Title: Quantum Theory Lecture - 100323

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Renormalization of φ^3

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$

LSZ: $\langle\Omega|\varphi(x)|\Omega\rangle = 0$

$\langle\vec{k}|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$

} need to shift + rescale

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 - Z_g\frac{g}{3!}\varphi^3 - Y\varphi$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_0 = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\mathcal{L}_{int} = -\frac{g}{3!}\varphi^3$$

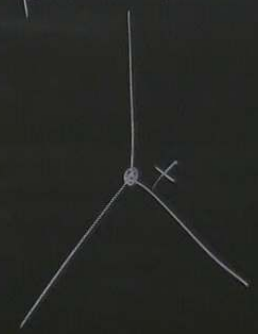
$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)\varphi^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 - \frac{1}{3!}(Z_g - 1)g\varphi^3 + Y\varphi$$

counterterms

Expect: $Z_i = 1 + O(g)$

$Y = 0 + O(g)$

position space



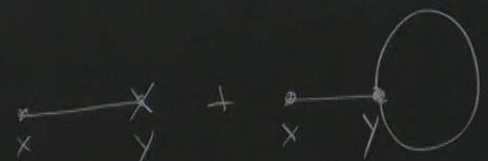
$= -i Z_g g \int d^4x$

$= -i Y \int d^4x$

momentum space

$= i p^2 (Z_\varphi - 1) - i (Z_m - 1) m^2$

$\langle \Omega | \varphi(x) | \Omega \rangle = 0$

$\langle \Omega | T\varphi(x) | \Omega \rangle =$  $+ O(g^2)$

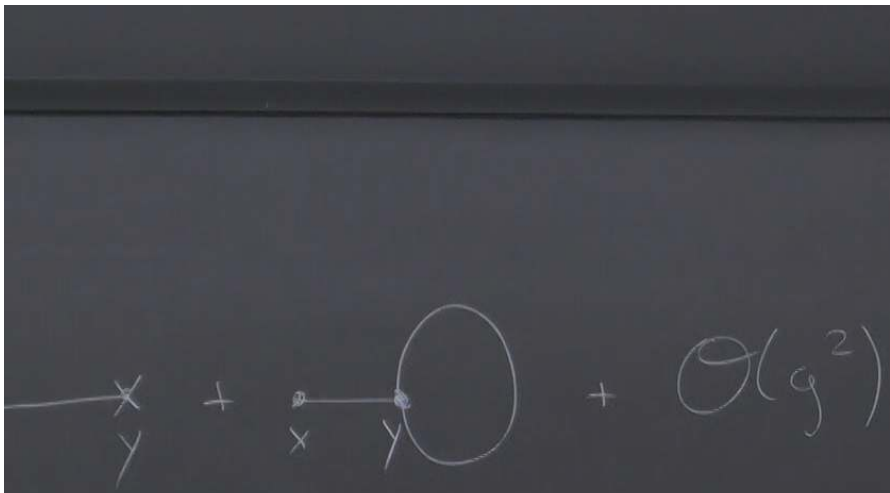
$O(g) \qquad O(g)$

$= -iY \int d^4y \, \Delta_{xy} + \frac{-iZ_g g}{2} \int d^4y \, \Delta_{xy} \Delta_{yy} + O(g^2)$

$= \left(-iY - \frac{ig}{2} \Delta_{00}\right) \int d^4y \, \Delta_{xy}$

$Z_g = 1 + O(g)$

$Y = \frac{-g}{2} \Delta_F(0)$



O(g) O(g)

$\int d^4y \Delta_{xy} + \frac{-iZ_g g}{2} \int d^4y \Delta_{xy} \Delta_{yy} + O(g^3)$

$\left(Y - \frac{ig}{2}\Delta_{00}\right) \int d^4y \Delta_{xy}$

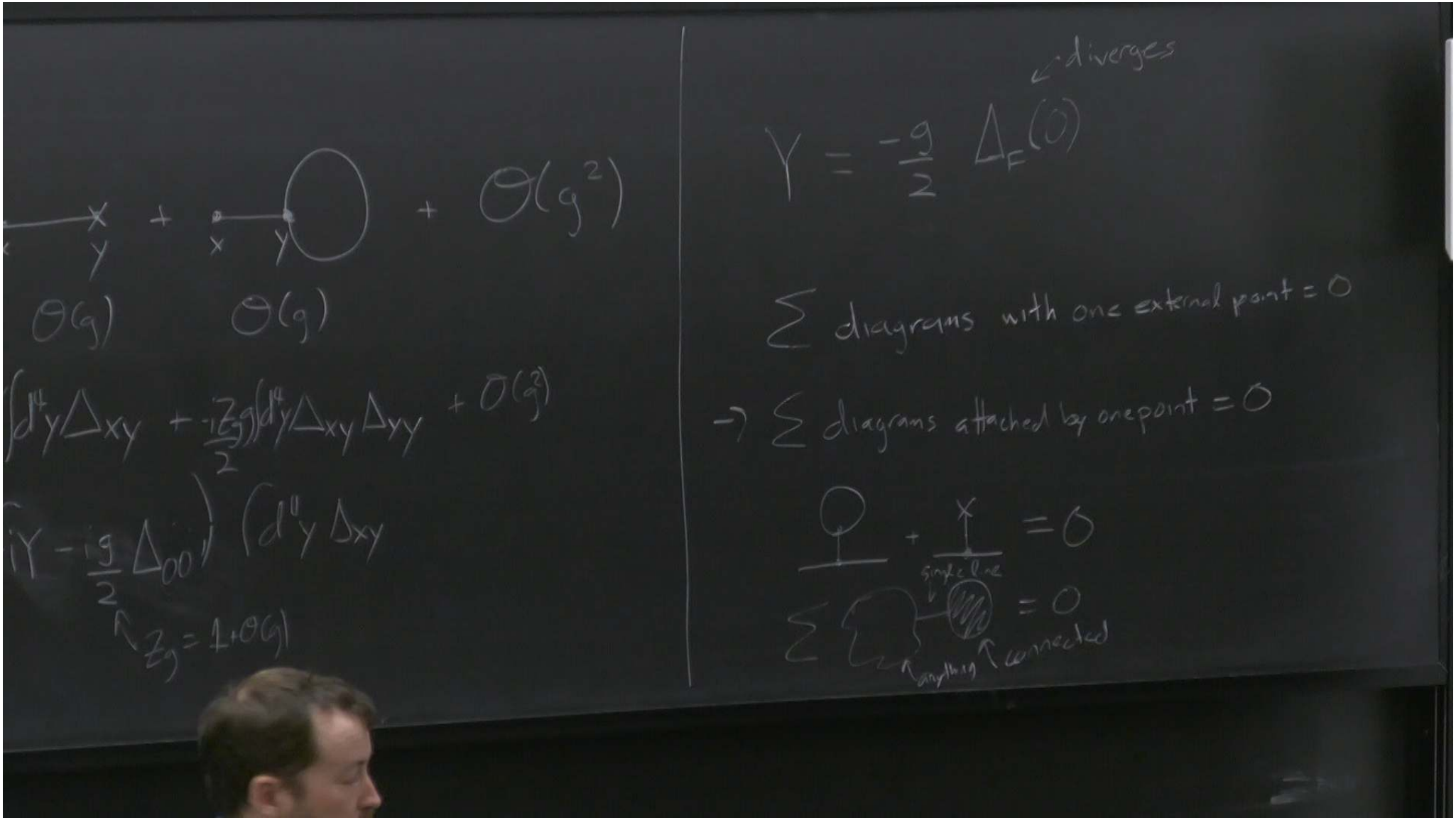
$Z_g = 1 + O(g)$

← diverges

$Y = \frac{-g}{2} \Delta_F(0)$

$\sum$ diagrams with one external point = 0

→ $\sum$ diagrams attached by one point = 0


x
y
+
x
y
+ O(g²)
O(g)
O(g)
∫d⁴y Δxy + (−iZ_g g/2) ∫d⁴y Δxy Δyy + O(g²)
(iγ − (ig/2) Δ00) ∫d⁴y Δxy
Z_g = 1 + O(g)
γ = −(g/2) Δ_F(0)
← diverges
Σ diagrams with one external point = 0
→ Σ diagrams attached by one point = 0
+
= 0
single line
= 0
anything
connected

$$\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$$

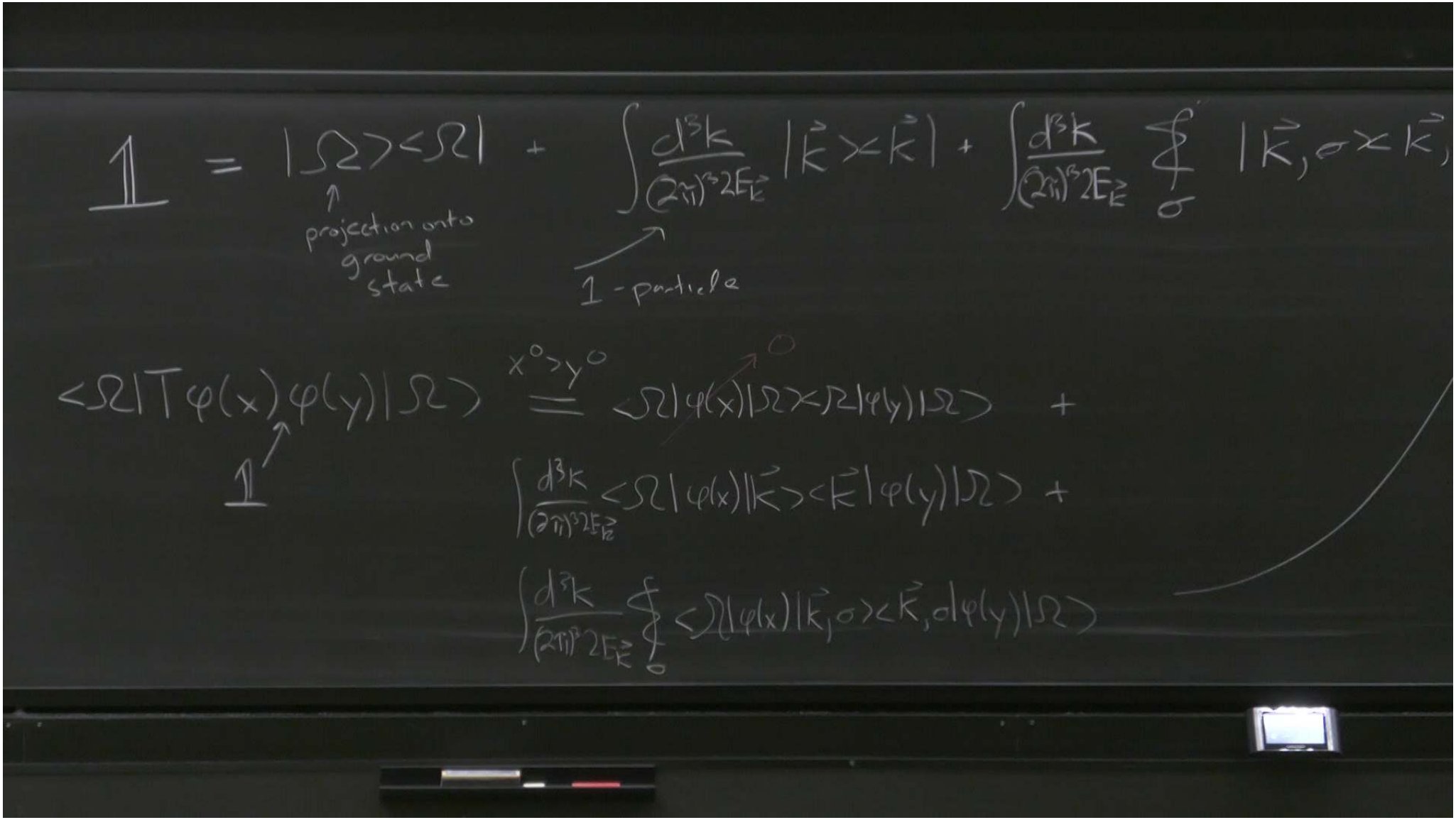
$$G(p,p') = \int d^4x \int d^4x' \langle\Omega|T\varphi(x)\varphi(x')|\Omega\rangle e^{i(p\cdot x + p'\cdot x')}$$

↑ momentum space 2-point function

$$= (2\pi)^4 \delta^4(p+p') G_2(p)$$

↑ compute

Källe


$\mathbb{1} = |\Omega\rangle\langle\Omega| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} |\vec{k}\rangle\langle\vec{k}| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \oint_\sigma |\vec{k},\sigma\rangle\langle\vec{k},$
projection onto ground state
1-particle
$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle \overset{x^0>y^0}{=} \langle\Omega|\varphi(x)|\Omega\rangle\langle\Omega|\varphi(y)|\Omega\rangle +$
$\mathbb{1}$
$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \langle\Omega|\varphi(x)|\vec{k}\rangle\langle\vec{k}|\varphi(y)|\Omega\rangle +$
$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \oint_\sigma \langle\Omega|\varphi(x)|\vec{k},\sigma\rangle\langle\vec{k},\sigma|\varphi(y)|\Omega\rangle$

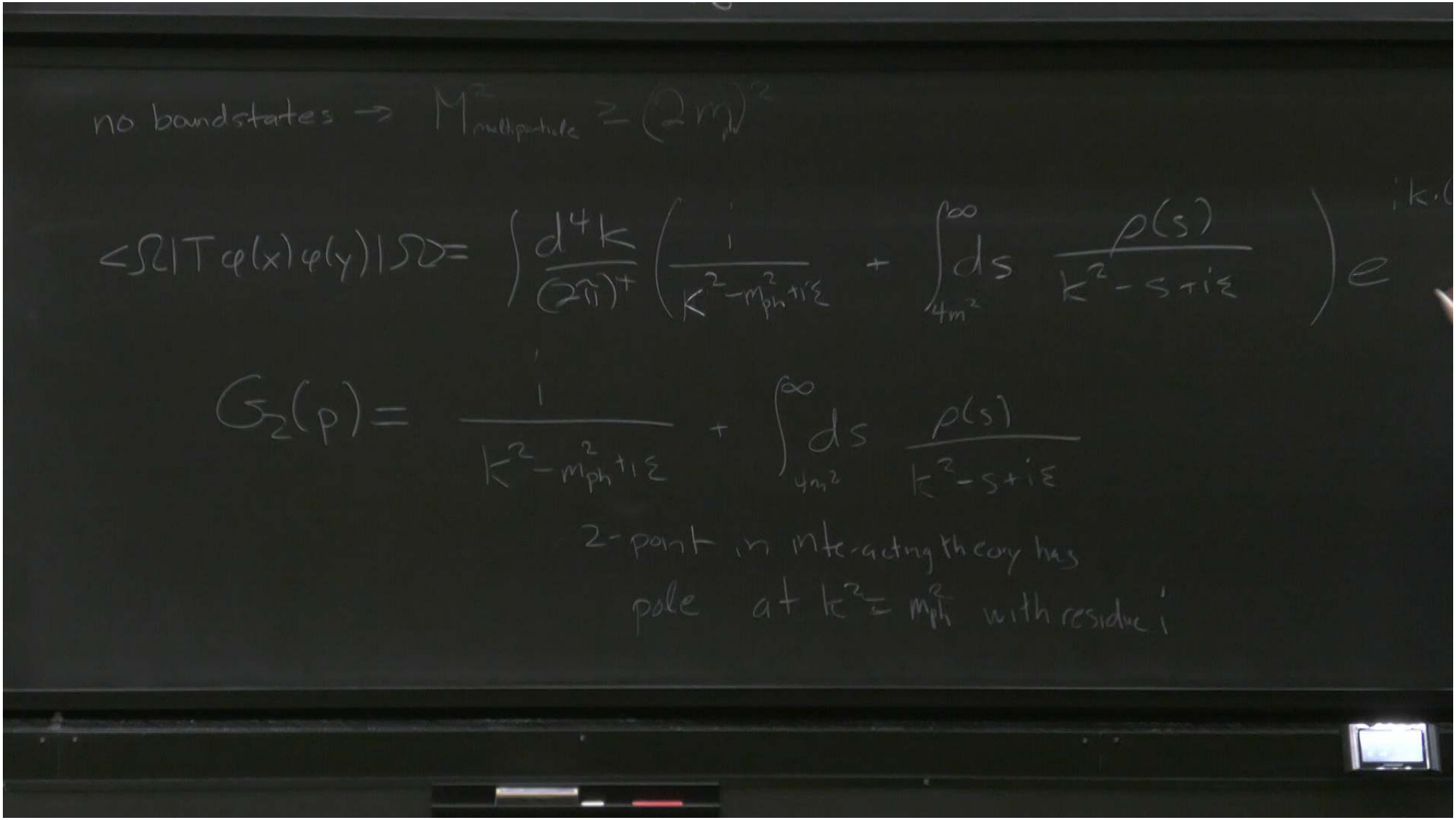
$\vec{k}, \sigma \times \vec{k}, \sigma |$

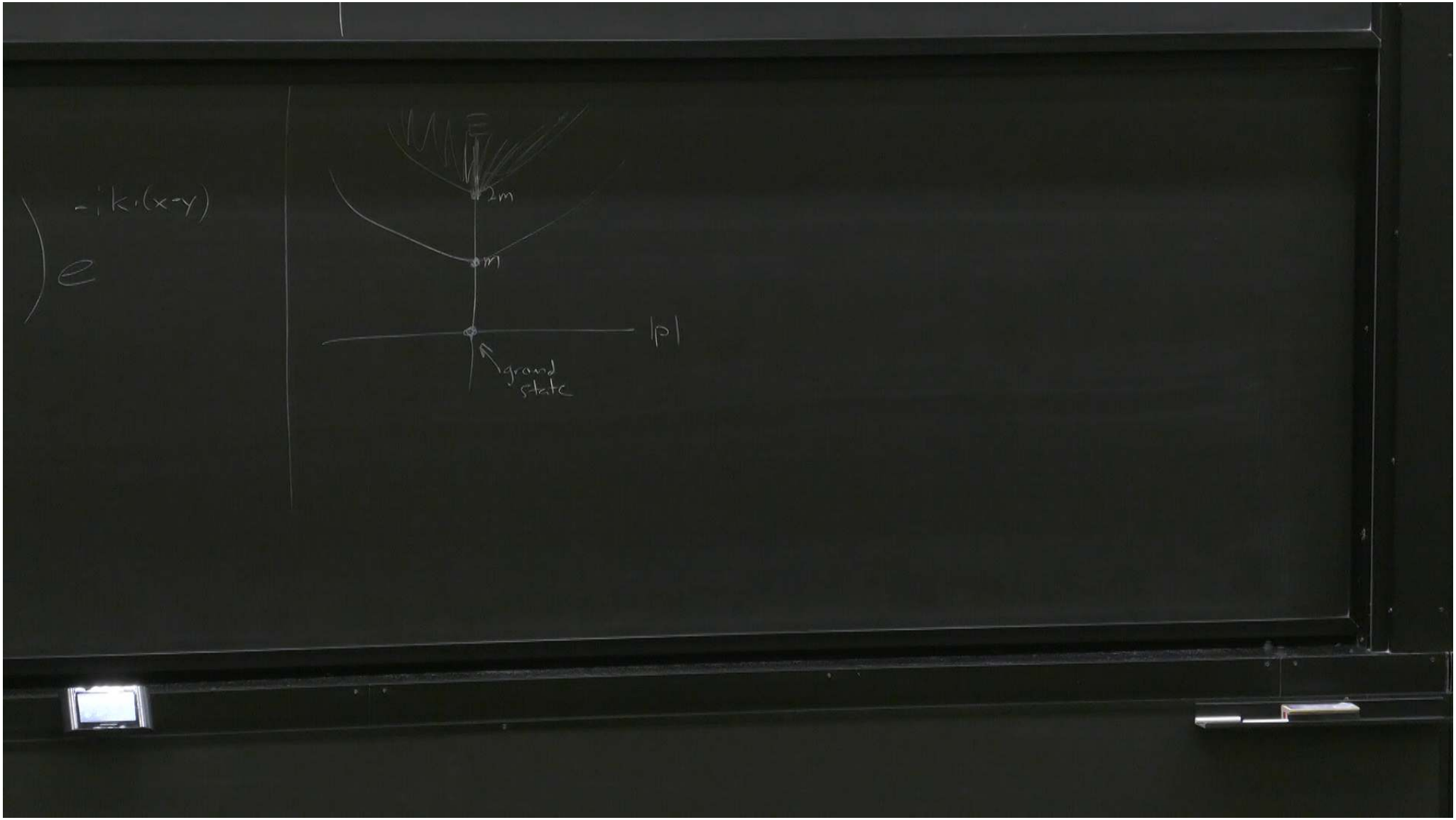
$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} e^{-ik(x-y)} + \text{multiparticle term}$$

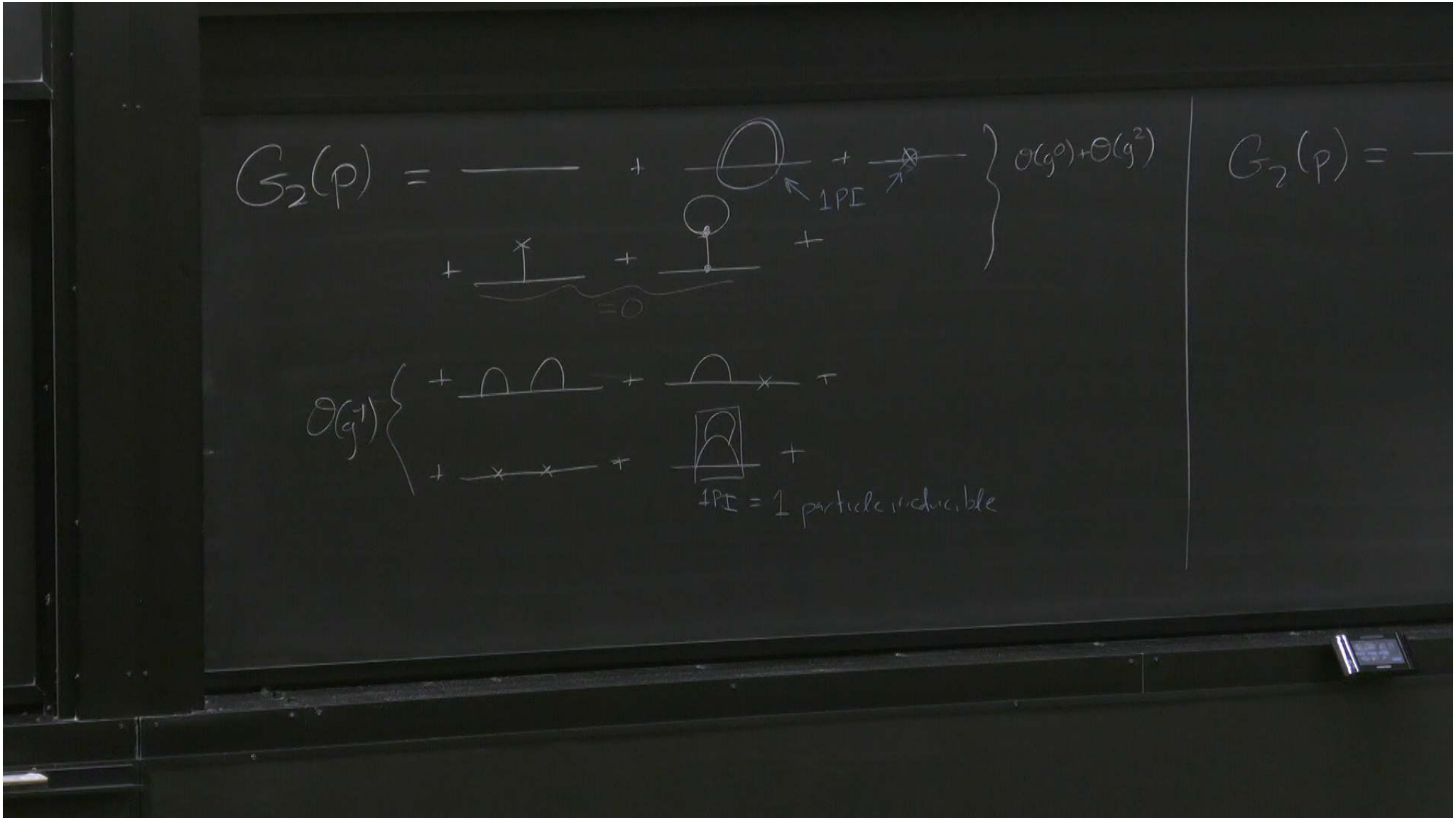
↑ part of Feynman propagator

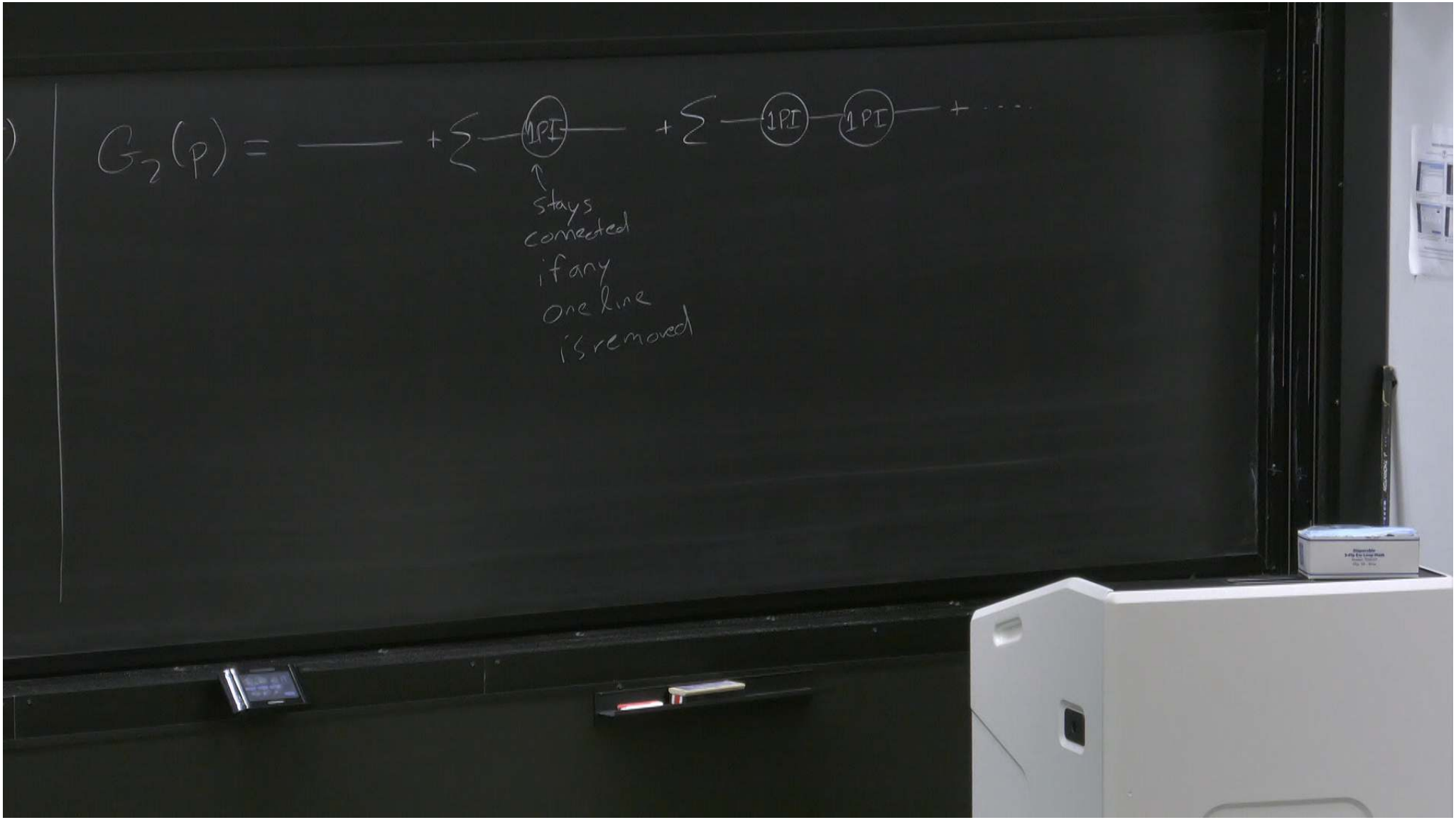
$$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} 2\pi i \, \delta(k^2 - m_{ph}^2) \, \Theta(k^0)$$

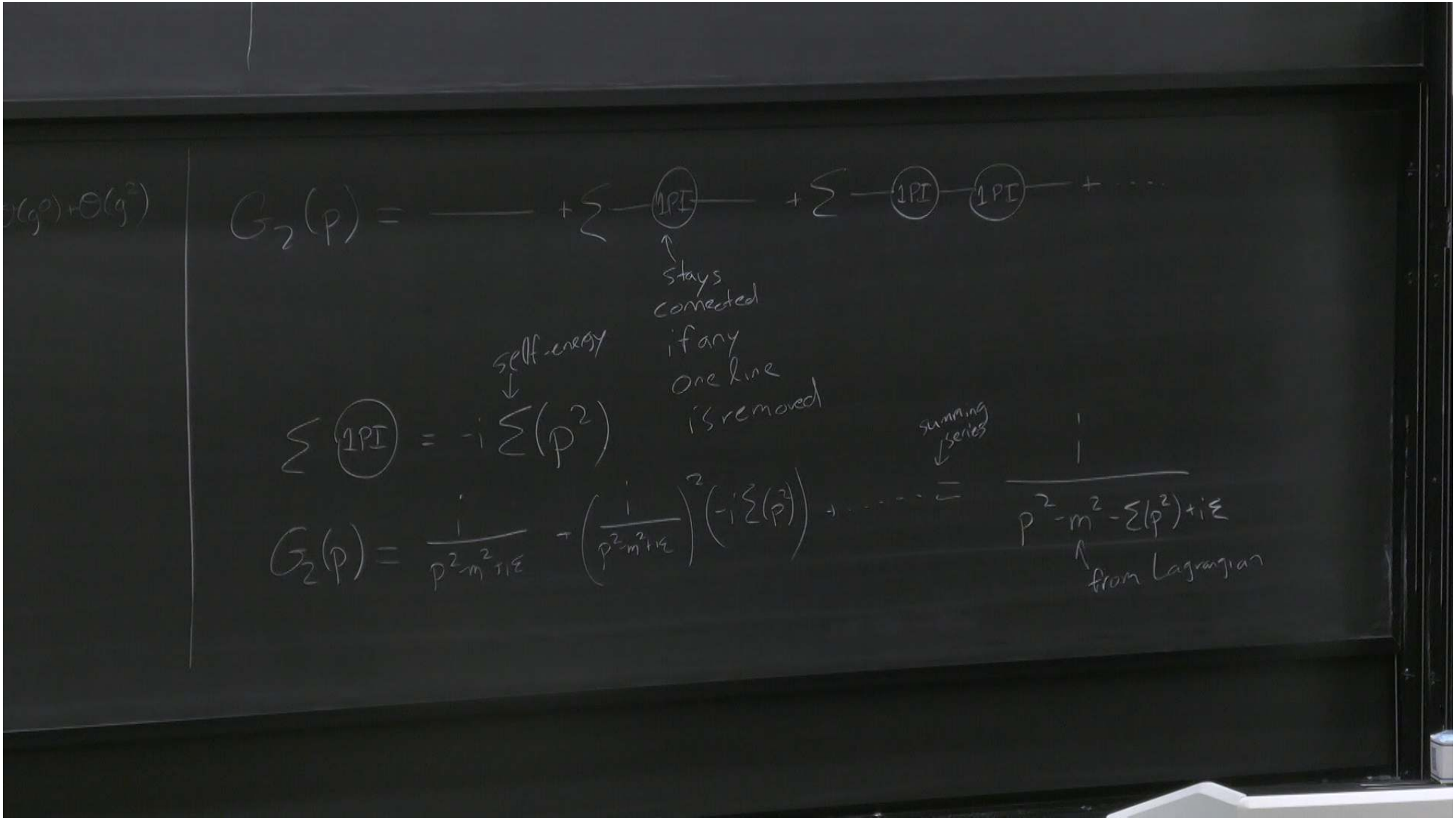
↑ physical mass!


no boundstates → $M^2_{\text{multiparticle}} = (2m_\phi)^2$
$\langle \Omega | T\,\varphi(x)\varphi(y) | \Omega \rangle = \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{k^2 - m_{ph}^2 + i\varepsilon} + \int_{4m^2}^{\infty} ds\, \frac{\rho(s)}{k^2 - s + i\varepsilon} \right) e^{ik\cdot}$
$G_2(p) = \frac{i}{k^2 - m_{ph}^2 + i\varepsilon} + \int_{4m^2}^{\infty} ds\, \frac{\rho(s)}{k^2 - s + i\varepsilon}$
2-point in interacting theory has
pole at $k^2 = m_{ph}^2$ with residue i


$-ik\cdot(x-y)$
e
2m
m
|p|
grand state


$G_2(p)$ =
1PI
$O(g^0)+O(g^2)$
$O(g^1)$
1PI = 1 particle irreducible
$G_2(p)$ =


$G_2(p) =$
1PI
1PI
1PI
stays connected if any one line is removed



$\mathcal{O}(g^0) + \mathcal{O}(g^2)$

$$G_2(p) = \text{———} + \sum \text{—}\,1PI\,\text{—} + \sum \text{—}\,1PI\,\text{—}\,1PI\,\text{—} + \cdots$$

stays connected if any one line is removed

self-energy

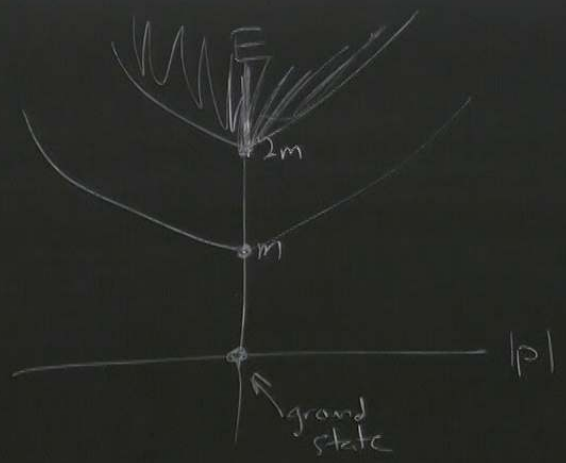
$$\sum 1PI = -i\,\Sigma(p^2)$$

summing series

$$G_2(p) = \frac{i}{p^2 - m^2 + i\varepsilon} + \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^2 \left(-i\,\Sigma(p^2)\right) + \cdots = \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\varepsilon}$$

from Lagrangian

2 normalization



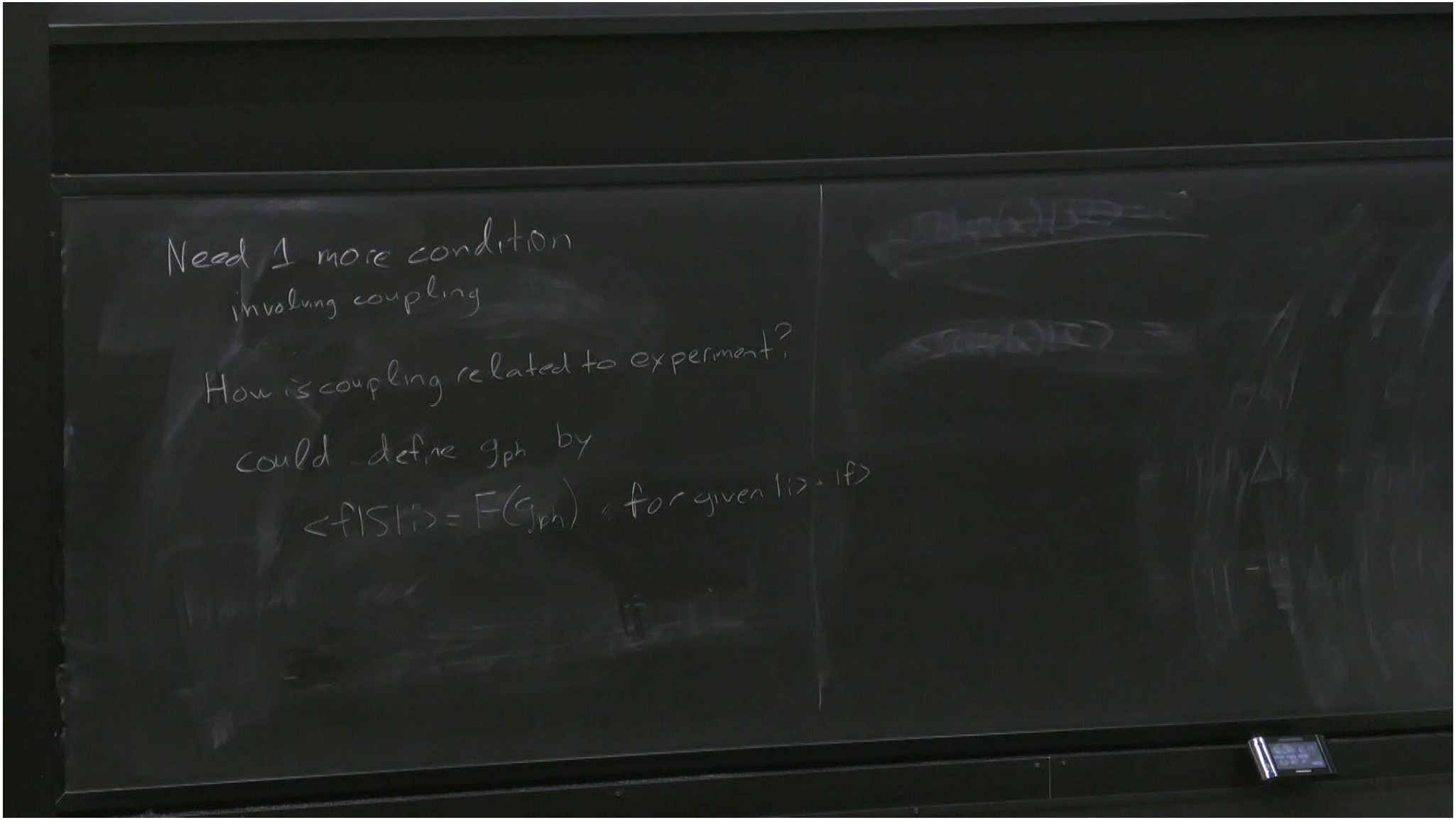
correct pole location

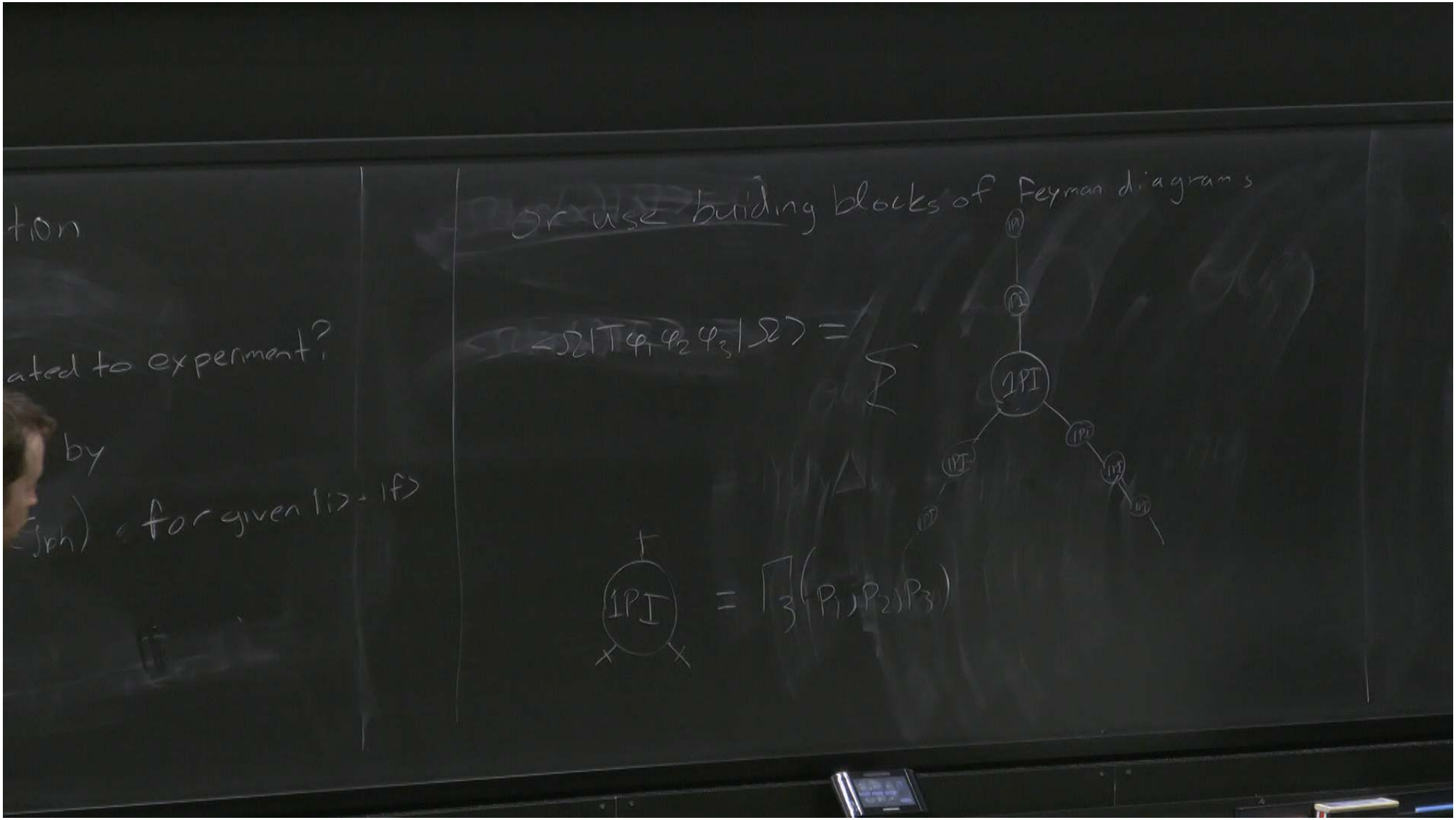
$$\Sigma(m_{ph}^2) = 0$$

connects m_{ph}^2 and m_{Lag}^2

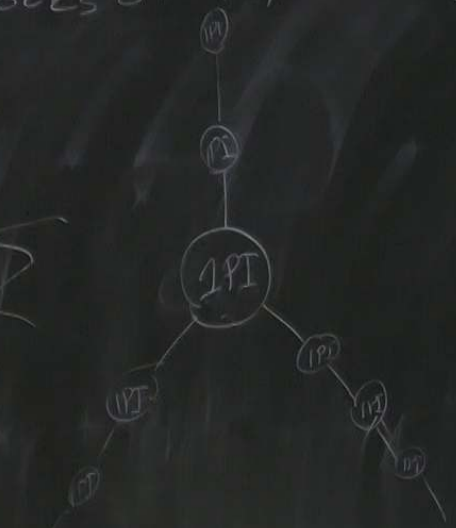
correct residue

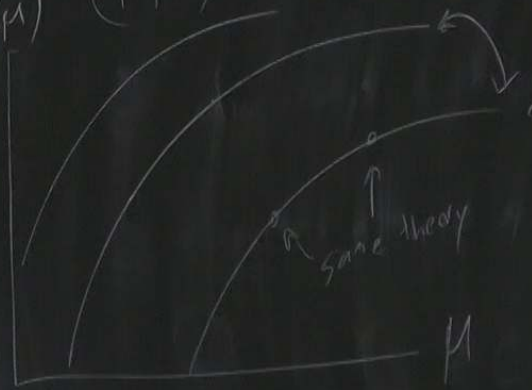
$$\Sigma'(m_{ph}^2) = 0$$


Need 1 more condition
involving coupling
How is coupling related to experment?
could define g_{ph} by
$\langle f|S|i\rangle = F(g_{ph})$ for given $|i\rangle$, $|f\rangle$


tion
ated to experiment?
by
for given |i> + |f>
or use buiding blocks of Feyman diagrams
<Ω|Tφ1φ2φ3|Ω> =
1PI
= Γ3(p1,p2,p3)

buiding blocks of Feyman diagrams



$\Gamma_3(p_1^{ref}, p_2^{ref}, p_3^{ref}) \equiv g(\mu)$ arbitrary!

$g(\mu)$ $(p_1^{ref})^2 = \mu^2$ μ is arbitrary