

Title: Quantum Theory Lecture - 100323

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## Renormalization of $\varphi^3$

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$

$$\text{LSZ: } \left. \begin{aligned} \langle \Omega | \varphi(x) | \Omega \rangle &= 0 \\ \langle \vec{k} | \varphi(x) | \Omega \rangle &= e^{ik \cdot x} \end{aligned} \right\} \text{ need to shift + rescale}$$

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 - Z_g \frac{g}{3!}\varphi^3 - Y\varphi$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_0 = \frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2$$

$$\mathcal{L}_{\text{int}} = -\frac{g}{3!} \varphi^3$$

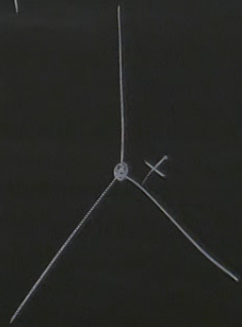
$$\mathcal{L}_{\text{ct}} = \frac{1}{2} (Z_\varphi - 1) \varphi^2 - \frac{1}{2} (Z_m - 1) m^2 \varphi^2 - \frac{1}{3!} (Z_g - 1) g \varphi^3 + Y \varphi$$

↑  
counterterms

Expect:  $Z_i = 1 + \mathcal{O}(g)$

$$Y = 0 + \mathcal{O}(g)$$

position space



$$= -iZg \int d^4x$$

$$= -iY \int d^4x$$

Momentum space

$$= i p^2 (Z_q - 1) - i (Z_m - 1) m^2$$

$$\langle \Omega | \varphi(x) | \Omega \rangle = 0$$

$$\langle \Omega | \varphi(x) | \Omega \rangle = \underbrace{\begin{array}{c} x \quad x \\ \hline x \quad y \end{array}}_{\mathcal{O}(g)} + \underbrace{\begin{array}{c} \bullet \quad \bullet \\ \hline x \quad y \end{array}}_{\mathcal{O}(g)} + \mathcal{O}(g^2)$$

$$= -iY \int d^4y \Delta_{xy} + \frac{-iz_3}{2} \int d^4y \Delta_{xy} \Delta_{yy} + \mathcal{O}(g^2)$$

$$= \left( -iY - \frac{ig}{2} \Delta_{00} \right) \int d^4y \Delta_{xy}$$

$\leftarrow z_3 = 4 \cdot 0g$

$$Y = -\frac{ig}{2} \Delta_{00}(0)$$

$\leftarrow \text{div}$

$$\frac{x}{y} + \frac{x}{y} \circlearrowleft + \mathcal{O}(g^2)$$

$\mathcal{O}(g) \quad \mathcal{O}(g)$

$$d^4y \Delta_{xy} + \frac{-iz_g}{2} d^4y \Delta_{xy} \Delta_{yy} + \mathcal{O}(g^2)$$

$$Y = \frac{-ig}{2} \Delta_{00} (d^4y \Delta_{xy})$$

$\uparrow z_g = 1 + \mathcal{O}(g)$

$$Y = \frac{-g}{2} \Delta_F(0) \leftarrow \text{diverges}$$

$\sum$  diagrams with one external point = 0

$\rightarrow \sum$  diagrams attached by one point = 0

$$\circlearrowleft + \frac{x}{y} = 0$$

$$\begin{array}{c}
 \text{---} \times \\
 \text{y} \\
 \Theta(g)
 \end{array}
 + 
 \begin{array}{c}
 \text{---} \times \text{---} \circ \\
 \text{x} \quad \text{y} \\
 \Theta(g)
 \end{array}
 + \Theta(g^2)$$

$$(d^4 y \Delta_{xy} + \frac{-iZ_3 g}{2} d^4 y \Delta_{xy} \Delta_{xy} + \mathcal{O}(g^3))$$

$$(Y - \frac{i g}{2} \Delta_{00}) (d^4 y \Delta_{xy})$$

$Z_3 = 1 + \mathcal{O}(g)$

$$Y = \frac{-g}{2} \Delta_F(0) \quad \leftarrow \text{diverges}$$

$\sum$  diagrams with one external point = 0

$\rightarrow \sum$  diagrams attached by one point = 0

$$\begin{array}{c} \circ \\ \text{---} \end{array}
 + 
 \begin{array}{c} \times \\ \text{---} \end{array}
 = 0$$

single line

$$\sum \left( \text{---} \circ \right) = 0$$

anything connected

$$\underline{\langle K | \varphi(x) | \Omega \rangle = e^{ik \cdot x}}$$

$$G(p, p') = \int d^4x \int d^4x' \langle \Omega | T \varphi(x) \varphi(x') | \Omega \rangle e^{i(p \cdot x + p' \cdot x')}$$

↑  
momentum space

2-point function

$$= (2\pi)^4 \delta^4(p+p') G_2(p)$$

↑  
compute

Källa



$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \int \frac{d^3k}{(2\pi)^3 2E_k} |\vec{k}\rangle\langle\vec{k}| + \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_{\sigma} |\vec{k}, \sigma\rangle\langle\vec{k}, \sigma|$$

↑  
projection onto  
ground  
state
↑  
1-particle

$$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle \stackrel{x^0 > y^0}{=} \langle\Omega|\varphi(x)|\Omega\rangle\langle\Omega|\varphi(y)|\Omega\rangle + \int \frac{d^3k}{(2\pi)^3 2E_k} \langle\Omega|\varphi(x)|\vec{k}\rangle\langle\vec{k}|\varphi(y)|\Omega\rangle + \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_{\sigma} \langle\Omega|\varphi(x)|\vec{k}, \sigma\rangle\langle\vec{k}, \sigma|\varphi(y)|\Omega\rangle$$

↑  
1

$$\langle \dots, \sigma \chi \vec{k}, \sigma | \dots \rangle \rightarrow = \int \frac{d^3k}{(2\pi)^3 2E_k} e^{-ik(x-y)} + \text{multiparticle term}$$

$\uparrow$   
 part of Feynman propagator

$$\int \frac{d^3k}{(2\pi)^3 2E_k} = \int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^2 - m_{ph}^2) \Theta(k^0)$$

$\uparrow$   
 physical mass!

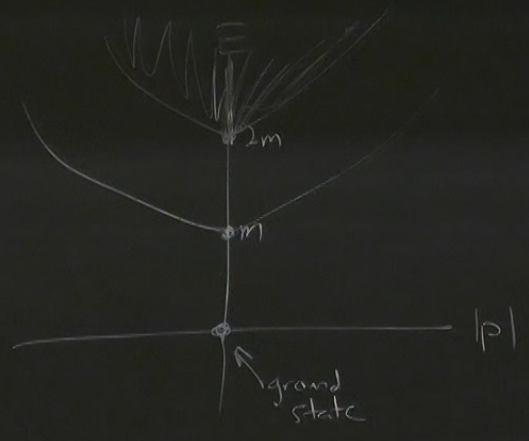
no boundstates  $\rightarrow M_{\text{multiparticle}} \geq (2m_p)^2$

$$\langle \Omega | T \varphi(x) \varphi(y) | \Omega \rangle = \int \frac{d^4 k}{(2\pi)^4} \left( \frac{i}{k^2 - m_{ph}^2 + i\epsilon} + \int_{4m^2}^{\infty} ds \frac{\rho(s)}{k^2 - s + i\epsilon} \right) e^{ik \cdot (x-y)}$$

$$G_2(p) = \frac{i}{k^2 - m_{ph}^2 + i\epsilon} + \int_{4m^2}^{\infty} ds \frac{\rho(s)}{k^2 - s + i\epsilon}$$

2-point in interacting theory has  
pole at  $k^2 = m_{ph}^2$  with residue  $i$

$$e^{-ik \cdot (x-y)}$$



$$G_2(p) = \text{---} + \text{---} \left. \vphantom{\text{---}} \right\} \mathcal{O}(g^0) + \mathcal{O}(g^2)$$

(The first term is a simple line. The second term is a line with a loop. The third term is a line with a cross. The fourth term is a line with a vertical line and a circle on top. The fifth term is a line with a vertical line and a circle on top, with a wavy line underneath labeled = 0. An arrow labeled 1PI points to the loop diagram.)

$$G_2(p) = \text{---}$$

$$\mathcal{O}(g^1) \left\{ \begin{array}{l} + \text{---} + \text{---} \\ + \text{---} + \text{---} \end{array} \right.$$

(The first row contains two diagrams: a line with two humps and a line with one hump and a cross. The second row contains two diagrams: a line with two crosses and a line with a square box containing a hump. Below the second row is the text "1PI = 1 particle irreducible".)

$$G_2(p) = \text{---} + \sum \text{---} \textcircled{1PI} \text{---} + \sum \text{---} \textcircled{1PI} \textcircled{1PI} \text{---} + \dots$$

↑  
 stays  
 connected  
 if any  
 one line  
 is removed

$O(g^0) + O(g^2)$

$$G_2(p) = \text{---} + \sum \text{---} \textcircled{\text{1PI}} \text{---} + \sum \text{---} \textcircled{\text{1PI}} \textcircled{\text{1PI}} \text{---} + \dots$$

self-energy  
↓

$$\sum \textcircled{\text{1PI}} = -i \Sigma(p^2)$$

↑ stays connected if any one line is removed

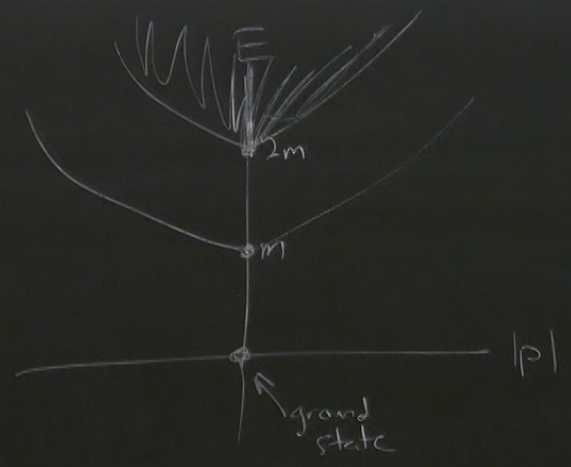
↑ summing series

$$G_2(p) = \frac{i}{p^2 - m^2 + i\epsilon} + \left( \frac{i}{p^2 - m^2 + i\epsilon} \right)^2 (-i \Sigma(p^2)) + \dots = \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\epsilon}$$

↑ from Lagrangian

4 parameters  
 2 normalization

(-y)



correct pole location

$$\sum (m_{ph}^2) = 0$$

connects  $m_{ph}^2$  and  $m_{Lag}^2$

correct residue

$$\sum' (m_{ph}^2) = 0$$



Need 1 more condition  
involving coupling

How is coupling related to experiment?

could define  $g_{ph}$  by

$$\langle f|S|i\rangle = F(g_{ph}) \quad \text{for given } |i\rangle \rightarrow |f\rangle$$

tion

ated to experiment?

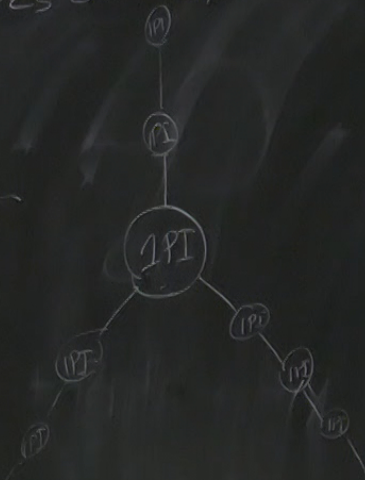
by

( $S_{ph}$ ) for given  $(i, j + i, f)$

or use building blocks of Feynman diagrams

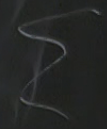
$$\langle S_{ph} | \varphi_1 \varphi_2 \varphi_3 | S_{ph} \rangle =$$

$$\text{1PI} = \sqrt{3} (P_1, P_2, P_3)$$



building blocks of Feynman diagrams

$$\langle \psi_3 | \mathcal{O} \rangle =$$



$$= \Gamma_3^{\text{ref}}(P_1, P_2, P_3)$$

$$\Gamma_3^{\text{ref}}(P_1, P_2, P_3) \equiv g(\mu) \text{ arbitrary!}$$

$$(P_i^{\text{ref}})^2 = \mu^2$$

$\mu$  is arbitrary

