

Title: Quantum Theory Lecture - 091123

Speakers: Bindiya Arora, Dan Wohns

Collection: Quantum Theory 2023/24

Date: September 11, 2023 - 10:45 AM

URL: <https://pirsa.org/23090041>

Perturbative Solution

Approximation - Weak Perturbation

First Order Contribution $c_m^{(1)} = \sum_n \int_0^{t_f} dt' e^{i\omega_{mn}t'} \frac{V_{mn}}{i\hbar} c_n(0)$ at $t = 0$, $c_n(0) = \delta_{n,i}$

Transition Probability for $i \rightarrow f$

$$|c_f^{(1)}(t)|^2 = P_{i \rightarrow f}(t) = \frac{V_{fi}^2}{(E_f - E_i)^2} \sin^2\left(\frac{\omega_{fi}t}{2}\right)$$

Fermi's Golden Rule

Energy levels are not discrete but has a finite width

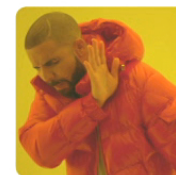
$$\sum_f P_{i \rightarrow f}(t) = \int P_{i \rightarrow f}(t) \overbrace{\rho(E_f)}^{\text{No. of states per unit energy around } E_f} dE_f$$

↓
Over Entire Spectrum

$$= \frac{2\pi |V_{fi}|^2 \rho(E_f) t}{\hbar}$$

Rate of transition
Independent of time
Fermi's Golden Rule

Discrete Width



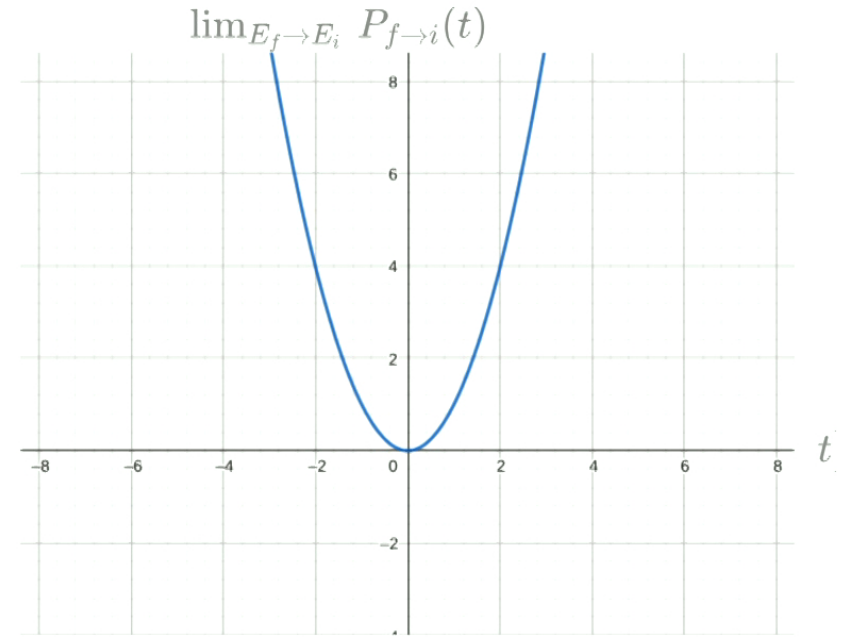
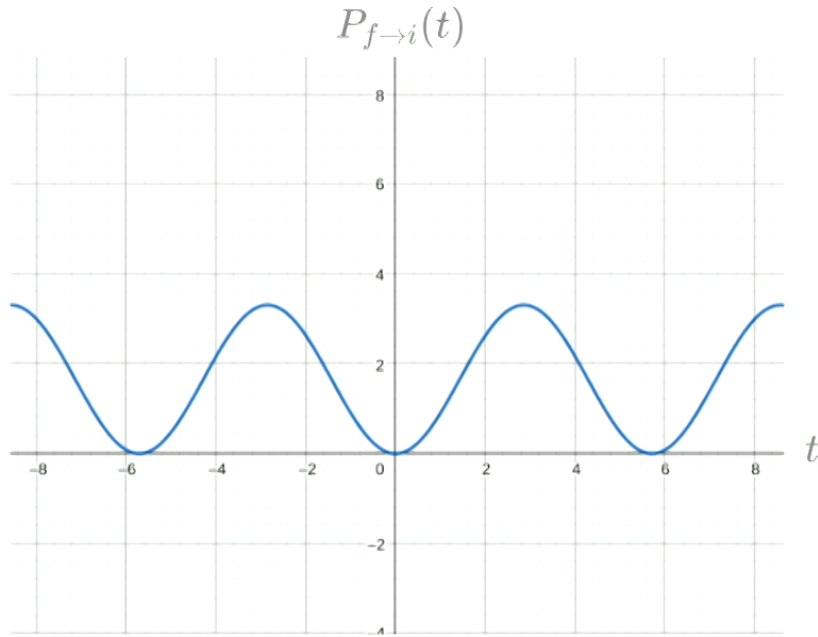
Finite Width



Variation Of Probability

Probability as a function of time

$$\lim_{E_f \rightarrow E_i} P_{f \rightarrow i}(t) = \frac{|V_{fi}|^2 t^2}{\hbar}$$



C.P. Phys
 Thursday 9AM
 Office hours
 (if you have concerns)
 =
 Homework:
 handwritten Ok
 if intelligible
 (if you take pictures
 please compress them!)

$$P_{i \rightarrow f} = \frac{4|V_{fi}|^2}{\hbar^2} \frac{\sin^2\left(\frac{\omega_{fi} t}{2}\right)}{\omega_{fi}^2}$$

$$\sum_f P_i$$

- I $\omega_{fi} \neq 0$ $f \equiv \equiv$
- II $\omega_{fi} = 0$ $i \text{ --- }$

$$P_{i \rightarrow f} \propto t^2 \Rightarrow P_{i \rightarrow f} \rightarrow t$$

$$W \propto t$$

$$\frac{1}{\omega_{fi}^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right)$$

f \equiv ΔE_f

i

$\propto t^2 \Rightarrow P_{i \rightarrow f} \rightarrow t$

x t

$$\sum_f P_{i \rightarrow f} = \int P_{i \rightarrow f}(t) \underbrace{\rho(\epsilon_f)}_{\substack{\text{no. density} \\ \text{no. of state per unit} \\ \text{energy around } \epsilon_f}} d\epsilon_f$$

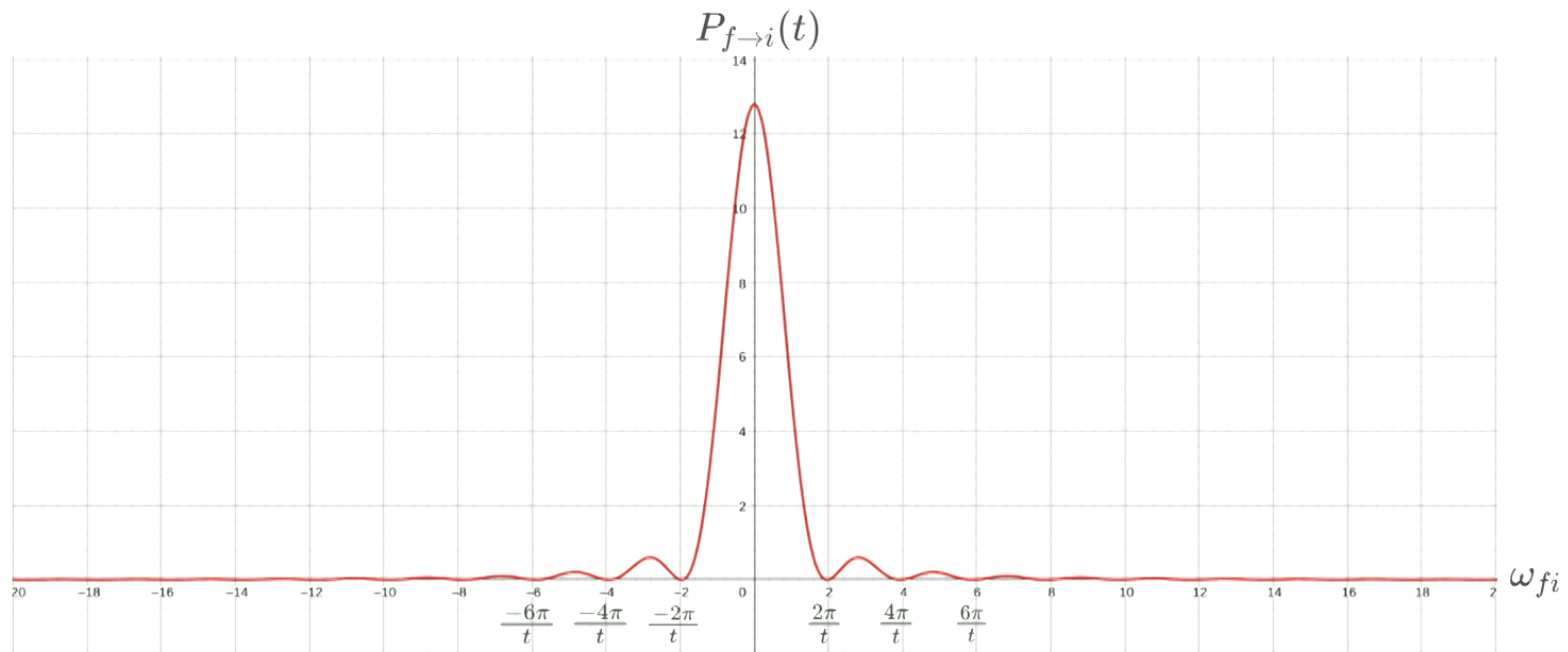
$$\sum_f P_{i \rightarrow f} = \int \frac{4|V_{fi}|^2}{\hbar^2} \frac{\sin^2\left(\frac{\omega_{fi} t}{2}\right)}{\omega_{fi}^2} \rho(\epsilon_f) d\epsilon_f$$

max cont $\rightarrow \omega_{fi} = 0 \quad \epsilon_f = \epsilon_i$

$$= \frac{4|V_{fi}|^2}{\hbar^2} \rho(\epsilon_f) \int \frac{\sin^2\left(\frac{\omega_{fi} t}{2}\right)}{\omega_{fi}^2} d\epsilon_f$$

Variation Of Probability

Probability as a function of energy difference



Brave File Edit View History Bookmarks People Tab Window Help

Quantum Theory Lecture 3 - Google Two Level System (Probability vs Time) Two Level System (Probability vs Time)

geogebra.org/m/nevcyngq

Guru Nanak Dev U... Qiskit Student My Drive - Google... Untitled0.ipynb -... Slack | 2022-ap-r... Data - ATOM Web... Application List, A... Atom Website Tra... Documentation fo... https://www1.udel... Bindya Arora Web... Data - Dropbox

GeoGebra ASSIGN

Two Level System (Probability vs Energy Difference)

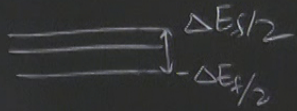
Author: VIPuL Badhan

The plot displays a red curve representing probability versus energy difference. The x-axis is labeled from -20 to 24 in increments of 2. The y-axis is labeled from -2 to 16 in increments of 2. The curve has a primary peak at x=0 with a height of approximately 13. There are smaller oscillations on either side, with a secondary peak at x ≈ ±2.5 with a height of approximately 1. A slider at the top left is labeled 't = 1.6'.

Mac OS dock icons: Finder, Launchpad, Terminal, Notes, Safari, Firefox, Adobe Acrobat, Zoom, Music, and a white cup icon.

Integrand \propto

$$\frac{\sin^2\left(\frac{\omega_{fi} t}{2}\right)}{\omega_{fi}^2 t^2} d\omega_{fi} = \frac{\pi \hbar t}{2}$$

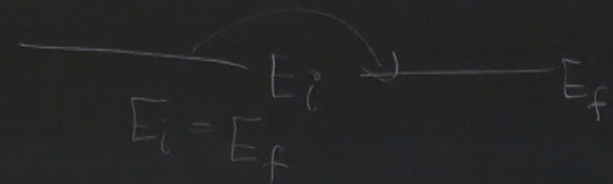
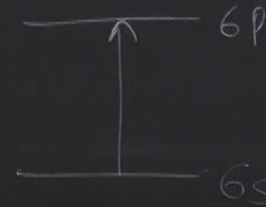


$$\sum_f P_{i \rightarrow f} = \frac{4 |V_{fi}|^2 \rho(E_f) \pi \hbar t}{\hbar^2} \propto t$$

$\propto t$

$W = \text{constant}$

$$E_p = E_i$$



Brave File Edit View History Bookmarks People Tab Window Help

Quantum Theory Lecture 3 - Google Docs presentation/d/1-fwQUXbKaoMcWwoHV22GaRxCXKuJrmSQ1MPdsq9vNrk/edit#slide=id.g2609f83558d_2_106

Quantum Theory Lecture 3

Variation Of Probability

Probability as a function of energy difference

Let us plot the integrand as a function of the integration variable ω_{fi} (energy difference). The result is shown in and exhibits a main central lobe followed by symmetrically arranged lobes of

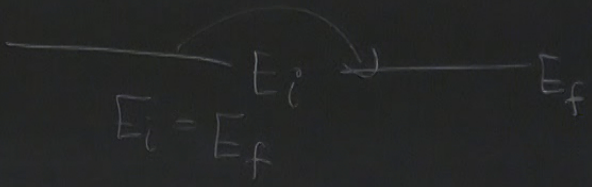
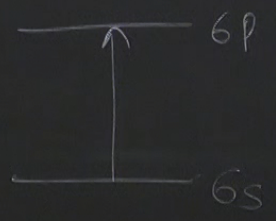
Mac OS Dock: Finder, Launchpad, Terminal, Notes, Safari, Firefox, Adobe Acrobat, Zoom, Music, Trash

$$\propto \int_{E_f - \Delta E_f/2}^{E_f + \Delta E_f/2} \frac{\sin^2\left(\frac{\omega_{fi} t}{2}\right)}{\omega_{fi}^2 t^2} d\omega_{fi} = \frac{\pi \hbar t}{2}$$

$$= \frac{4 |V_{fi}|^2 P(E_f) \pi \hbar t}{\hbar^2}$$

$\propto t$
 $= \text{constant}$

$$E_f = E_i \Rightarrow P(E_i)$$



- ⇒ Single/composite system QM.
- ⇒ Problem, with QM.
- ⇒ Density matrix/operators
define.
properties
time evolution
- ⇒ Density matrix for spin (pure)
⇒ for mixture of spins

Single $\Rightarrow |\psi\rangle, A, \text{S.E.}, \langle m|\psi\rangle^2$
 $= \sum_i c_n |m\rangle$

Composite (2 particles)

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

$$|\psi_A\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \checkmark$$

$$|\psi_B\rangle = |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \checkmark$$

$$|\psi\rangle_{AB} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

operators (local)

$$\hat{O} = \hat{A} \otimes \hat{I}_B + \hat{I}_A \otimes \hat{B}$$

$$\begin{aligned} \hat{A} &= |1\rangle\langle 0| \\ \hat{O} &= \hat{A} \otimes \hat{I}_B \\ &= |1\rangle\langle 0| \otimes \hat{I}_B \\ &\quad \downarrow \quad \downarrow \\ &\quad (|\psi_A\rangle \otimes |\psi_B\rangle) \end{aligned}$$

C.P. Phys
Thursday 9AM
Office hours
(if you have concerns)

=
Homework
handwritten Ok
if intelligible
(if you take pictures
please caption them!)

$$H = H_A \otimes I_B + I_A \otimes H_B$$
$$U = e^{-iHt/\hbar} = e^{-iH_A t/\hbar} \otimes e^{-iH_B t/\hbar}$$

$$|\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

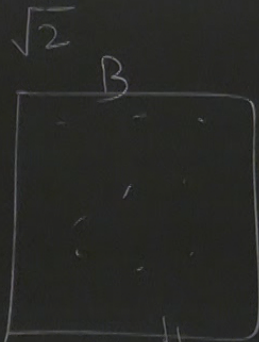
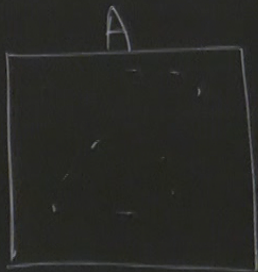
$$= |0\rangle_A |0\rangle_B \Rightarrow \text{Joint measurement}$$

Partial measurement

Partial measurement

Singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle^A \otimes |-\rangle^B - |-\rangle^A \otimes |+\rangle^B)$$



Classical mix of $|+\rangle |-\rangle$

Alice's own corr	Joint state	Bob's state
$\Pr[+]=\frac{1}{2}$	$ +\rangle_x^A \otimes -\rangle_x^B$	$ -\rangle_x$
$\Pr[-]=\frac{1}{2}$	$ -\rangle_x^A \otimes +\rangle_x^B$	$ +\rangle_x$

$$|\psi\rangle^B \neq \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

Ensemble

$$\left\{ \frac{1}{2}, |+\rangle \right\}, \left\{ \frac{1}{2}, |-\rangle \right\}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \checkmark$$

Operators (local)

$$\hat{O} = \hat{A} \otimes \hat{I}_B + \hat{I}_A \otimes \hat{B}$$

Alice's own coin	Joint state	Bob's state
$\Pr[+] = \frac{1}{2}$	$ +\rangle_x^A \otimes -\rangle_x^B$	$ -\rangle_x$
$\Pr[-] = \frac{1}{2}$	$ -\rangle_x^A \otimes +\rangle_x^B$	$ +\rangle_x$

$$|\psi\rangle^B = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

ensemble

$$\left\{ \frac{1}{2}, |+\rangle \right\}, \left\{ \frac{1}{2}, |-\rangle \right\}$$

$$\Rightarrow \left\{ \frac{1}{2}, |+\rangle_x \right\}, \left\{ \frac{1}{2}, |-\rangle_x \right\}$$

$$|+\rangle_z, |-\rangle_z \Rightarrow Z$$

$$|+\rangle_x, |-\rangle_x \Rightarrow X$$

$$|+\rangle_y, |-\rangle_y \Rightarrow Y$$

Bob's ^{State} \rightarrow $\left\{ \frac{1}{\sqrt{2}} |+\rangle \right\}$ $\left\{ \frac{1}{\sqrt{2}} |-\rangle \right\}$ $\left\{ \frac{1}{\sqrt{2}} |+\rangle \right\}$ $\left\{ \frac{1}{\sqrt{2}} |-\rangle \right\}$

② $|\psi\rangle = \frac{1}{\sqrt{2}} |+\rangle + e^{i\phi} \frac{1}{\sqrt{2}} |-\rangle$

ϕ_1 ϕ_2 --- $\phi_{100} \Rightarrow$ not good coherence.

$$\left\{ \frac{1}{4} |-\rangle \right\}$$

⇒ not good coherence

③ pure states \Rightarrow $|+\rangle$, $|-\rangle$
mixtures $\frac{|+\rangle + |-\rangle}{\sqrt{2}}$
maximally mixed states
50% $|+\rangle$ 50% $|-\rangle$

Density matrix formulation

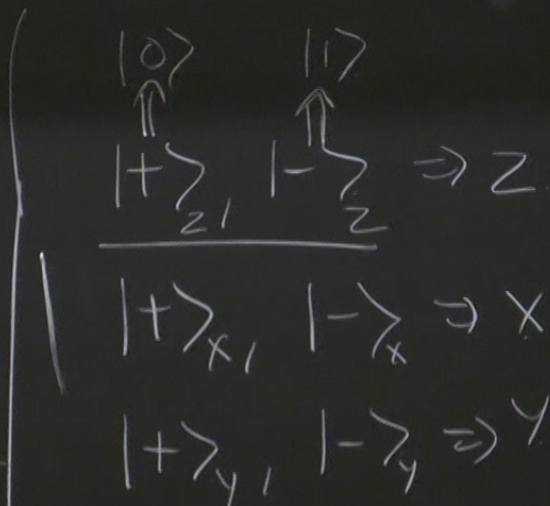
$|\psi\rangle\langle\psi| \Rightarrow$ for pure
all of same kind

⇒ Single/com
⇒ Problem
⇒ Density m
⇒ Density matrix

Joint state	Bob's state
$+\frac{A}{\sqrt{x}} \otimes -\frac{B}{\sqrt{x}}\rangle$	$ -\frac{B}{\sqrt{x}}\rangle$
$-\frac{A}{\sqrt{x}} \otimes +\frac{B}{\sqrt{x}}\rangle$	$ +\frac{B}{\sqrt{x}}\rangle$

$$\frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

$$\left\{ \frac{1}{\sqrt{2}}, |+\rangle \right\}, \left\{ \frac{1}{\sqrt{2}}, |-\rangle \right\}$$



$$\Rightarrow \left\{ \frac{1}{\sqrt{2}}, |+\rangle_x \right\}, \left\{ \frac{1}{\sqrt{2}}, |-\rangle_x \right\}$$

for mixed

N

$\rho =$

Pure

mixed

for mixed states

$N \rightarrow$ subsystems $|\psi_i\rangle$ $W_i \Rightarrow$ classical prob.

$$\rho = \sum_i W_i |\psi_i\rangle\langle\psi_i|$$

Pure \rightarrow $|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\rho = \frac{(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)}{2}$

mixed \rightarrow $\left\{ \frac{1}{2} |0\rangle, \frac{1}{2} |1\rangle \right\}$ $\rho = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$

$$\rho = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\langle 1 | = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

observables

$$|\psi_i\rangle = \sum_n c_n^i |n\rangle$$

$$\langle A \rangle = \text{Tr}(\rho A)$$

$$\langle \hat{A} \rangle_i = \langle \psi_i | \hat{A} | \psi_i \rangle$$

$$\begin{aligned} \langle A \rangle &= \sum_i w_i \langle \psi_i | \hat{A} | \psi_i \rangle \\ &= \sum_i w_i \sum_n \sum_{n'} c_n^{i*} c_{n'}^i \langle n' | A | n \rangle \\ &= \sum_n \sum_{n'} \langle n | \rho | n' \rangle \langle n' | A | n \rangle \\ &= \sum_n \langle n | \rho A | n \rangle = \text{Tr}(\rho A) \end{aligned}$$

$|\psi_i\rangle$ $W_i \Rightarrow$ classical prob.

$\langle \psi_i | \circ$

$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$\left. \begin{matrix} \\ \frac{1}{2} |1\rangle \end{matrix} \right\}$

$|\psi_i\rangle = i=1,2$

$50\% |0\rangle$ $50\% |1\rangle$
 \uparrow \uparrow
 $|\psi_1\rangle$ $|\psi_2\rangle$

$$\rho = \frac{(|0\rangle + |1\rangle)(\langle 11| + \langle 01|)}{2} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\rho = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1| = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

observables

$$\langle \hat{A} \rangle_i = \langle \psi_i | \hat{A} | \psi_i \rangle$$

$$\langle A \rangle = \sum_i W_i \langle \psi_i | \hat{A} | \psi_i \rangle = \sum_i W_i \sum_n \sum_{n'} C_n^{i*} C_{n'}^i \langle n | \hat{A} | n' \rangle$$

$$= \sum_n \sum_{n'} \langle n | \rho | n' \rangle \langle n' | \hat{A} | n \rangle = \sum_n \langle n | \rho \hat{A} | n \rangle$$

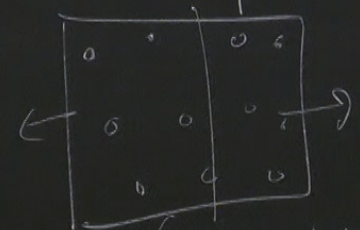
es

$$|\psi_1\rangle = \sqrt{\frac{2}{3}}|+\rangle + \sqrt{\frac{1}{3}}|-\rangle, \quad |\psi_2\rangle = \sqrt{\frac{4}{5}}|+\rangle + \sqrt{\frac{1}{5}}|-\rangle$$

$$|\psi_i\rangle = i=1,2$$

systems $|\psi_i\rangle$ $W_i \Rightarrow$ classical prob

$$W_i |\psi_i\rangle \langle \psi_i|$$



$50\% |0\rangle$ $50\% |1\rangle$
 \uparrow
 $|\psi_1\rangle$ $|\psi_2\rangle$

$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\rho = \frac{(|0\rangle + |1\rangle)(\langle 1| + \langle 0|)}{2} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\left\{ \frac{1}{2} |0\rangle, \frac{1}{2} |1\rangle \right\}$$

$$\rho = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1| = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Observable

$$\langle \hat{A} \rangle_i =$$

$$\langle A \rangle =$$

$$= \sum_i$$

$$= \sum_n$$

$$=$$

Properties

$$1) \rho = \left(\sum W_i |\psi_i\rangle \langle \psi_i| \right)^\dagger = \rho$$

$$2) \text{Tr}(\rho) = 1$$

$$= \sum_n \langle n | \rho | n \rangle$$

$$= \sum_{n,i} W_i \langle n | \psi_i \rangle \langle \psi_i | n \rangle$$

$$= \sum_i W_i \langle \psi_i | \psi_i \rangle = \sum_i W_i = 1$$

$$3) \text{Tr}(\rho^2) =$$

$$3) \text{Tr}(P^2) \leq (\text{Tr}(P))^2$$

$$\text{Tr}(P^2) = \sum_n \langle n | P^2 | n \rangle$$

$$= \sum_n \langle n | P P | n \rangle$$

$$= \sum_{i,j} w_i w_j \langle n | \psi_i \rangle \langle \psi_i | \psi_j \rangle \langle \psi_j | n \rangle$$

$$= \sum_{i,j} w_i w_j |\langle \psi_i | \psi_j \rangle|^2 \leq \sum_i w_i^2 |\langle \psi_i | \psi_i \rangle|^2 = (\text{tr}(P))^2$$

$$\text{tr}(\rho^2) = (\text{tr}(\rho))^2$$

↓
pure states

$$\text{tr}(\rho^2) < \frac{1}{2} < \text{tr}(\rho)$$

$$|\langle \psi_j | \psi_j \rangle|^2 \leq \sum_i W_i^2 |\langle \psi_i | \psi_i \rangle|^2 = (\text{tr}(\rho))^2$$

$$I_{st} \quad |\psi\rangle = |0\rangle \rightarrow \left(\frac{2}{3}\right)_{\text{Prob}}$$

$$|\psi_2\rangle = |1\rangle \rightarrow \frac{1}{2}$$

$$P = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

I_{nd}

$$|\psi_1\rangle = \sqrt{\frac{2}{3}}|+\rangle + \sqrt{\frac{1}{3}}|-\rangle$$

Prob $\frac{1}{2}$

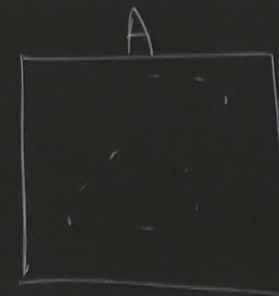
$$|\psi_2\rangle = \sqrt{\frac{2}{3}}|+\rangle - \sqrt{\frac{1}{3}}|-\rangle$$

Prob $\frac{1}{2}$

Partial measurement

① Singlet state

$$|\psi\rangle = \frac{|+\rangle^A \otimes |-\rangle^B - |-\rangle^A \otimes |+\rangle^B}{\sqrt{2}}$$



Ind

Prob

$$|\psi_1\rangle = \sqrt{\frac{2}{3}}|+\rangle + \sqrt{\frac{1}{3}}|-\rangle$$

Prob $\frac{1}{2}$

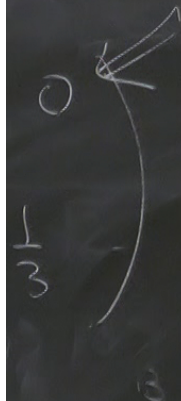
$$|\psi_2\rangle = \sqrt{\frac{2}{3}}|+\rangle - \sqrt{\frac{1}{3}}|-\rangle$$

Prob $\frac{1}{2}$

Density for pure spin $\frac{1}{2}$

$$|\psi\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\rangle$$

$$|\psi\rangle\langle\psi| = \begin{bmatrix} \cos^2\frac{\theta}{2} & \cos\frac{\theta}{2}\sin\frac{\theta}{2}e^{-i\phi} \\ \cos\frac{\theta}{2}\sin\frac{\theta}{2}e^{i\phi} & \sin^2\frac{\theta}{2} \end{bmatrix}$$



3

$|+\rangle |-\rangle$

Proj 10)

$$\frac{1}{2} \Rightarrow |+\rangle\langle +|_x = \frac{I}{2} + \frac{\vec{X} \cdot \vec{\sigma}}{2}$$

$$\frac{1}{2} \Rightarrow |-\rangle\langle -|_x = \frac{I}{2} - \frac{\vec{X} \cdot \vec{\sigma}}{2} \quad \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\rho_B = \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1|$$
$$= \left(\frac{I}{2} \right)$$

$$\text{Tr}(\rho^2) \leq (\text{Tr}(\rho))^2$$

$$\text{Tr}(\rho^2) = \sum_n \langle n | \rho^2 | n \rangle$$

$$= \sum_{n \neq m} \langle n | \rho | m \rangle \langle m | \rho | n \rangle$$
$$= \sum_{i,j} w_i w_j \langle n |$$

$$= \sum_{i,j} w_i w_j \langle$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Tr

$$\frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1| \Rightarrow \text{Bob mix in Z-basis}$$

$$\frac{1}{2} |+\rangle_x \langle +| + \frac{1}{2} |-\rangle_x \langle -| \Rightarrow \text{X-basis}$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \frac{|0\rangle+|1\rangle}{\sqrt{2}} & \frac{|0\rangle-|1\rangle}{\sqrt{2}} \end{array}$$

$$|\psi_i\rangle^2 = (\text{tr}(P))^2$$