Title: Quantum Theory Lecture - 100323

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Renormalization of φ^3

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$

$$\left.\begin{aligned} \text{LSZ:} \quad \langle\Omega|\varphi(x)|\Omega\rangle &= 0 \\ \langle\vec{k}|\varphi(x)|\Omega\rangle &= e^{ik\cdot x} \end{aligned}\right\} \text{ need to shift + rescale}$$

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 - Z_g\frac{g}{3!}\varphi^3 - Y\varphi$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_0 = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

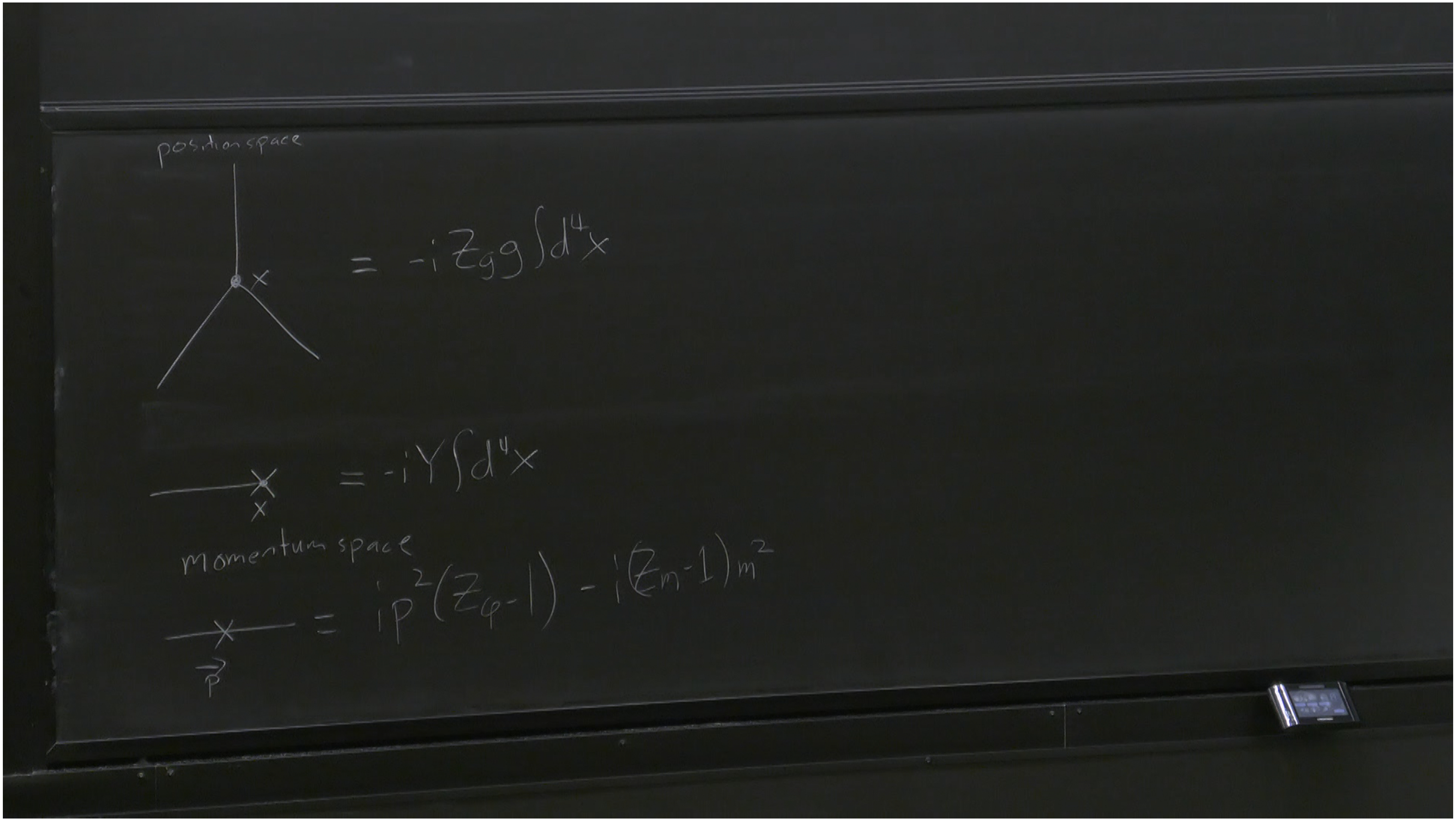
$$\mathcal{L}_{int} = -\frac{g}{3!}\varphi^3$$

$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)\varphi^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 - \frac{1}{3!}(Z_g - 1)g\varphi^3 + Y\varphi$$

counterterms

Expect: $Z_i = 1 + \mathcal{O}(g)$

$Y = 0 + \mathcal{O}(g)$


position space
$= -i Z_g g \int d^4x$
x
$= -iY \int d^4x$
x
momentum space
$= i p^2 (Z_\varphi - 1) - i (Z_m - 1) m^2$
$\vec{p}$

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$

$$\langle\Omega|T\varphi(x)|\Omega\rangle = \quad + \quad + \mathcal{O}(g^2)$$

$x \quad y \qquad x \quad y$

$\mathcal{O}(g) \qquad \mathcal{O}(g)$

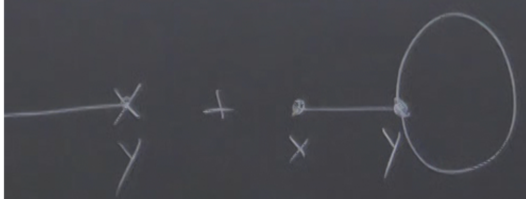
$$= -iY\int d^4y\,\Delta_{xy} + \frac{-iZ_g g}{2}\int d^4y\,\Delta_{xy}\Delta_{yy} + \mathcal{O}(g^2)$$

$$= \left(-iY - \frac{ig}{2}\Delta_{00}\right)\int d^4y\,\Delta_{xy}$$

$Z_g = 1 + \mathcal{O}(g)$

$$Y = -\frac{g}{2}\Delta_F(0)$$

← dive



$+ \mathcal{O}(g^2)$

$\mathcal{O}(g) \qquad \mathcal{O}(g)$

$\int d^4y\, \Delta_{xy} + \frac{-iZ_g g}{2} \int d^4y\, \Delta_{xy} \Delta_{yy} + \mathcal{O}(g^2)$

$\left(Y - \frac{ig}{2} \Delta_{00}\right) \int d^4y\, \Delta_{xy}$

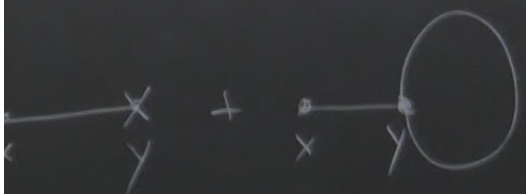
$Z_g = 1 + \mathcal{O}(g)$

diverges

$$Y = \frac{-g}{2} \Delta_F(0)$$

$\sum$ diagrams with one external point $= 0$

$\rightarrow \sum$ diagrams attached by one point $= 0$



$$+ \mathcal{O}(g^2)$$

$\mathcal{O}(g)$ $\mathcal{O}(g)$

$$\int d^4y\,\Delta_{xy} + \frac{-iZ_g g}{2}\int d^4y\,\Delta_{xy}\Delta_{yy} + \mathcal{O}(g^2)$$

$$\left(iY - \frac{ig}{2}\Delta_{00}\right)\int d^4y\,\Delta_{xy}$$

$$Z_g = 1 + \mathcal{O}(g)$$

diverges

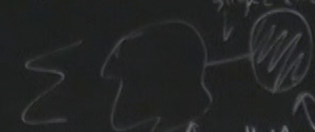
$$Y = -\frac{g}{2}\Delta_F(0)$$

$\sum$ diagrams with one external point $= 0$

$\rightarrow \sum$ diagrams attached by one point $= 0$

$= 0$

single line

$\sum$  $= 0$

anything

connected

$$\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$$

Källe

$$G(p,p') = \int d^4x \int d^4x' \langle\Omega| T\varphi(x)\varphi(x')|\Omega\rangle e^{i(p\cdot x + p'\cdot x')}$$

momentum space 2-point function

$$= (2\pi)^4 \delta^4(p+p')\, G_2(p)$$

compute

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} |\vec{k}\rangle\langle\vec{k}| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \sum_{\sigma} |\vec{k},\sigma\rangle\langle\vec{k},$$

projection onto ground state

1-particle

$$\langle\Omega|T\varphi(x)\,\mathbb{1}\,\varphi(y)|\Omega\rangle \overset{x^0>y^0}{=} \langle\Omega|\varphi(x)|\Omega\rangle\langle\Omega|\varphi(y)|\Omega\rangle + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \langle\Omega|\varphi(x)|\vec{k}\rangle\langle\vec{k}|\varphi(y)|\Omega\rangle + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \sum_{\sigma} \langle\Omega|\varphi(x)|\vec{k},\sigma\rangle\langle\vec{k},\sigma|\varphi(y)|\Omega\rangle$$

$\vec{k},\sigma\rangle\langle\vec{k},\sigma|$

$$= \int \frac{d^3k}{(2\pi)^3\, 2E_{\vec{k}}}\, e^{-ik(x-y)} \quad + \text{ multiparticle term}$$

part of Feynman propagator

$$\int \frac{d^3k}{(2\pi)^3\, 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4}\, 2\pi\, \delta(k^2 - m_{ph}^2)\, \Theta(k^0)$$

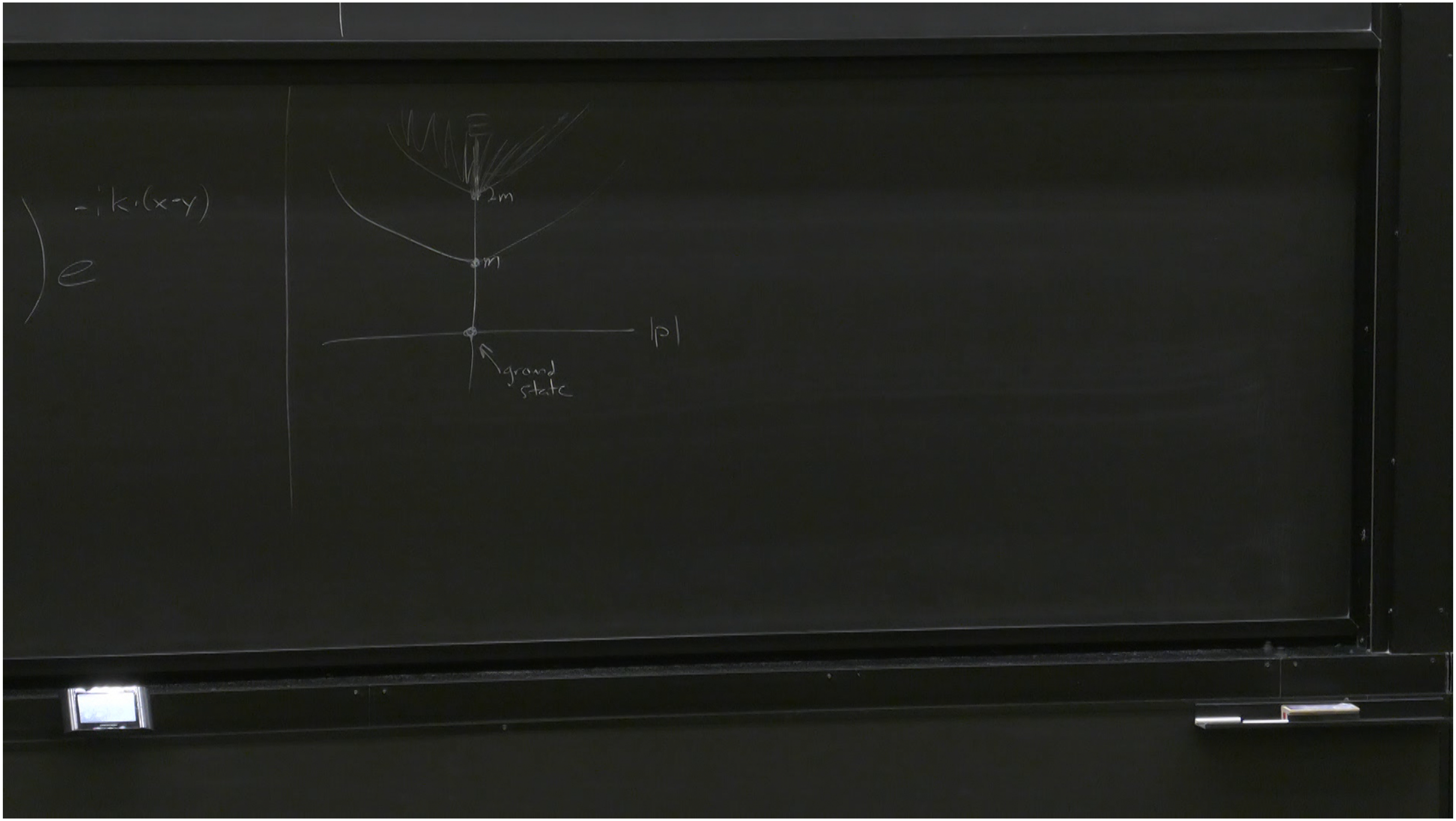
physical mass!

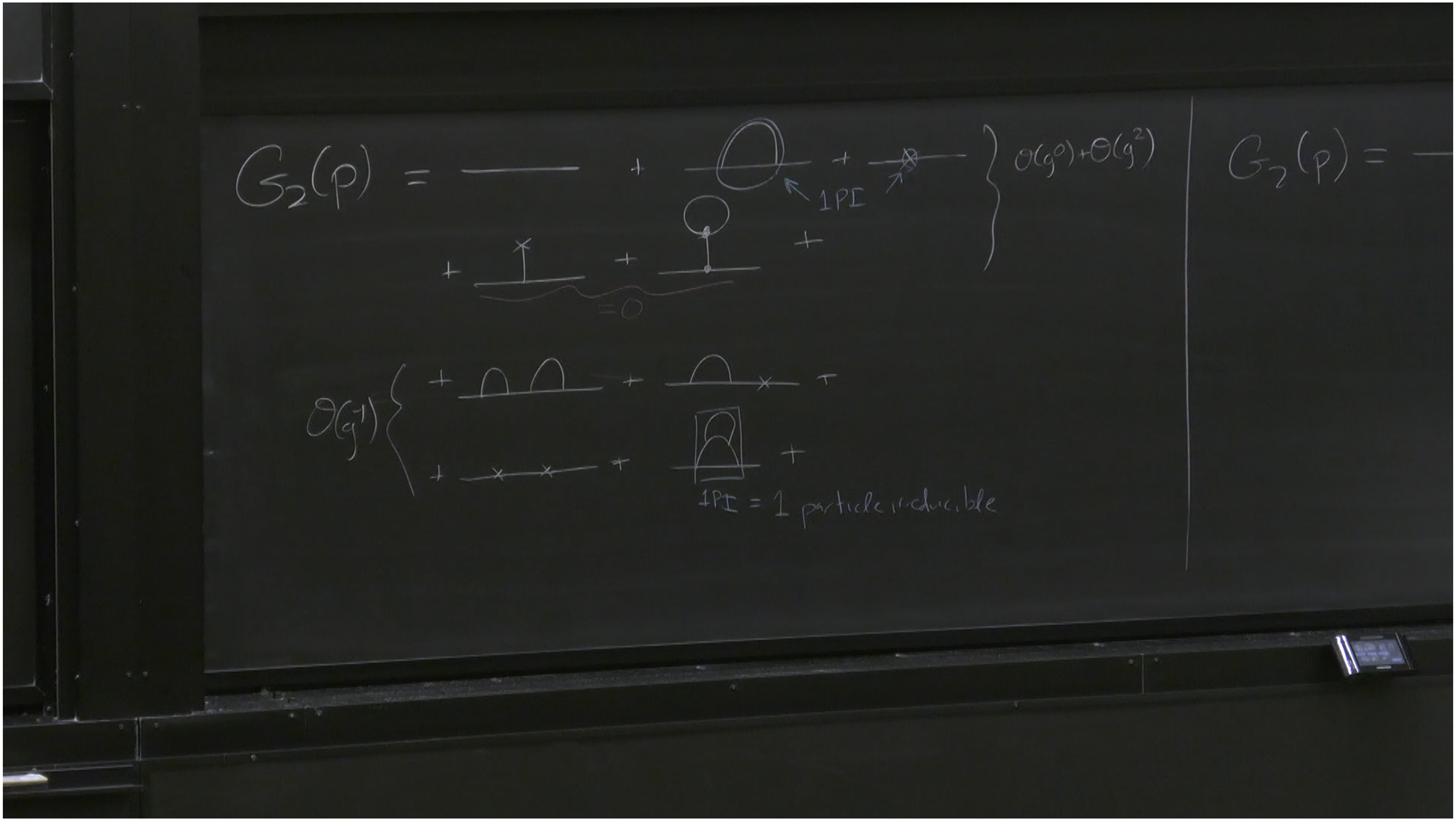
no boundstates $\rightarrow$ $M^2_{\text{multiparticle}} \geq (2m_{ph})^2$

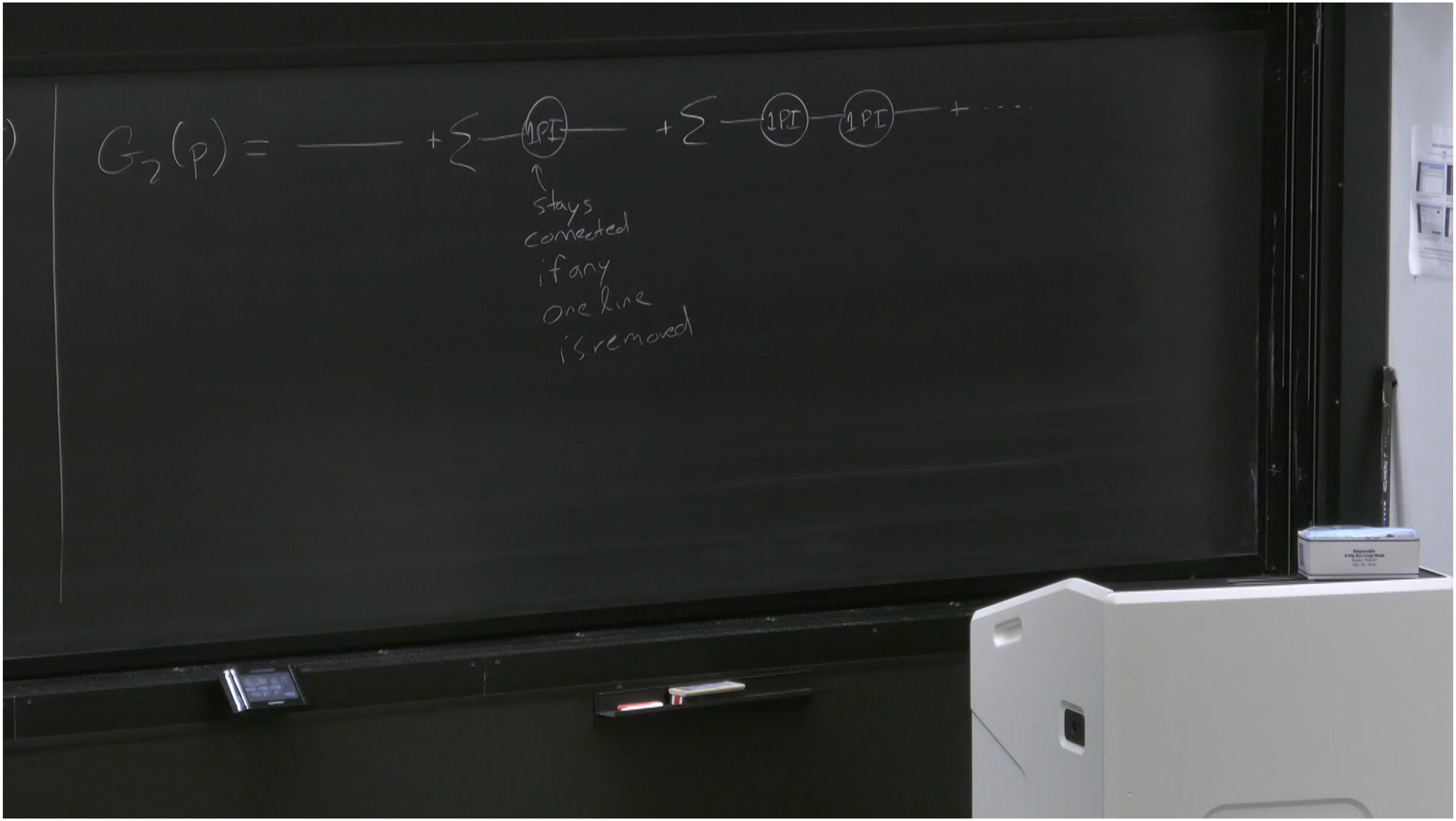
$$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle = \int \frac{d^4k}{(2\pi)^4}\left(\frac{i}{k^2-m_{ph}^2+i\varepsilon} + \int_{4m^2}^{\infty} ds \, \frac{\rho(s)}{k^2-s+i\varepsilon}\right)e^{ik\cdot(\cdots)}$$

$$G_2(p) = \frac{i}{k^2-m_{ph}^2+i\varepsilon} + \int_{4m^2}^{\infty} ds \, \frac{\rho(s)}{k^2-s+i\varepsilon}$$

2-point in interacting theory has pole at $k^2 = m_{ph}^2$ with residue i


-ik·(x-y)
e
2m
m
|p|
ground state


G₂(p) =
1PI
O(g⁰)+O(g²)
=0
O(g¹)
1PI = 1 particle irreducible
G₂(p) =



$G_2(p) =$ —— $+ \sum$ —(1PI)—— $+ \sum$ —(1PI)—(1PI)— $+ \cdots$
stays connected if any one line is removed

$\mathcal{O}(g^p)+\mathcal{O}(g^2)$

$$G_2(p) = \text{———} + \sum \text{—(1PI)—} + \sum \text{—(1PI)—(1PI)—} + \cdots$$

stays connected if any one line is removed

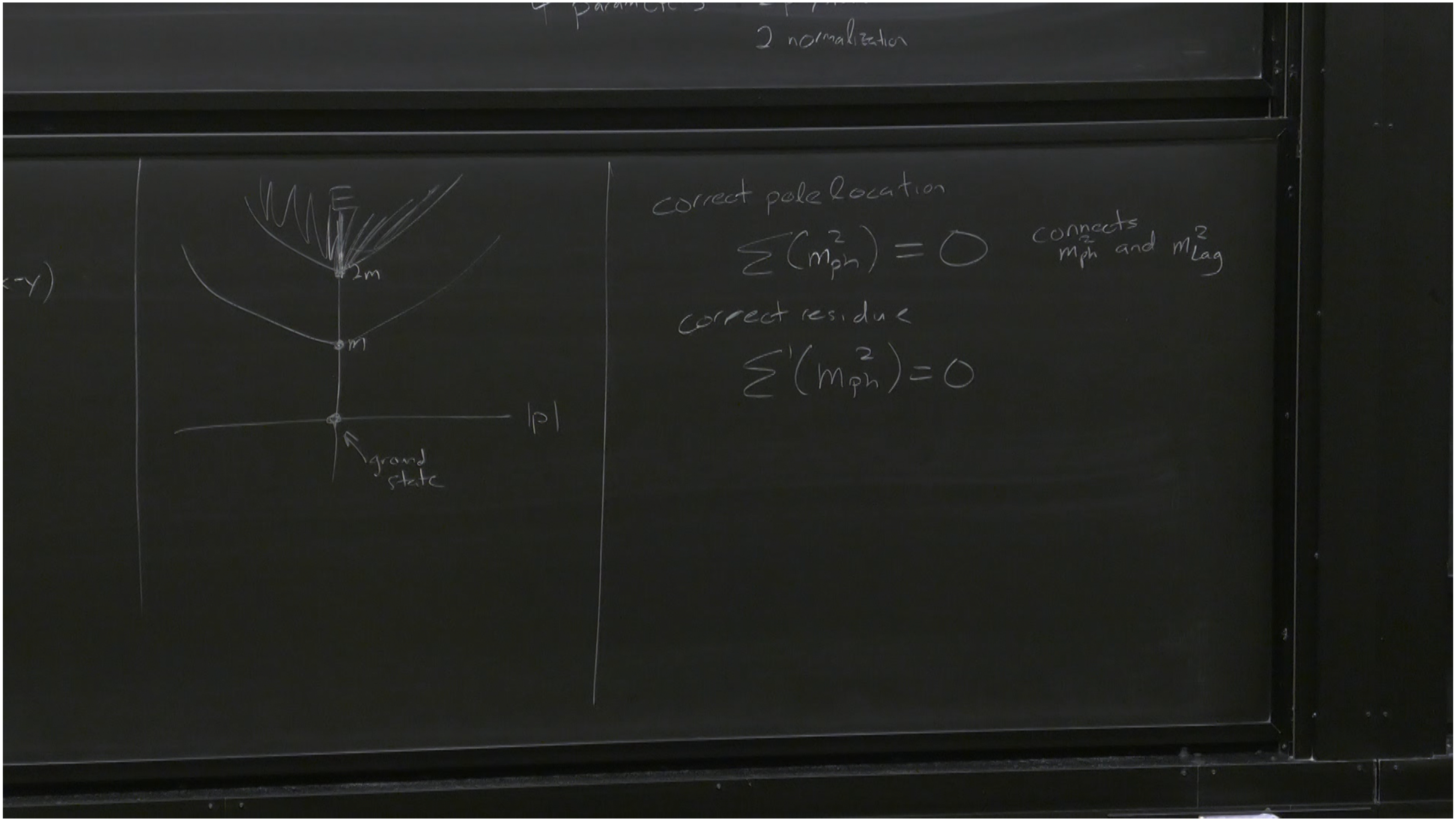
self-energy

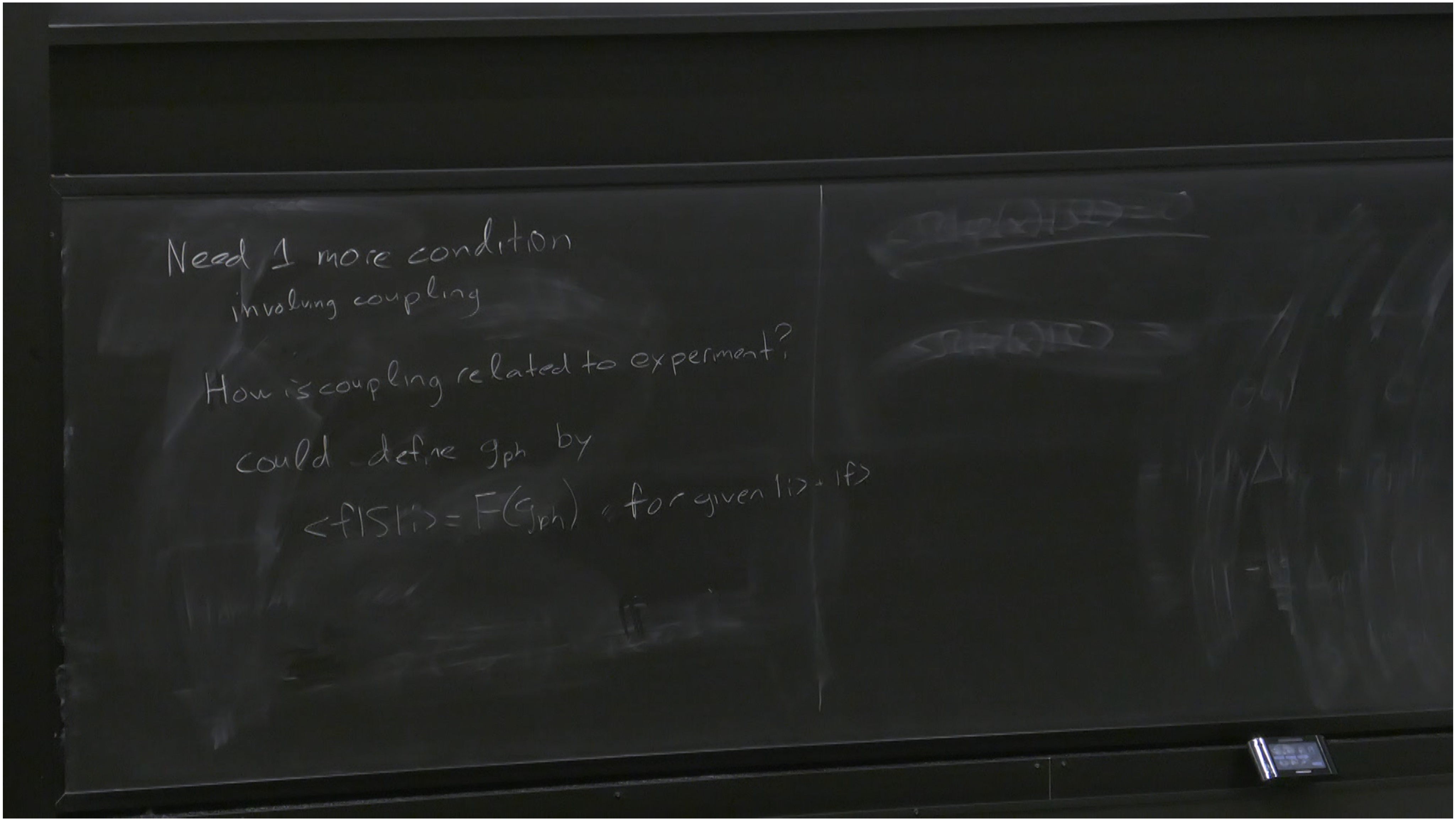
$$\sum \text{(1PI)} = -i\,\Sigma(p^2)$$

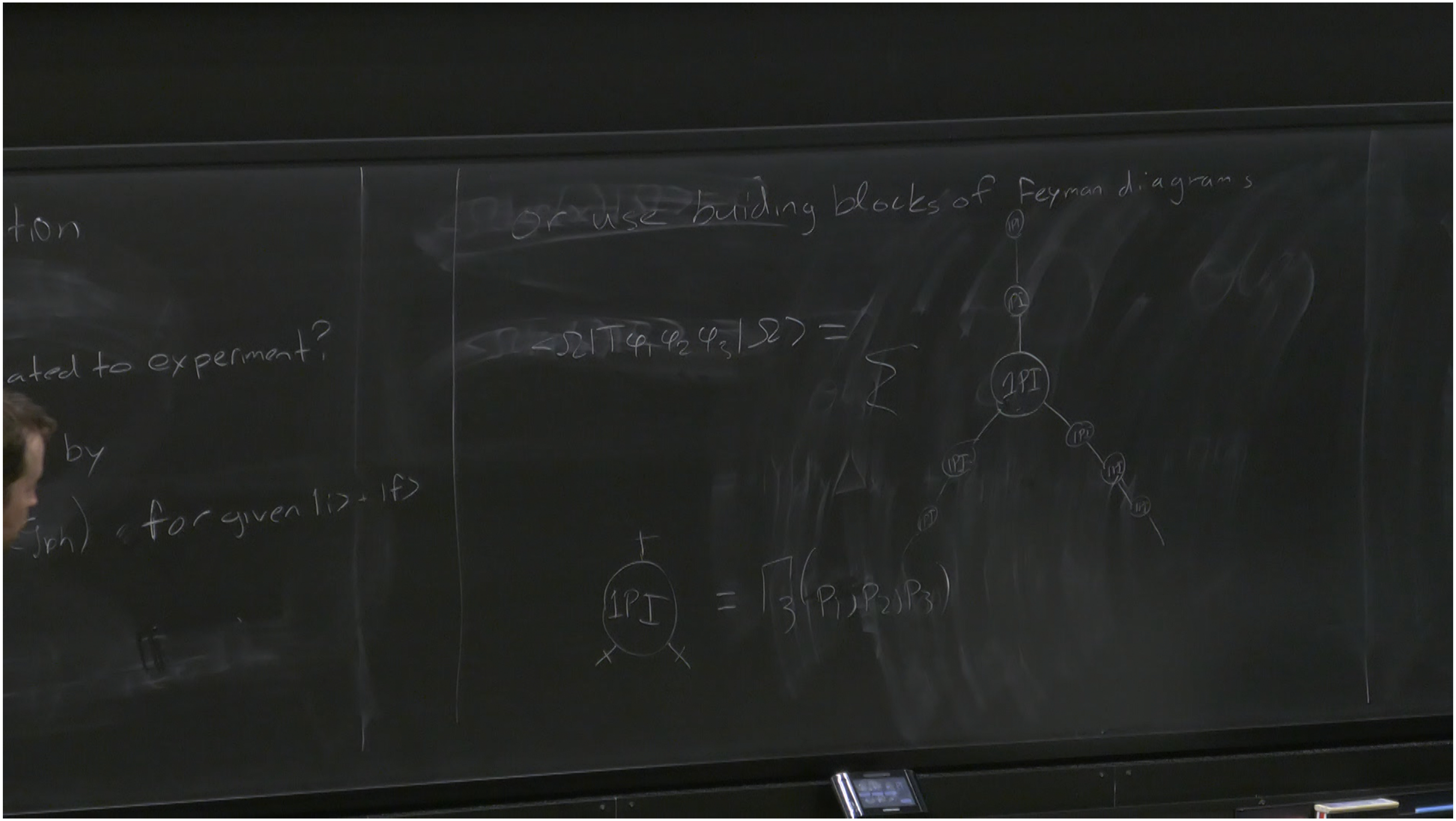
summing series

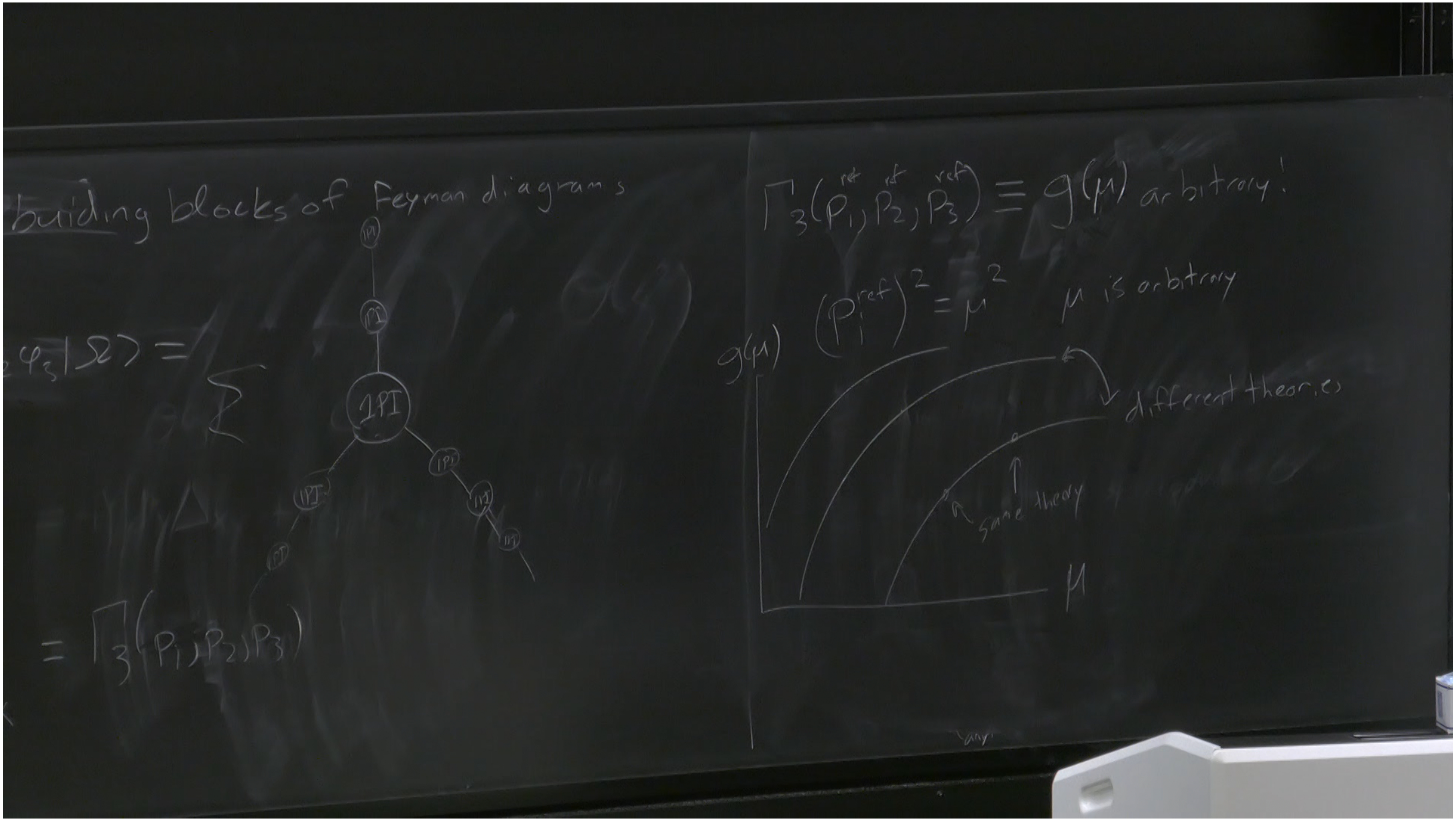
$$G_2(p) = \frac{i}{p^2-m^2+i\varepsilon} + \left(\frac{i}{p^2-m^2+i\varepsilon}\right)^2\left(-i\,\Sigma(p^2)\right) + \cdots = \frac{i}{p^2-m^2-\Sigma(p^2)+i\varepsilon}$$

from Lagrangian


2 normalization
E
2m
m
|p|
ground state
correct pole location
Σ(m_ph^2) = 0
connects m_ph^2 and m_Lag^2
correct residue
Σ'(m_ph^2) = 0


Need 1 more condition
involving coupling
How is coupling related to experiment?
could define g_ph by
<f|S|i> = F(g_ph) for given |i> + |f>


tion
ated to experiment?
by
for given |i> + |f>
or use building blocks of Feyman diagrams
<Ω|T φ1 φ2 φ3|Ω> =
1PI
= Γ3(P1, P2, P3)


building blocks of Feyman diagrams
1PI
$\varphi_3|\Omega\rangle =$
$\sum$
$= \Gamma_3(p_1, p_2, p_3)$
$\Gamma_3(p_1^{ref}, p_2^{ref}, p_3^{ref}) \equiv g(\mu)$ arbitrary!
$(p_i^{ref})^2 = \mu^2$
μ is arbitrary
$g(\mu)$
different theories
same theory
μ