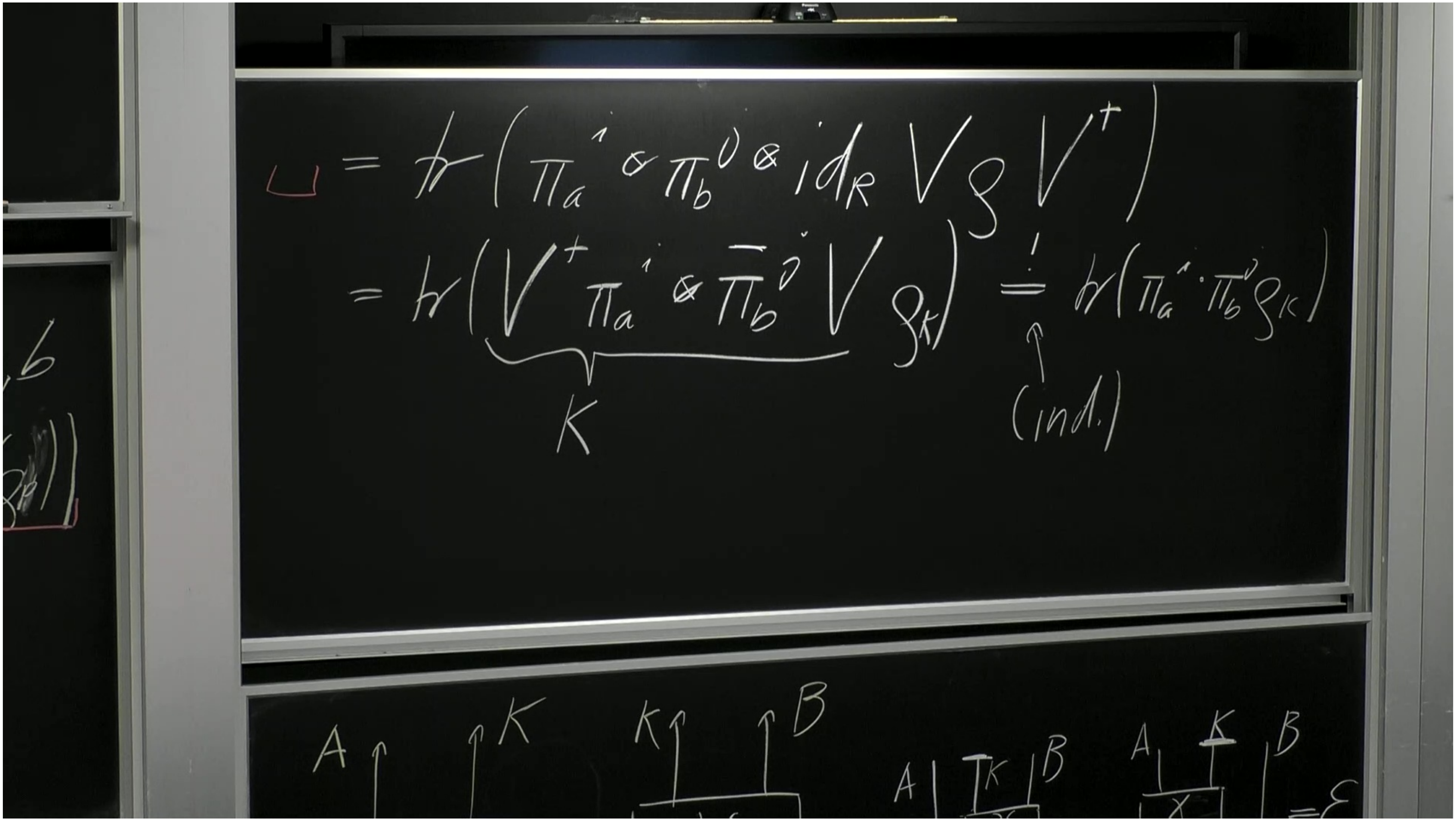

(comm.)
(fact.)

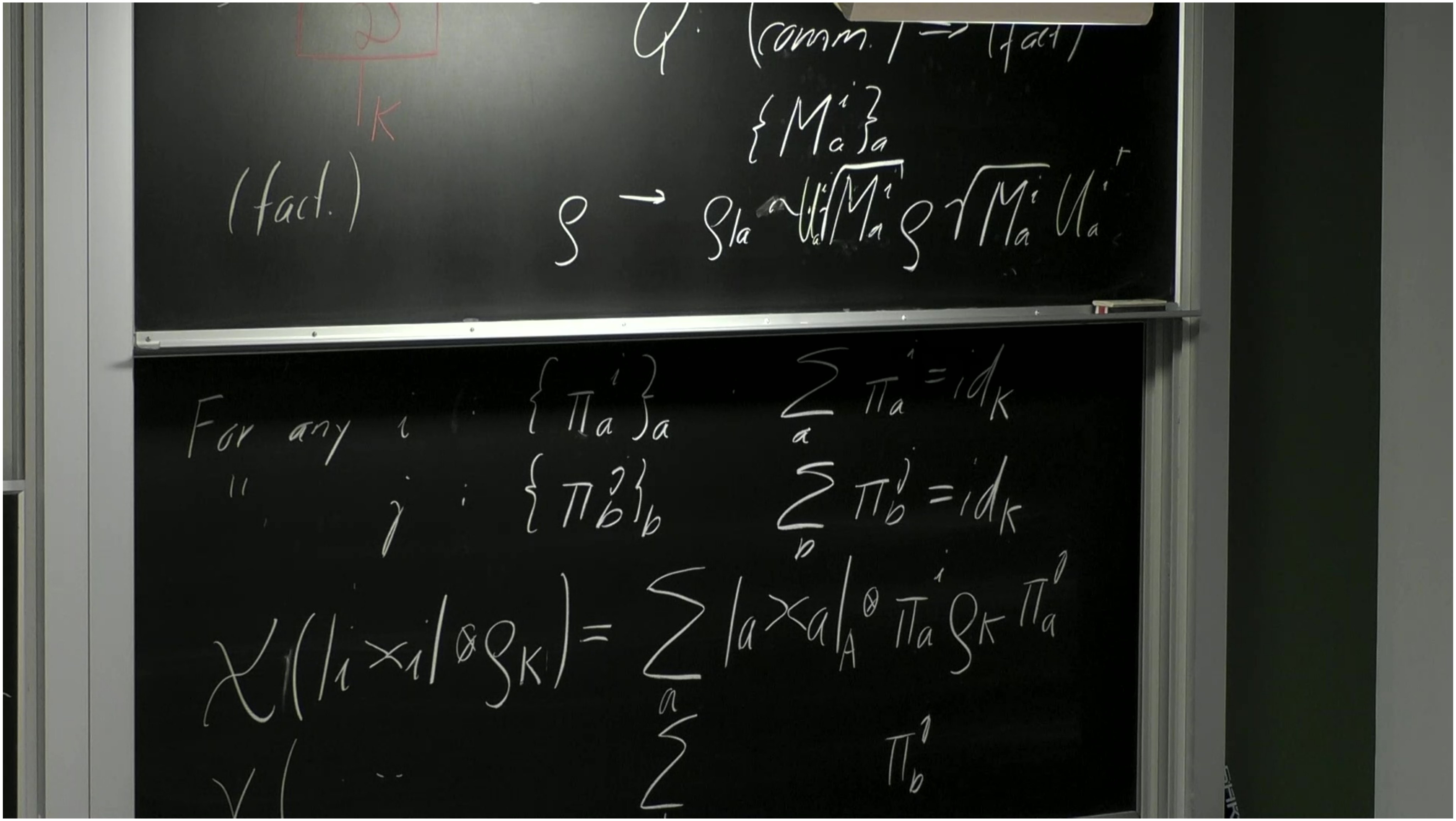

(comm.)
(fact.)
K' ⊗ K''
K
Q: (comm.) ⇒? (fact)
K' ≃ K'' ≃ K

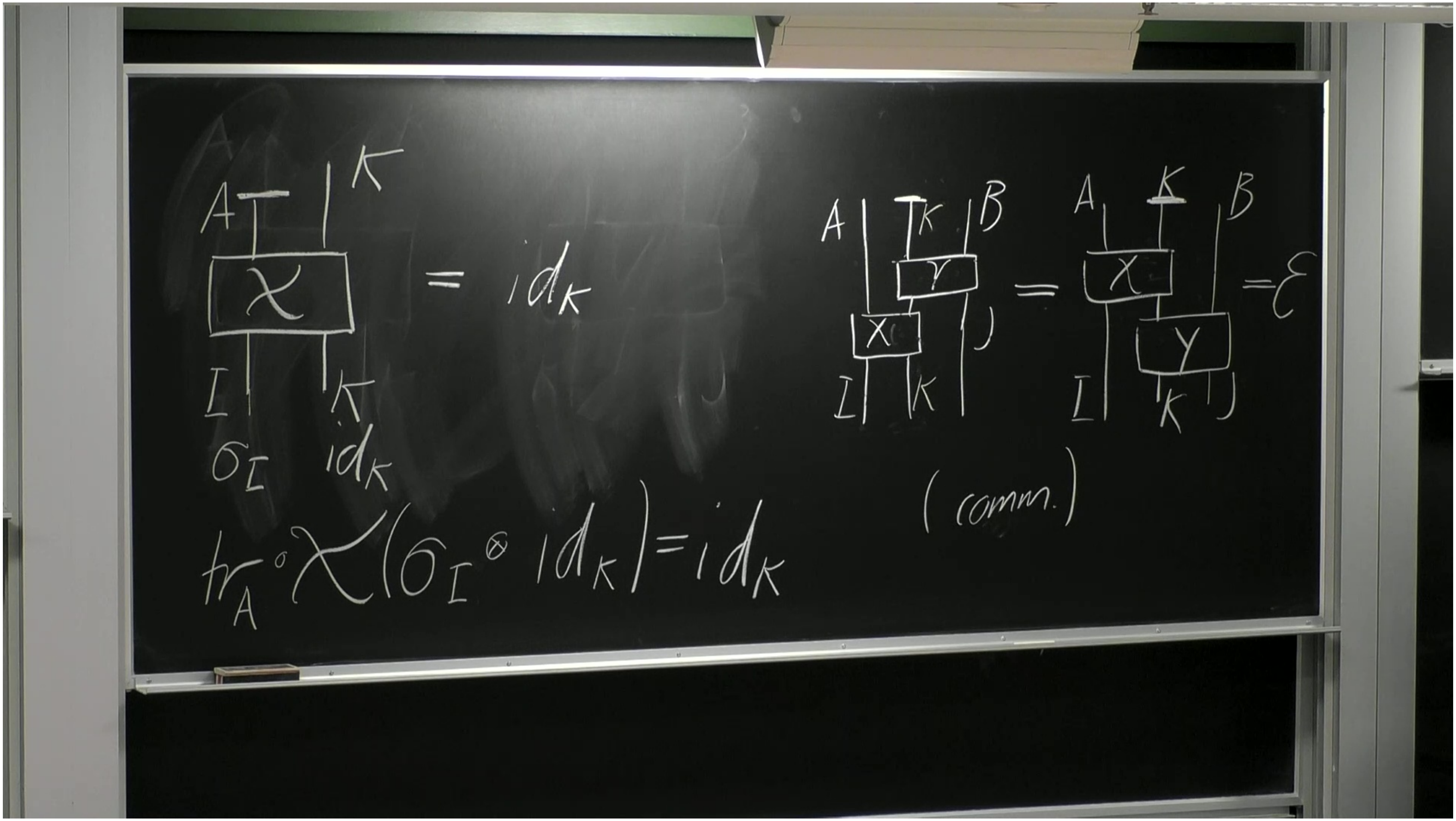

For any i : {Π^i_a}_a   Σ_a Π^i_a = id_K
" j : {Π^j_b}_b   Σ_b Π^j_b = id_K
χ(|i⟩⟨i| ⊗ ρ_K) = Σ_a |a⟩⟨a|_A ⊗ Π^i_a ρ_K Π^j_a
γ(...   Σ_b   Π^j_b


(comm) $\Longleftrightarrow$ $[\pi_a^i, \pi_b^j] = 0 \quad \forall i,j,a,b$
(ind.) $\Longleftrightarrow$ $\sum_{a,b} |a\rangle\langle a| \otimes |b\rangle\langle b| \, \mathrm{tr}_{K'K''}(\pi_a^i \otimes \pi_b^j \, \mathcal{D}(\rho_K))$
on K'
on K''
$= \mathcal{E}(|i\rangle\langle i| \otimes |j\rangle\langle j| \otimes \rho_K)$


= tr(π_a^i ⊗ π_b^j ⊗ id_R V ρ V†)
= tr(V† π_a^i ⊗ π̄_b^j V ρ_K)
K
A
K
K
B
A
K
B
A
K
B
= ε


= tr(π_a^i ⊗ π_b^j ⊗ id_R V ρ V†)
= tr(V† π_a^i ⊗ π̄_b^j V ρ_K) =! tr(π_a^i · π_b^j ρ_K)
K
(ind.)
A K K B
A K B A K B


Q: (comm.) ⇒ (fact.)
{M^i_a}_a
(fact.)
ρ → ρ|a ~ U^i_a √M^i_a ρ √M^i_a U^i_a†
For any i : {Π^i_a}_a   Σ_a Π^i_a = id_K
" j : {Π^j_b}_b   Σ_b Π^j_b = id_K
χ(|i⟩⟨i| ⊗ ρ_K) = Σ_a |a⟩⟨a|_A ⊗ Π^i_a ρ_K Π^i_a


A
K
χ
= id_K
I
K
σ_I
id_K
tr_A ∘ χ(σ_I ⊗ id_K) = id_K
A
K
B
Y
X
I
K
J
=
A
K
B
X
= ε
Y
I
K
J
(comm.)