Title: Asymptotic structure and the characterisation of gravitational radiation at infinity

Speakers: Jose Senovilla

Series: Quantum Gravity

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Abstract: With the main purpose of identifying the existence of gravitational radiation at infinity (scri), a novel approach to the asymptotic structure of spacetime is presented, focusing mainly in cases with non-negative cosmological constant. The basic idea is to consider the strength of tidal forces experienced by scri. To that end I will introduce the asymptotic (radiant) super-momentum, a causal vector defined at scri with remarkable properties that, in particular, provides an innovative characterization of gravitational radiation valid for the general case with ? >= 0 (and which has been proven to be equivalent when ? = 0 to the standard one based on the News tensor). This analysis is also shown to be supported by the initial- (or final-) value Cauchy-type problem defined at scri. The implications are discussed in some detail. The geometric structure of scri, and of its cuts, is clarified. The question of whether or not a News tensor can be defined in the presence of a positive cosmological constant is addressed. Several definitions of asymptotic symmetries are presented. Conserved charges that may detect gravitational radiation are exhibited. Balance laws that might be useful as diagnostic tools to test the accuracy of model waveforms discussed. An interpretation of the Geroch `rho' tensor is found. The whole thing will be complemented with a series of illustrative examples based on exact solutions. In particular we will see that exact solutions with black holes will be radiative if, and only if, they are accelerated.

Zoom link: https://pitp.zoom.us/j/95879544523?pwd=MFl5UEtUZ0hUcU1hNk1SZ2R4MThxUT09

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PRD 102 (2020) 101502

CQG 39 (2022) 10LT01

CQG 39 (2022) 165011

CQG 39 (2022) 165012

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$(\hat{M}, \hat{g})$ Completion, $M \supset \hat{M}$, $g = \Omega^2 \hat{g}$ on $\hat{M}$, $\Omega > 0$ on $\hat{M}$

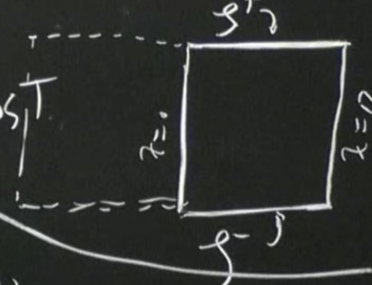
$n_\alpha = \nabla_\alpha \Omega$ Gauge: $\Omega \longrightarrow \Omega \omega$, $\omega \neq 0$ $\mathcal{J}$ "scri": $\Omega = 0$

dS: $ds^2 = -dt^2 + \cosh^2 t\,\underbrace{\left(dx + \sin^2\chi\, d\Omega^2\right)}$

Einstein Cylinder $ds_E^2 = -dt^2 + d\Omega_3$

$$= \cosh^2 t\left(-\frac{dt^2}{\cosh^2 t} + d\Omega_3^2\right) = \frac{1}{\cos^2 T}\left(-dT^2 + d\Omega_3^2\right)$$

$\tan T = \sinh t$ $\quad \Omega = \cos T$

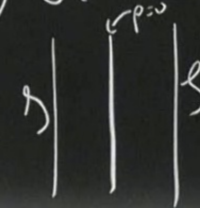


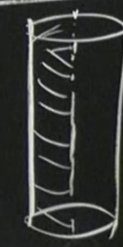
AdS: $ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2$ $\quad \rho \in (0,\infty)$

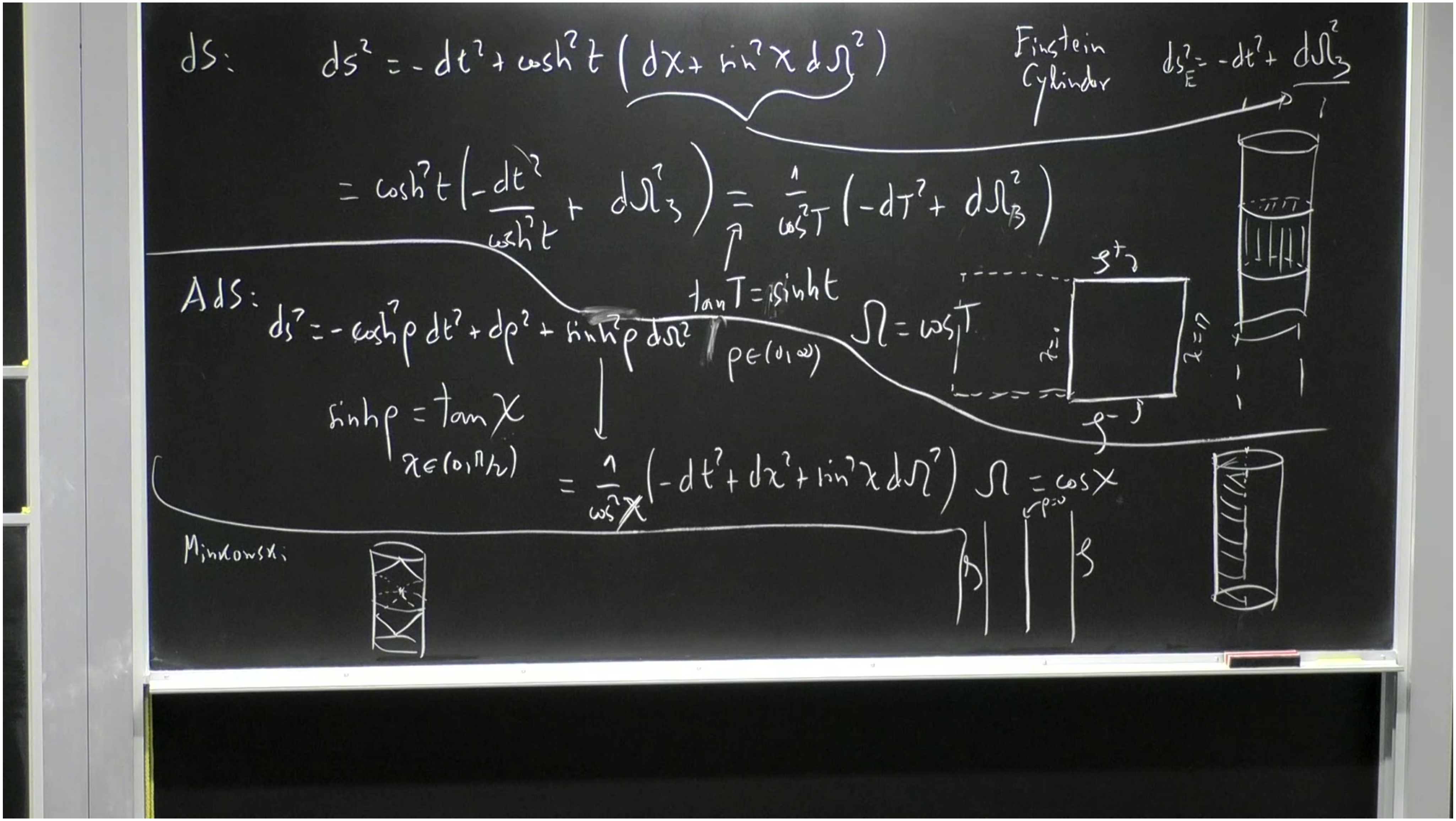
$\sinh\rho = \tan\chi$

$\chi \in (0, \pi/2)$

$$= \frac{1}{\cos^2\chi}\left(-dt^2 + d\chi^2 + \sin^2\chi\, d\Omega^2\right) \quad \Omega = \cos\chi$$






dS: ds² = - dt² + cosh² t (dX + sin² X dΩ²)
Einstein Cylinder ds²_E = - dt² + dΩ₃²
= cosh² t (- dt²/cosh² t + dΩ₃²) = 1/cos² T (- dT² + dΩ₃²)
tan T = sinh t
Ω = cos T
AdS: ds² = - cosh² ρ dt² + dρ² + sinh² ρ dΩ²
ρ ∈ (0, ∞)
sinh ρ = tan X
X ∈ (0, π/2)
= 1/cos² X (- dt² + dX² + sin² X dΩ²) Ω = cos X
Minkowski:

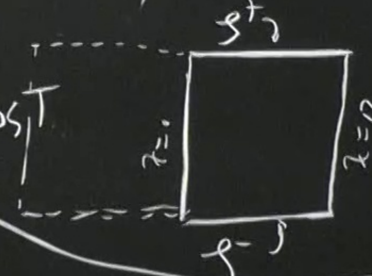
dS: $ds^2 = -dt^2 + \cosh^2 t\,\underbrace{\left(d\chi^2 + \sin^2\chi\, d\Omega^2\right)}$

$= \cosh^2 t\left(-\frac{dt^2}{\cosh^2 t} + d\Omega_3^2\right) = \frac{1}{\cos^2 T}\left(-dT^2 + d\Omega_3^2\right)$

$\tan T = \sinh t$

$\Omega = \cos T$

Einstein Cylinder $ds_E^2 = -dt^2 + d\Omega_3^2$



AdS: $ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2$

$\rho \in (0,\infty)$

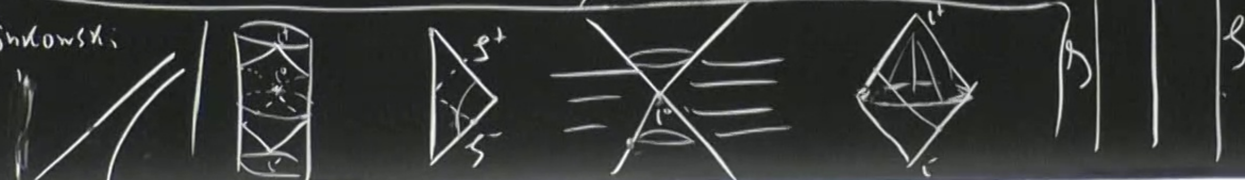
$\sinh\rho = \tan\chi$

$\chi \in (0, \pi/2)$

$= \frac{1}{\cos^2\chi}\left(-dt^2 + d\chi^2 + \sin^2\chi\, d\Omega^2\right)$ $\quad \Omega = \cos\chi$

$\rho = 0$

Minkowski:



$(\hat{M}, \hat{g})$ Completion, $M \supset \hat{M}$, $g = \Omega^2 \hat{g}$ on $\hat{M}$, $\Omega > 0$ on $\hat{M}$

$n_\alpha = \nabla_\alpha \Omega$ Gauge: $\Omega \to \Omega\omega,\ \omega \neq 0$ $\mathcal{J}$ "scri": $\Omega = 0$

$n_\alpha n^\alpha \overset{\mathcal{J}}{\equiv} -\frac{\Lambda}{3}$ $g^{\alpha\beta} n_\alpha n_\beta$ $\nabla_\alpha n_\beta \overset{\mathcal{J}}{\equiv} \frac{1}{4} \nabla_\rho n^\rho g_{\alpha\beta}$ fix $\nabla_\rho n^\rho \overset{\mathcal{J}}{=} 0 \Rightarrow \nabla_\alpha n_\beta \overset{\mathcal{J}}{=} 0$

$\underline{n^\rho \nabla_\rho \omega \overset{\mathcal{J}}{=} 0}$

$(\hat{M}, \hat{g})$ Completion, $M \supset \hat{M}$, $g = \Omega^2 \hat{g}$ on $\hat{M}$, $\Omega > 0$ on $\hat{M}$

$n_\alpha = \nabla_\alpha \Omega$

Gauge: $\Omega \longrightarrow \Omega \omega, \ \omega \neq 0$

$\mathscr{J}$ "scri": $\Omega = 0$

$n_\alpha n^\alpha \overset{\mathscr{J}}{=} -\frac{\Lambda}{3}$

$g^{\alpha\beta} n_\alpha n_\beta$

$\nabla_\alpha n_\beta \overset{\mathscr{J}}{=} \frac{1}{4} \nabla_\rho n^\rho g_{\alpha\beta}$

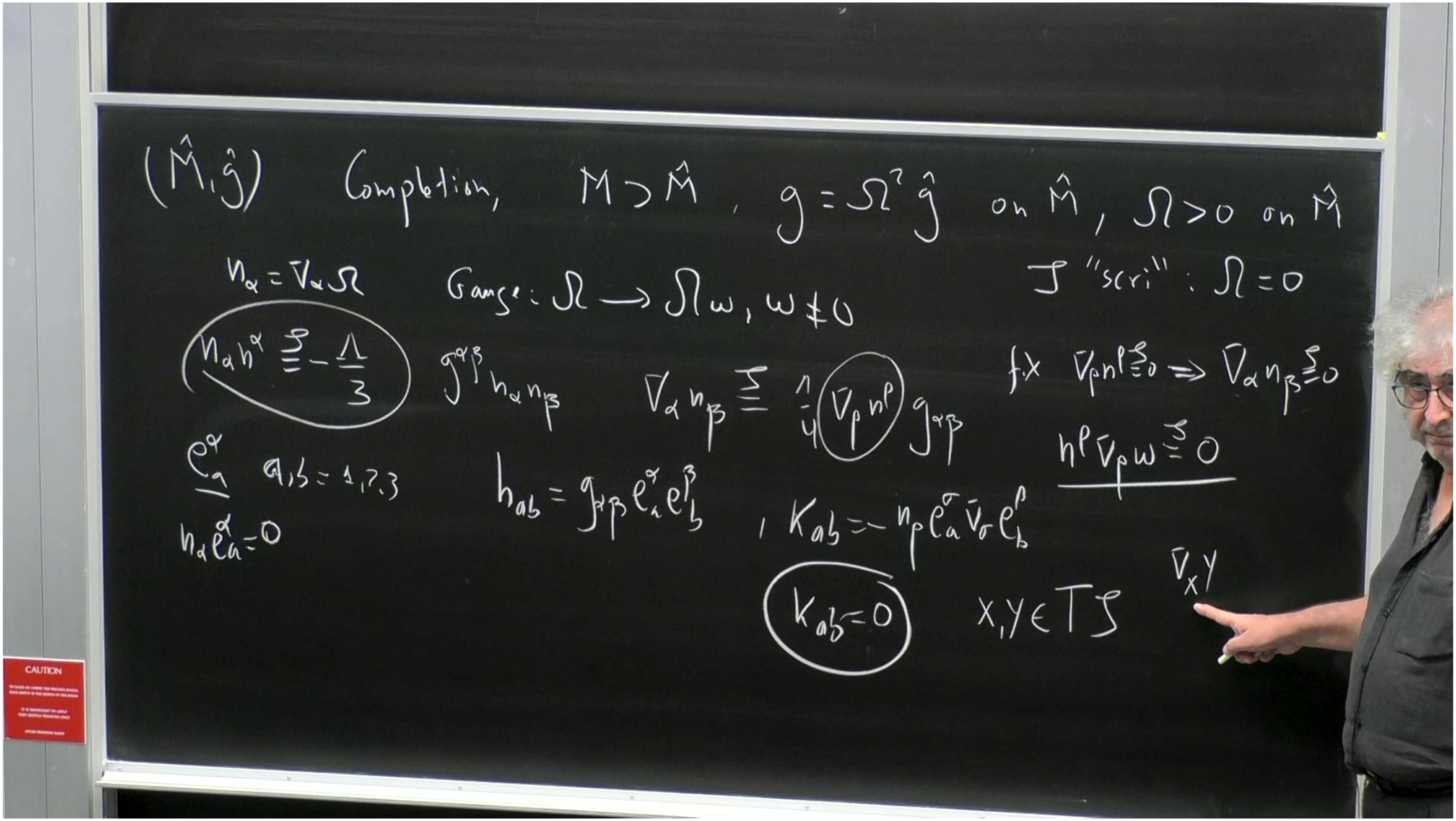
fix $\nabla_\rho n^\rho \overset{\mathscr{J}}{=} 0 \Rightarrow \nabla_\alpha n_\beta \overset{\mathscr{J}}{=} 0$

$n^\rho \nabla_\rho \omega \overset{\mathscr{J}}{=} 0$

e^α_a $\quad a, b = 1, 2, 3$

$n_\alpha e^\alpha_a = 0$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$, $\quad K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$


$(\hat M, \hat g)$ Completion, $M \supset \hat M$, $g = \Omega^2 \hat g$ on $\hat M$, $\Omega > 0$ on $\hat M$
$n_\alpha = \nabla_\alpha \Omega$
Gauge: $\Omega \to \Omega \omega$, $\omega \neq 0$
$\mathcal{J}$ "scri": $\Omega = 0$
$n_\alpha n^\alpha \overset{\mathcal{J}}{=} -\frac{\Lambda}{3}$
$g^{\alpha\beta} n_\alpha n_\beta$
$\nabla_\alpha n_\beta \overset{\mathcal{J}}{=} \frac{1}{4} \nabla_\rho n^\rho g_{\alpha\beta}$
fix $\nabla_\rho n^\rho \overset{\mathcal{J}}{=} 0 \Rightarrow \nabla_\alpha n_\beta \overset{\mathcal{J}}{=} 0$
$n^\rho \nabla_\rho \omega \overset{\mathcal{J}}{=} 0$
e^α_a $a,b = 1,2,3$
$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$, $K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$
$n_\alpha e^\alpha_a = 0$
$K_{ab} = 0$
$X, Y \in T\mathcal{J}$
$\nabla_X Y$
CAUTION

$n_\alpha n^\alpha \overset{\Sigma}{=} -\frac{\Lambda}{3}$

$g^{\alpha\beta} n_\alpha n_\beta$

$\nabla_\alpha n_\beta \overset{\Sigma}{=} \frac{1}{4} \nabla_\rho n^\rho g_{\alpha\beta}$

fix $\nabla_\rho n^\rho \overset{\Sigma}{=} 0 \Rightarrow \nabla_\alpha n_\beta \overset{\Sigma}{=} 0$

$n^\rho \nabla_\rho w \overset{\Sigma}{=} 0$

$e^\alpha_a \quad a,b = 1,2,3$

$n_\alpha e^\alpha_a = 0$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b \quad , \; K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$

$\nabla_X Y = \bar{\nabla}_X Y$

$K_{ab} = 0 \qquad X, Y \in T\Sigma$

$\bar{\nabla}_c h_{ab} = 0$

$\bar{\nabla}, \; \bar{\Gamma}^a_{bc}$

$(\hat{M}, \hat{g})$ Completion, $M \supset \hat{M}$, $g = \Omega^2 \hat{g}$ on $\hat{M}$, $\Omega > 0$ on $\hat{M}$

$n_\alpha = \nabla_\alpha \Omega$

Gauge: $\Omega \longrightarrow \Omega \omega$, $\omega \neq 0$

$\mathcal{J}$ "scri" : $\Omega = 0$

$n_\alpha n^\alpha \stackrel{\mathcal{J}}{\equiv} -\frac{\Lambda}{3}$ $\quad g^{\alpha\beta} n_\alpha n_\beta$ $\quad \nabla_\alpha n_\beta \stackrel{\mathcal{J}}{\equiv} \frac{1}{4} \nabla_\rho n^\rho g_{\alpha\beta}$

fix $\nabla_\rho n^\rho \stackrel{\mathcal{J}}{=} 0 \Rightarrow \nabla_\alpha n_\beta \stackrel{\mathcal{J}}{=} 0$

$n^\rho \nabla_\rho \omega \stackrel{\mathcal{J}}{=} 0$

e^α_a $\quad a, b = 1, 2, 3$ $\quad h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$, $K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$

$n_\alpha e^\alpha_a = 0$

$\nabla_X Y = \bar{\nabla}_X Y$

$K_{ab} = 0$ $\quad X, Y \in T\mathcal{J}$

$\bar{\nabla}_c h_{ab} = 0$ $\quad \bar{\nabla}$, $\bar{\Gamma}^a_{bc}$

$-n_\alpha \epsilon_{abc} = \nabla \eta_{\alpha\rho\rho\sigma} e^\rho_a e^\rho_b e^\sigma_c$, $\bar{\nabla}_d \epsilon_{abc} = 0$

$\Lambda \neq 0$

(M, g) Completion, $M \supset \hat{M}$, $g = \Omega^2 \hat{g}$ on $\hat{M}$, $\Omega > 0$ on $\hat{M}$

$n_\alpha = \nabla_\alpha \Omega$ Gauge: $\Omega \to \Omega\omega$, $\omega \neq 0$ $\mathscr{I}$ "scri": $\Omega = 0$

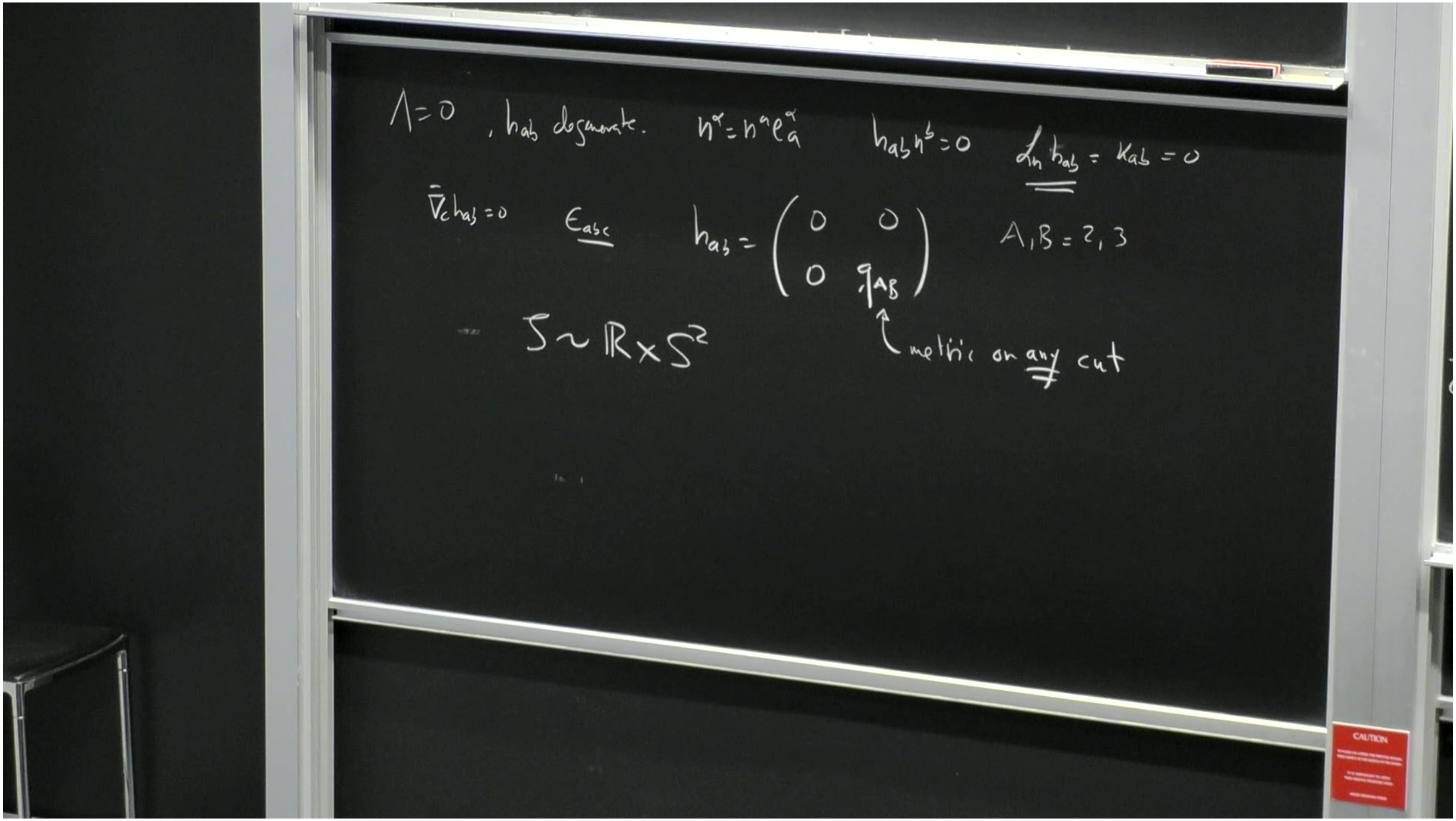
$n_\alpha n^\alpha \stackrel{\mathscr{I}}{=} -\frac{\Lambda}{3}$ $g^{\alpha\beta} n_\alpha n_\beta$ $\nabla_\alpha n_\beta \stackrel{\mathscr{I}}{=} \frac{1}{4} (\nabla_\rho n^\rho) g_{\alpha\beta}$ fix $\nabla_\rho n^\rho \stackrel{\mathscr{I}}{=} 0 \Rightarrow \nabla_\alpha n_\beta \stackrel{\mathscr{I}}{=} 0$

e^α_a $a, b = 1,2,3$ $h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$, $K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$ $n^\rho \nabla_\rho \omega \stackrel{\mathscr{I}}{=} 0$

$n_\alpha e^\alpha_a = 0$ $\nabla_X Y = \bar{\nabla}_X Y$

CUTS: any 2-dimensional (spacelike) submanifold of $\mathscr{I}$ $\bar{\nabla}_c h_{ab} = 0$ $K_{ab} = 0$ $X, Y \in T\mathscr{I}$ $\bar{\nabla}$, $\bar{\Gamma}^a_{bc}$

$-n_\alpha \epsilon_{abc} = \nabla\ \eta_{\alpha\gamma\rho\sigma} e^\gamma_a e^\rho_b e^\sigma_c$, $\bar{\nabla}_d \epsilon_{abc} = 0$


$\Lambda = 0$, h_{ab} degenerate.
$n^\alpha = n^a e^\alpha_a$
$h_{ab} n^b = 0$
$\mathcal{L}_n h_{ab} = K_{ab} = 0$
$\bar{\nabla}_c h_{ab} = 0$
ϵ_{abc}
$h_{ab} = \begin{pmatrix} 0 & 0 \\ 0 & q_{AB} \end{pmatrix}$
$A, B = 2, 3$
$S \sim \mathbb{R} \times S^2$
metric on any cut

$\Lambda \neq 0$ $\quad h_{ab}$ is positive def. $\quad \Lambda > 0$; $\underline{K_{ab} = 0}$

$\qquad\qquad\qquad$ Lorentzian $\quad \Lambda < 0$

$\bar{\nabla}_c h_{ab} = 0$ Levi-Civita

dS $\to S^3$

Kerr-dS $\to \mathbb{R} \times S^2$ $\quad \Big|\ \Lambda > 0$

Kottler with $k \leq 0$ $\ \mathbb{R}^3$

$n_\alpha = \nabla_\alpha \Omega$ Gauge: $\Omega \longrightarrow \Omega\omega,\ \omega \neq 0$ $\mathscr{I}$ "scri": $\Omega = 0$

$n_\alpha n^\alpha \overset{\mathscr{I}}{=} -\frac{\Lambda}{3}$ $g^{\alpha\beta} n_\alpha n_\beta$ $\nabla_\alpha n_\beta \overset{\mathscr{I}}{=} \frac{1}{4}(\nabla_\rho n^\rho)\, g_{\alpha\beta}$ fix $\nabla_\rho n^\rho \overset{\mathscr{I}}{=} 0 \Longrightarrow \nabla_\alpha n_\beta \overset{\mathscr{I}}{=} 0$

$n^\rho \nabla_\rho \omega \overset{\mathscr{I}}{=} 0$

e^α_a $a, b = 1, 2, 3$ $h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$, $K_{ab} = -n_\rho e^\sigma_a \nabla_\sigma e^\rho_b$

$n_\alpha e^\alpha_a = 0$ $\nabla_X Y = \bar\nabla_X Y$ $\Lambda \neq 0$

TS: any 2-dimensional (spacelike) submanifold of $\mathscr{I}$ $\bar\nabla_c h_{ab} = 0$ $K_{ab} = 0$ $X, Y \in T\mathscr{I}$ $\bar\nabla$, $\bar\Gamma^a_{bc}$

$-n_\alpha \epsilon_{abc} = \nabla\, \eta_{\alpha\mu\rho\sigma} e^\mu_a e^\rho_b e^\sigma_c$, $\bar\nabla_d \epsilon_{abc} = 0$

$$C^\alpha{}_{\beta\lambda\mu} \overset{\mathscr{I}}{=} 0 \quad \Rightarrow \quad d^\alpha{}_{\beta\lambda\mu} = \frac{1}{\Omega} C^\alpha{}_{\beta\lambda\mu}$$ good limit at $\mathscr{I}$

$$\nabla_\alpha d^\alpha{}_{\beta\lambda\mu} \overset{\hat{M}}{=} \frac{1}{\Omega} \nabla_\alpha C^\alpha{}_{\beta\lambda\mu}$$

$$S_{\alpha\beta} = \frac{1}{2}\left(R_{\alpha\beta} - \frac{R}{6} g_{\alpha\beta}\right)$$

$$n_\rho d^\rho{}_{\beta\lambda\mu} + 2 \nabla_{[\lambda} S_{\mu]\beta} \overset{\mathscr{I}}{=} 0$$

$(j_{\alpha\beta})$

$F_{\alpha\beta}$, $\quad T_{\alpha\beta} = F_{\alpha\rho}F_\beta{}^\rho - \frac{1}{4} g_{\alpha\beta} F_{\rho\sigma}F^{\rho\sigma}$, $\quad T_{\alpha\beta}u^\alpha v^\beta \geq 0$

$= F_{\alpha\rho}F_\beta{}^\rho + {}^*F_{\alpha\rho}{}^*F_\beta{}^\rho \quad T^\lambda{}_\lambda = 0 \quad \forall\, u^\alpha, v^\beta$ future

$$T_{\alpha\beta\lambda\mu} = C_{\alpha\rho\lambda\sigma}C_\beta{}^\rho{}_\mu{}^\sigma + {}^*C_{\alpha\rho\lambda\sigma}{}^*C_\beta{}^\rho{}_\mu{}^\sigma \qquad {}^*F_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\beta\lambda\mu}F^{\lambda\mu}$$

$$T_{\alpha\beta\lambda\mu} = T_{(\alpha\beta\lambda\mu)},\quad T^\alpha{}_{\alpha\lambda\mu} = 0,\quad \nabla_\alpha T^\alpha{}_{\beta\lambda\mu} = 0 \qquad \left({}^*C_{\alpha\beta\lambda\mu} = \frac{1}{2}\eta_{\alpha\beta\rho\sigma}C^{\rho\sigma}{}_{\lambda\mu} \right.$$

$T_{\alpha\beta\lambda\mu}u^\alpha v^\beta w^\lambda z^\mu \geq 0 \quad u, v, w, z$ future.

$$F_{\alpha\beta}, \quad T_{\alpha\beta} = F_{\alpha\rho}F_\beta{}^\rho - \frac{1}{4} g_{\alpha\beta} F_{\rho\sigma}F^{\rho\sigma}, \quad T_{\alpha\beta}u^\alpha v^\beta \geq 0$$

$$= F_{\alpha\rho}F_\beta{}^\rho + \overset{*}{F}_{\alpha\rho}\overset{*}{F}_\beta{}^\rho \quad T^\lambda{}_\lambda = 0 \qquad \forall\, u^\alpha, v^\beta \text{ future}$$

$$T_{\alpha\beta\lambda\mu} = C_{\alpha\rho\lambda\sigma}C_\beta{}^\rho{}_\mu{}^\sigma + \overset{*}{C}_{\alpha\rho\lambda\sigma}\overset{*}{C}_\beta{}^\rho{}_\mu{}^\sigma \qquad \overset{*}{F}_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\beta\lambda\mu}F^{\lambda\mu}$$

$$T_{\alpha\beta\lambda\mu} = T_{(\alpha\beta\lambda\mu)}, \quad T^\alpha{}_{\alpha\lambda\mu} = 0, \quad \nabla_\alpha T^\alpha{}_{\beta\lambda\mu} = 0 \qquad \overset{*}{C}_{\alpha\beta\lambda\mu} = \frac{1}{2}\eta_{\alpha\beta\rho\sigma}C^{\rho\sigma}{}_{\lambda\mu}$$

$$T_{\alpha\beta\lambda\mu}u^\alpha v^\beta w^\lambda z^\mu \geq 0 \quad u, v, w, z \text{ future.}$$

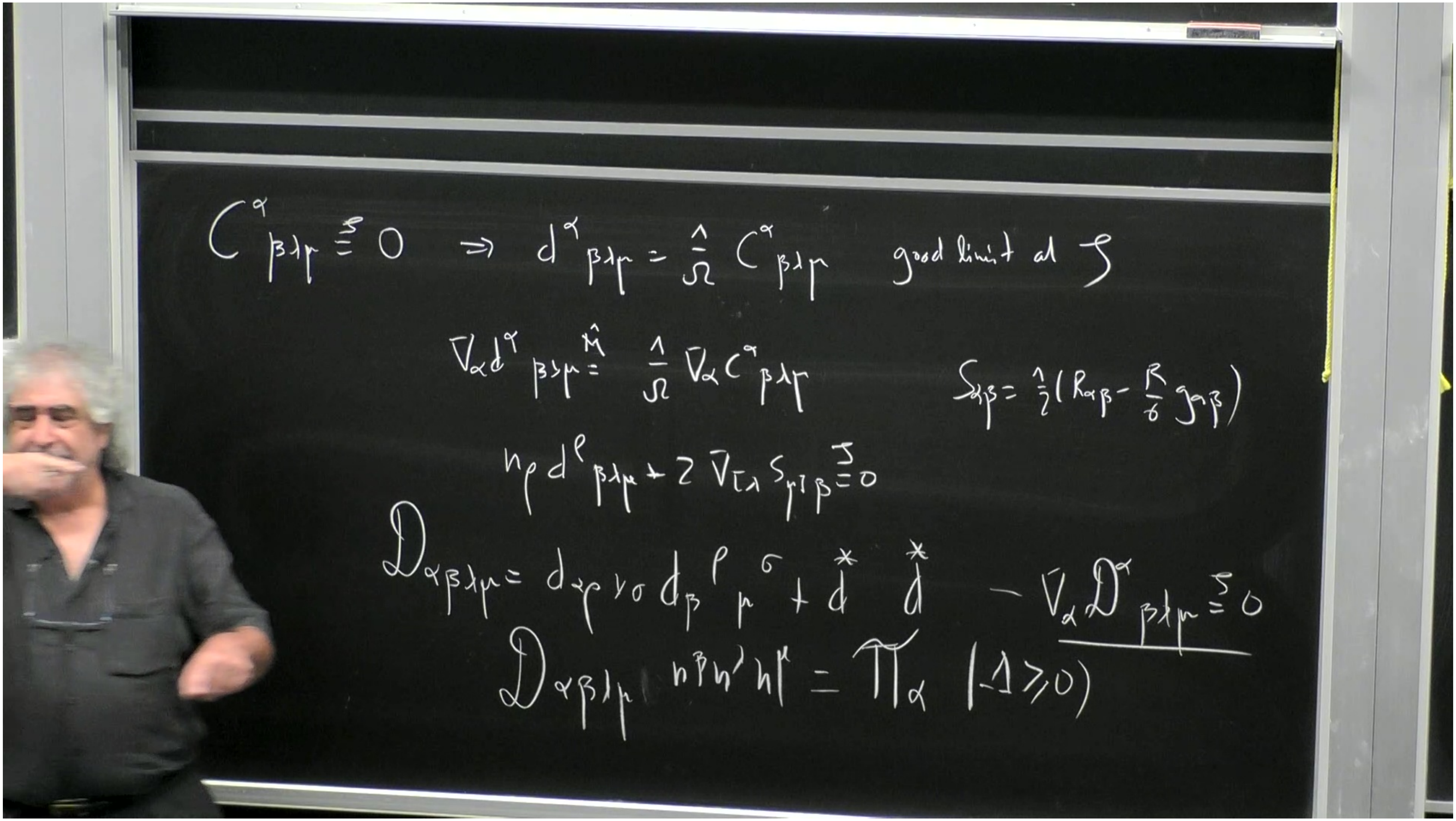
$\nabla_\alpha T^\alpha{}_\beta = -F_\beta{}^\rho J_\rho$ $\quad T_{\alpha\beta} = F_{\alpha\rho} F_\beta{}^\rho - \frac{1}{4} g_{\alpha\beta} F_{\rho\sigma} F^{\rho\sigma}$, $\quad T_{\alpha\beta} u^\alpha v^\beta \geq 0$

$= F_{\alpha\rho} F_\beta{}^\rho + \overset{*}{F}_{\alpha\rho} \overset{*}{F}_\beta{}^\rho \qquad T^\lambda{}_\lambda = 0 \qquad \forall\, u^\alpha, v^\beta$ future

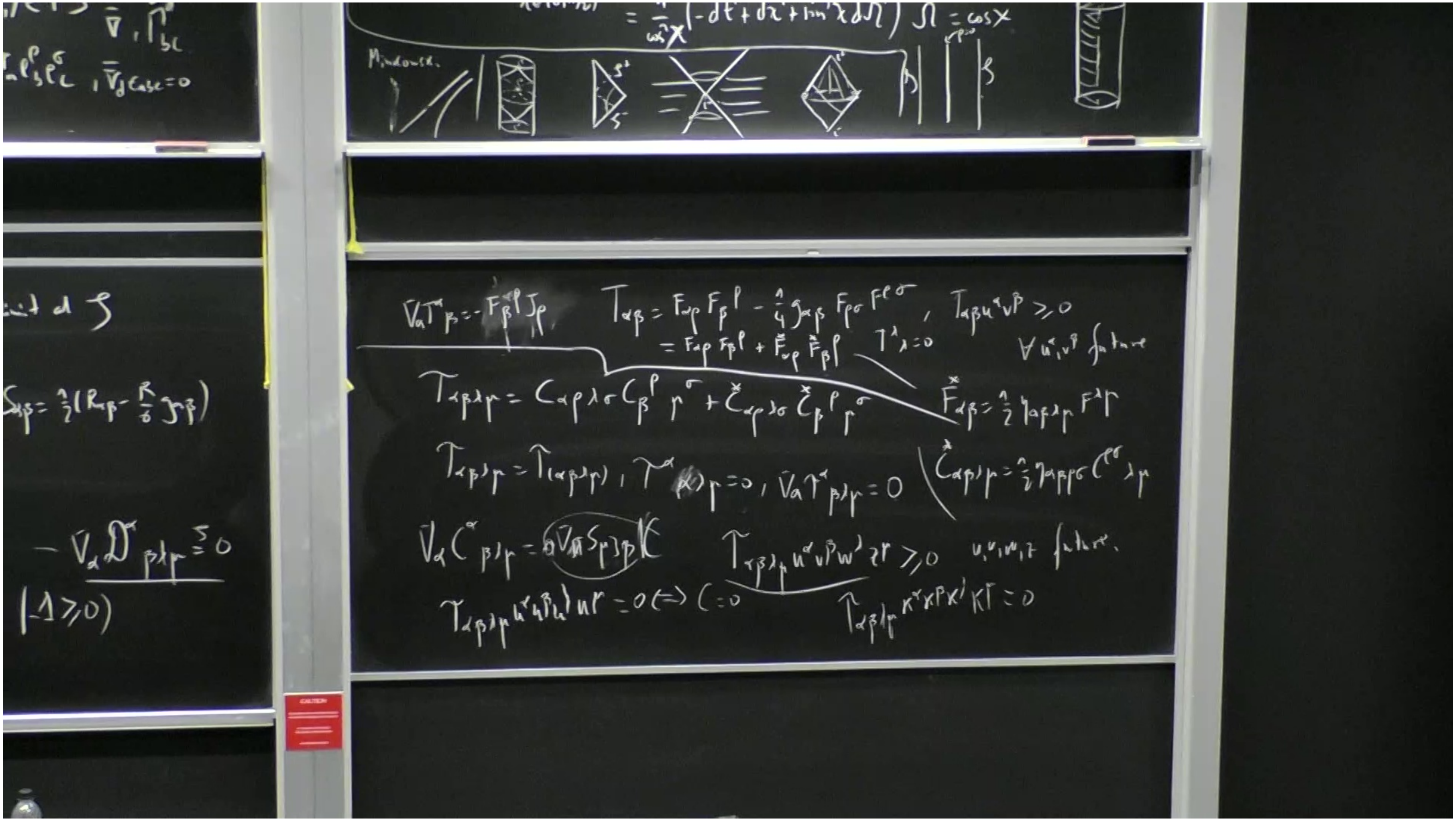
$$T_{\alpha\beta\lambda\mu} = C_{\alpha\rho\lambda\sigma} C_\beta{}^\rho{}_\mu{}^\sigma + \overset{*}{C}_{\alpha\rho\lambda\sigma} \overset{*}{C}_\beta{}^\rho{}_\mu{}^\sigma \qquad \overset{*}{F}_{\alpha\beta} = \frac{1}{2} \eta_{\alpha\beta\lambda\mu} F^{\lambda\mu}$$

$$T_{\alpha\beta\lambda\mu} = T_{(\alpha\beta\lambda\mu)}, \quad T^\alpha{}_{\alpha\lambda\mu} = 0, \quad \nabla_\alpha T^\alpha{}_{\beta\lambda\mu} = 0 \qquad \overset{*}{C}_{\alpha\beta\lambda\mu} = \frac{1}{2} \eta_{\alpha\beta\rho\sigma} C^{\rho\sigma}{}_{\lambda\mu}$$

$$\nabla_\alpha C^\alpha{}_{\beta\lambda\mu} = \nabla_{[\lambda} S_{\mu]\beta} \qquad T_{\alpha\beta\lambda\mu} u^\alpha v^\beta w^\lambda z^\mu \geq 0 \quad u, v, w, z \text{ future.}$$



$C^{\alpha}{}_{\beta\lambda\mu} \overset{\mathscr{I}}{=} 0 \Rightarrow d^{\alpha}{}_{\beta\lambda\mu} = \frac{1}{\Omega} C^{\alpha}{}_{\beta\lambda\mu}$ good limit at $\mathscr{I}$
$\nabla_{\alpha} d^{\alpha}{}_{\beta\lambda\mu} \overset{\hat{M}}{=} \frac{1}{\Omega} \nabla_{\alpha} C^{\alpha}{}_{\beta\lambda\mu}$
$S_{\alpha\beta} = \frac{1}{2}\left(R_{\alpha\beta} - \frac{R}{6} g_{\alpha\beta}\right)$
$n_{\rho} d^{\rho}{}_{\beta\lambda\mu} + 2 \nabla_{[\lambda} S_{\mu]\beta} \overset{\mathscr{I}}{=} 0$
$D_{\alpha\beta\lambda\mu} = d_{\alpha\rho\nu\sigma} d_{\beta}{}^{\rho}{}_{\mu}{}^{\sigma} + \overset{*}{d} \; \overset{*}{d}$
$\underline{-\nabla_{\alpha} D^{\alpha}{}_{\beta\lambda\mu} \overset{\mathscr{I}}{=} 0}$
$D_{\alpha\beta\lambda\mu} n^{\beta} n^{\lambda} n^{\mu} = \Pi_{\alpha} \quad (\Lambda \geq 0)$



Minkowski.
$\Omega = \cos\chi$
$S_{\alpha\beta} = \frac{1}{2}(R_{\alpha\beta} - \frac{R}{6} g_{\alpha\beta})$
$\nabla_\alpha T^\alpha{}_\beta = -F_\beta{}^\rho J_\rho$
$T_{\alpha\beta} = F_{\alpha\rho} F_\beta{}^\rho - \frac{1}{4} g_{\alpha\beta} F_{\rho\sigma} F^{\rho\sigma}$, $T_{\alpha\beta} u^\alpha v^\beta \geq 0$
$= F_{\alpha\rho} F_\beta{}^\rho + \star F_{\alpha\rho} \star F_\beta{}^\rho$ $T^\lambda{}_\lambda = 0$ $\forall u^\alpha, v^\beta$ future
$T_{\alpha\beta\lambda\mu} = C_{\alpha\rho\lambda\sigma} C_\beta{}^\rho{}_\mu{}^\sigma + \star C_{\alpha\rho\lambda\sigma} \star C_\beta{}^\rho{}_\mu{}^\sigma$
$\star F_{\alpha\beta} = \frac{1}{2} \eta_{\alpha\beta\lambda\mu} F^{\lambda\mu}$
$T_{\alpha\beta\lambda\mu} = T_{(\alpha\beta\lambda\mu)}$, $T^\alpha{}_{\alpha\lambda\mu} = 0$, $\nabla_\alpha T^\alpha{}_{\beta\lambda\mu} = 0$
$\star C_{\alpha\beta\lambda\mu} = \frac{1}{2} \eta_{\alpha\beta\rho\sigma} C^{\rho\sigma}{}_{\lambda\mu}$
$\nabla_\alpha C^\alpha{}_{\beta\lambda\mu} = 2\nabla_{[\lambda} S_{\mu]\beta}$
$T_{\alpha\beta\lambda\mu} u^\alpha v^\beta w^\lambda z^\mu \geq 0$ u, v, w, z future.
$T_{\alpha\beta\lambda\mu} u^\alpha u^\beta u^\lambda u^\mu = 0 \Leftrightarrow C = 0$
$T_{\alpha\beta\lambda\mu} K^\alpha K^\beta K^\lambda K^\mu = 0$

$$C^\alpha{}_{\beta\lambda\mu} \overset{\mathscr{I}}{=} 0 \quad \Rightarrow \quad d^\alpha{}_{\beta\lambda\mu} = \frac{1}{\Omega} C^\alpha{}_{\beta\lambda\mu}$$ good limit at $\mathscr{I}$

Π_α is $\perp$ to all cuts
$\Leftrightarrow$ no radiation

$$\nabla_\alpha d^\alpha{}_{\beta\lambda\mu} \overset{\hat{M}}{=} \frac{1}{\Omega} \nabla_\alpha C^\alpha{}_{\beta\lambda\mu}$$

$$S_{\alpha\beta} = \frac{1}{2}\left(R_{\alpha\beta} - \frac{R}{6} g_{\alpha\beta}\right)$$

$$n_\rho d^\rho{}_{\beta\lambda\mu} + 2 \nabla_{[\lambda} S_{\mu]\beta} \overset{\mathscr{I}}{=} 0$$

n^α is principal vector of $d^\mu{}_{\beta\lambda\mu}$

$$\mathcal{D}_{\alpha\beta\lambda\mu} = d_{\alpha\rho\nu\sigma} d_\beta{}^\rho{}_\mu{}^\sigma + \overset{*}{d}\,\overset{*}{d} \qquad \underline{\nabla_\alpha \mathcal{D}^\alpha{}_{\beta\lambda\mu} \overset{\mathscr{I}}{=} 0}$$

$$\mathcal{D}_{\alpha\beta\lambda\mu} n^\beta n^\lambda n^\mu = \Pi_\alpha \quad (\Lambda \geq 0)$$

$$\underline{\underline{\quad}}_{bc} \quad (h_{ab}, n^a) \qquad h_{ab} n^b = 0, \qquad \bar{\nabla}_a n^b = 0$$

$$\bar{R}^a{}_{bcd} \quad \bar{R}^a{}_{bcd} = -R^a{}_{bdc}, \quad R^a{}_{[bcd]} = 0 \quad \bar{\nabla}_{[e} R_{cd]}{}^a{}_b = 0$$

$$\bar{R}_{bd} = \bar{R}^c{}_{bcd} \; (= \bar{R}_{db}) \qquad n^b \bar{R}^a{}_{bcd} = 0, \quad n^b \bar{R}_{bd} = 0$$

$$\mathcal{L}_n \bar{\Gamma}^a{}_{bc} = -\bar{R}^a{}_{bcd} n^d, \quad \bar{R}_{abcd} = h_{ae} \bar{R}^e{}_{bcd} \Rightarrow n^a \bar{R}_{abcd} = 0$$

Riemann properties

$$\mathcal{L}_n \bar{R}_{abcd} = 0, \quad \underline{\mathcal{L}_n K = 0}$$

$$\bar{R}_{abcd} = K(h_{ac} h_{bd} - h_{ad} h_{bc})$$

$$\Gamma^a{}_{bc} \;\; [illegible] \;\; (h_{ab}, n^a) \qquad h_{ab}n^b = 0, \qquad \bar\nabla_a n^b = 0$$

$$\bar R^a{}_{bcd} \;\; \bar R^a{}_{bcd} = -\bar R^a{}_{bdc}, \quad \bar R^a{}_{[bcd]} = 0, \quad \bar\nabla_{[e} \bar R_{cd]}{}^a{}_b = 0$$

$$\bar R_{bd} = \bar R^c{}_{bcd} \;\; (= \bar R_{db}) \qquad n^b \bar R^a{}_{bcd} = 0, \quad n^b \bar R_{bd} = 0$$

$$\mathcal{L}_n \bar\Gamma^a{}_{bc} = -\bar R^a{}_{bcd}\, n^d, \qquad \bar R_{abcd} = h_{ae} \bar R^e{}_{bcd} \Rightarrow n^a \bar R_{abcd} = 0$$

Riemann properties

$$\mathcal{L}_n \bar R_{abcd} = 0, \quad \underline{\mathcal{L}_n K = 0}$$

$$\bar R_{abcd} = K(h_{ac}h_{bd} - h_{ad}h_{bc})$$

$$S_{\mu\nu} e^\mu_a e^\nu_b = \bar S_{ab}, \quad n^a \bar S_{ab} = 0, \quad h^{ab} \bar S_{ab} = K$$

$$\mathcal{L}_n \bar{\Gamma}^a_{bc} = n^a \left[\bar{S}_{bc} - \frac{1}{2} h_{bc} \left(h^{ef} \bar{R}_{ef} - 3K \right) \right]$$

$$g^{\mu\nu} \nabla_\nu w = w^c e^\mu_c$$

$w_a = \bar{\nabla}_a w$

$$\bar{S}_{ab} \longrightarrow \bar{S}_{ab} - \overset{a}{\frac{1}{w}} \bar{\nabla}_a \bar{\nabla}_b w + \overset{a}{\frac{2}{w^2}} \bar{\nabla}_a w \bar{\nabla}_b w - \overset{a}{\frac{1}{2w^2}} h_{ab} w^c \bar{\nabla}_c w$$

Result: t_{ab}, $t_{ab} = t_{ba}$, $n^a t_{ab} = 0$, $\hat{t}_{ab}$

$$\underline{\tilde{\bar{\bar{\nabla}}}_{[c} \tilde{t}_{a]b} = \bar{\nabla}_{[c} t_{a]b}} + \frac{1}{w} w_{[c} h_{a]b} \left(aK - h^{ef} t_{ef} \right)$$

$$\mathcal{L}_n \bar{\Gamma}^a_{bc} = n^a\left[\bar{S}_{bc} - \tfrac{1}{2} h_{bc}\left(h^{ef}\bar{R}_{ef} - 3K\right)\right]$$

$g^{\mu\nu}\nabla_\nu \omega = \omega^c e^\mu_c$

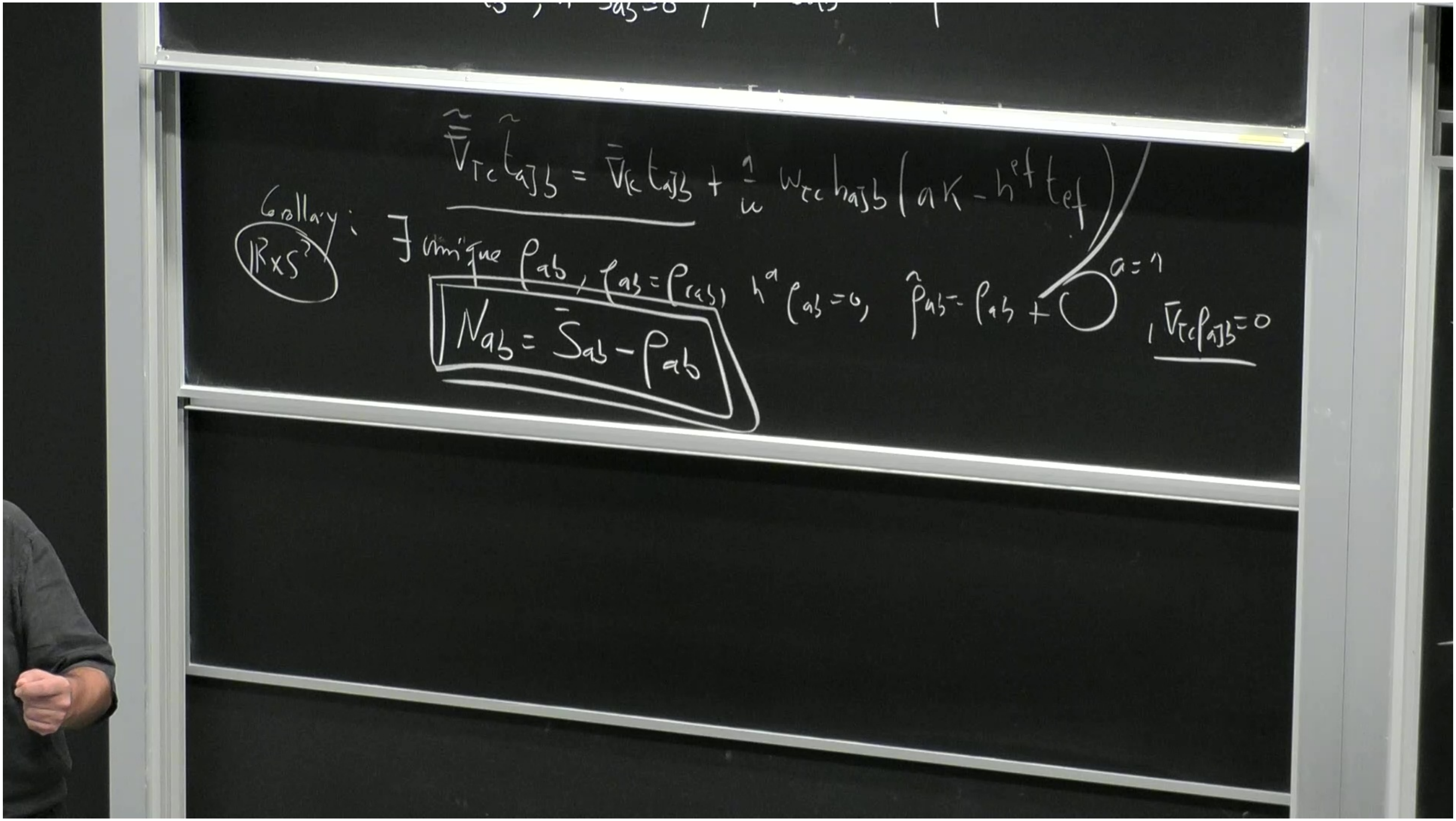
$\omega_a = \bar{\nabla}_a \omega$

$$\bar{S}_{ab} \longrightarrow \bar{S}_{ab} - \left(\overset{a}{\tfrac{1}{\omega}} \bar{\nabla}_a \bar{\nabla}_b \omega + \overset{a}{\tfrac{2}{\omega^2}} \bar{\nabla}_a \omega \bar{\nabla}_b \omega - \overset{a}{\tfrac{1}{2\omega^2}} h_{ab}\, \omega^c \bar{\nabla}_c \omega\right)$$

Result: t_{ab}, $t_{ab} = t_{ba}$, $n^a t_{ab} = 0$, $\hat{t}_{ab}$

$$\tilde{\bar{\nabla}}_{[c} \hat{t}_{a]b} = \bar{\nabla}_{[c} t_{a]b} + \tfrac{1}{\omega}\, \omega_{[c} h_{a]b}\left(aK - h^{ef} t_{ef}\right)$$

Corollary: $\mathbb{R}\times S^2$: $\exists$ unique ρ_{ab}, $\rho_{ab} = \rho_{(ab)}$, $n^a \rho_{ab} = 0$, $\hat{\rho}_{ab} = \rho_{ab} + \bigcirc^{a=1}$, $\bar{\nabla}_{[c}\rho_{a]b} = 0$



$\tilde{\bar{\nabla}}_{[c}\tilde{t}_{a]b} = \bar{\nabla}_{[c}t_{a]b} + \frac{1}{n} w_{[c}h_{a]b}(\alpha K - h^{ef}t_{ef})$
Corollary:
$\mathbb{R}\times S^2$
$\exists$ unique ρ_{ab}, $\rho_{ab}=\rho_{(ab)}$, $n^a\rho_{ab}=0$, $\hat{\rho}_{ab}=\rho_{ab}+$
$\alpha=1$
$\bar{\nabla}_{[c}\rho_{a]b}=0$
$N_{ab}=\bar{S}_{ab}-\rho_{ab}$

$$g_{AB}, \quad g = \frac{1}{k}\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \qquad \underline{D_A}$$

$$D_B D_A n_{(i)} = -\frac{1}{k} g_{AB} \, n_{(i)} \qquad \left(\Delta n_{(i)} = -2k \, n_{(i)} \right)$$

$$n_{(i)} = (\sin\theta\cos\varphi, \cos\theta \quad , \cos\theta)$$

$$\Pi_{(\mu)} = (1, n_{(i)})$$

$$\eta^{\mu\nu} \Pi_{(\mu)} \Pi_{(\nu)} = 0$$

$$\left[D_A D_B \Pi_{(\mu)} - \frac{1}{2} \Delta \Pi_{(\mu)} \, g_{AB} = 0 \ ? \right.$$

Π_α is $\perp$ to all cuts

$\Longleftrightarrow$ no radiation

n^α is principal vector of $d^\mu{}_{\beta\lambda\mu}$

$$\nabla_\alpha d^\gamma{}_{\beta\lambda\mu} \overset{\hat{M}}{=} \frac{1}{\Omega} \nabla_\alpha C^\gamma{}_{\beta\lambda\mu}$$

$$S_{\alpha\beta} = \frac{1}{2}\left(R_{\alpha\beta} - \frac{R}{6} g_{\alpha\beta}\right)$$

$$\tilde{D}_A \tilde{D}_B \tilde{\Pi}_{(\mu)} - \frac{1}{2} \tilde{q}_{AB} \tilde{\Delta} \tilde{\Pi}_{(\mu)} - \frac{\tilde{K}}{2} q_{AB} \tilde{\Pi}_{(\mu)} =$$

$$\omega\Big[(\text{same}) \quad + \Pi_{(\mu)} \big(D_A D_B \omega - \frac{2}{\omega} D_A \omega D_B \omega + \frac{1}{2\omega} q^{CE} D_E \omega D_C \omega \, q_{AB} \big) \Big]$$

$$\tilde{\Pi}_{(\mu)} = \omega \Pi_{(\mu)}$$

$$\tilde{\rho}_{AB} - \rho_{AB}$$

$$\Big(D_A D_B \Pi_{(\mu)} - \frac{1}{2} \Delta \Pi_{(\mu)} q_{AB} = 0$$