Title: Quantum Theory Lecture - 092023

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Collection: Quantum Theory 2023/24

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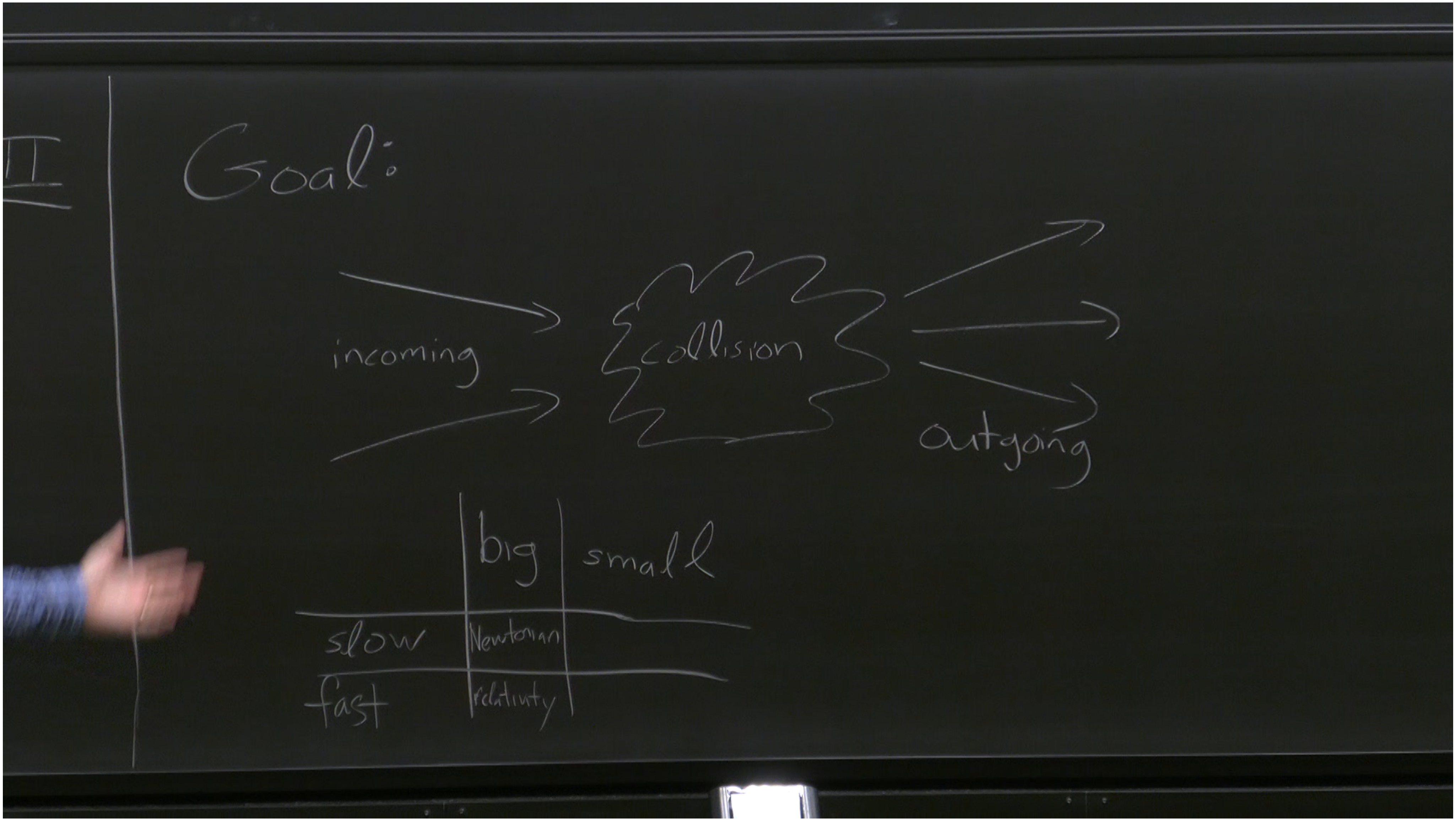
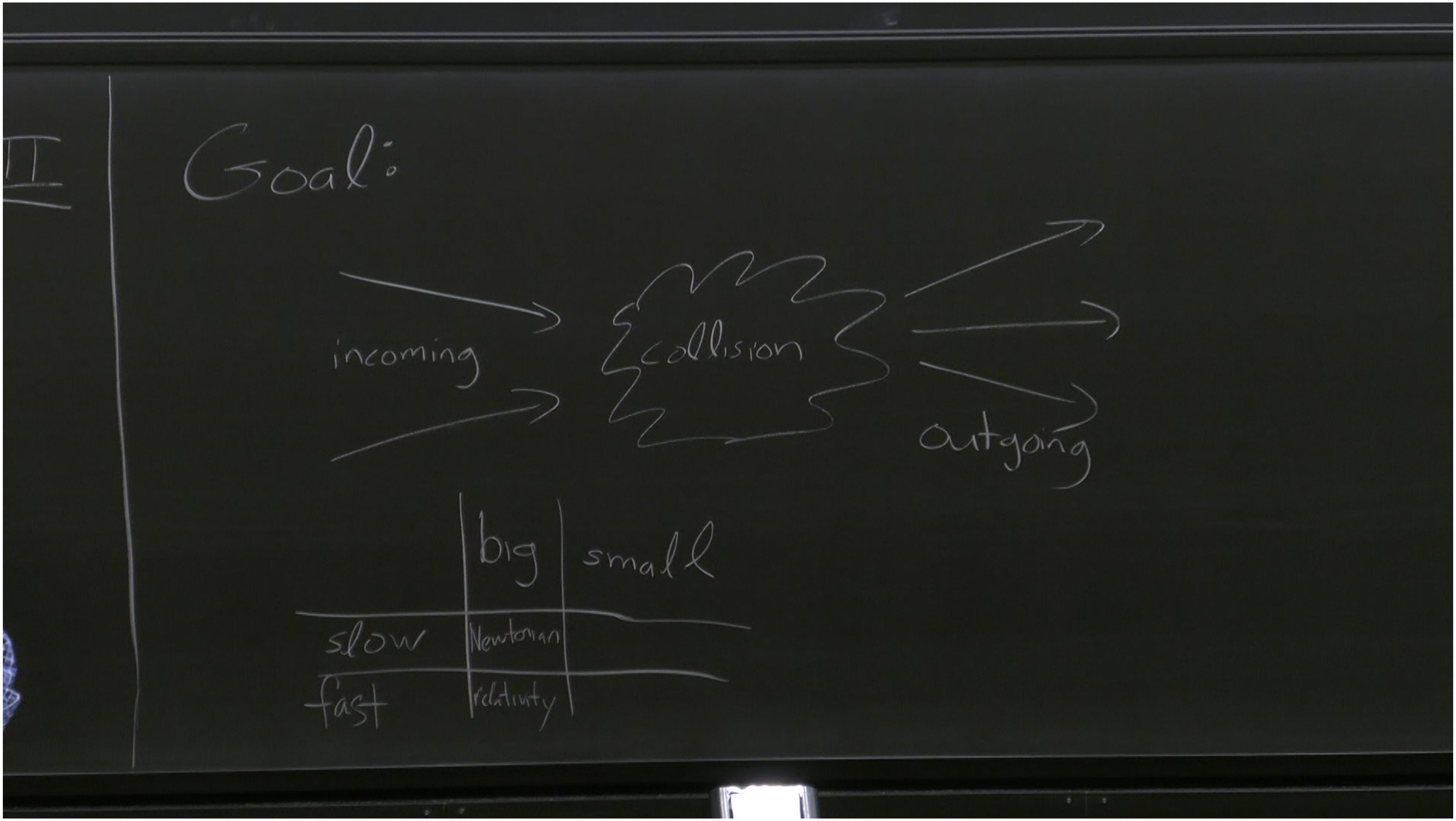

Goal:
incoming
collision
outgoing
big
small
slow
Newtonian
fast
relativity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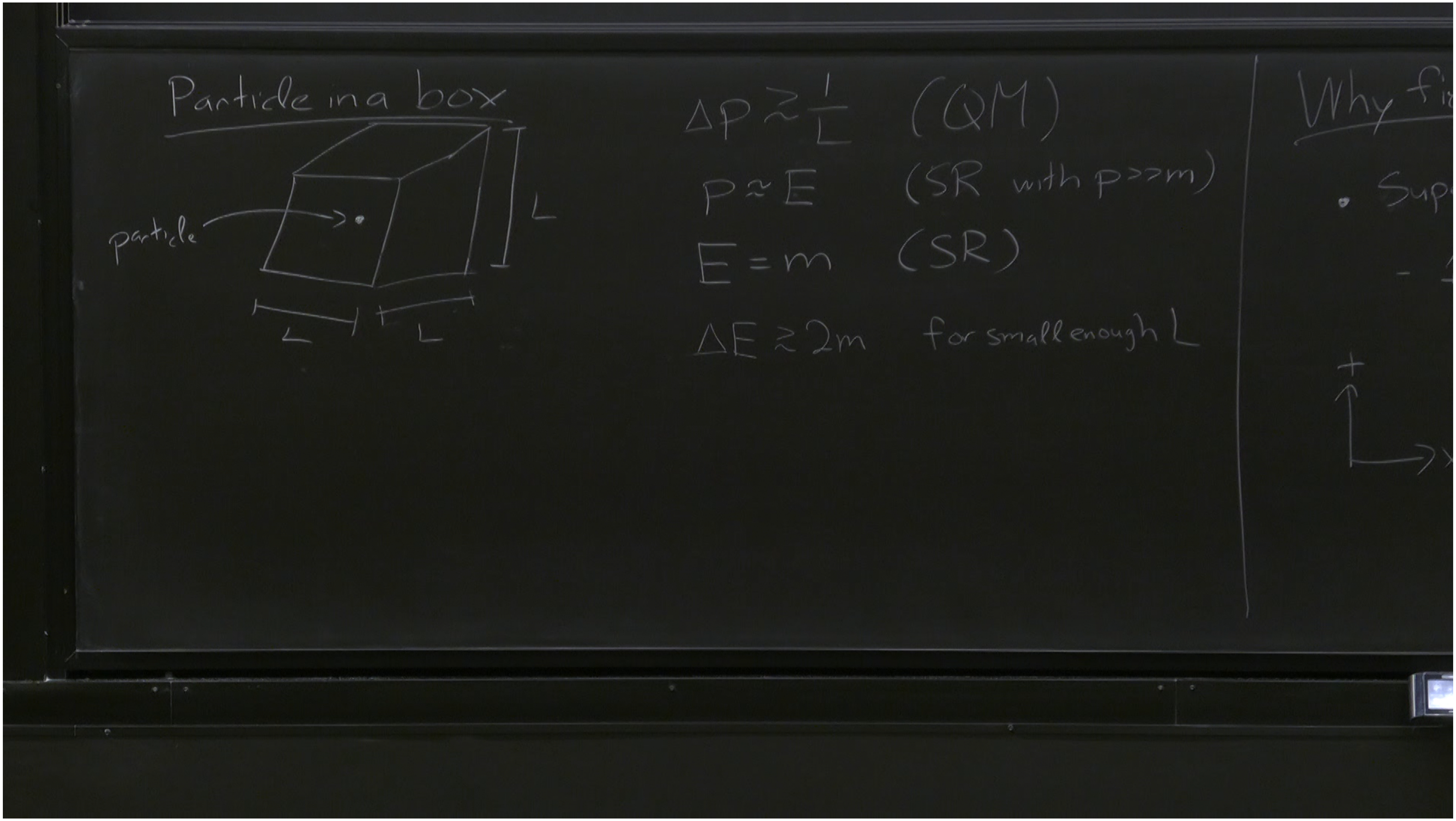
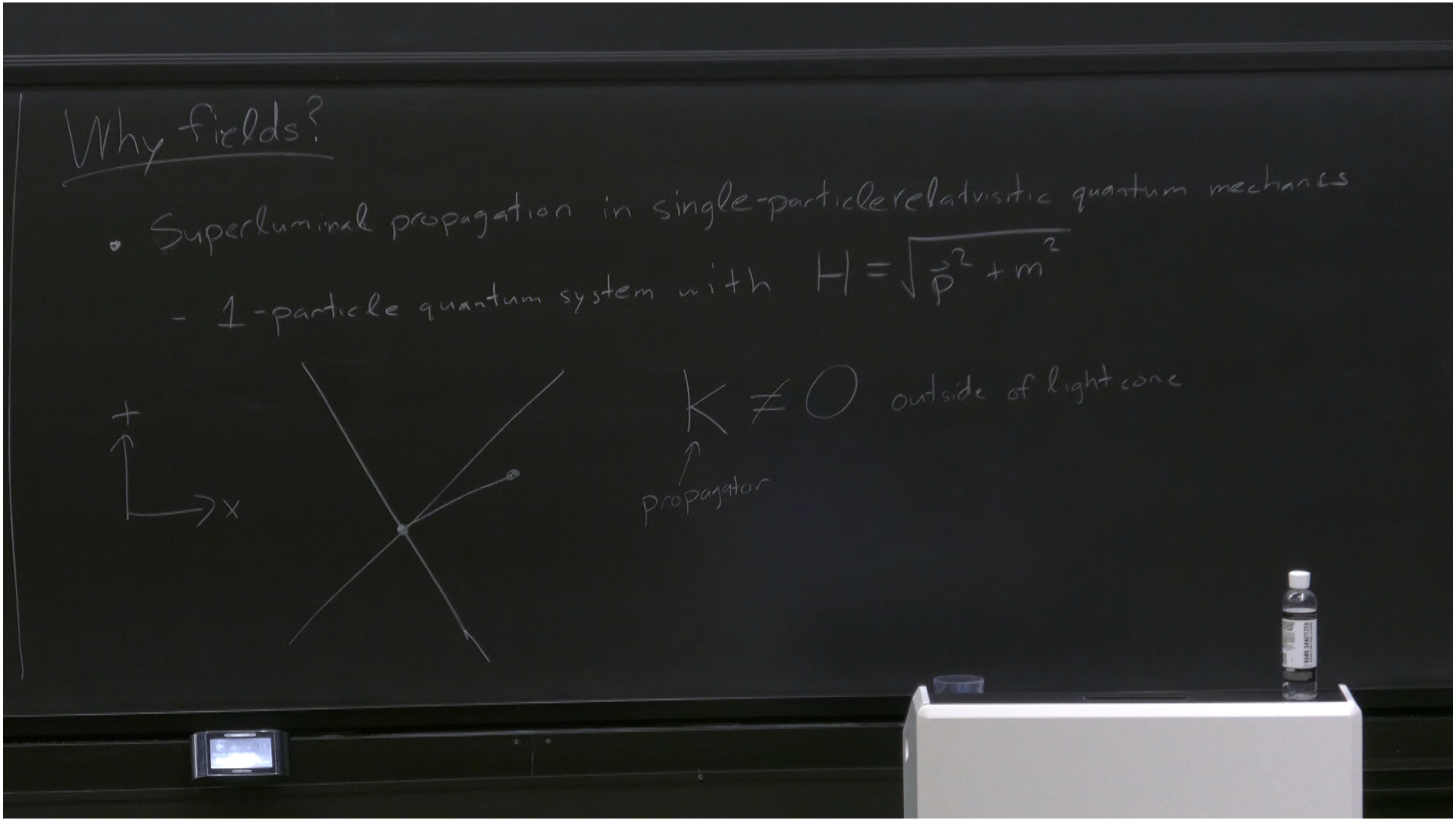

Particle in a box
particle
L
L
L
Δp ≳ 1/L (QM)
p ≈ E (SR with p>>m)
E = m (SR)
ΔE ≳ 2m for small enough L
Why
Sup


Why fields?
• Superluminal propagation in single-particle relativistic quantum mechanics
– 1-particle quantum system with $H = \sqrt{\vec{p}^2 + m^2}$
$K \neq 0$ outside of light cone
propagator
t
x

Quantization

- $\phi \to \int \mathcal{D}\phi$ (QT + QFT II) path integrals
- $\phi \to \hat{\phi}$ (QT + QFT I) canonical

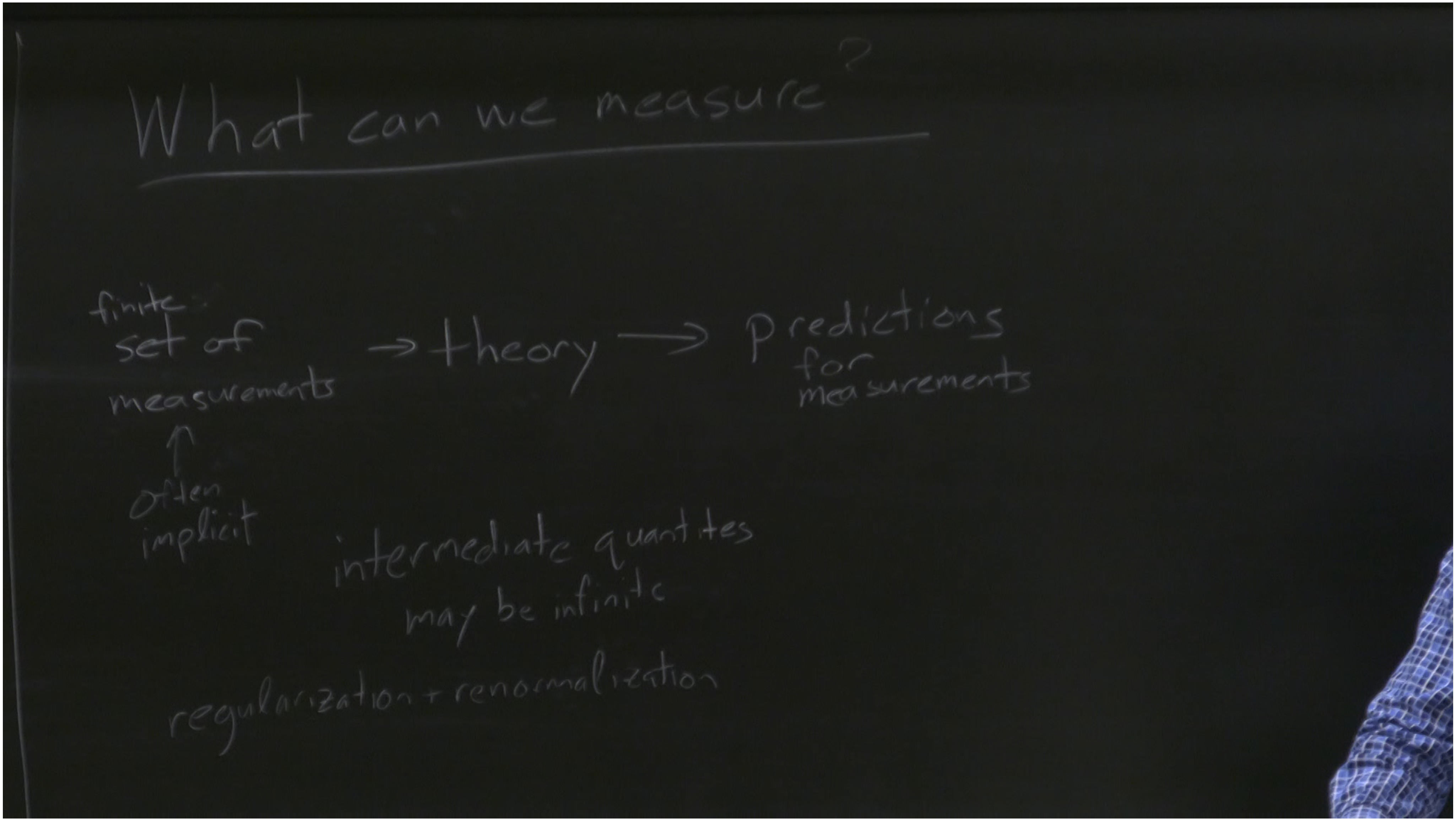
Position + Time

time parameter
length parameter

$$\hat{x}, t \xrightarrow{\text{promote}} \hat{x}, \hat{t} \xrightarrow{\text{string theory}} X^{\mu}(\tau, \sigma)$$

demote $\hat{x}$

$$x, t \to \text{no } \hat{x}$$


What can we measure?
finite set of measurements → theory → predictions for measurements
often implicit
intermediate quantites may be infinite
regularization + renormalization

...cal fields

...on: $\varphi(\vec{x},t) = \varphi(x)$

scalar field

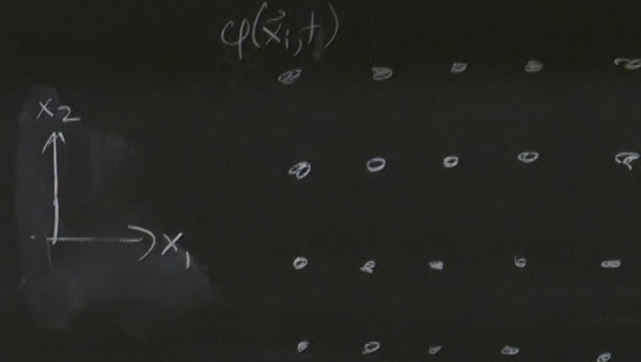
real for now

...ors $\vec{x}$, d^3x, bold

...rs x, d^4x, italics

$$\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad ds^2 = dt^2 - d\vec{x}^{\,2}$$

discrete space



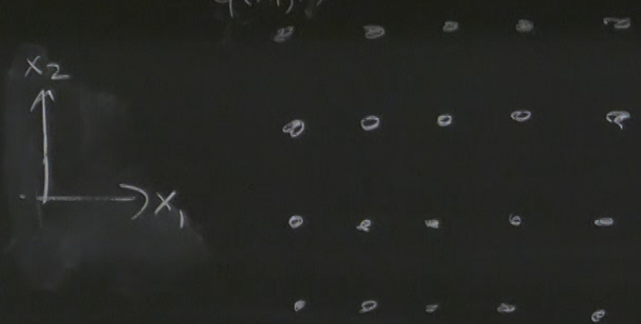
system of many particles

$q_a(t)$

index indicating which particle

discrete space

$\varphi(\vec{x}_i, t)$



system of many particles

$q_a(t)$

index indicating which particle

Analogy

Field theory	Many particles
φ	q
$\vec{x}$	a

Lagrangian

$$S = \int L(q_a, \dot{q}_a)\, dt$$

Lagrangian density

$$S = \int L\, dt = \int \underline{d^3\vec{x}\, dt}\, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$$

$$S = \int d^4x\, d^4y\, \cancel{\mathcal{L}(\varphi(x), \varphi(y), \partial_\mu\varphi(x), \partial_\mu\varphi(y))}$$

$\to$ not local

$\varphi(x)\varphi(y)$

Hamiltonian

$$q_a, p_b \longrightarrow \hat{q}_a, \hat{p}_b$$

co

$$\varphi(\vec{x}), \tilde{\pi}(\vec{x}) \longrightarrow \hat{\varphi}(\vec{x}), \tilde{\pi}(\vec{x})$$

Hamiltonian

$q_a, P_b \longrightarrow \hat{q}_a, \hat{P}_b$

conjugate momentum $\neq$ physical momentum

$\varphi(\vec{x}), \tilde{\pi}(\vec{x}) \longrightarrow \hat{\varphi}(\vec{x}), \tilde{\pi}(\vec{x})$

$$\tilde{\pi}(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}(x)}$$

← density

time derivate

Klein-Gordon Theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$$

EOM: $$(\partial_\mu\partial^\mu + m^2)\varphi = 0$$

$$\varphi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3}\,\tilde{\varphi}(k,t)\,e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{\tilde{\varphi}}(\vec{k},t) - (\vec{k}^2 + m^2)\tilde{\varphi}(\vec{k},t) = 0$$

$$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

$$\varphi(\vec{x},t) = \varphi^*(\vec{x},t)$$

algebra $\Big($ $A(\vec{k}) = B^*(-\vec{k})$ $\qquad \underbrace{-ik\cdot x}$ with $k^0 = E_{\vec{k}}$ $\qquad$ same sign ☹

$$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left(A(\vec{k})\, e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}} + \underbrace{A^*(-\vec{k})\, e^{+iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}_{\text{change of variables } \vec{k} \to -\vec{k}} \right)$$

3-vector ☹

$$+ A^*(\vec{k})\, e^{+ik\cdot x}$$

even minus signs from measure + limits

$+iE_{\vec{k}}t$

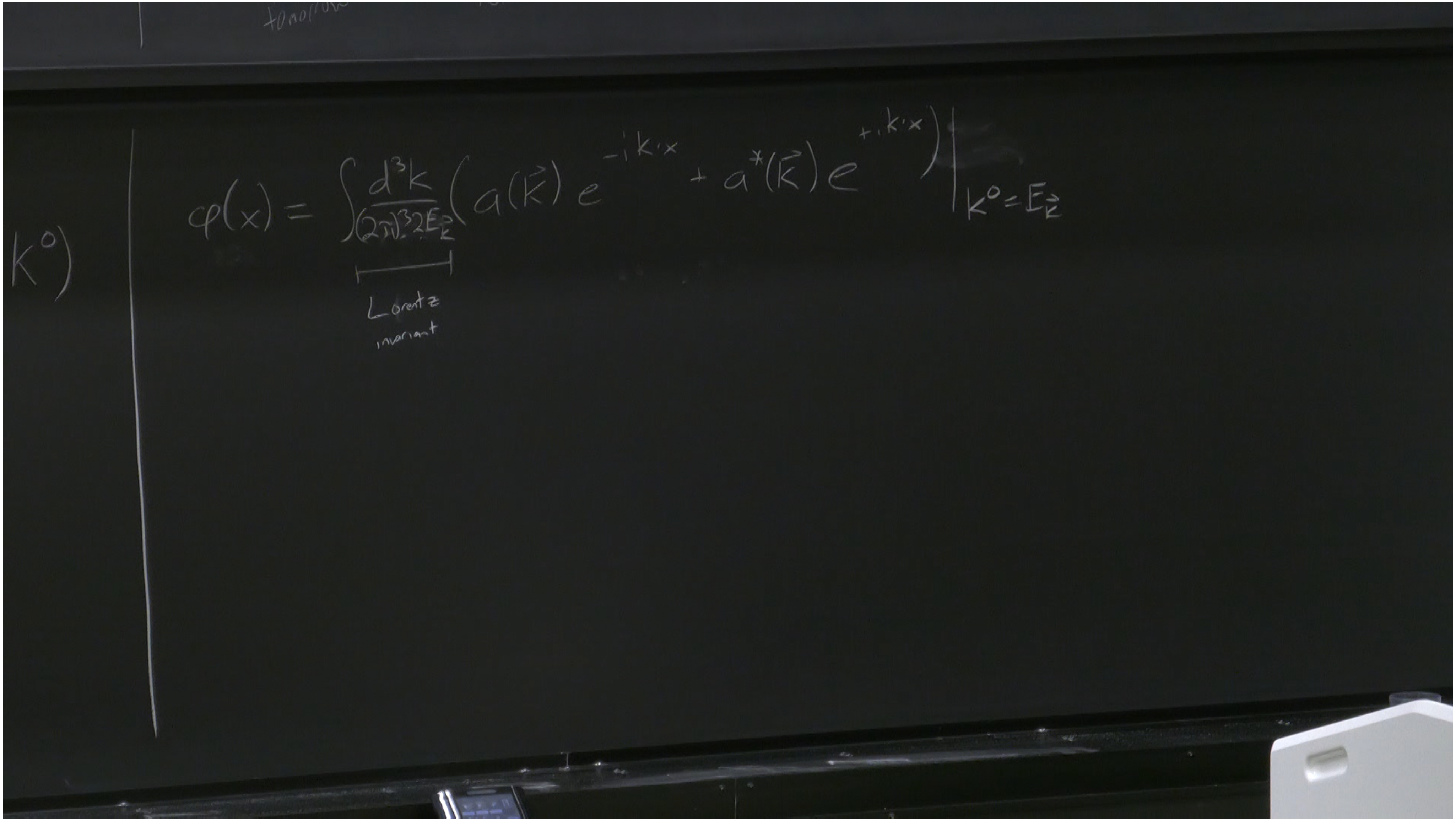
$)e$

$$= \int \frac{d^4k}{(2\pi)^4}\, \delta(\underbrace{k^2 - m^2}_{k^{0\,2} - \vec{k}^2 - m^2})\, \frac{1}{2E_{\vec{k}}}\, \Theta(k^0)$$

$$\delta(f(x)) = \sum_{\substack{\text{roots} \\ x_i}} \frac{\delta(x - x_i)}{|f'(x_i)|}$$

$$A(\vec{k}) = \frac{a(\vec{k})}{2E_{\vec{k}}}$$

$$\varphi(x) = \int d$$


$\varphi(x) = \int \underbrace{\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}}_{\text{Lorentz invariant}} \left(a(\vec{k})\, e^{-ik\cdot x} + a^*(\vec{k})\, e^{+ik\cdot x} \right) \Big|_{k^0 = E_{\vec{k}}}$