Title: Quantum Theory Lecture - 092723

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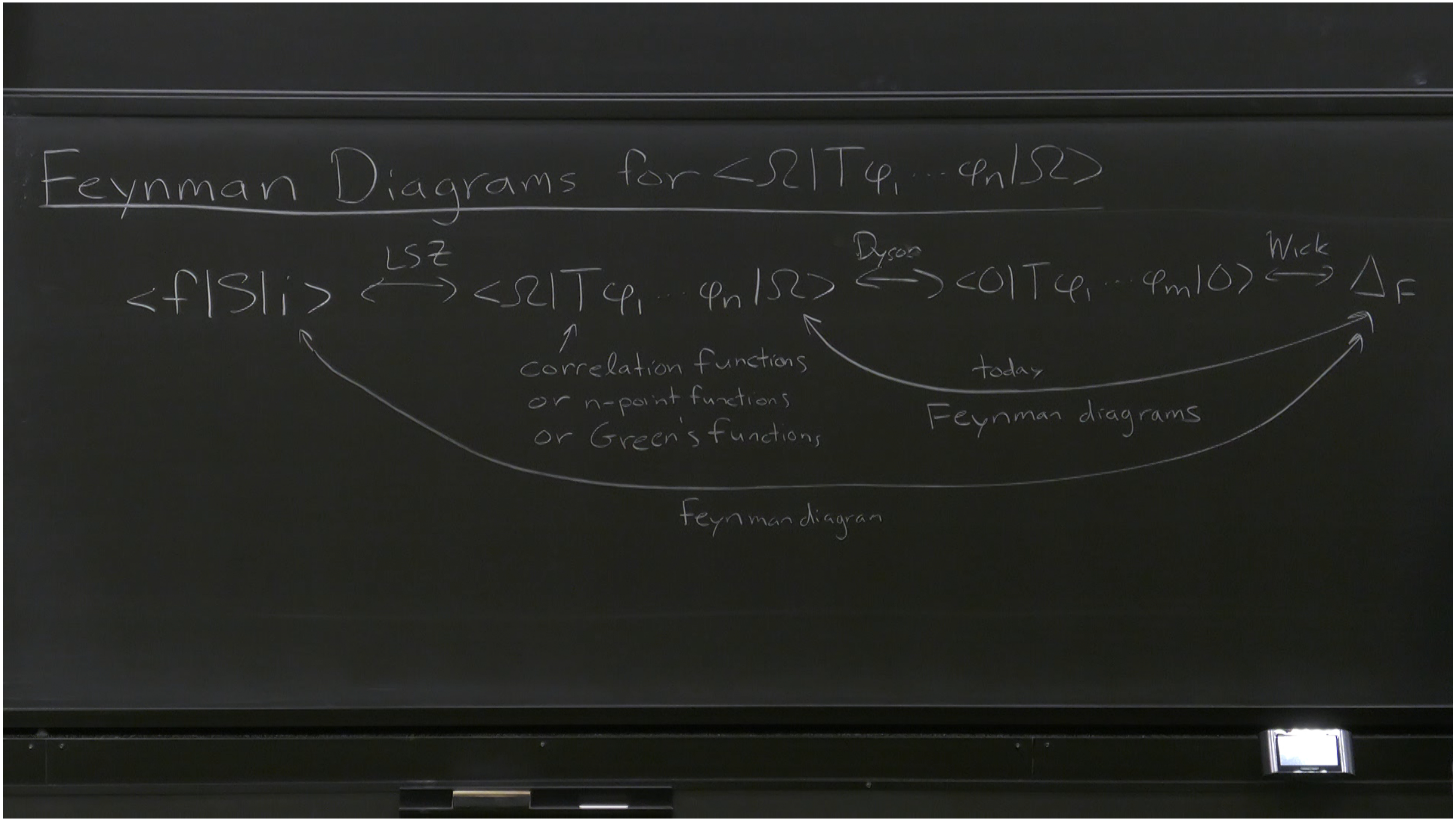
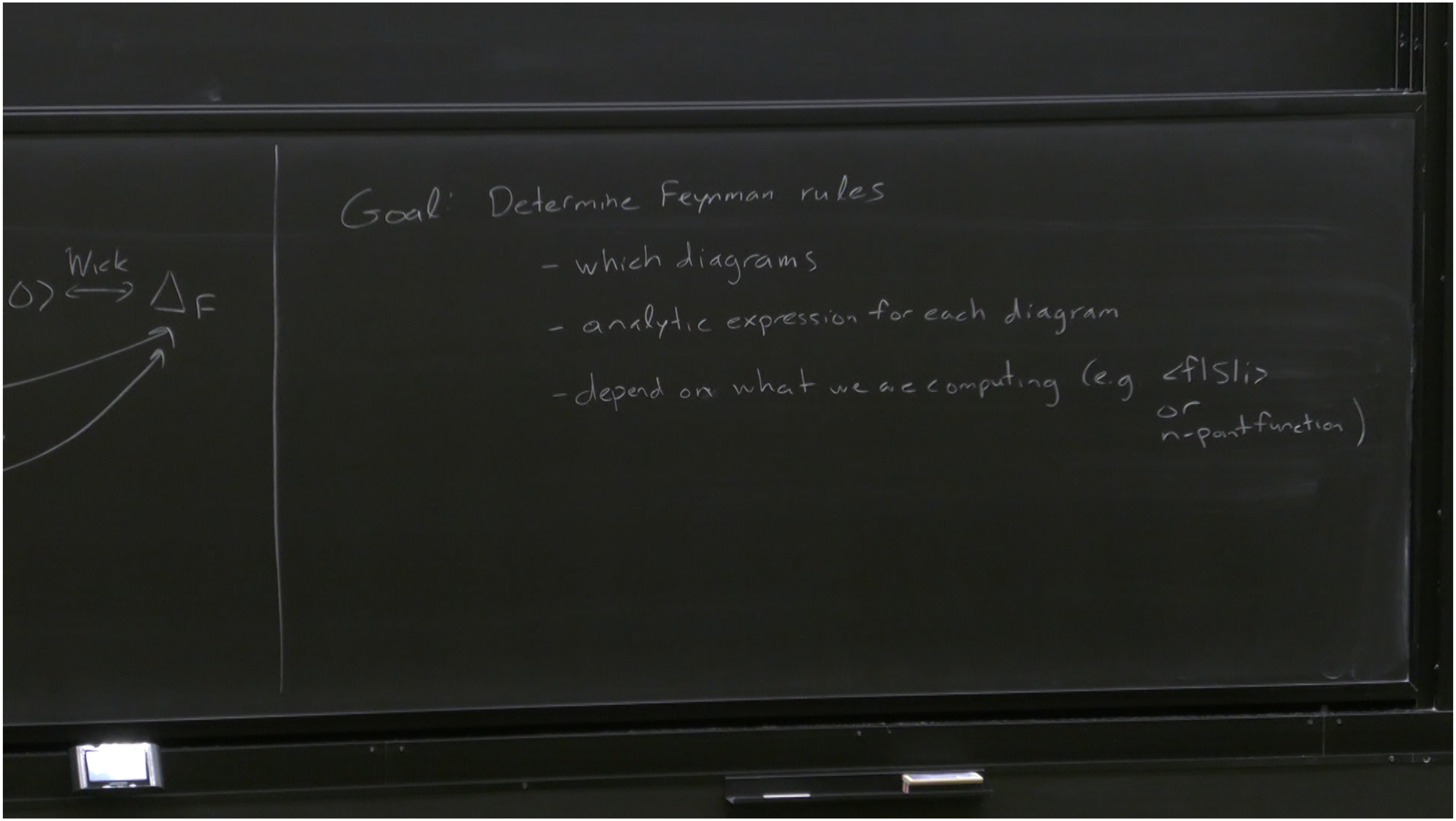
Collection: Quantum Theory 2023/24

Date: September 27, 2023 - 10:45 AM

URL: https://pirsa.org/23090048


Feynman Diagrams for <Ω|T φ1 ··· φn|Ω>
LSZ
Dyson
Wick
<f|S|i> <——> <Ω|T φ1 ··· φn|Ω> <——> <0|T φ1 ··· φm|0> <——> ΔF
correlation functions
or n-point functions
or Green's functions
today
Feynman diagrams
Feynman diagram

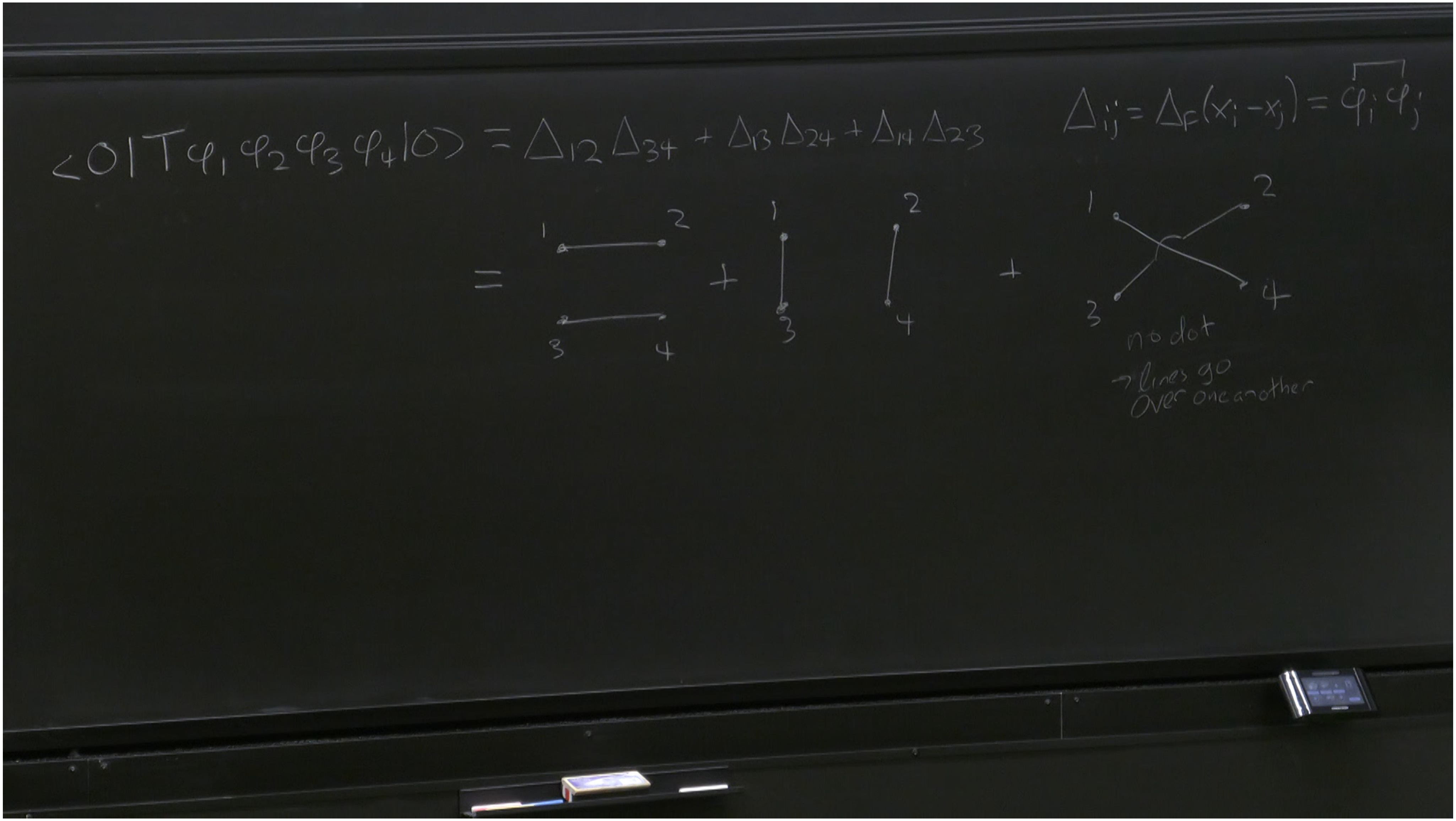

Wick
0⟩ ⟷ Δ_F
Goal: Determine Feynman rules
- which diagrams
- analytic expression for each diagram
- depend on what we are computing (e.g. ⟨f|S|i⟩ or n-point function)

Goal: Determine Feynman rules

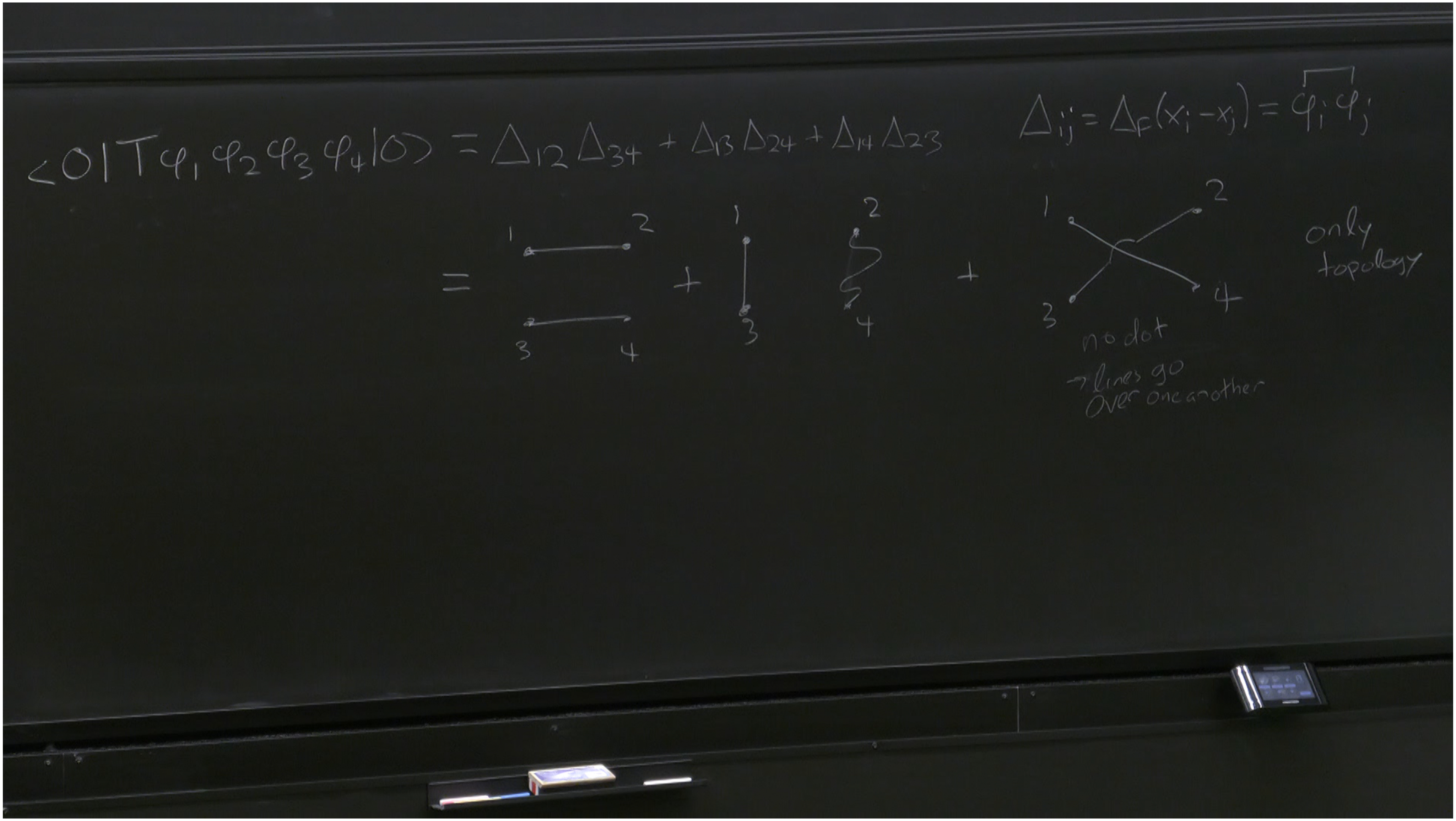
- which diagrams
- analytic expression for each diagram
- depend on what we are computing (e.g. $\langle f|S|i\rangle$ or n-point function)

Basic idea: lines —— $= \Delta_F$

points • represent a point in spacetime



$\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$
$\Delta_{ij} = \Delta_F(x_i - x_j) = \varphi_i \varphi_j$
1 2 3 4
no dot
→ lines go over one another


$\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$
$\Delta_{ij} = \Delta_F(x_i - x_j) = \varphi_i \varphi_j$
1 2 3 4
no dot
→ lines go over one another
only topology

$) = \varphi_i \varphi_j$ (contraction)

only topology

her

Example: $\langle \Omega | T \varphi_1 \varphi_2 | 0 \rangle$ in $\mathcal{L}_{int} = -\frac{g}{4!}\varphi^4$

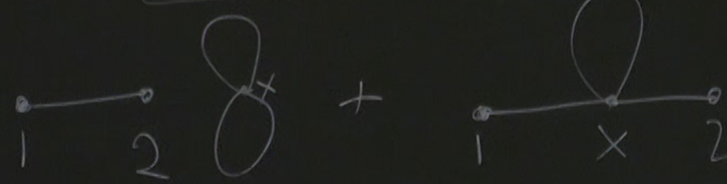
$$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \varphi_2 \exp\left[i \int d^4x \, \mathcal{L}_{int} \right] | 0 \rangle}{\langle 0 | T \exp\left[i \int d^4x \, \mathcal{L}_{int} \right] | 0 \rangle}$$

numerator $= \langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\frac{ig}{4!}\int d^4x\,\varphi_x^4|0\rangle$

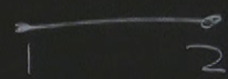
$\langle 0|T\varphi_1\varphi_2|0\rangle =$ 

$\frac{ig}{4!}\langle 0|T\varphi_1\varphi_2\int d^4x\,\varphi_x|0\rangle = 3\,\frac{ig}{4!}\int d^4x\;\varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x + 12\,\frac{ig}{4!}\int d^4x\;\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x$

4 1 3

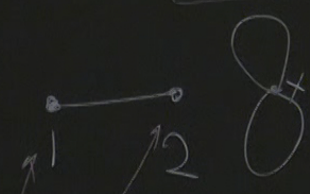
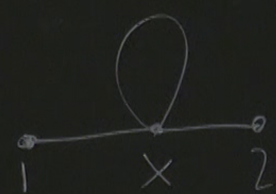
$=$ 

numerator $= \langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\frac{ig}{4!}\int d^4x\,\varphi_x^4|0\rangle$

$\langle 0|T\varphi_1\varphi_2|0\rangle =$ 

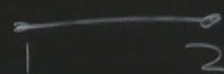
$\frac{ig}{4!}\langle 0|T\varphi_1\varphi_2\int d^4x\,\varphi_x|0\rangle = 3\varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x + 12\frac{ig}{4!}\int d^4x\,\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x$

(annotations: $\frac{ig}{4!}\int d^4x$; 4; 1; 3)

$=$  $+$ 

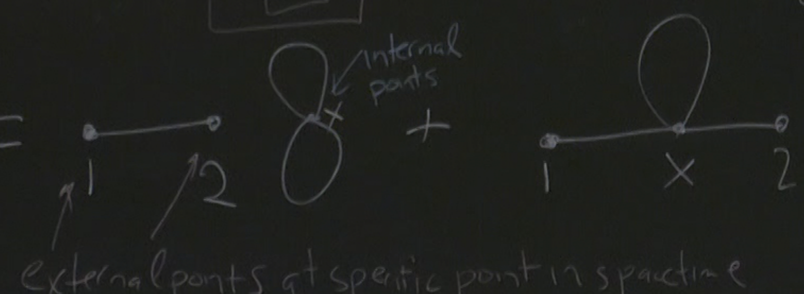
external points at specific point in spacetime

$$\text{numerator} = \langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2 \tfrac{ig}{4!}\int d^4x\, \varphi_x^4|0\rangle$$

$$\langle 0|T\varphi_1\varphi_2|0\rangle =$$ 

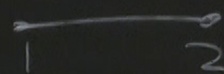
$$\frac{ig}{4!}\langle 0|T\varphi_1\varphi_2\int d^4x\, \varphi_x^4|0\rangle = 3\,\frac{ig}{4!}\int d^4x\; \varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x + 12\,\frac{ig}{4!}\int d^4x\; \varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x$$

(contraction counts: 4, 1, 3)

$=$ 

numerator $= \langle 0|T\varphi_1\varphi_2|0\rangle + \langle 0|T\varphi_1\varphi_2\frac{-ig}{4!}\int d^4x\,\varphi_x^4|0\rangle$

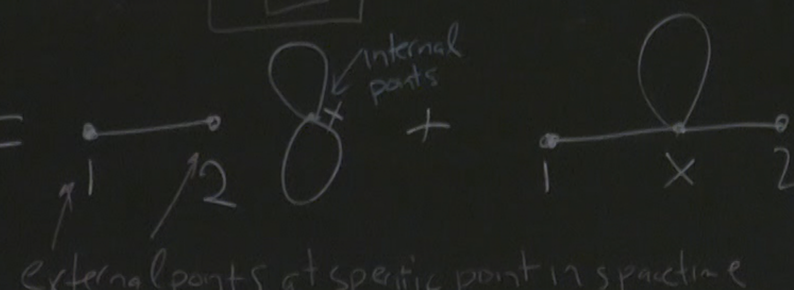
$\langle 0|T\varphi_1\varphi_2|0\rangle =$



$\frac{ig}{4!}\langle 0|T\varphi_1\varphi_2\int d^4x\,\varphi_x^4|0\rangle = 3\varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x + 12\frac{-ig}{4!}\int d^4x\,\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x$

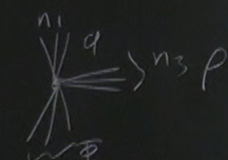
(annotations: $\frac{-ig}{4!}\int d^4x$; 4; 1; 3)

$=$



Feynman rules for numerator

numerator $= \langle 0 | T \varphi_1 \cdots \varphi_n \exp[i \int d^4x \, \mathcal{L}_{int}] | 0 \rangle =$ all diagrams with n external points



1. $\circ\!\!-\!\!\!-\!\!\circ$ (x to y) $= \Delta_{xy}$

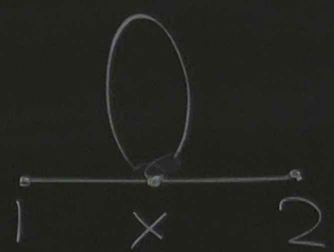
2. $\leftarrow$ (x) $= 1$

3. $\times$ $= -ig \int d^4x$ (depends on theory)

$$\left(\mathcal{L}_{int} = -\frac{\lambda \varphi^{n_1} \Phi^{n_2} \rho^{n_3}}{n_1! \, n_2! \, n_3!} \right)$$

4. Divide by symmetry factor (requires normalization)

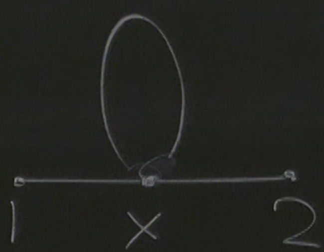
symmetry factor $S =$ number of ways to map diagram to itself with external points held fixed



$S = 2$ ← swapping ends internal line

$$\frac{1}{S} = \frac{1}{2} = \frac{\#\text{Wick contractions}}{(\text{factorials from exp})(\text{factorials from } \mathcal{L}_{int})} = \frac{12}{1!\,4!} = \frac{1}{2}$$

symmetry factor $S =$ number of ways to map diagram to itself with external points held fixed

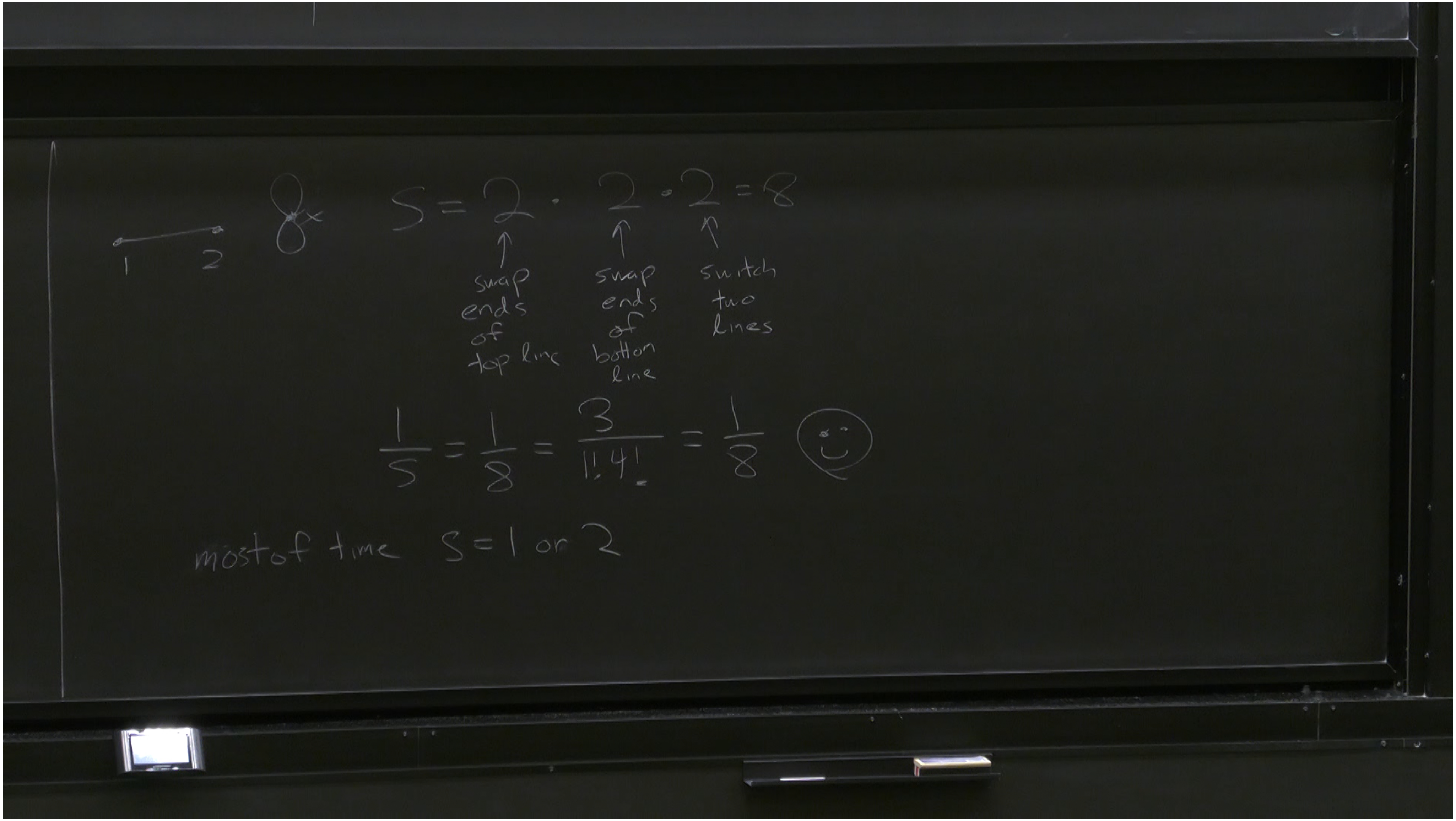


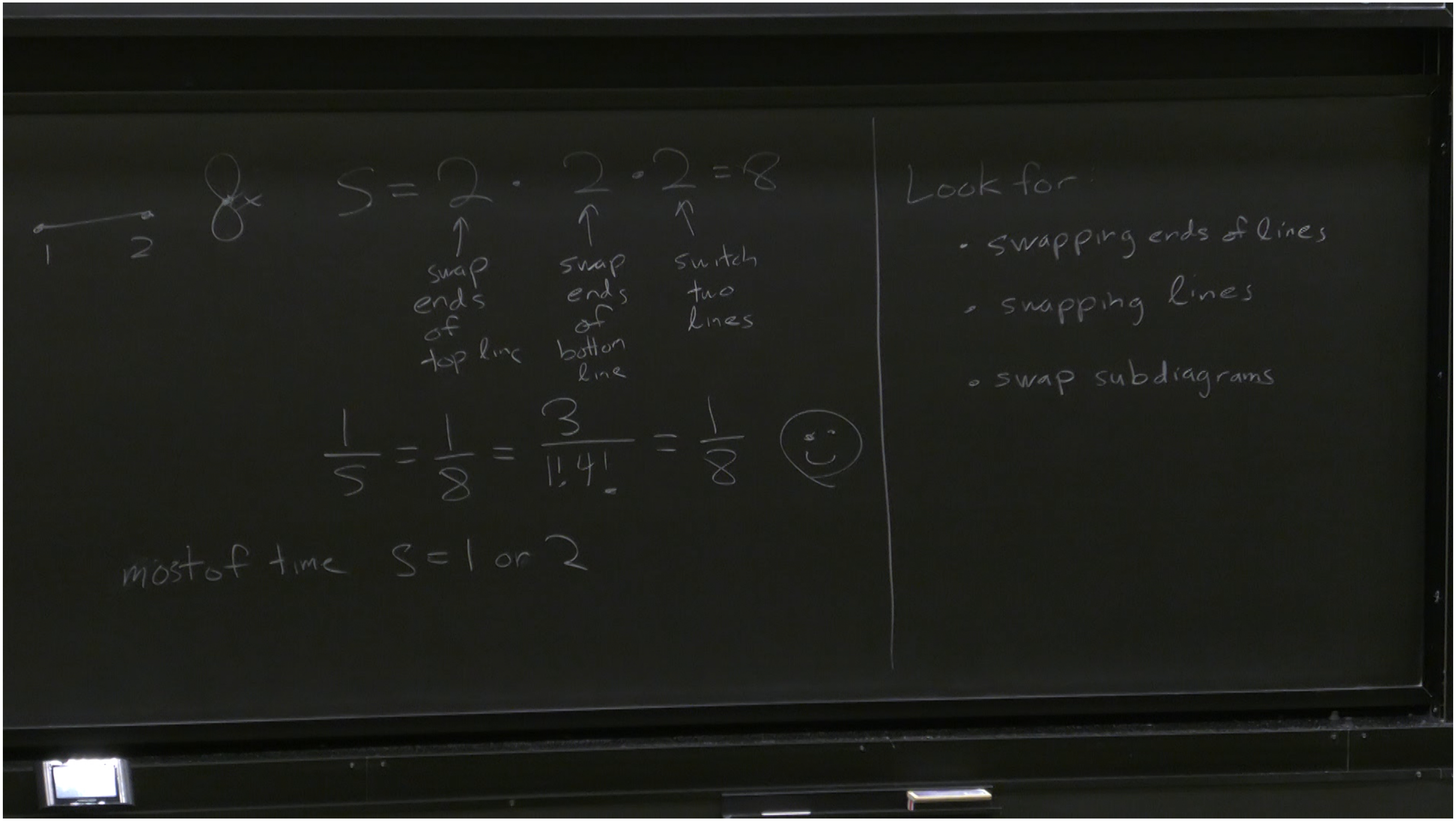
$S = 2$

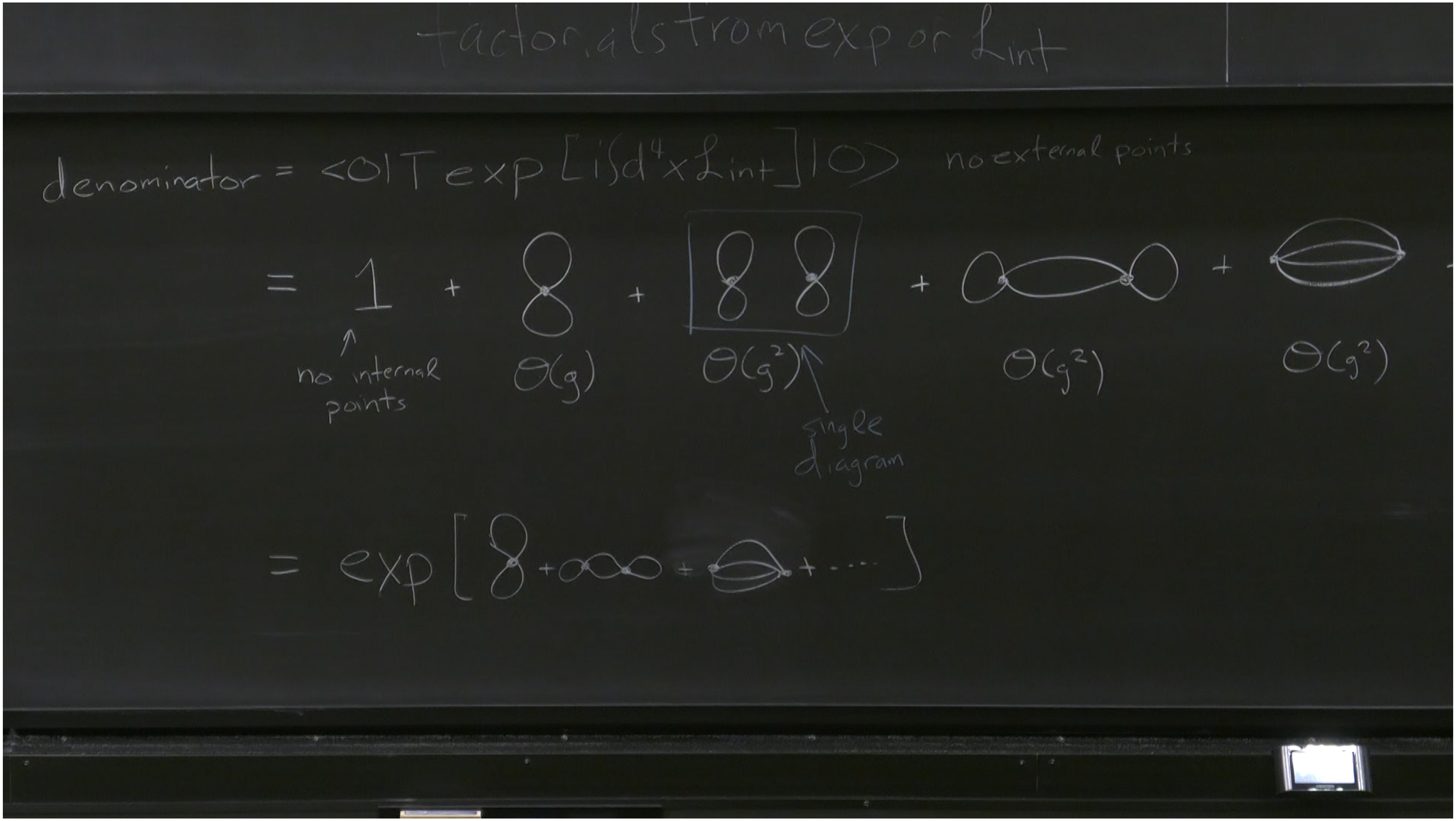
↑ swapping ends internal line

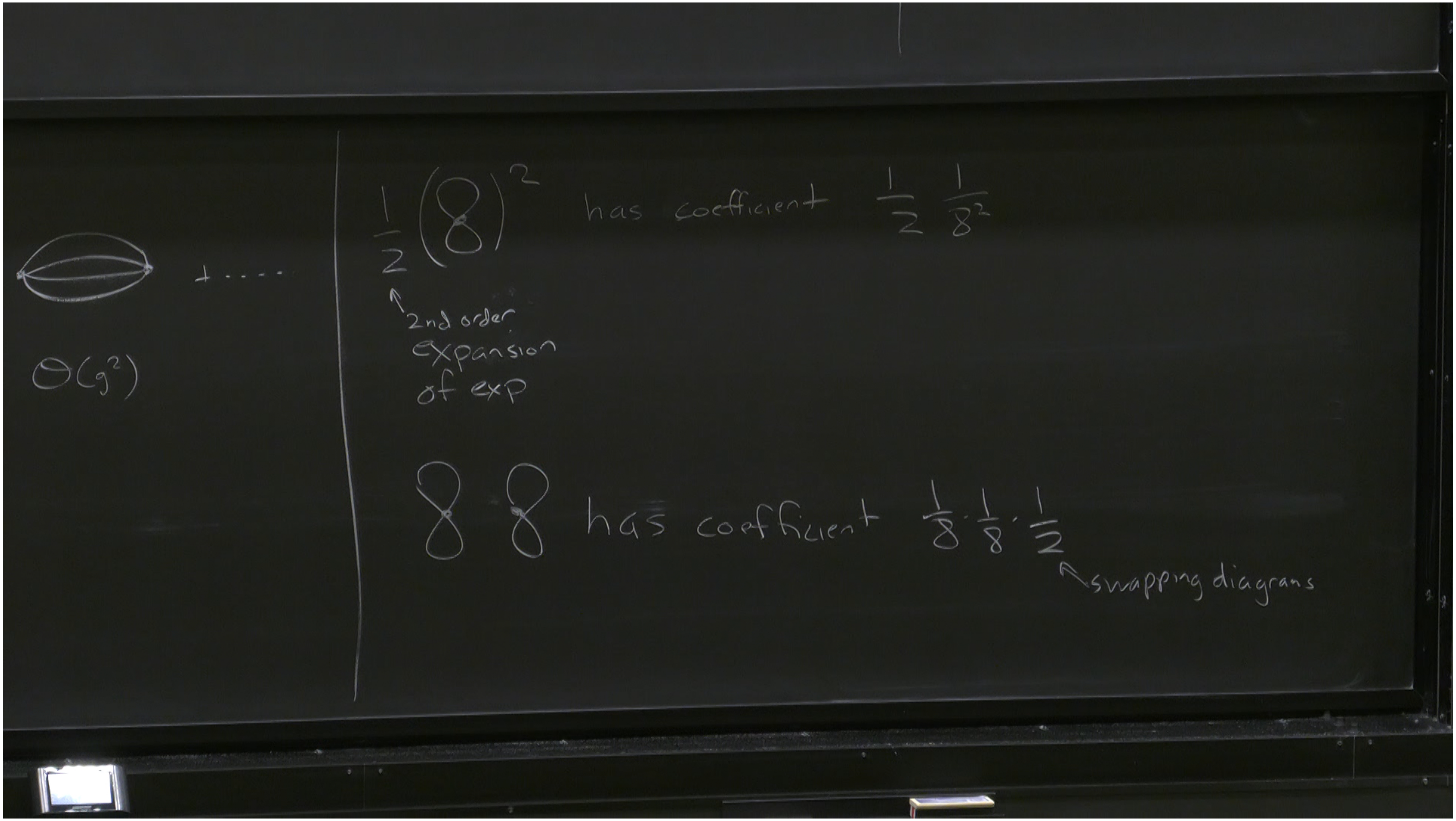
$$\frac{1}{S} = \frac{1}{2} = \frac{\#\text{Wick contractions}}{(\text{factorials from exp})(\text{factorials from } \mathcal{L}_{int})} = \frac{12}{1!\,4!} = \frac{1}{2}$$

factorials from exp or $\mathcal{L}_{int}$


1 2
S = 2 · 2 · 2 = 8
swap ends of top line
swap ends of bottom line
switch two lines
1/S = 1/8 = 3/(1! 4!) = 1/8
most of time S = 1 or 2

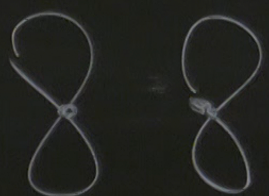

1
2
S = 2 · 2 · 2 = 8
swap ends of top line
swap ends of bottom line
switch two lines
1/S = 1/8 = 3/(1! 4!) = 1/8
most of time S = 1 or 2
Look for:
· swapping ends of lines
· swapping lines
· swap subdiagrams


factorials from exp or $\mathcal{L}_{int}$
denominator = $\langle 0 | T \exp[i \int d^4x \, \mathcal{L}_{int}] | 0 \rangle$ no external points
= 1 + ...
no internal points
$\mathcal{O}(g)$
$\mathcal{O}(g^2)$
single diagram
$\mathcal{O}(g^2)$
$\mathcal{O}(g^2)$
= exp [... + ...]

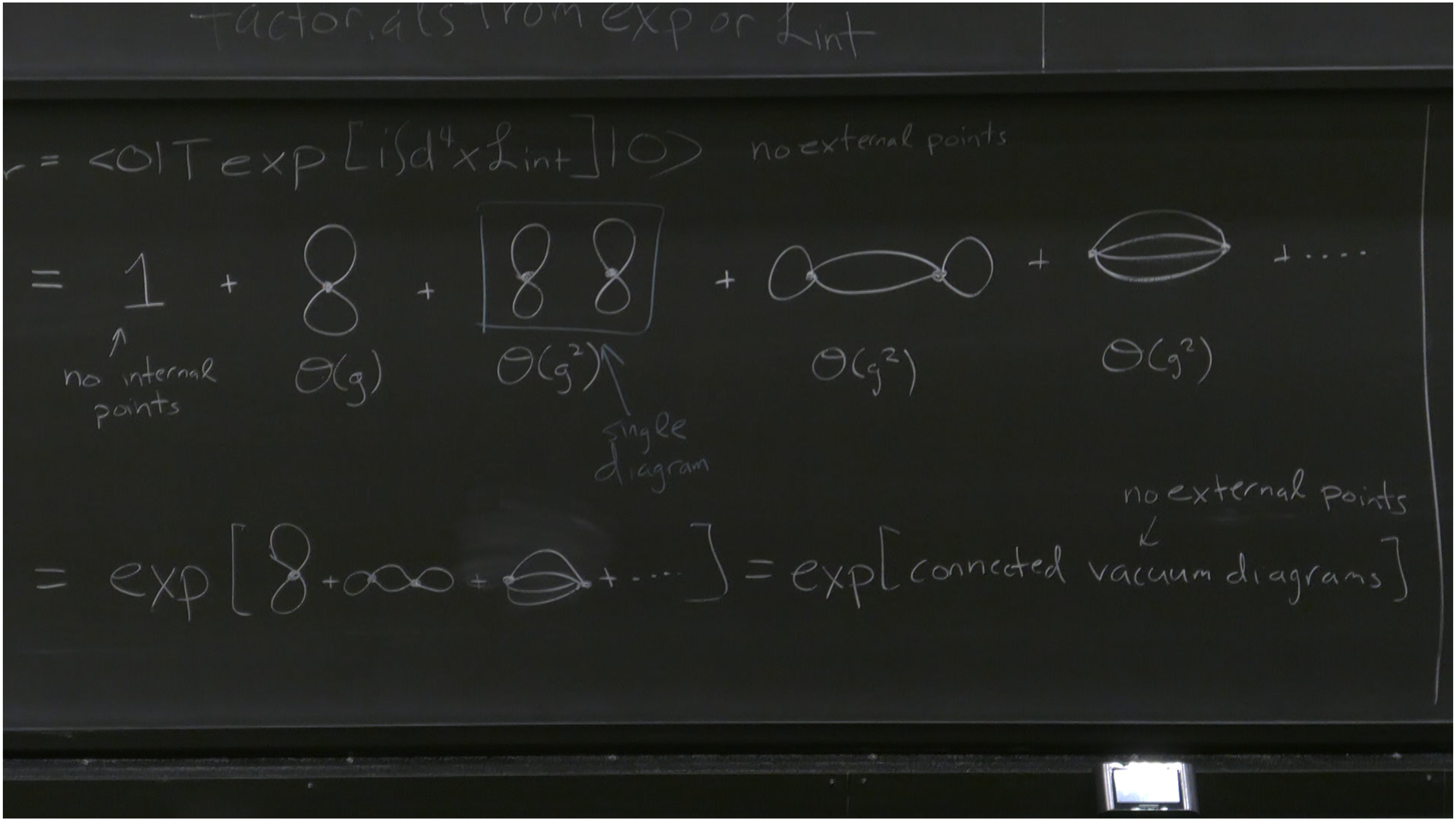

has coefficient $\frac{1}{2}\frac{1}{8^2}$
2nd order expansion of exp
$O(g^2)$
has coefficient $\frac{1}{8}\cdot\frac{1}{8}\cdot\frac{1}{2}$
swapping diagrams

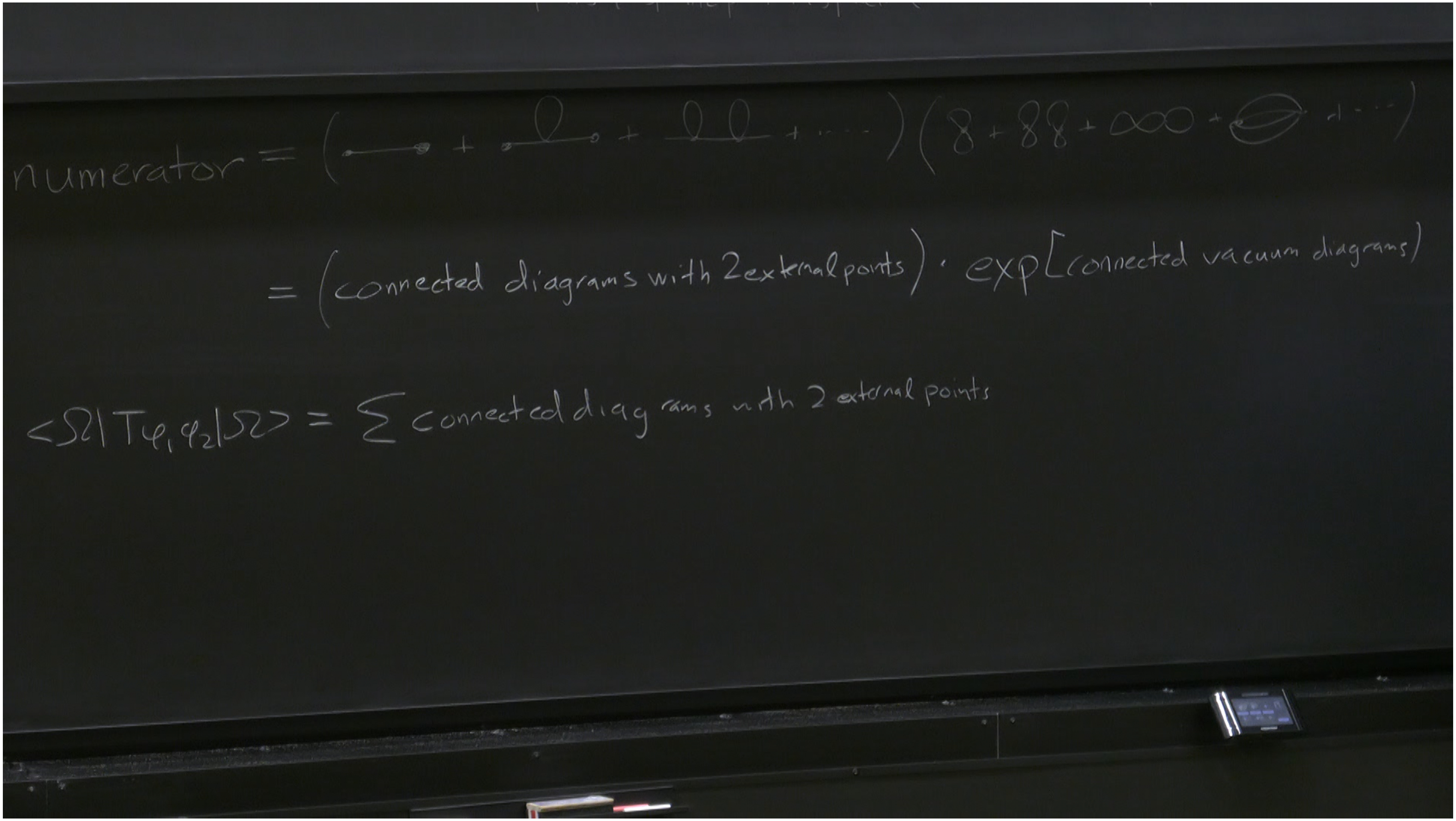
$\frac{1}{2}$ ()^2 has coefficient $\frac{1}{2}\frac{1}{8^2}$

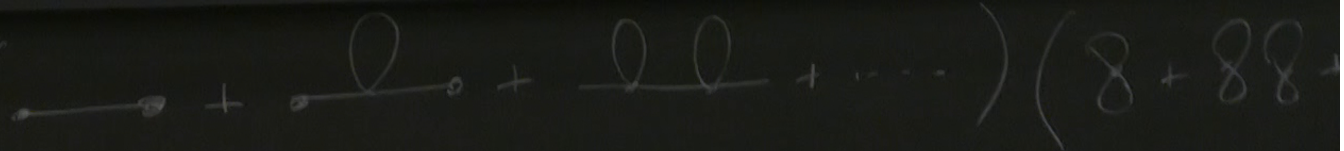
↑ 2nd order expansion of exp

 has coefficient $\frac{1}{8}\cdot\frac{1}{8}\cdot\frac{1}{2}$

↑ swapping diagrams

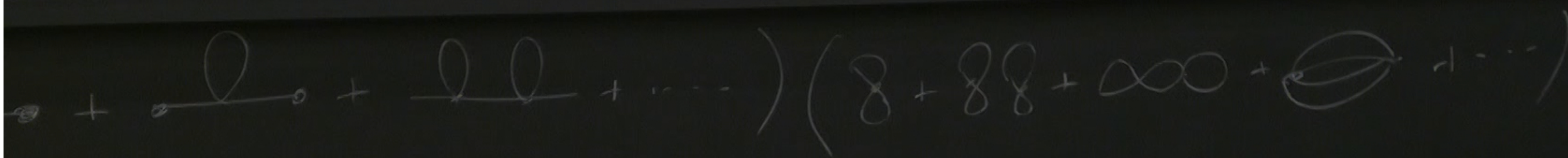

factorials from exp or L_int
= <0|T exp[i∫d⁴x L_int]|0>
no external points
= 1 +
O(g)
O(g²)
O(g²)
O(g²)
no internal points
single diagram
no external points
= exp[...] = exp[connected vacuum diagrams]


numerator =
= (connected diagrams with 2 external points) · exp[connected vacuum diagrams)
<Ω|T φ1 φ2|Ω> = Σ connected diagrams with 2 external points

numerator = () (...

= (connected diagrams with 2 external points) · exp[...

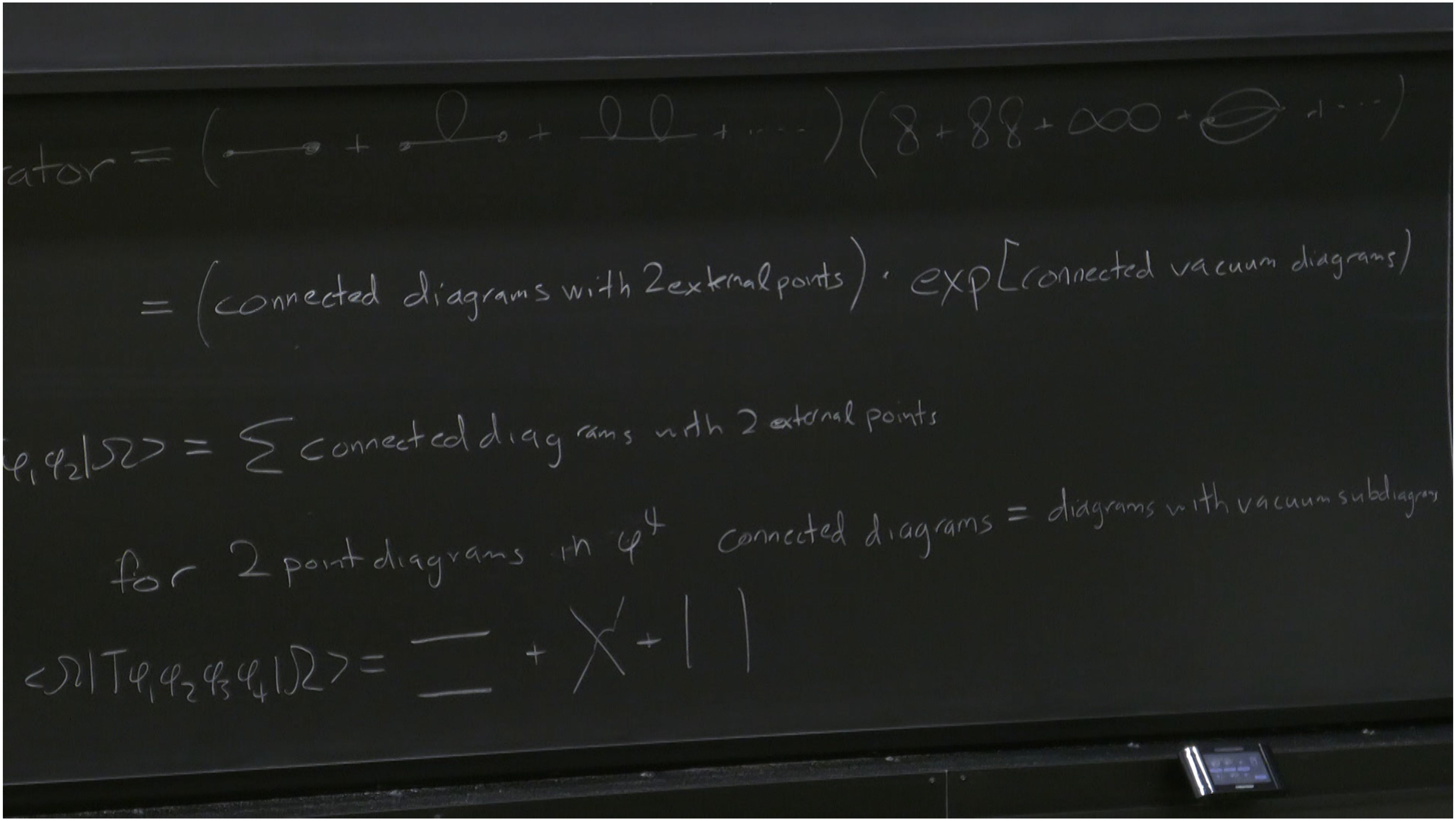
$\langle\Omega| T\varphi_1\varphi_2 |\Omega\rangle = \sum$ connected diagrams with 2 external points



Fe

...ected diagrams with 2 external points) · exp[connected vacuum diagrams)

connected diagrams with 2 external points


ator =
= (connected diagrams with 2 external points) · exp[connected vacuum diagrams)
φ1 φ2|Ω> = Σ connected diagrams with 2 external points
for 2 point diagrams in φ4 connected diagrams = diagrams with vacuum subdiagrams
<Ω|T φ1 φ2 φ3 φ4|Ω> =

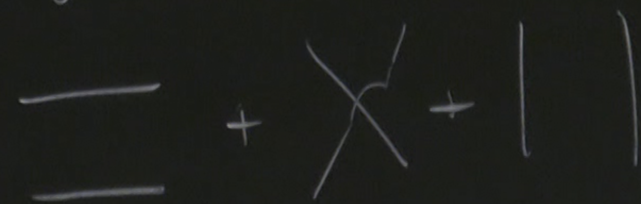
numerator = (

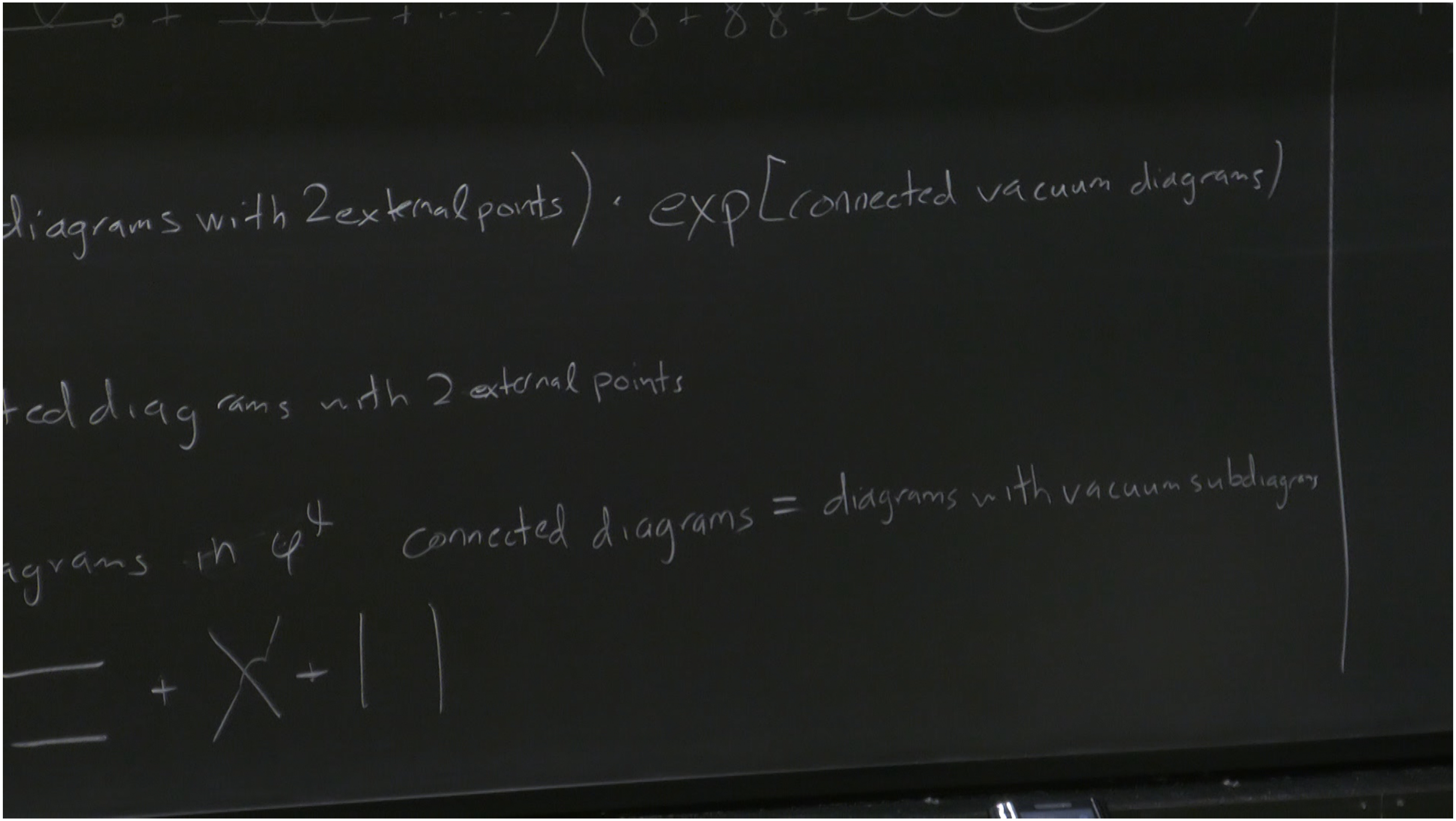
= (connected diagrams with 2 external points)

$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle = \sum$ connected diagrams with 2 external po

for 2 point diagrams in φ^4 connected di

$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$

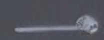



diagrams with 2 external points) · exp[connected vacuum diagrams)
ted diagrams with 2 external points
grams in φ⁴
connected diagrams = diagrams with vacuum subdiagrams

Feynman rules for $\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle$

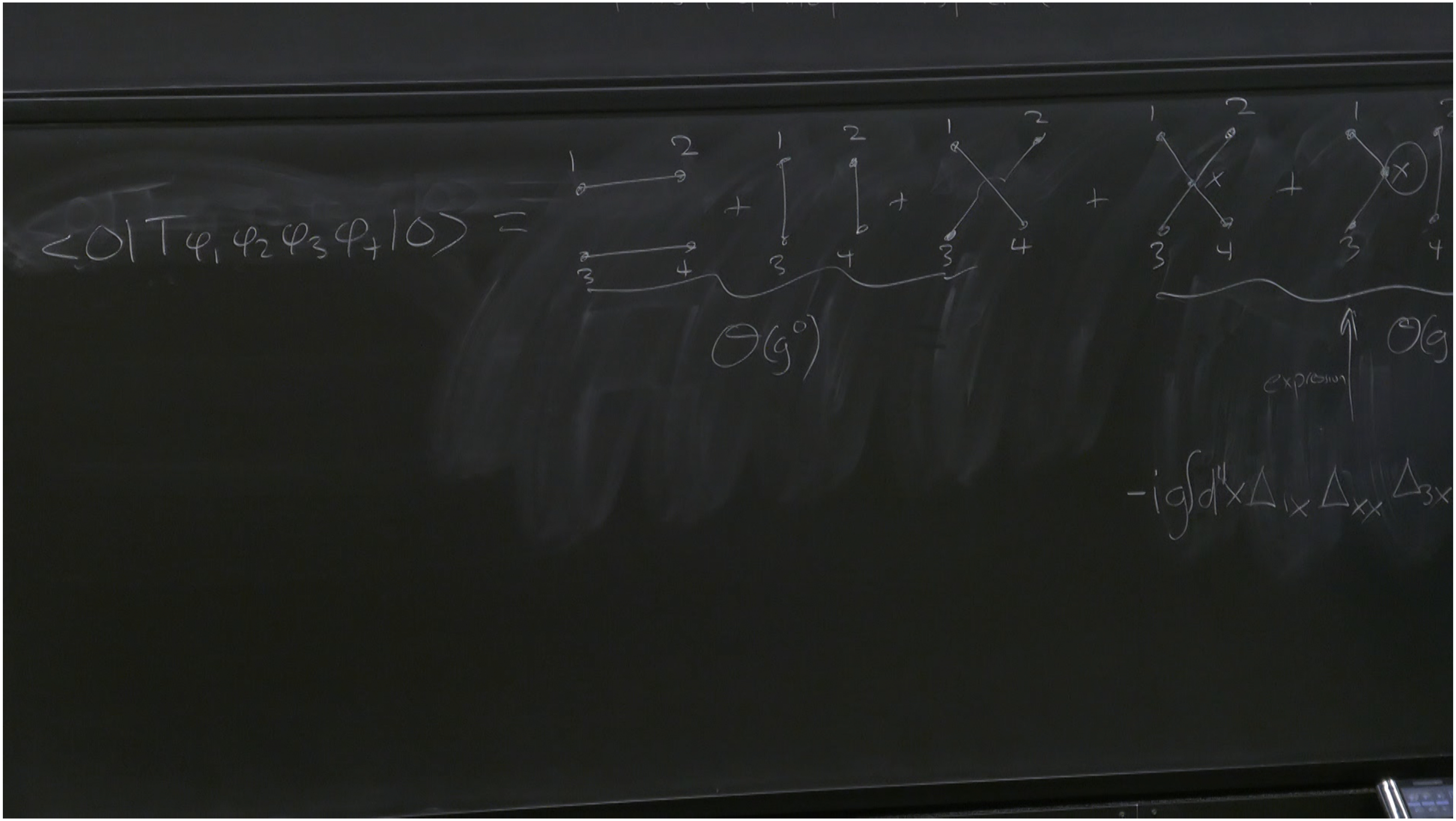
$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = \sum$ Feynman diagrams with no vacuum subdiagrams

1. —— $= \Delta_F$

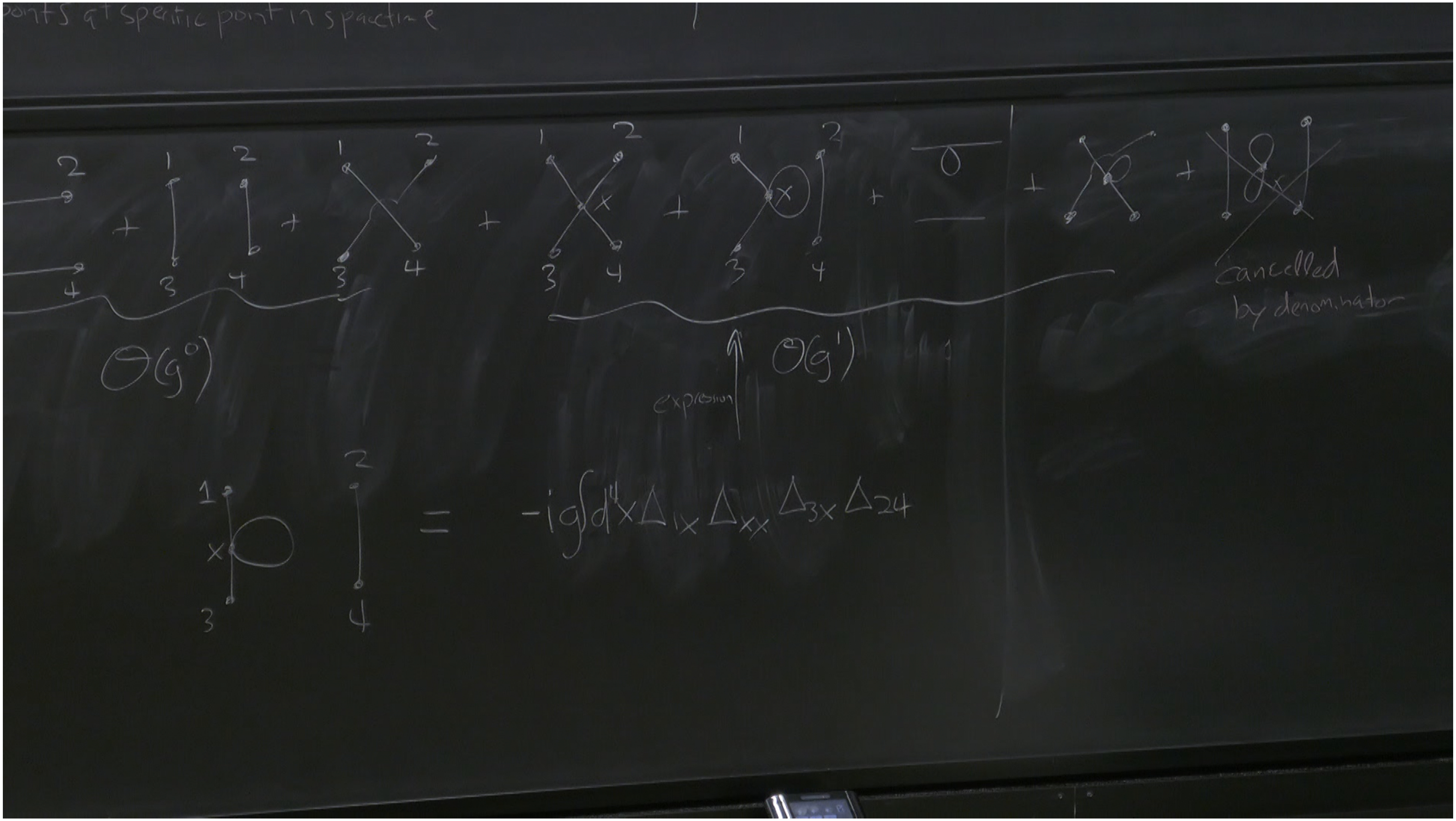
2.  $= 1$

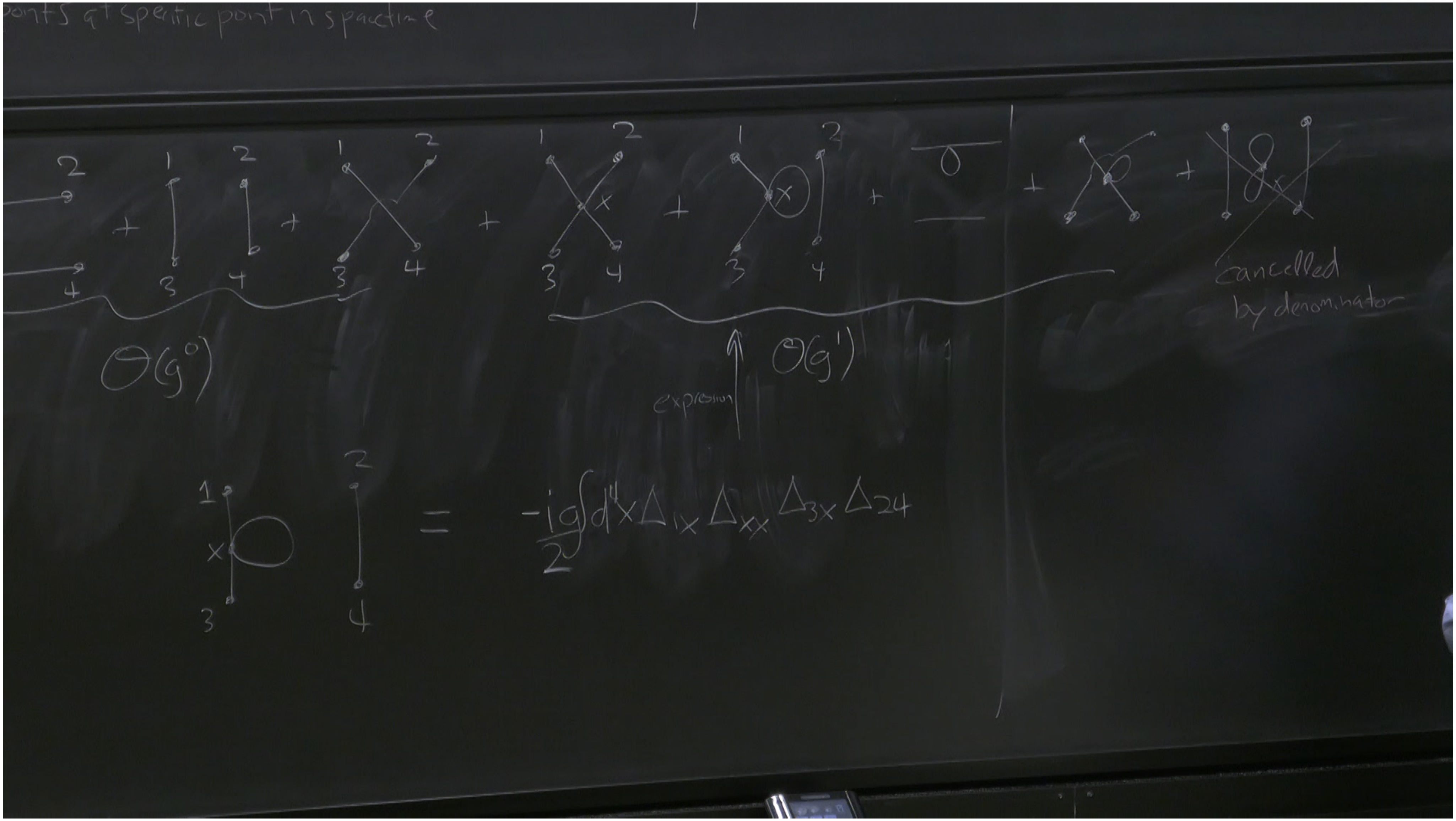
3.  $= -ig\int d^4x$

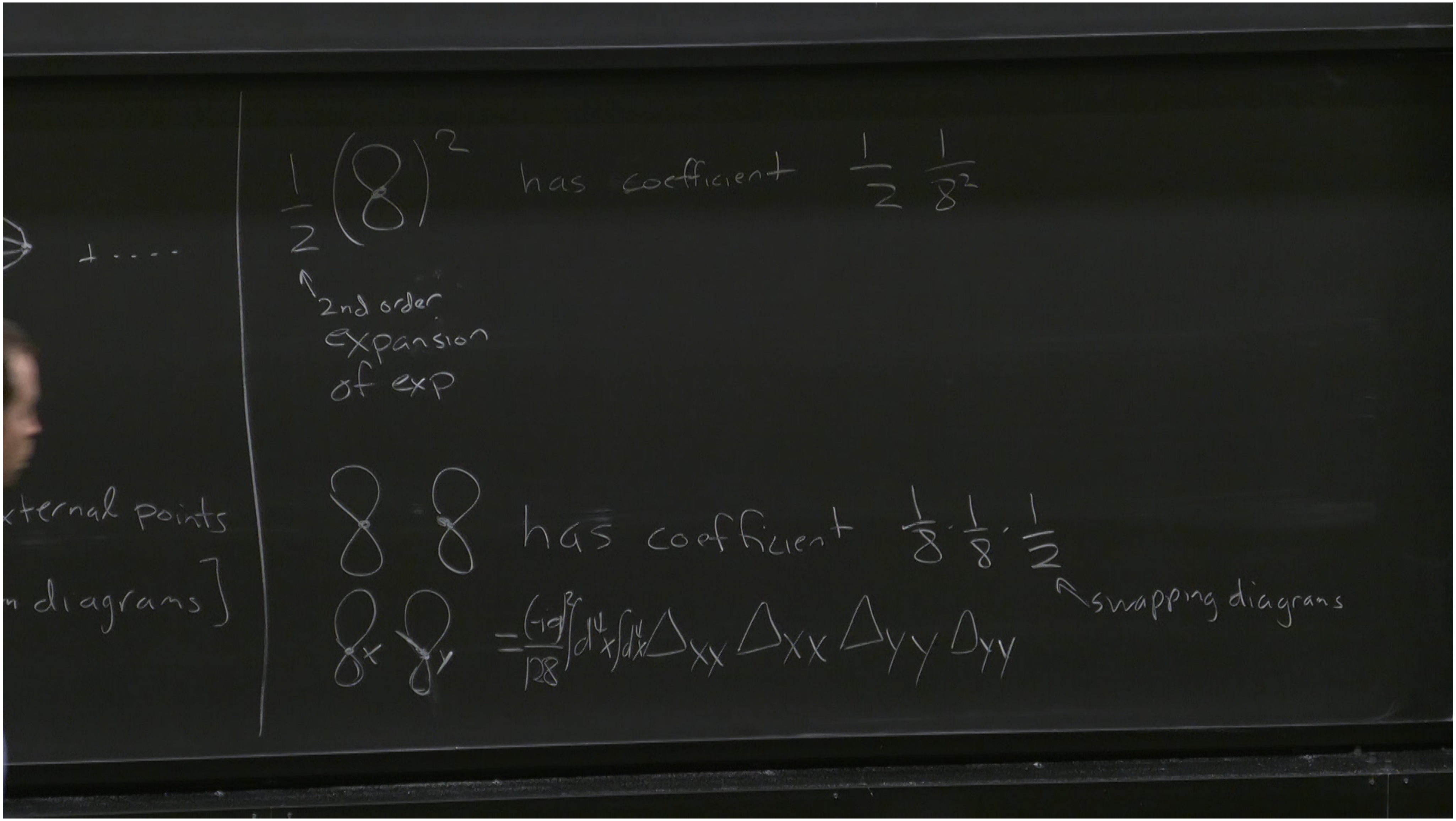
4. Divide by S

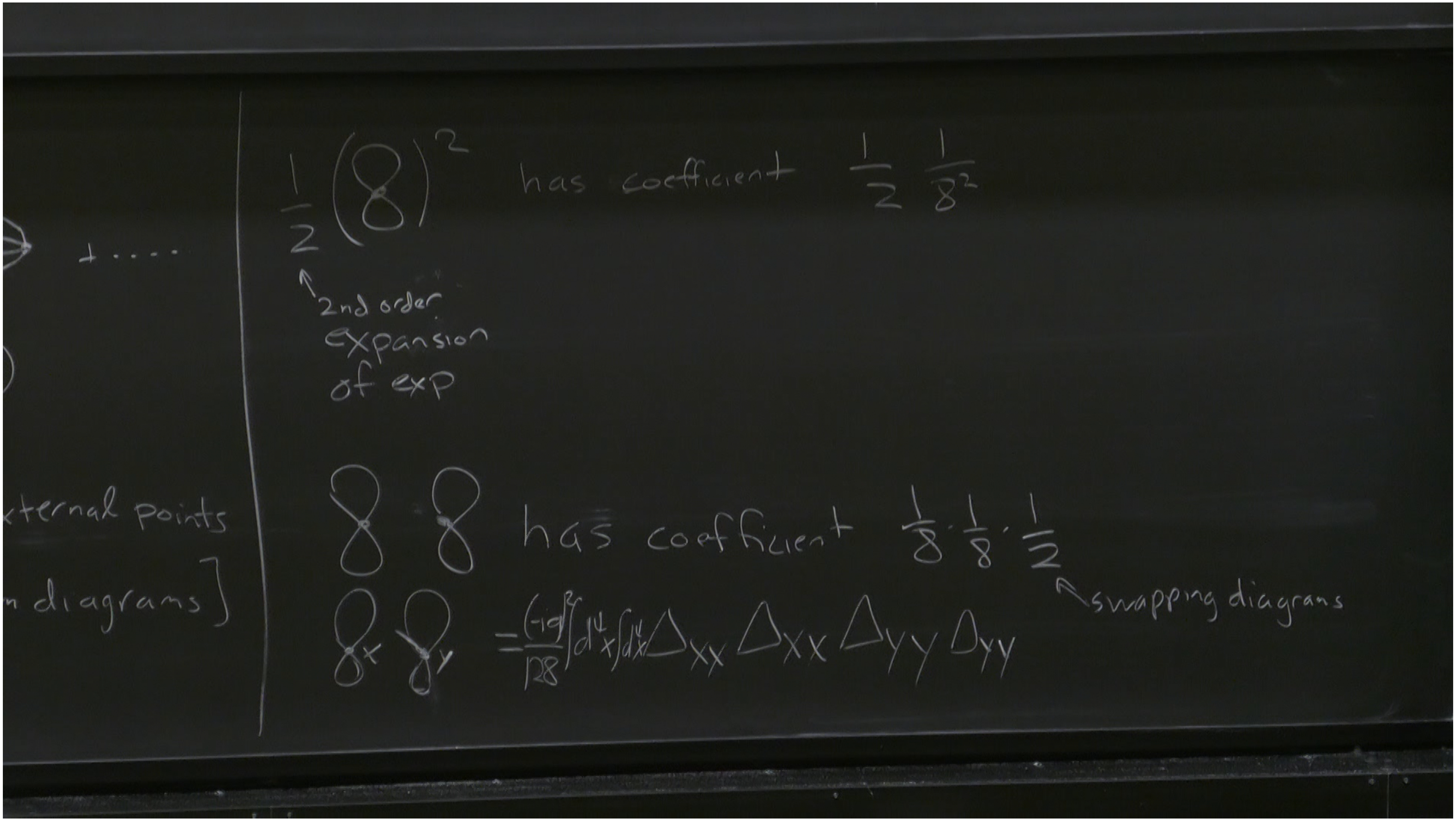


<0|Tφ1φ2φ3φ4|0> =
1 2
3 4
+ + + +
O(g^0)
O(g
expression
-ig∫d^4x Δ1x Δxx Δ3x

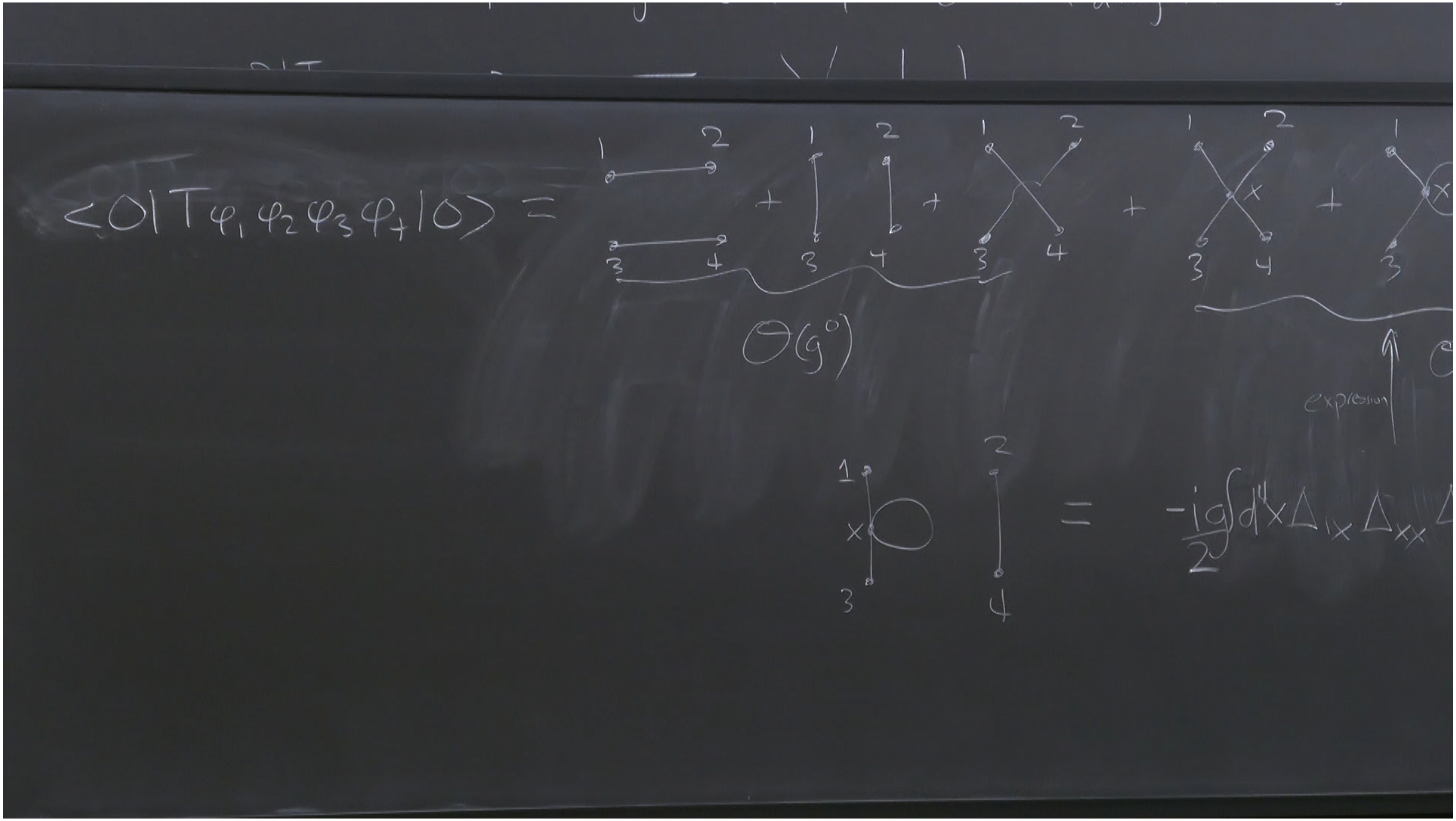

points at specific point in spacetime
cancelled by denominator
O(g°)
O(g')
expression
= -ig∫d⁴x Δ1x Δxx Δ3x Δ24

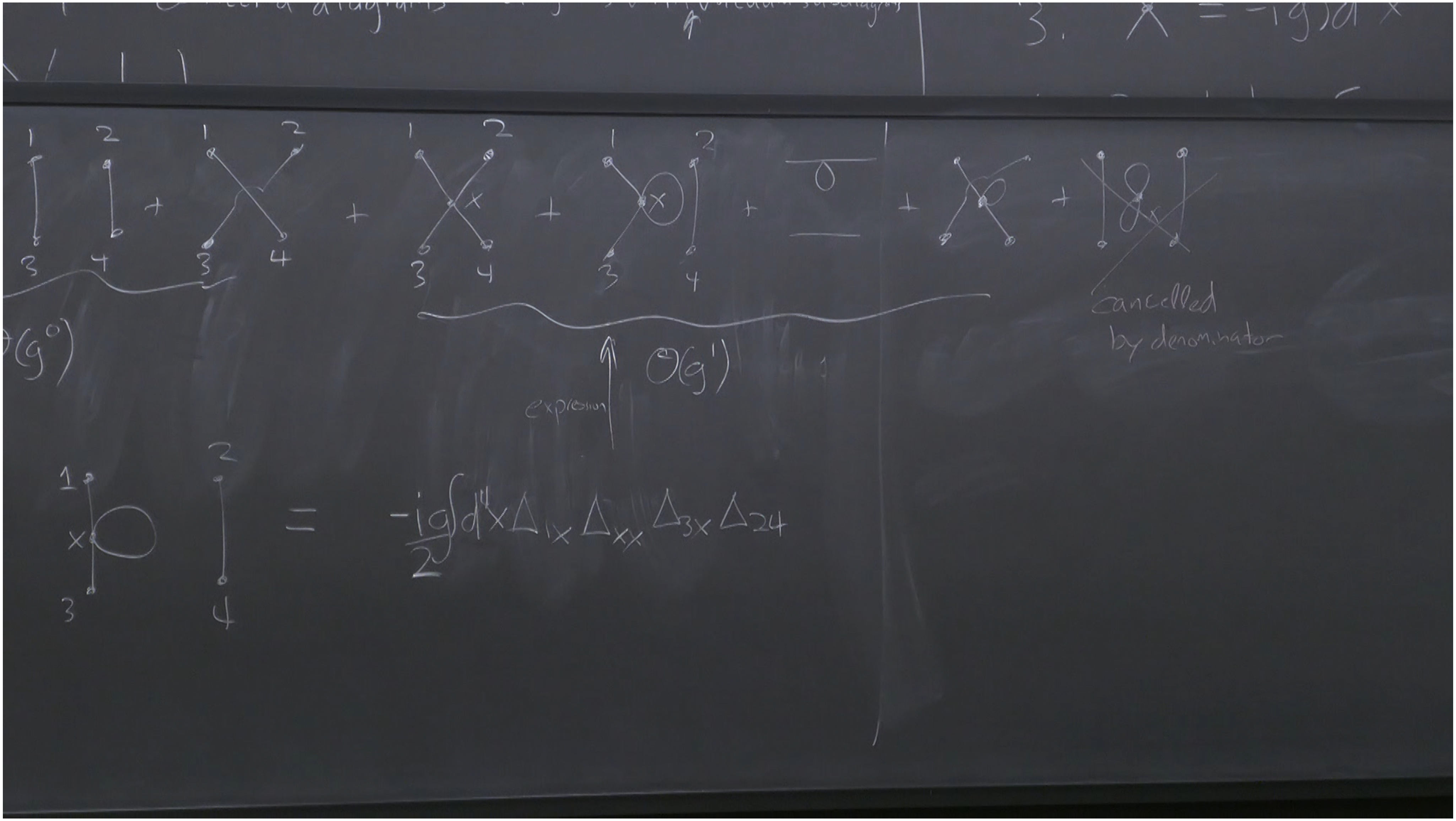

points at specific point in spacetime
cancelled by denominator
O(g^0)
O(g^1)
expression
= -ig/2 ∫d^4x Δ1x Δxx Δ3x Δ24

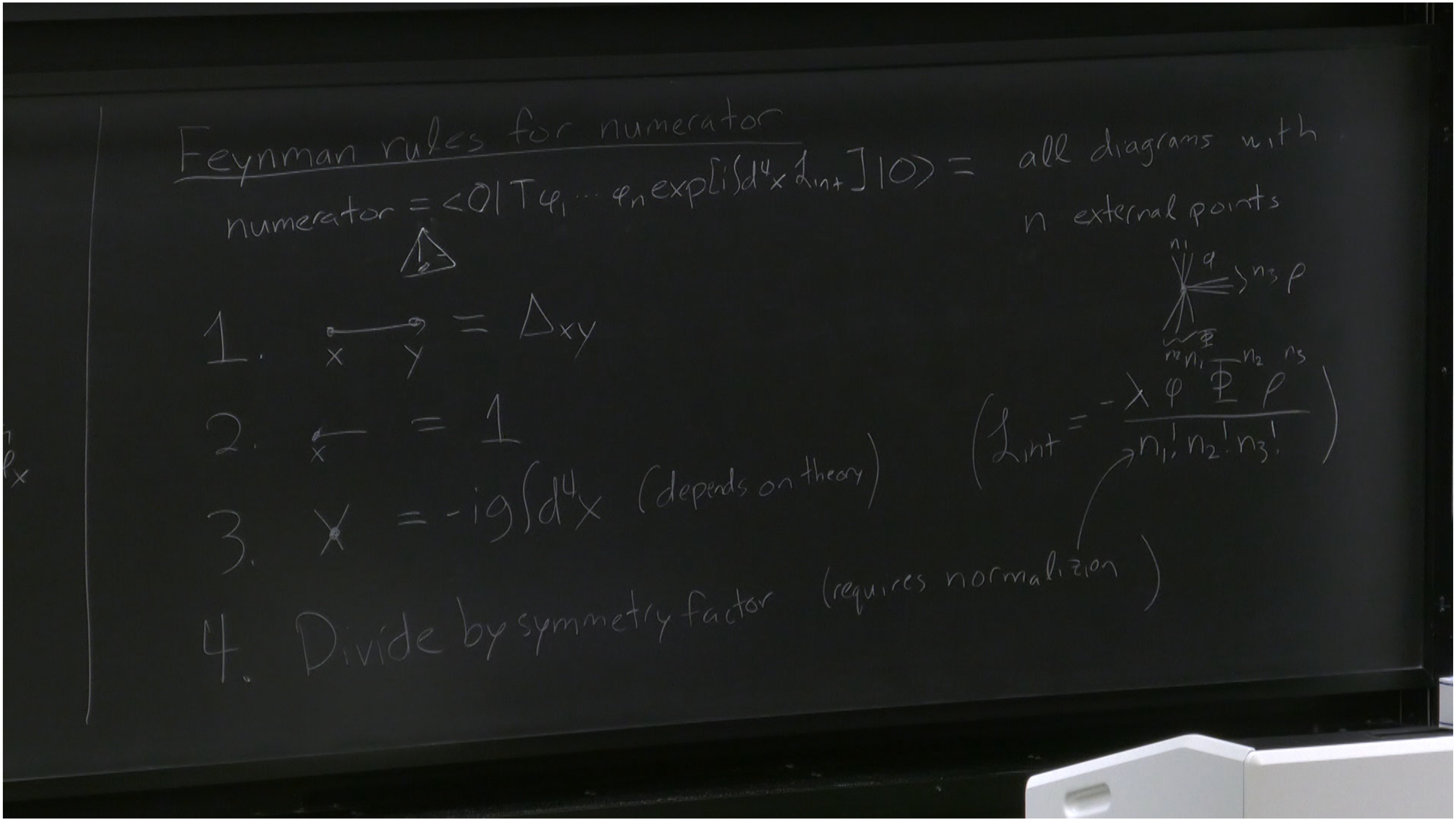

has coefficient 1/2 1/8²
2nd order expansion of exp
+ . . . .
xternal points
n diagrams]
has coeffient 1/8 · 1/8 · 1/2
swapping diagrams

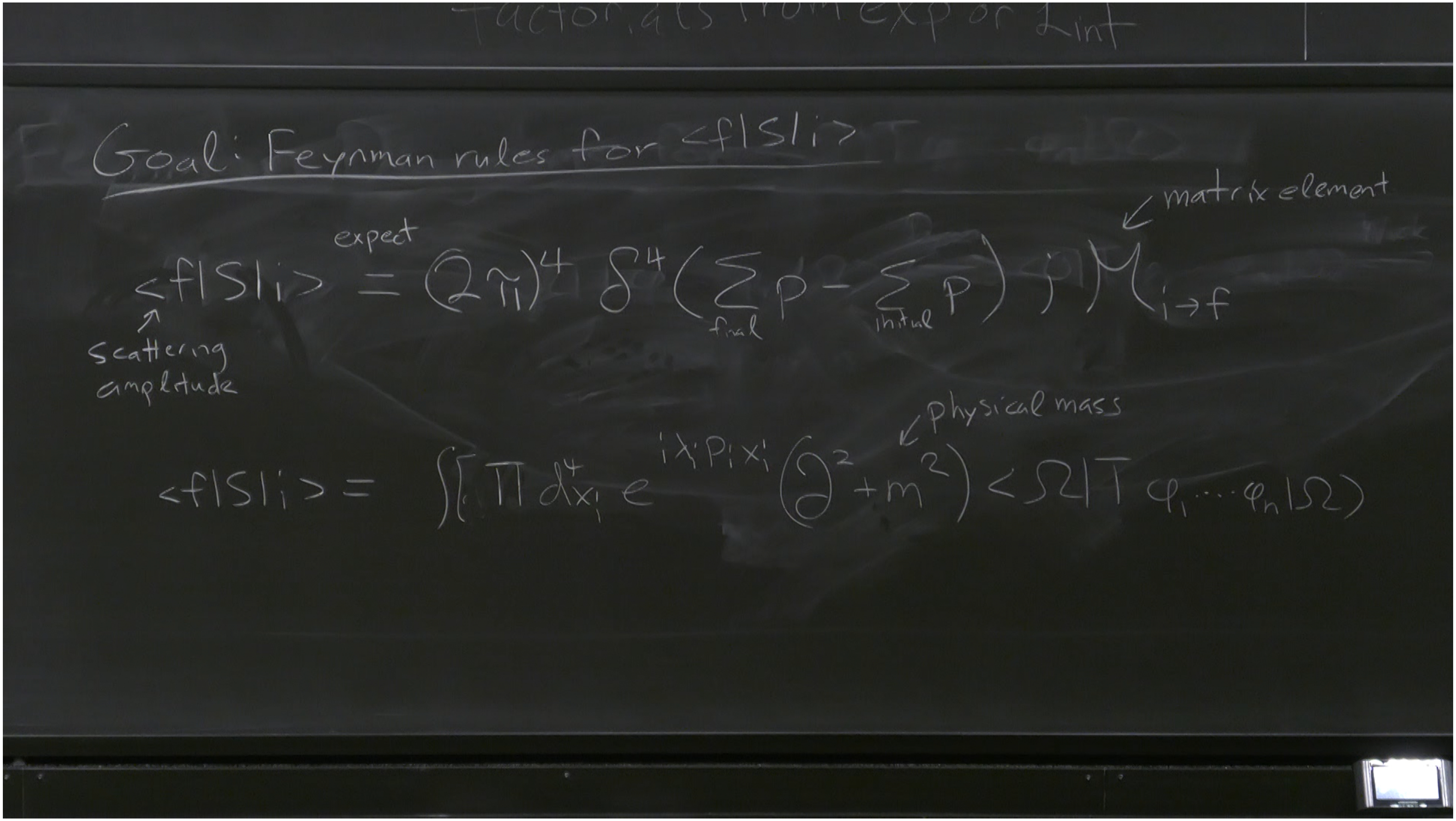


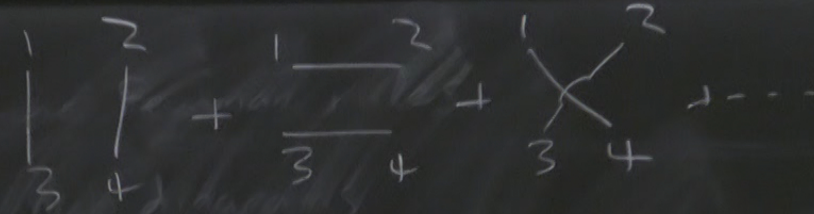
1/2 (8)^2 has coefficient 1/2 1/8^2
+ . . . .
2nd order expansion of exp
xternal points
n diagrams]
8 8 has coefficient 1/8 · 1/8 · 1/2
swapping diagrams
= (-ig)^2/128 ∫d^4x ∫d^4y Δxx Δxx Δyy Δyy

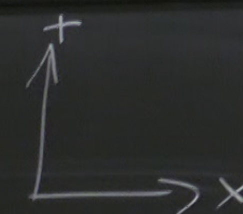

⟨0|T φ1 φ2 φ3 φ4|0⟩ =
1 2 3 4
+
O(g^0)
expression
= −ig/2 ∫ d^4x Δ1x Δxx


cancelled
by denominator
expression
O(g')
= -ig/2 ∫d^4x Δ_1x Δ_xx Δ_3x Δ_24


Feynman rules for numerator
numerator = $\langle 0 | T \varphi_1 \cdots \varphi_n \exp[i\int d^4x \, \mathcal{L}_{int}] | 0 \rangle$ = all diagrams with n external points
1. x y = Δ_{xy}
2. x = 1
3. = $-ig\int d^4x$ (depends on theory)
4. Divide by symmetry factor (requires normalization)
$\left(\mathcal{L}_{int} = -\frac{\lambda \varphi^{n_1} \Phi^{n_2} \rho^{n_3}}{n_1! \, n_2! \, n_3!}\right)$


factorials from exp or $\mathcal{L}_{int}$
Goal: Feynman rules for <f|S|i>
matrix element
expect
$\langle f|S|i\rangle = (2\pi)^4 \delta^4\left(\sum_{final} p - \sum_{initial} p\right) i \mathcal{M}_{i\to f}$
scattering amplitude
physical mass
$\langle f|S|i\rangle = \int \prod_i d^4x_i \, e^{i x_i p_i x_i} (\partial^2 + m^2) \langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle$

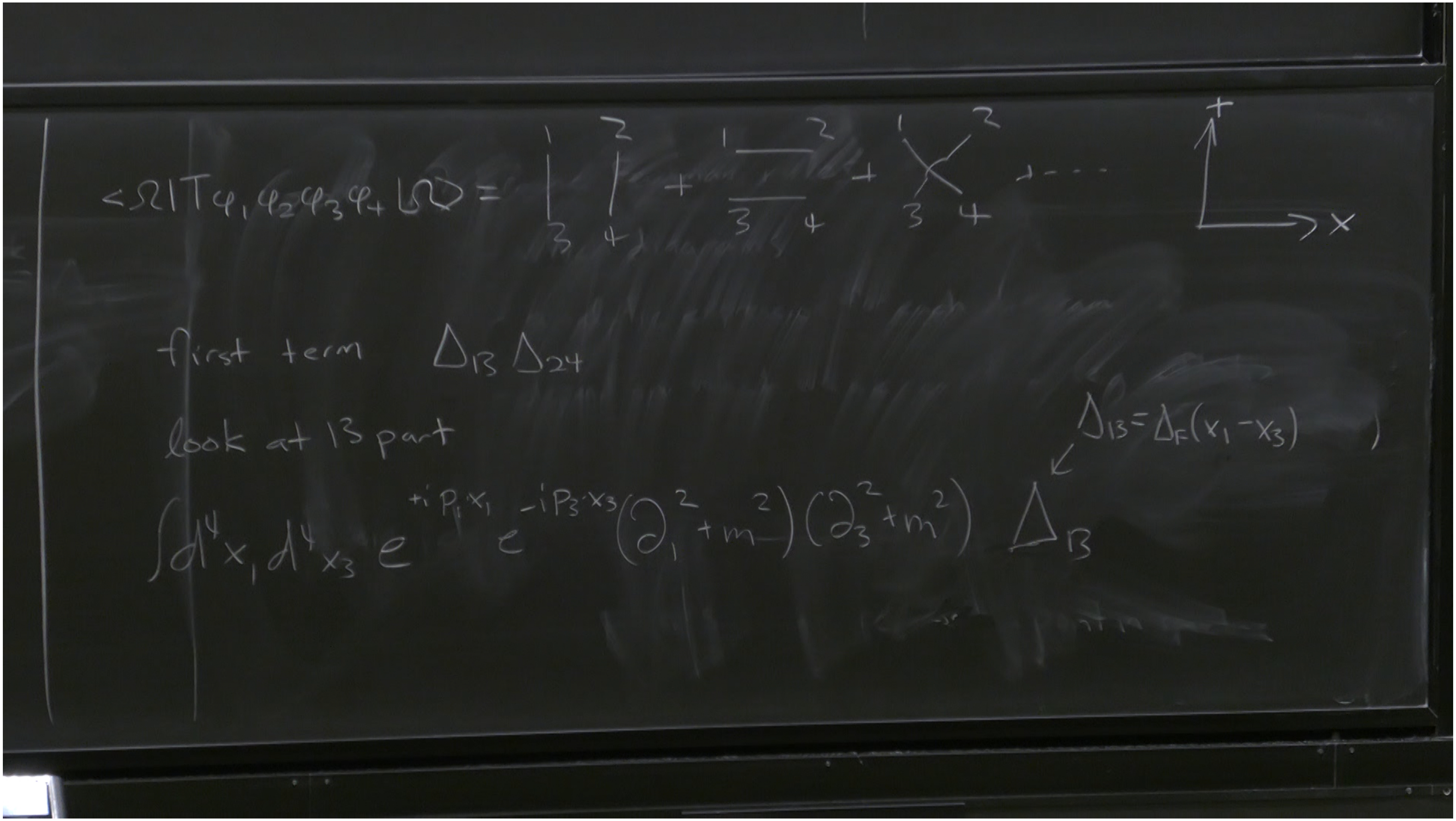
$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$ 



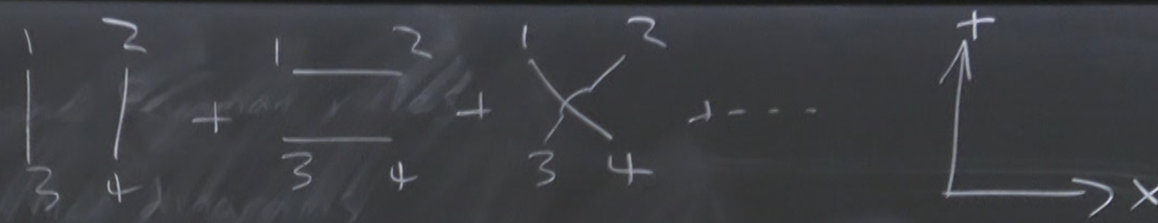
first term $\Delta_{13}\Delta_{24}$

look at 13 part

$$\int d^4x_1\, d^4x_3\, e^{+ip_1\cdot x_1} e^{-ip_3\cdot x_3} (\partial_1^2+m^2)(\partial_3^2+m^2)\, \Delta_{13}$$


$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$
1 2 3 4 + 1 2 3 4 + 1 2 3 4 + ...
t
x
first term $\Delta_{13} \Delta_{24}$
look at 13 part
$\Delta_{13} = \Delta_F(x_1 - x_3)$
$\int d^4x_1 \, d^4x_3 \, e^{+ip_1 \cdot x_1} e^{-ip_3 \cdot x_3} (\partial_1^2 + m^2)(\partial_3^2 + m^2) \, \Delta_{13}$

$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$



first term $\Delta_{13} \Delta_{24}$

look at 13 part

$$P_{13} = \frac{P_1 + P_3}{2} \qquad X_{13} = X_1 + X_3$$

$$\bar{P}_{13} = \frac{P_1 - P_3}{2} \qquad \bar{X}_{13} = X_1 - X_3$$

$$\Delta_{13} = \Delta_F(x_1 - x_3)$$

$$\int d^4x_1\, d^4x_3\, e^{+ip_1\cdot x_1} e^{-ip_3\cdot x_3} (\partial_1^2 + m^2)(\partial_3^2 + m^2)\, \Delta_{13}$$

$$= \frac{1}{2^4}\int d^4X_{13}\, d^4\bar{X}_{13}\, e^{i(\bar{P}_{13}\cdot X_{13} + P_{13}\cdot \bar{X}_{13})}\, \Delta(\bar{X}_{13})$$

$$= \frac{1}{2^4} \int d^4 x_{13} \, e^{i \bar{p}_{13} \cdot x_{13}} \, \tilde{\Delta}_F(p_{13})$$

$$= (2\pi)^4 \, \delta^{(4)}(p_1 - p_3) \, \hat{\Delta}_F(p_1 - p_3)$$

Every connected component will have a 4-momentum conserving delta!

$$= \frac{1}{2^4} \int d^4 x_{13}\, e^{i p_{13} \cdot x_{13}}\, \tilde{\Delta}_F(p_{13})$$

$$= (2\pi)^4\, \delta^{(4)}(p_1 - p_3)\, \hat{\Delta}_F(p_1 + p_3)$$

Every connected component will have a 4-momentum conserving delta!