Title: Quantum Theory Lecture - 090823

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Collection: Quantum Theory 2023/24

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Schrödinger.

$$\langle A(t)\rangle = \langle \psi_S(t) | A_S | \psi(t)_S\rangle$$

$$= \langle \psi(0) | \underline{U^\dagger A_S U} | \psi(0)\rangle$$

$$|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle$$

$$\hat{A}|a\rangle = a|a\rangle$$

equ^n of motion

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

Heisenberg.

$$\langle A_H(t)\rangle = \langle \psi(0)| U^\dagger A_S U |\psi(0)\rangle$$

$$A_H(t) = U^\dagger A_S U$$

$$a \underbrace{U^\dagger|a\rangle}_{|a\rangle_H} = \hat{A} \underbrace{U^\dagger|a\rangle}_{|a_H\rangle}$$

$$\frac{d\hat{A}_H(t)}{dt} = \frac{1}{i\hbar}[A_H(t), H]$$

Free particle.

$$H = \frac{\hat{p}^2}{2m}$$

Schrödinger.

$$\langle A(t)\rangle = \langle \psi_s(t) | A_s | \psi(t)_s\rangle$$

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$$\frac{dA_H(t)}{dt} = \frac{1}{i\hbar}[A_H(t), H]$$

Free particle.

$$H = \frac{\hat{P}_x^2}{2m}$$

$$P_x, \; X$$

$$\frac{dP_x(t)}{dt} = \frac{1}{i\hbar}\left[P_x(t), \frac{P_x^2(t)}{2m}\right] = 0$$

$$\frac{dx}{dt} = \frac{1}{i\hbar}\left[X, \frac{P_x^2(x)}{2m}\right] \Rightarrow \frac{d}{d}$$

eisenberg.

$$\langle(t)\rangle = \langle\psi(0)|U^\dagger A_S U|\psi(0)\rangle$$

$$_H(t) = U^\dagger A_S U$$

$$a\,\underbrace{U^\dagger|a\rangle}_{|a\rangle_H} = \hat{A}\,\underbrace{U^\dagger|a\rangle}_{|a_H\rangle}$$

$$\frac{dA_H(t)}{dt} = \frac{1}{i\hbar}[A_H(t), H]$$

Free particle.

$$H = \frac{\hat{P}_x^2}{2m}$$

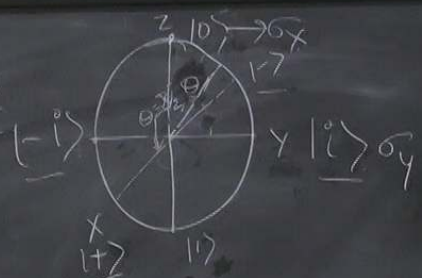
P_x, X

$$\frac{dP_x(t)}{dt} = \frac{1}{i\hbar}\left[P_x(t), \frac{P_x^2(t)}{2m}\right] = 0 \rightarrow \frac{dP_x}{dt} = 0 \quad P_x(t) = P_x(0)$$

$$\frac{dx}{dt} = \frac{1}{i\hbar}\left[X, \frac{P_x^2(t)}{2m}\right] \Rightarrow \frac{dx}{dt} = \frac{P_x(0)}{m}$$

Bloch sphere

$|0\rangle \quad |1\rangle$



σ_z

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

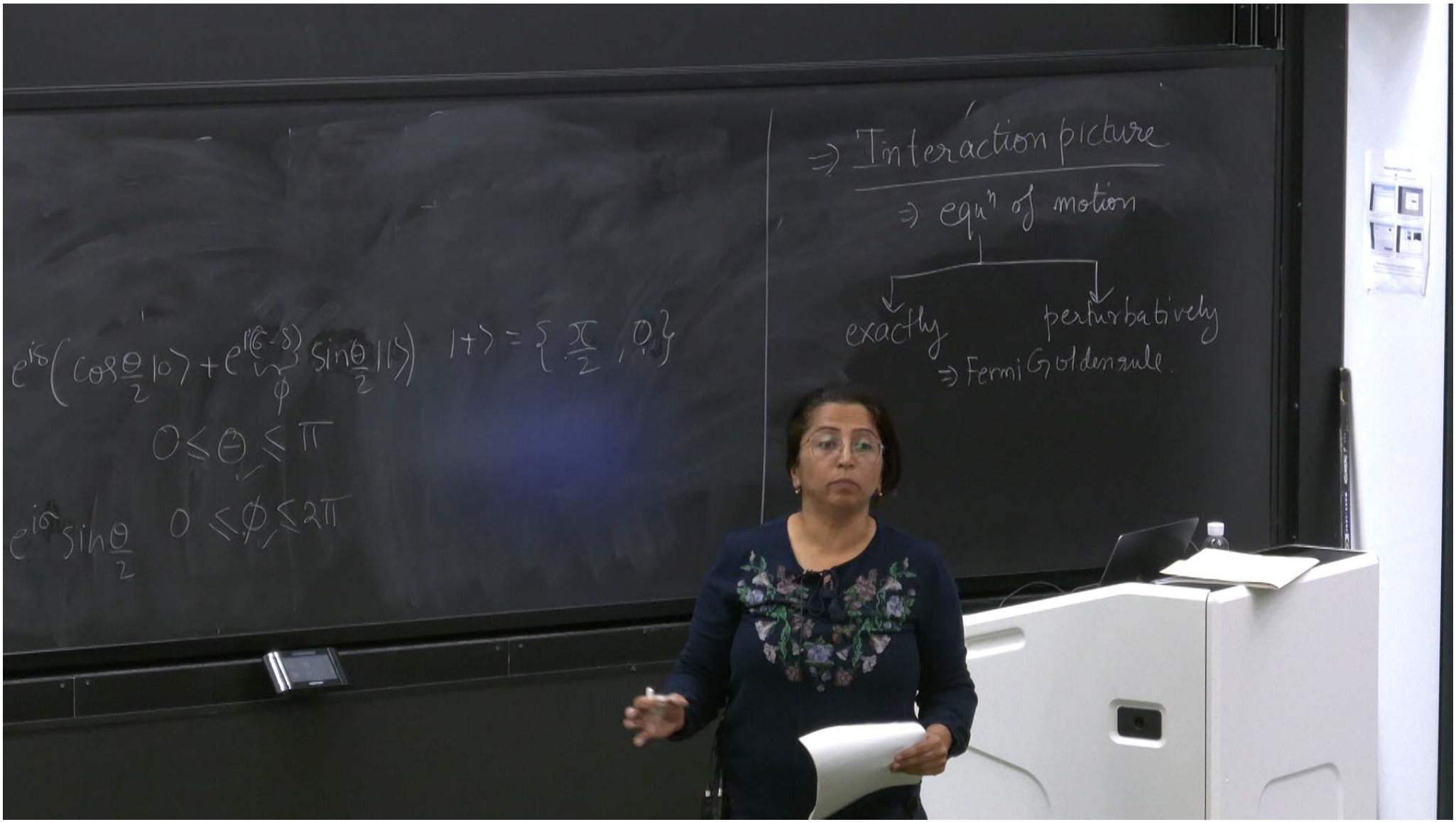
$$|\pm i\rangle = \frac{|0\rangle \pm i|1\rangle}{\sqrt{2}}$$

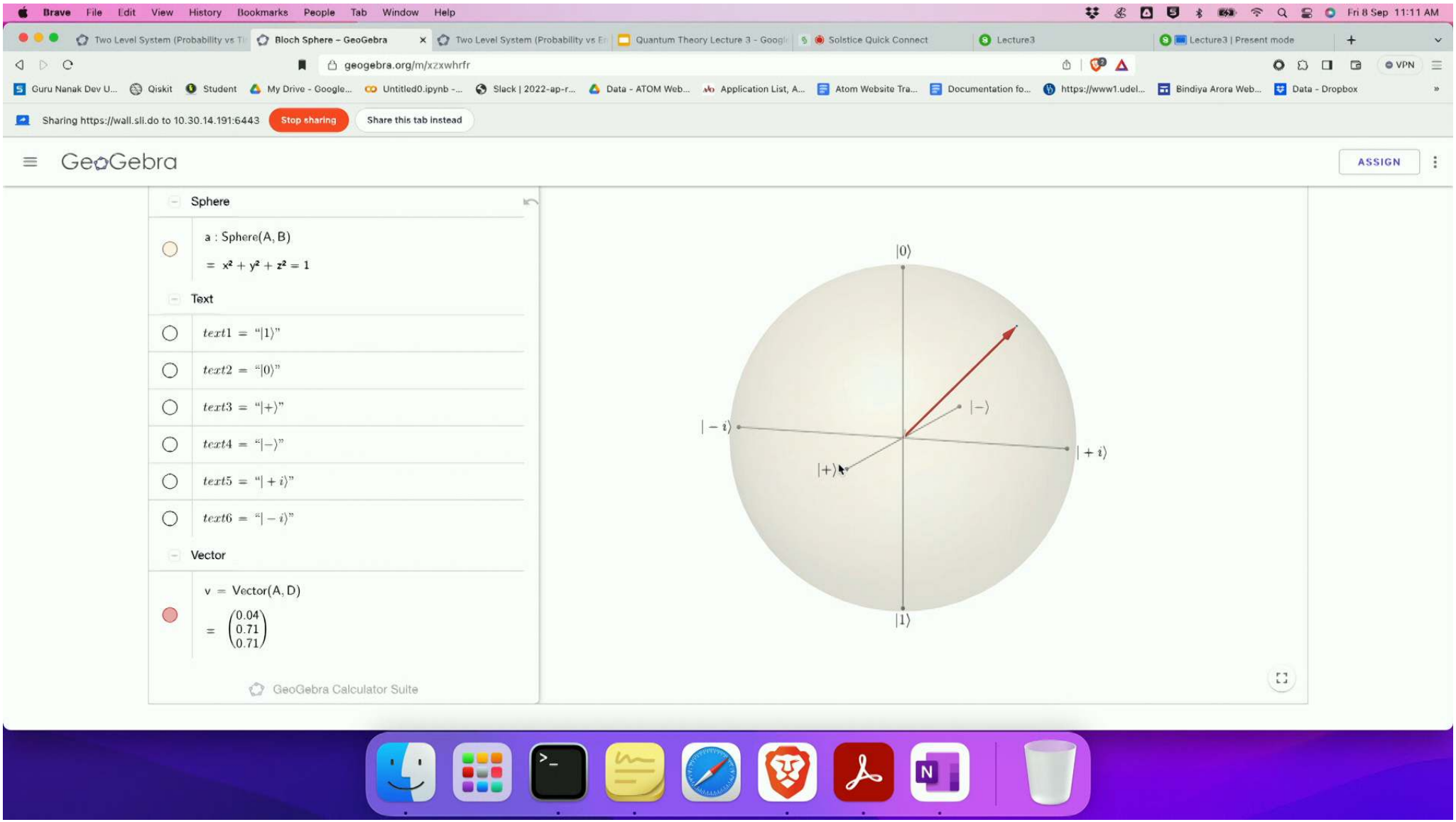
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \Rightarrow e^{i\delta}\left(\cos\frac{\theta}{2}|0\rangle + e^{i(\underbrace{\sigma-\delta}_{\phi})}\sin\frac{\theta}{2}|1\rangle\right) \qquad |+\rangle = \left\{\frac{\pi}{2}, \phi\right\}$$

$$|\alpha|^2 + |\beta|^2 = 1 \qquad 0 \leq \theta \leq \pi$$

By convention

$$\alpha = e^{i\delta}\cos\frac{\theta}{2} \qquad \beta = e^{i\sigma}\sin\frac{\theta}{2} \qquad 0 \leq \phi \leq 2\pi$$


$e^{i\delta}\left(\cos\frac{\theta}{2}|0\rangle + e^{i(\delta-\delta)}\sin\frac{\theta}{2}|1\rangle\right)$
ϕ
$|+\rangle = \{\frac{\pi}{2}, 0\}$
$0 \leq \theta \leq \pi$
$e^{i\delta}\sin\frac{\theta}{2}$
$0 \leq \phi \leq 2\pi$
⇒ Interaction picture
⇒ eqn of motion
exactly
perturbatively
⇒ Fermi Golden rule.


Bloch Sphere – GeoGebra
geogebra.org/m/xzxwhrfr
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GeoGebra
ASSIGN
Sphere
a : Sphere(A, B)
= x² + y² + z² = 1
Text
text1 = "|1⟩"
text2 = "|0⟩"
text3 = "|+⟩"
text4 = "|−⟩"
text5 = "|+i⟩"
text6 = "|−i⟩"
Vector
v = Vector(A, D)
= (0.04, 0.71, 0.71)
GeoGebra Calculator Suite
|0⟩
|−⟩
|−i⟩
|+i⟩
|+⟩
|1⟩

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A Quick Recap

Arrow of Time

Why we see time flowing in one direction ?

Time Evolution

Time evolution signifies the change of a state occurring due to the passage of time

In Quantum Mechanics

- In contr
- The te
  Opera

Schrödinger's Approach

The state of a system evolves with time

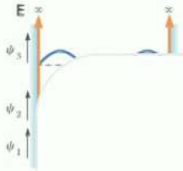
Schrodinger Equation

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Time Evolution

$$|\psi(x,t)\rangle = U(t,t_0)|\psi(x,t_0)\rangle$$

Stationary States



- All observables are independent of time
- Stationary states are the eigenstates of

Heisenberg's Approach

The operators (observables) evolves with time

Heisenbera's Eauation of Motion

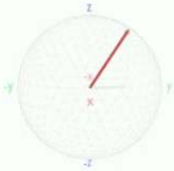
Spin Precession On Bloch Sphere

A representation of given quantum state of qubit

$$H = \omega S_z$$

Heisenberg's Approach

t=0	t=π/ω
$S_Z\{\|+\rangle_Z, \|-\rangle_Z\}$	$\rightarrow S_Z\{\|+\rangle_Z, \|-\rangle_Z\}$
$S_X\{\|+\rangle_X, \|-\rangle_X\}$	$\rightarrow -S_Y\{\|-\rangle_Y, \|+\rangle_Y\}$
$S_Y\{\|+\rangle_Y, \|-\rangle_Y\}$	$\rightarrow S_X\{\|+\rangle_X, \|-\rangle_X\}$



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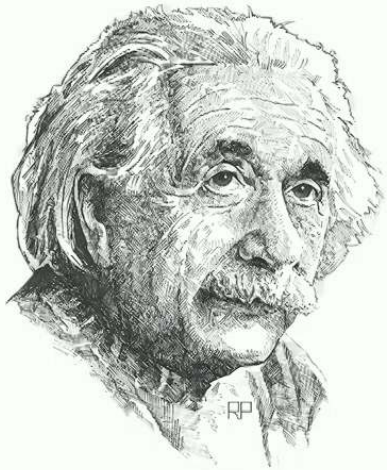

"We can't solve problems by using the same kind of thinking we used when we created them"

Albert Einstein

Image Source : Albert Einstein Drawing, favpng.com/png_view/transparent-albert-einstein-albert-einstein-drawing-png/PQyWim5D

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Time Dependent Perturbation

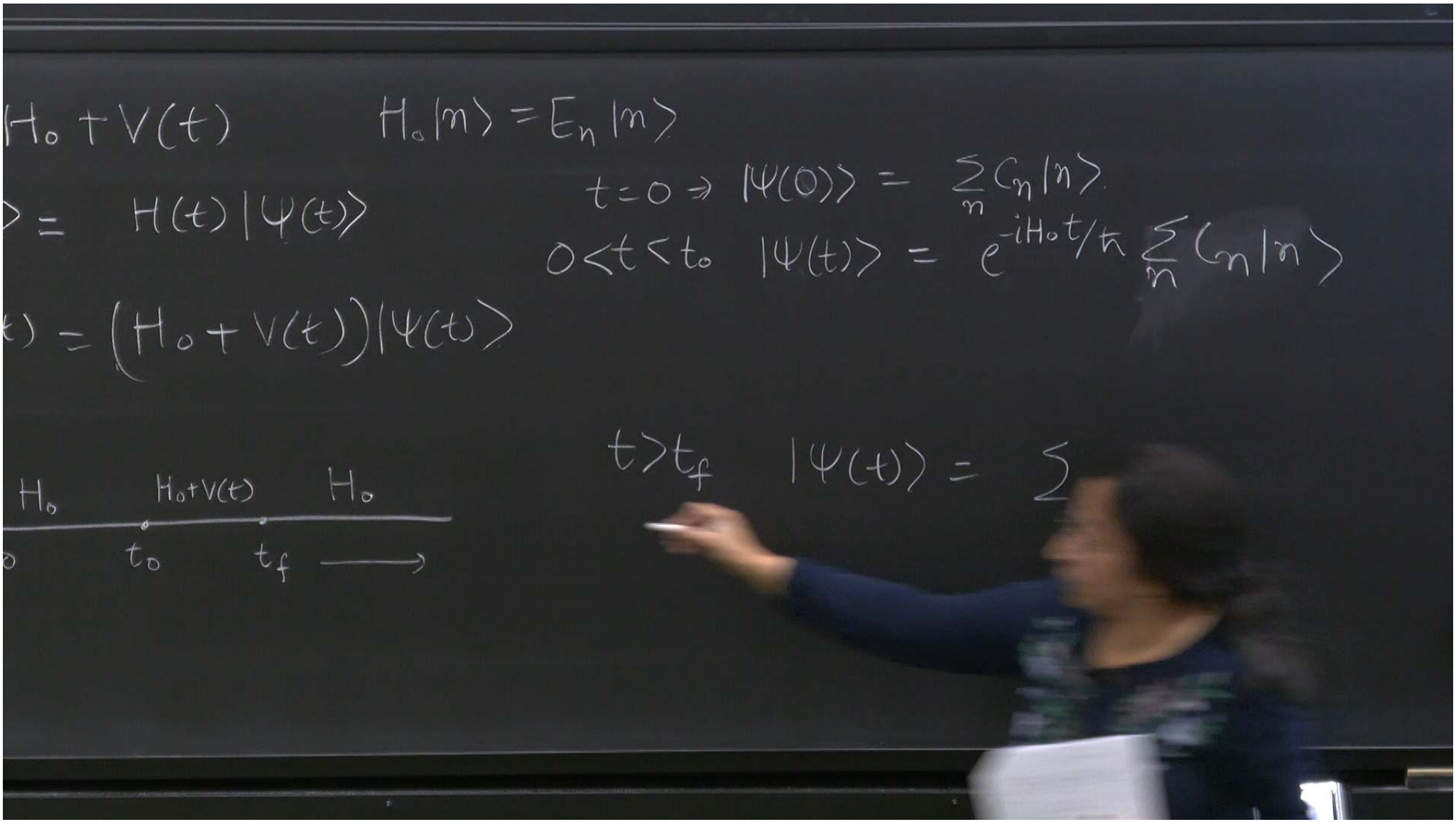
$$H^0 \qquad H^0 + V(t) \qquad H^0$$

$$t_0 \qquad t_f$$

Schrödinger Equation

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \underbrace{(H^0 + V(t))}_{H(t)} |\psi(t)\rangle$$

What are the probabilities of finding the system in different eigen states after the perturbation is turned off?


$H_0 + V(t)$
$H_0|n\rangle = E_n|n\rangle$
$\rangle = H(t)|\psi(t)\rangle$
$t=0 \Rightarrow |\psi(0)\rangle = \sum_n c_n|n\rangle$
$0<t<t_0 \quad |\psi(t)\rangle = e^{-iH_0 t/\hbar}\sum_n c_n|n\rangle$
$t) = (H_0 + V(t))|\psi(t)\rangle$
$t>t_f \quad |\psi(t)\rangle = \sum$
H_0
$H_0+V(t)$
H_0
t_0
t_f

$H_0 + V(t)$ $\qquad H_0|n\rangle = E_n|n\rangle$

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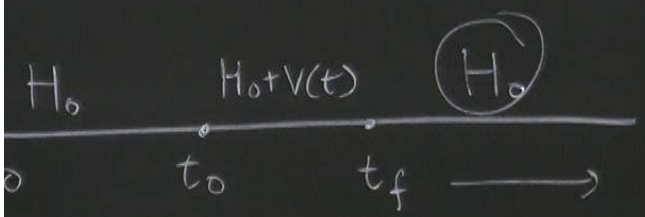
$t) = (H_0 + V(t))|\psi(t)\rangle$

$t>t_f \quad |\psi(t)\rangle = \sum c_n(t)|n\rangle$



(t) $\qquad H_0|n\rangle = E_n|n\rangle$

$(t)|\psi(t)\rangle$

$t=0 \Rightarrow |\psi(0)\rangle = \sum_n c_n|n\rangle$

$0<t<t_0 \quad |\psi(t)\rangle = e^{-iH_0t/\hbar}\sum_n c_n|n\rangle$

$H_0+V(t))|\psi(t)\rangle$

$t_0<t<t_f \quad |\psi(t)\rangle = \underline{e^{-iH_0t/\hbar}}\sum_n c_n(t)|n\rangle$

$t>t_f \quad |\psi(t)\rangle = e^{-iH_0t/\hbar}\sum \underline{\underline{c_n(t_f)}}|n\rangle$

$H_0+V(t)$ $\quad H_0$

$t_f \longrightarrow$

$$H(t) = H_0 + V(t) \qquad H_0|n\rangle = E_n|n\rangle$$

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle$$

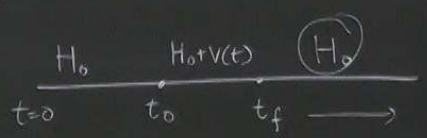
$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \left(H_0 + V(t)\right)|\psi(t)\rangle$$

$$t=0 \Rightarrow |\psi(0)\rangle = \sum_n c_n |n\rangle$$

$$0<t<t_0 \quad |\psi(t)\rangle = e^{-iH_0 t/\hbar} \sum_n c_n |n\rangle$$

$$t_0<t<t_f \quad |\psi(t)\rangle = e^{-iH_0(t-t_0)/\hbar} \sum_n c_n(t)|n\rangle$$

$$t>t_f \quad |\psi(t)\rangle = e^{-iH(t-t_0)/\hbar} \sum c_n(t)|n\rangle$$



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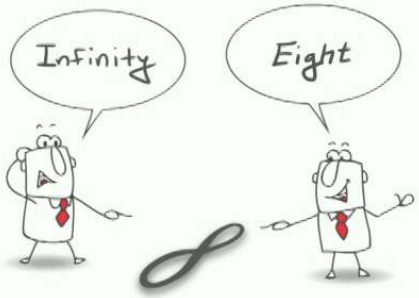
Interaction Picture

The both **states** and operators (**observables**) evolve with time

Shifting Frame of Reference $$|\psi_I(t)\rangle = e^{\frac{iH^0 t}{\hbar}}|\psi(t)\rangle$$

Schrödinger Equation $$i\hbar\frac{d}{dt}|\psi_I(t)\rangle = V_I(t)|\psi_I(t)\rangle$$



where $$V_I(t) = e^{\frac{iH^0 t}{\hbar}}\, V(t)\, e^{-\frac{iH^0 t}{\hbar}}$$

$$i\hbar \frac{\partial}{\partial t} |\psi_I(t)\rangle = V_I(t) |\psi_I(t)\rangle \qquad V_I = R^\dagger V R \qquad |\psi_I(t)\rangle = \sum_n c_n(t) |n\rangle$$

$$|\psi(t)\rangle_S = e^{-iH_0 t/\hbar} |\psi_I(t)\rangle$$

$$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t) |n\rangle = V_I(t) \sum_n c_n(t) |n\rangle$$

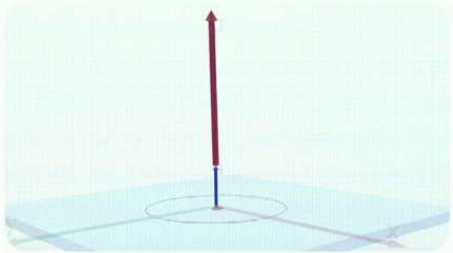
Example

Precession of spin for magnetic field in z direction

Particle with a magnetic moment inside a magnetic field

$$H^0 = \omega_0 . S_z$$

Larmor Frequency



Here's an example that illustrates how changing the way we look at things can make solving problems involving time-dependent Hamiltonians much easier.

Example

Precession of spin in rotated frame of reference

Particle with a magnetic moment inside a magnetic field

$$H^0 = \omega_0 . S_z$$

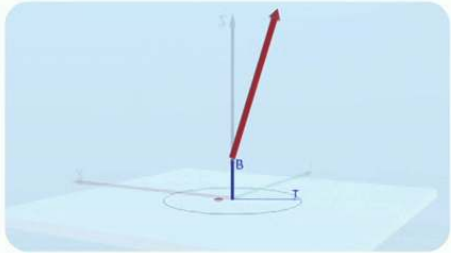
Larmor Frequency

Perturbation in Interaction Picture

$$V_I(t) = \Omega S_x$$

For small perturbation

$\Omega << \omega_0$



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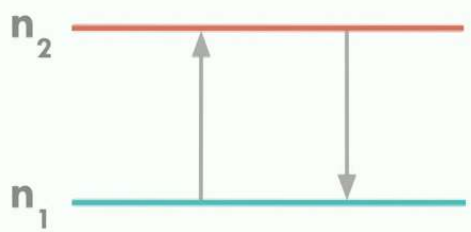
Time Evolution

Analyzing the effect of time-dependent potential on transition between eigenstates



Expanding Wave Function in Basis of H^0 $\quad |\psi_I(t)\rangle = \sum_n c_n(t)|n\rangle$

Equation for Coefficients $\quad i\hbar\frac{d}{dt}\sum_m c_m(t)|m\rangle = V_I(t)\sum_n c_n(t)|n\rangle$

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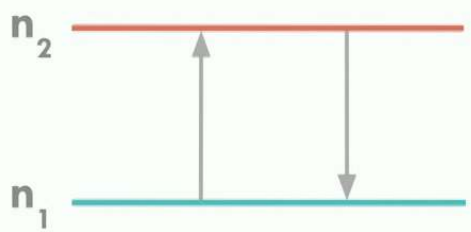
Analyzing the effect of time-dependent potential on transition between eigenstates



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Coupled Differential Equations

Using Orthogonality of eigen states, $V_{mn}(t) = \langle m|V(t)|n\rangle$ and $\omega_{mn} = \dfrac{E_m - E_n}{\hbar}$

$$i\hbar \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12}e^{i\omega_{12}t} & \cdots \\ V_{21}e^{i\omega_{21}t} & V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}$$

Exactly Perturbatively

$$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t) \langle m|n\rangle = \sum_n \underbrace{\langle m|V_I(t)|n\rangle}\, c_n(t)$$

$$i\hbar\, \dot{c}_m(t) = \sum_n c_n(t) \underbrace{\langle m|V(t)|n\rangle}_{V_{mn}} e^{i(E_m - E_n)t/\hbar}$$

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n Level System

Consider a **sudden violent perturbation** at t=0

$t = 0$

Perturbation $$V(t) = \begin{bmatrix} 0 & \alpha \\ \alpha^\star & 0 \end{bmatrix} \delta(t)$$

Coupled Differential Equation $$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn} t} V_{mn}(t) c_n(t)$$

$$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t) \langle m|n\rangle = \sum_n \underbrace{\langle m|V_I(t)|n\rangle}\, c_n(t)$$

$$i\hbar\, \dot{c}_m(t) = \sum_n c_n(t) \underbrace{\langle m|V(t)|n\rangle}_{V_{mn}} e^{i(E_m - E_n)t/\hbar}$$

<u>Exactly</u> at $t = 0_-$ $\quad c_1(0_-) = 1 \quad c_2(0_-) = 0$

$$i\hbar\, \dot{c}_1(t) = e^{i\omega_{12}t} V_{12} c_2(t) + \cancel{e^{i\omega_{11}t} V_{11} c_1(t)}$$

$$i\hbar\, \dot{c}_2(t) = e^{-i\omega_{12}t} V_{21} c_1(t) + \cancel{e^{i\omega_{22}t} V_{22} c_2(t)}$$

$= t$

$) =$

$C_2(t) \propto \delta(t)$

$C_1(t) \propto \delta(t)$

$$\delta(t) \to \Delta(t) = \begin{cases} \frac{1}{t_0} & t \in [0, t_0] \\ & t_0 \to 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f(t)\,\delta(t-a) = f(a)\,\delta(t-a)$$

$t)$

$$i\hbar\,\dot{c}_1(t) = \epsilon \qquad c_2(t) \propto \delta(t)$$

$$i\hbar\,\dot{c}_2(t) = \qquad c_1(t) \propto \delta(t)$$

$$\delta(t) \to \Delta(t) = \begin{cases} \frac{1}{t_0} & t \in [0, t_0 \\ 0 & \text{otherwise} \end{cases}$$

$$f(t)\,\delta(t-a) = f(a)\,\delta(t-$$

$$\left[\begin{aligned} c_1(t) &= \cos\frac{|\alpha|\,t}{\hbar t_0} \\ c_2(t) &= -i\frac{|\alpha|}{\alpha}\sin\frac{|\alpha|\,t}{\hbar t_0} \end{aligned}\right] \quad t \in [0, t_0]$$

$$c_1(t) = \cos\frac{|\alpha|}{\hbar} \qquad c_2(t) = -i\frac{|\alpha|}{\alpha}\sin\frac{|\alpha|}{\hbar} \Rightarrow t = t_0$$

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Perturbative Solution

$$c_m(t) = \overbrace{\delta_{mi}}^{\text{No Transitions}} + \overbrace{\frac{1}{i\hbar}\sum_n \int_{t_0}^{t_f} dt' c_n(t_0) V_{mn}(t') e^{i\omega_{mn}t'}}^{\text{Single Transitions}} + \underbrace{\left(\frac{1}{i\hbar}\right)^2 \sum_n \sum_l \int_{t_0}^{t_f} \int_{t_0}^{t'} dt' dt'' c_l(t'') V_{nl}(t'') e^{i\omega_{nl}t''} V_{mn}(t') e^{i\omega_{mn}t'}}_{\text{Double Transitions}} + \cdots$$

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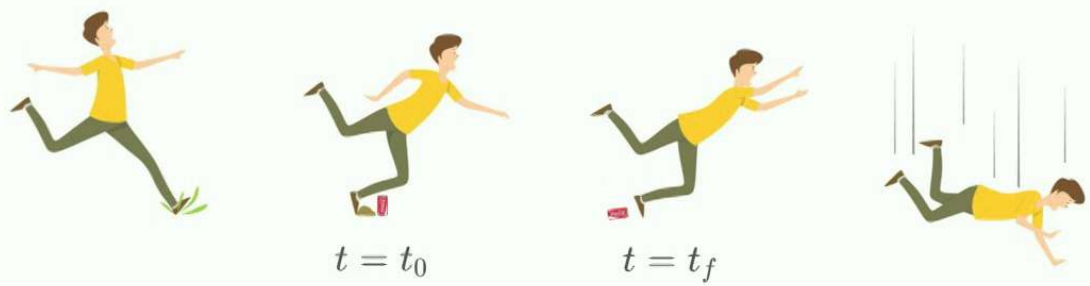
n Level System

Consider a **constant perturbation** turned on from $t=t_0$ to $t=t_f$



Perturbation $$V(t) = \begin{cases} 0 & otherwise \\ V & t_0 < t < t_f \end{cases}$$

Coupled Differential Equation $$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn}(t) c_n(t)$$

Upto First Order $$c_m(t) = \delta_{mi} + \frac{1}{i\hbar} \sum_n \int_{t_0}^{t_f} dt' c_n(t_0) V_{mn}(t') e^{i\omega_{mn}t'}$$

$C_i(0) = \delta_{mi}$ Constant

$$P_{i\to f}(t) = |C_f^{(1)}(t)|^2$$

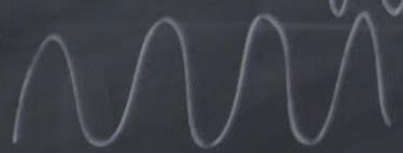
$$C_f^{(1)}(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t')\, e^{i\omega_{fi}t'}\, dt'$$

$$\boxed{P_{i\to f}(t) = \frac{4 V_{fi}^2}{\hbar^2 \omega_{fi}^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right)}$$

1) Energy separation →

II Energy non conserving trans

WRT t

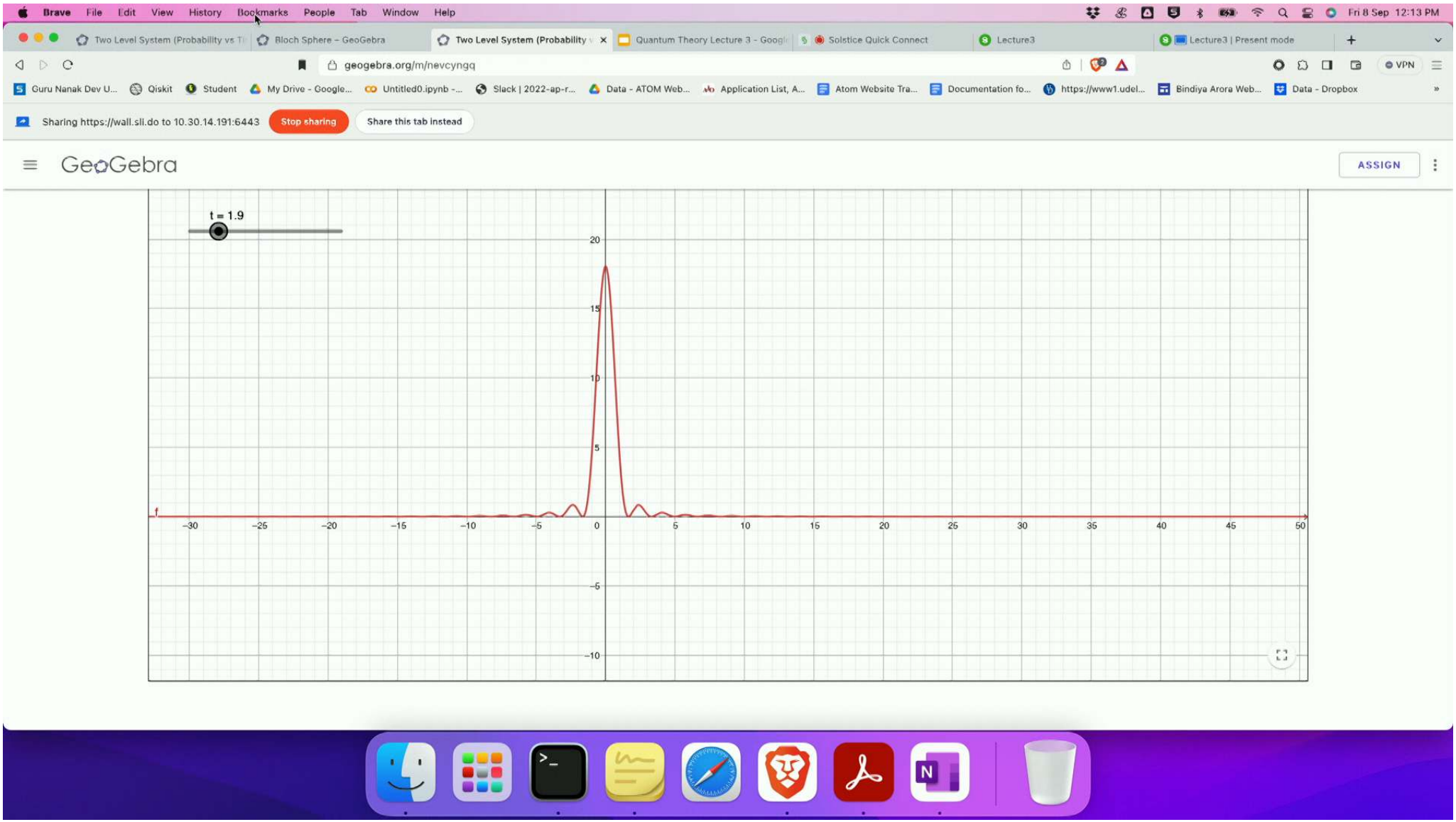


$E_f \neq E_i$ $\quad \omega_{fi} \neq 0$

Energy conserving

$\omega_{fi} = 0$

$$P_{i \to f} = \frac{|V_{fi}|^2 t^2}{\hbar^2}$$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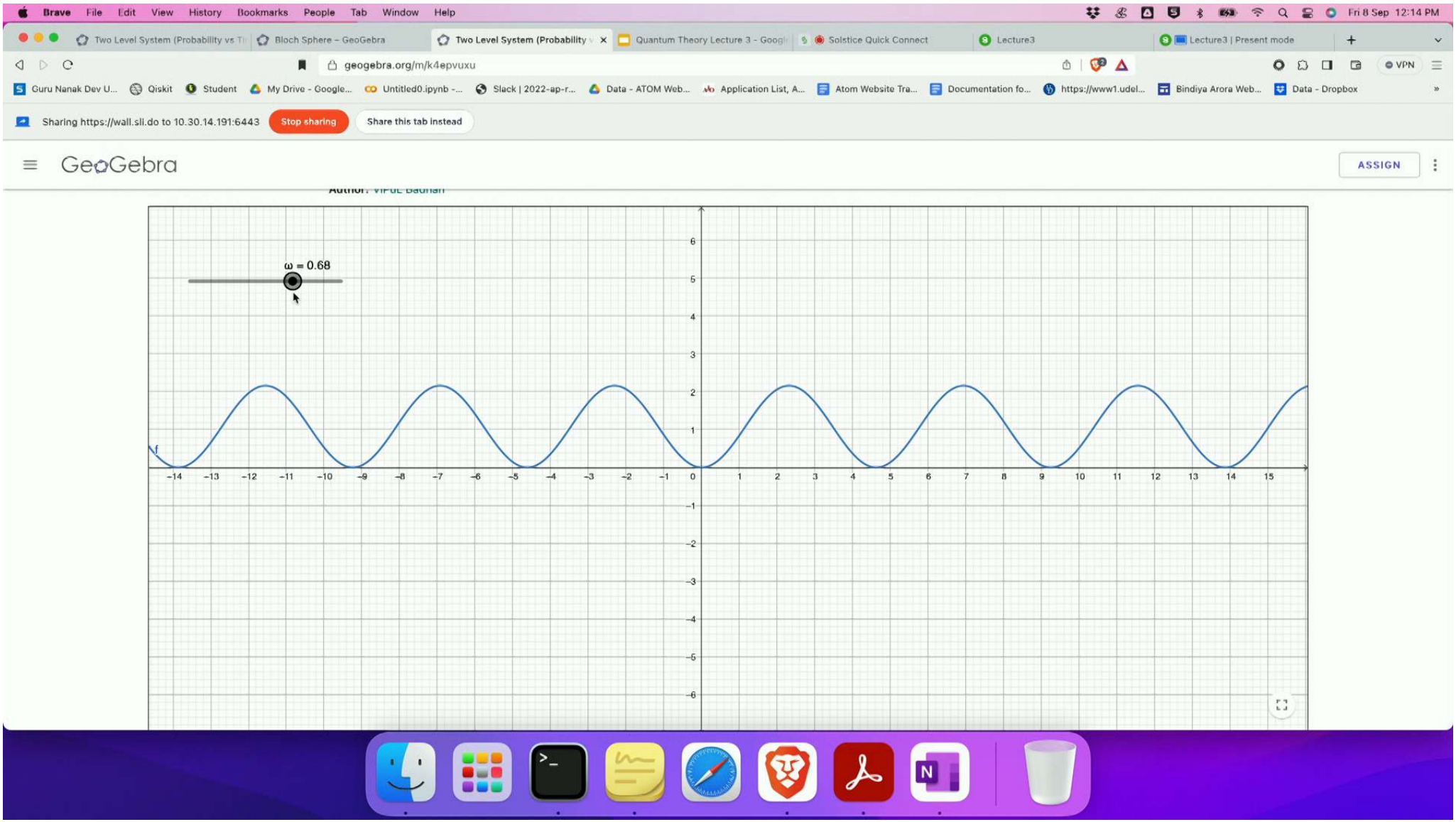
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Fermi's Golden Rule

Energy levels are not discrete but has a finite width

No. of states per unit energy around E_f

$$\sum_f P_{i\to f}(t) = \int P_{i\to f}(t)\,\rho(E_f)\,dE_f$$

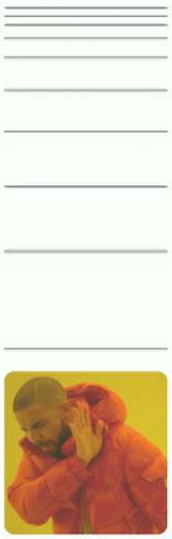
Over Entire Spectrum

$$= \frac{2\pi |V_{fi}|^2\,\rho(E_f)\,t}{\hbar}$$

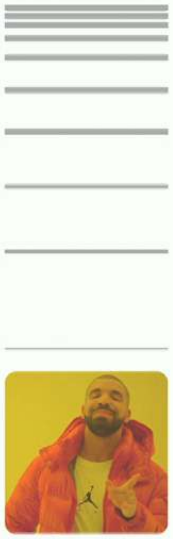
Rate of transition Independent of time

Fermi's Golden Rule

Discrete Width



Finite Width



$$W = \frac{P}{t} \propto t$$

1) Energy separation →

II Energy non conserving trans

wrt t $\quad E_f \neq E_i \quad \omega_{fi} \neq 0$

Energy conserving

$\omega_{fi} = 0$

$$P_{i \to f} = \frac{|V_{fi}|^2 t^2}{\hbar^2}$$