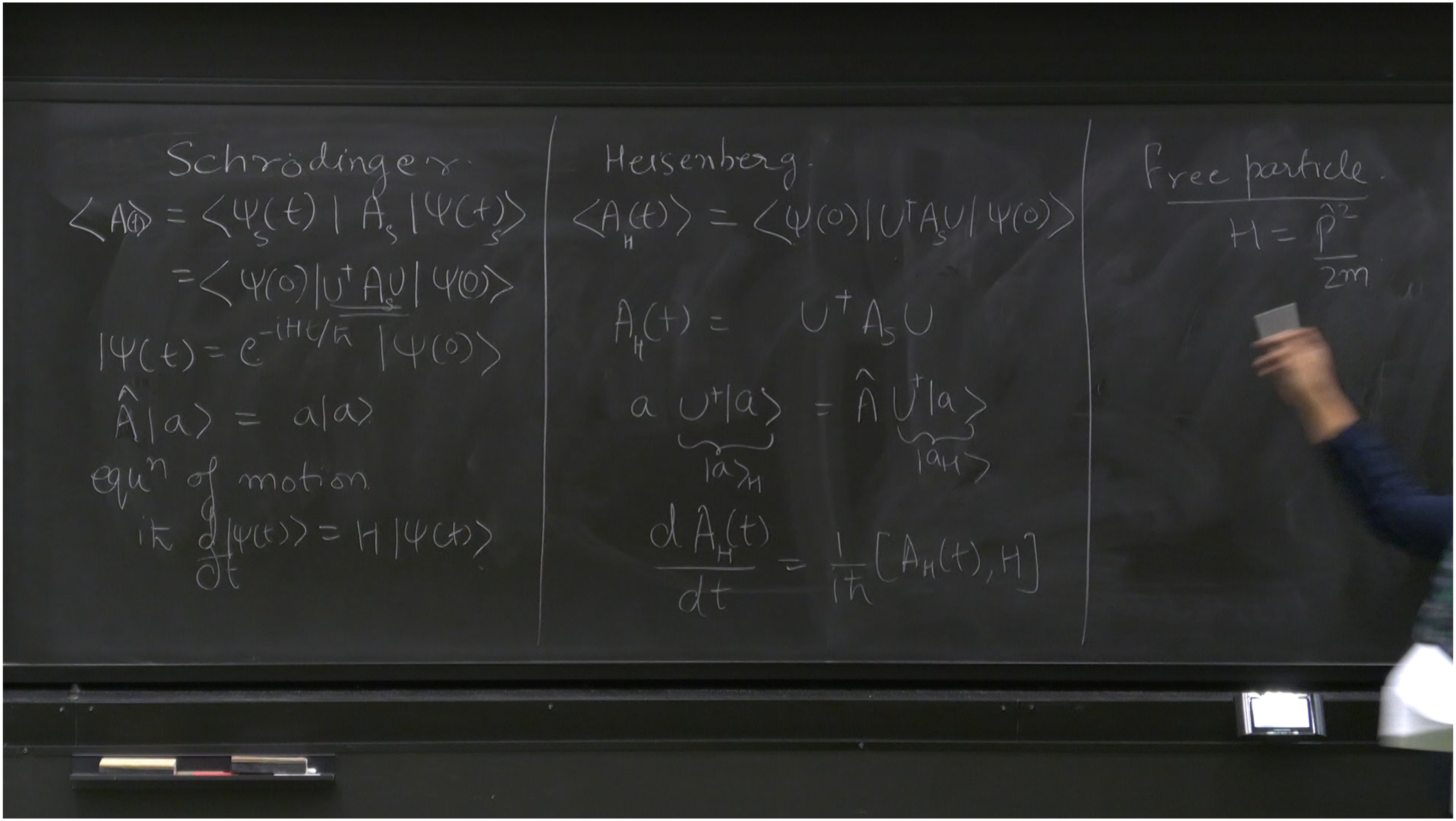
Title: Quantum Theory Lecture - 090823

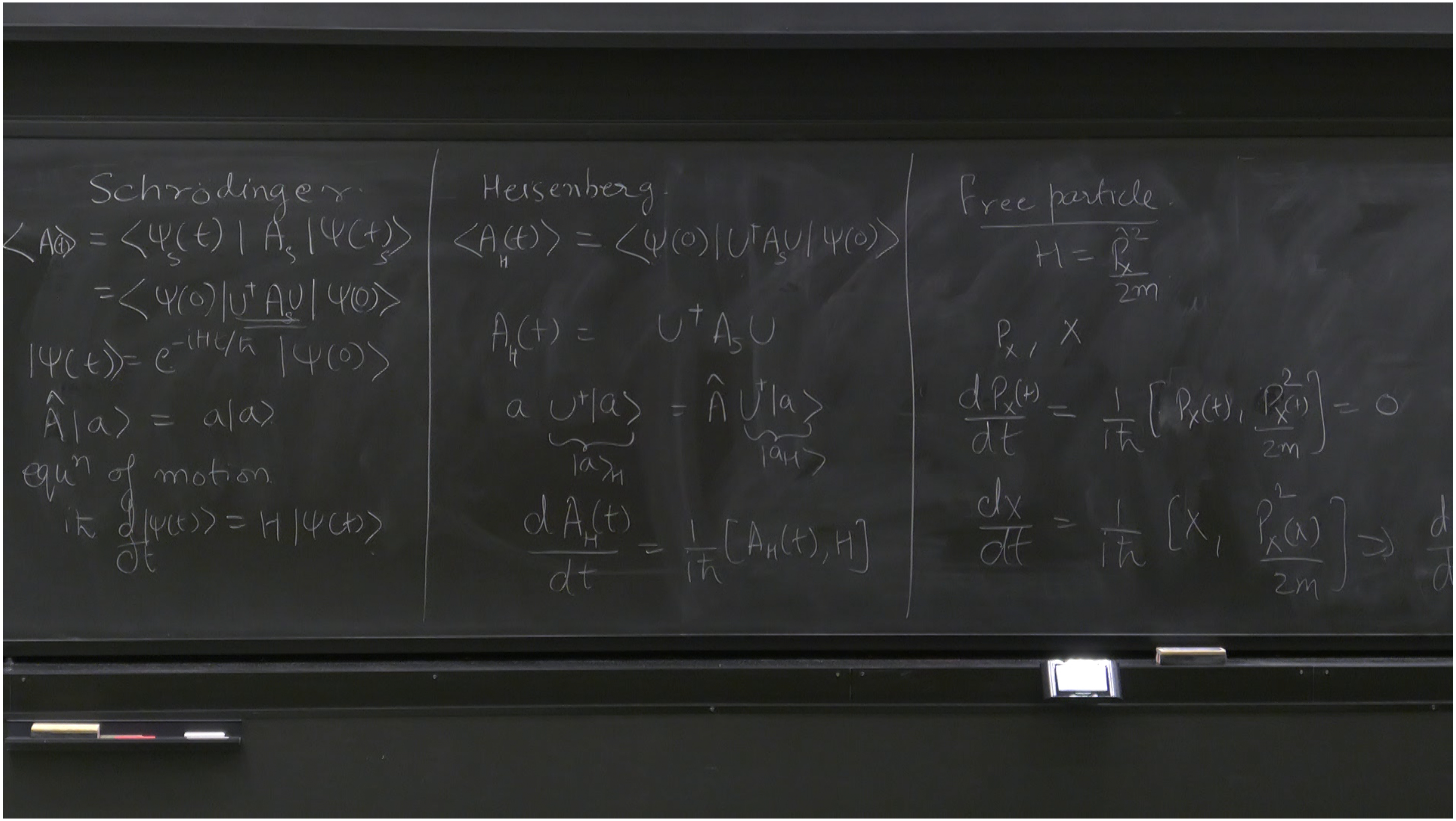
Speakers: Bindiya Arora, Dan Wohns

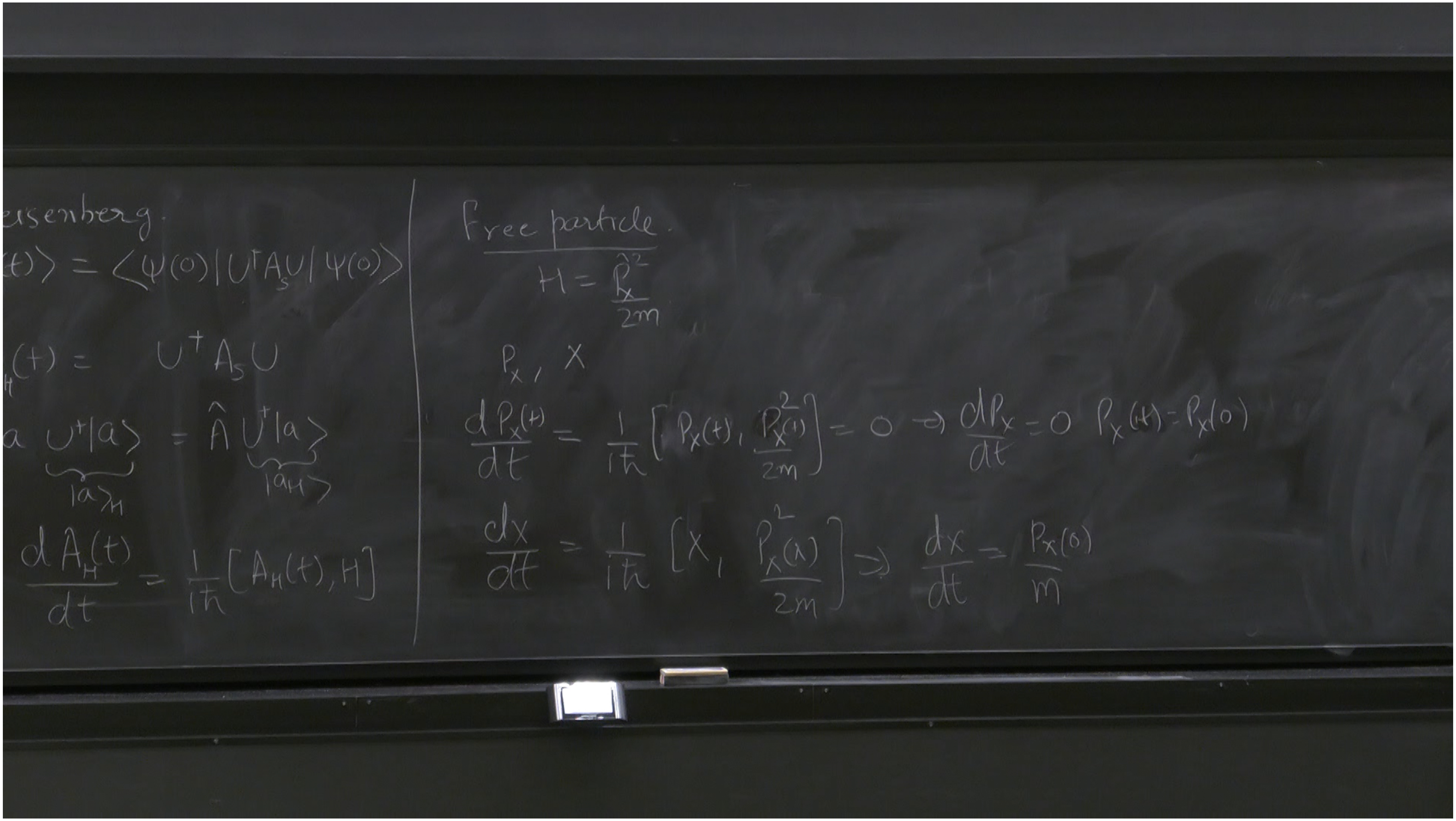
Collection: Quantum Theory 2023/24

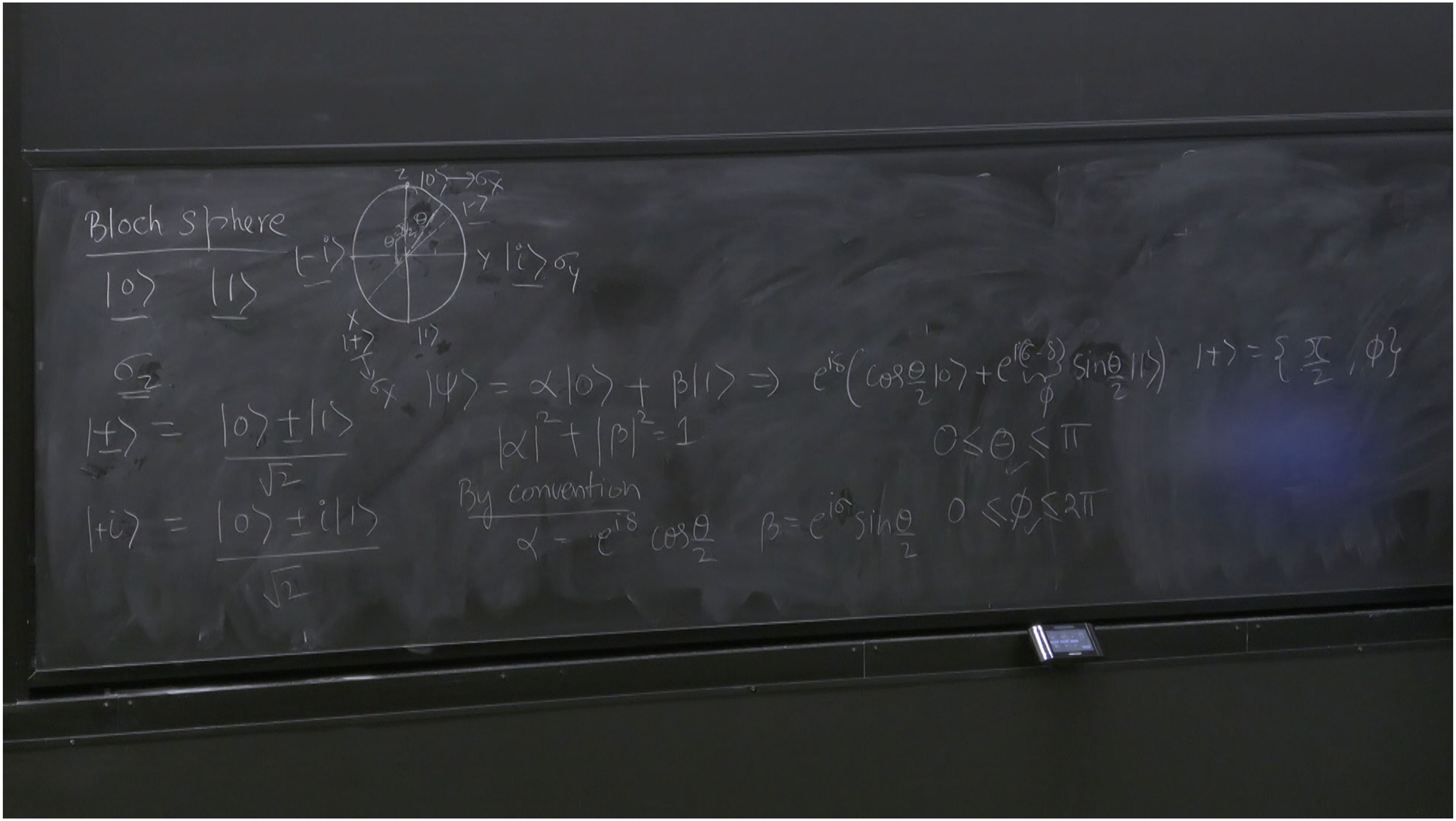
Date: September 08, 2023 - 10:45 AM

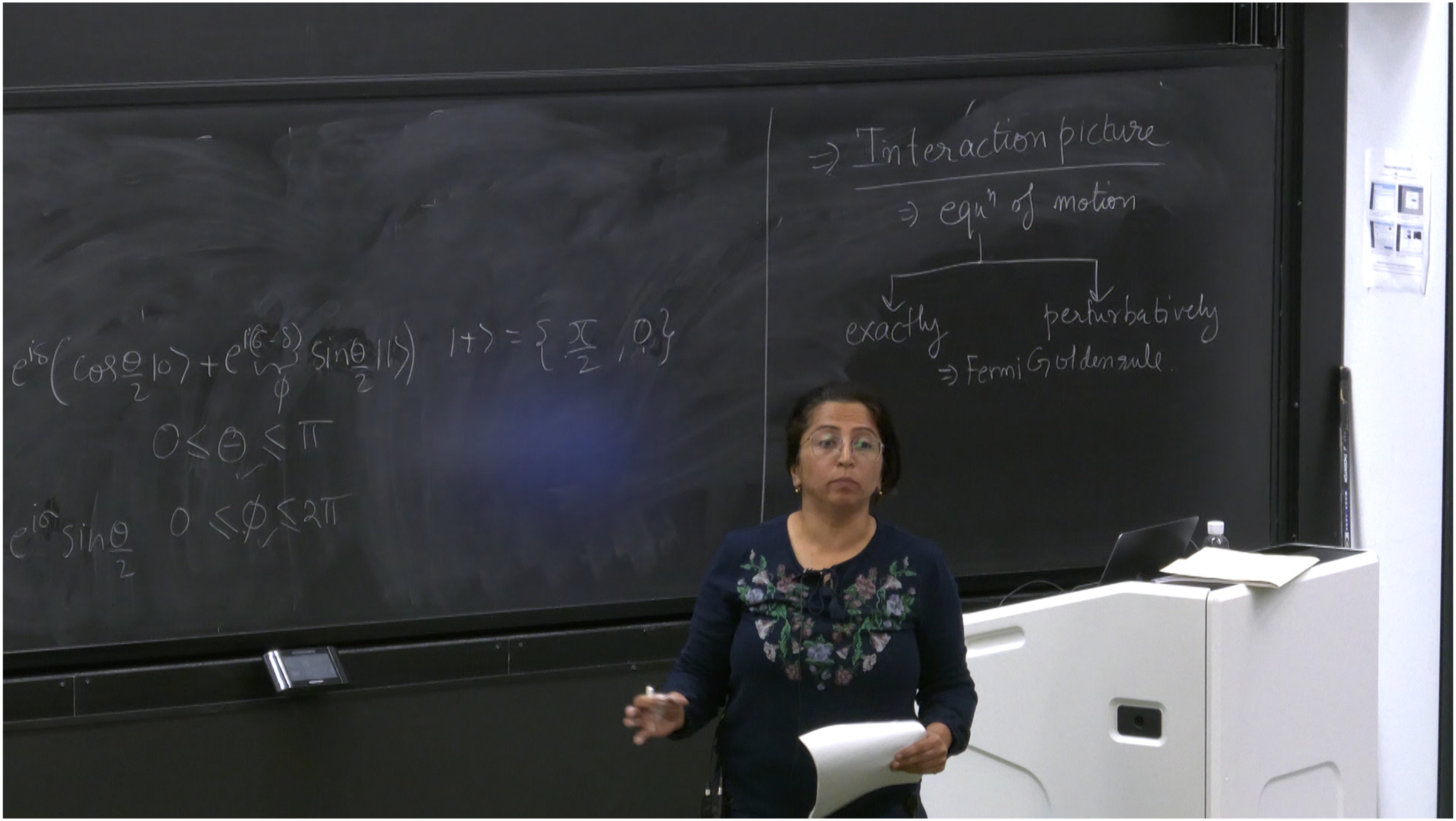
URL: https://pirsa.org/23090040

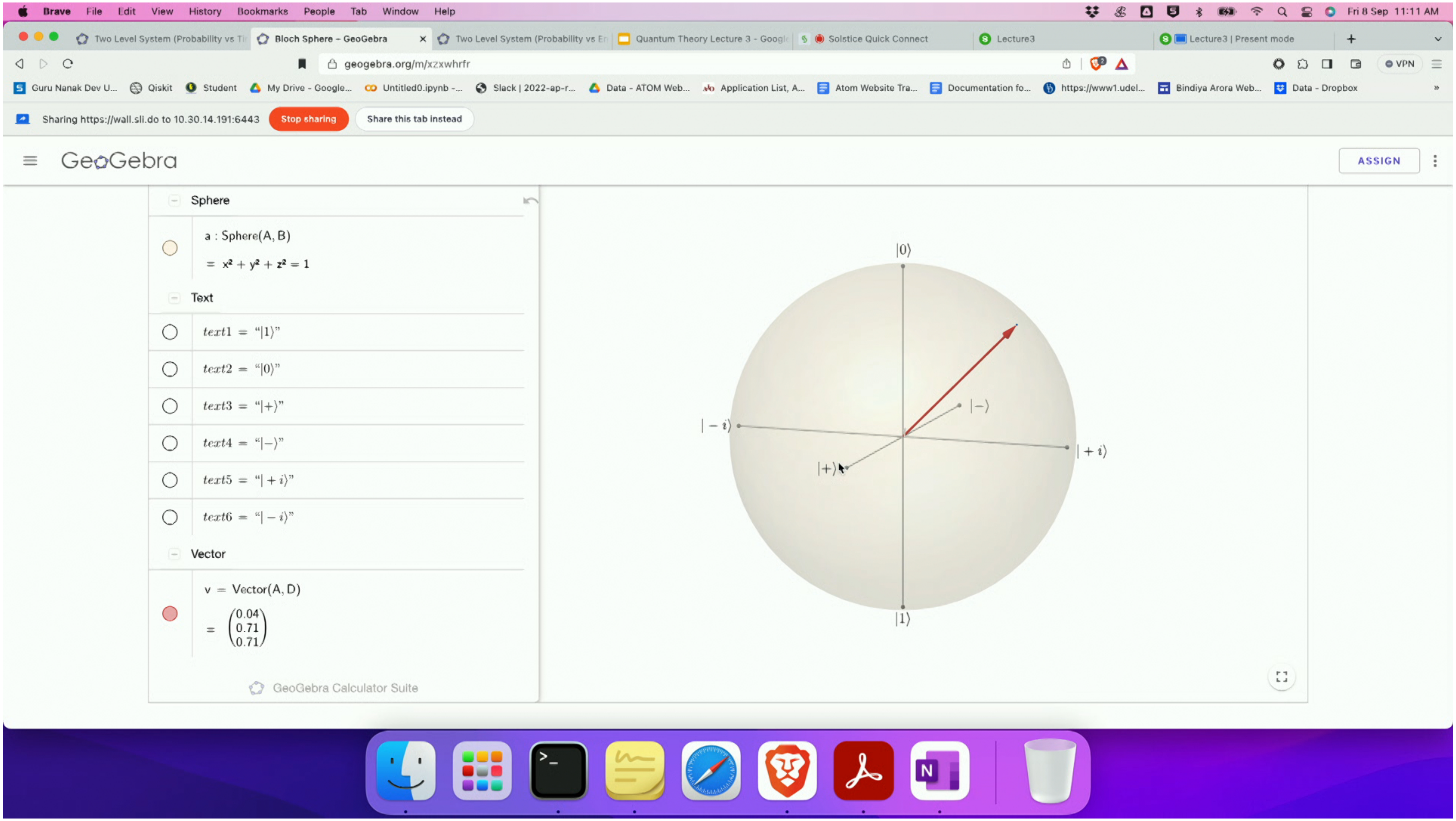

Schrödinger
⟨A(t)⟩ = ⟨Ψ_s(t) | A_s | Ψ(t)⟩_s
= ⟨Ψ(0)| U† A_s U | Ψ(0)⟩
|Ψ(t)⟩ = e^{-iHt/ħ} |Ψ(0)⟩
Â|a⟩ = a|a⟩
equ^n of motion
iħ d/dt |Ψ(t)⟩ = H |Ψ(t)⟩
Heisenberg
⟨A_H(t)⟩ = ⟨Ψ(0)| U† A_s U | Ψ(0)⟩
A_H(t) = U† A_s U
a U†|a⟩ = Â U†|a⟩
|a⟩_H
|a_H⟩
d A_H(t) / dt = 1/iħ [A_H(t), H]
Free particle
H = p̂^2 / 2m

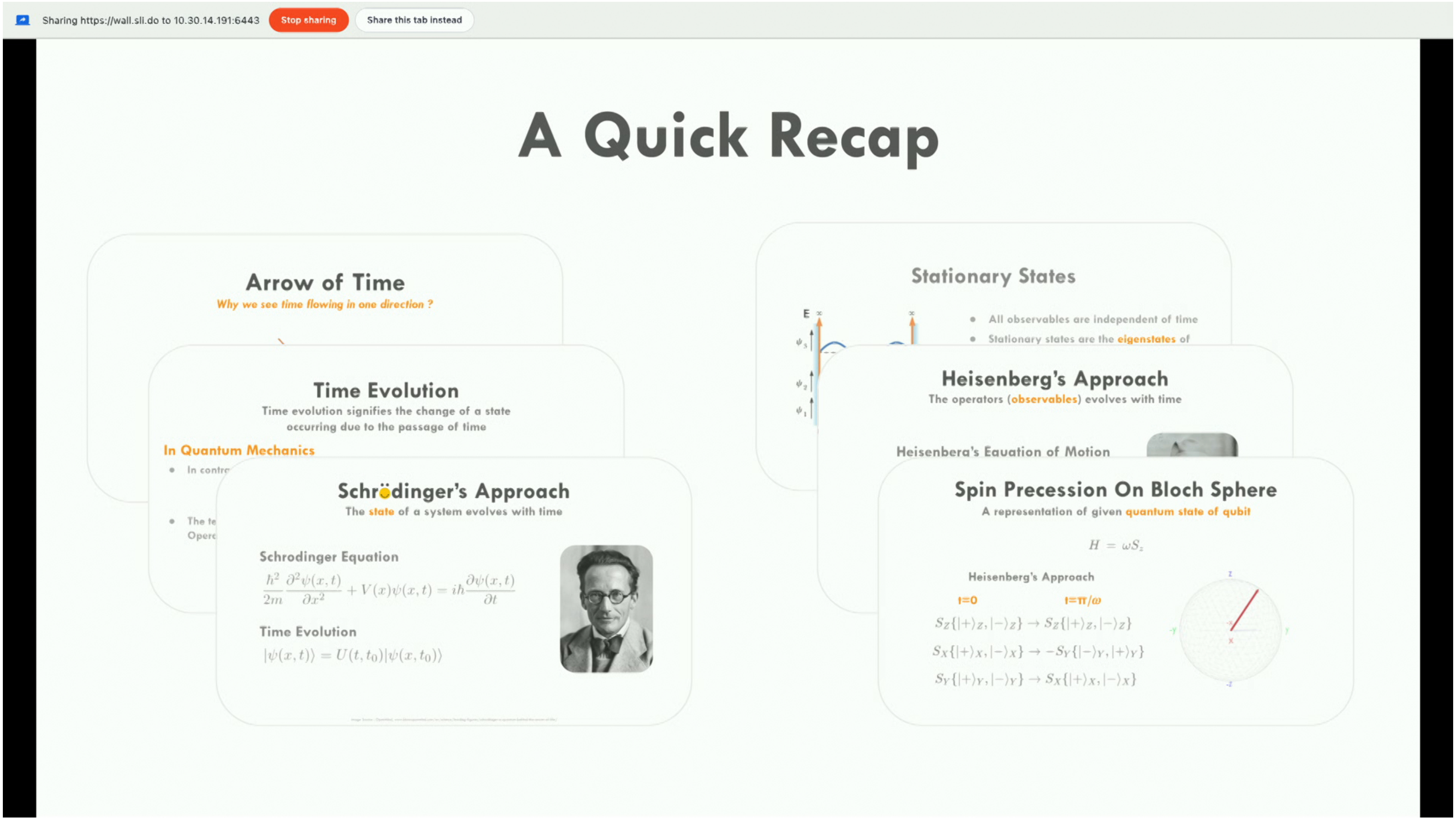

Schrödinger
⟨A(t)⟩ = ⟨ψ_s(t)| A_s |ψ(t)_s⟩
= ⟨ψ(0)| U† A_s U |ψ(0)⟩
|ψ(t)⟩ = e^{-iHt/ħ} |ψ(0)⟩
Â|a⟩ = a|a⟩
equ^n of motion
iħ d/dt |ψ(t)⟩ = H |ψ(t)⟩
Heisenberg
⟨A_H(t)⟩ = ⟨ψ(0)| U† A_s U |ψ(0)⟩
A_H(t) = U† A_s U
a U†|a⟩ = Â U†|a⟩
|a⟩_H
|a_H⟩
d A_H(t)/dt = 1/iħ [A_H(t), H]
Free particle
H = P_x^2/2m
P_x , X
dP_x(t)/dt = 1/iħ [P_x(t), P_x^2(t)/2m] = 0
dx/dt = 1/iħ [X, P_x^2(t)/2m] ⇒


eisenberg.
(t)> = <ψ(0)| U†A_S U |ψ(0)>
_H(t) = U† A_S U
a U†|a> = Â U†|a>
|a>_H
|a_H>
d A_H(t) / dt = 1/iħ [A_H(t), H]
Free particle.
H = P̂_x² / 2m
P_x , X
d P_x(t) / dt = 1/iħ [P_x(t), P_x²(t)/2m] = 0 → dP_x/dt = 0 P_x(t) = P_x(0)
dx/dt = 1/iħ [X, P_x²(t)/2m] ⇒ dx/dt = P_x(0)/m

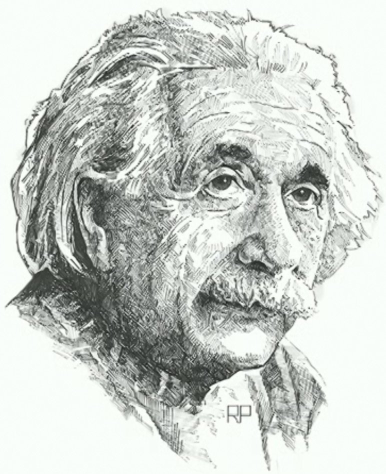

Bloch Sphere
|0⟩ |1⟩
σz
|±⟩ = (|0⟩ ± |1⟩)/√2
|±i⟩ = (|0⟩ ± i|1⟩)/√2
z |0⟩ → σx
|−⟩
|−i⟩
y |i⟩ σy
x |+⟩ → σx
|1⟩
|ψ⟩ = α|0⟩ + β|1⟩ ⇒ e^{iδ}(cos θ/2 |0⟩ + e^{i(φ−δ)} sin θ/2 |1⟩)
|α|² + |β|² = 1
|+⟩ = {π/2, φ}
0 ≤ θ ≤ π
By convention
α = e^{iδ} cos θ/2 β = e^{iδ} sin θ/2
0 ≤ φ ≤ 2π


⇒ Interaction picture
⇒ eqn of motion
exactly
perturbatively
⇒ Fermi Golden rule
0 ≤ θ ≤ π
0 ≤ φ ≤ 2π


geogebra.org/m/xzxwhrfr
Sharing https://wall.sli.do to 10.30.14.191:6443
Stop sharing
Share this tab instead
GeoGebra
ASSIGN
Sphere
a : Sphere(A, B)
= x² + y² + z² = 1
Text
text1 = "|1⟩"
text2 = "|0⟩"
text3 = "|+⟩"
text4 = "|−⟩"
text5 = "|+i⟩"
text6 = "|−i⟩"
Vector
v = Vector(A, D)
= (0.04, 0.71, 0.71)
GeoGebra Calculator Suite
|0⟩
|1⟩
|+⟩
|−⟩
|+i⟩
|−i⟩


Sharing https://wall.sli.do to 10.30.14.191:6443
Stop sharing
Share this tab instead
A Quick Recap
Arrow of Time
Why we see time flowing in one direction ?
Time Evolution
Time evolution signifies the change of a state occurring due to the passage of time
In Quantum Mechanics
Schrödinger's Approach
The state of a system evolves with time
Schrodinger Equation
$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$
Time Evolution
$|\psi(x,t)\rangle = U(t,t_0)|\psi(x,t_0)\rangle$
Stationary States
All observables are independent of time
Stationary states are the eigenstates of
Heisenberg's Approach
The operators (observables) evolves with time
Heisenberg's Equation of Motion
Spin Precession On Bloch Sphere
A representation of given quantum state of qubit
$H = \omega S_z$
Heisenberg's Approach
t=0
t=π/ω
$S_Z\{|+\rangle_Z, |-\rangle_Z\} \rightarrow S_Z\{|+\rangle_Z, |-\rangle_Z\}$
$S_X\{|+\rangle_X, |-\rangle_X\} \rightarrow -S_Y\{|-\rangle_Y, |+\rangle_Y\}$
$S_Y\{|+\rangle_Y, |-\rangle_Y\} \rightarrow S_X\{|+\rangle_X, |-\rangle_X\}$

Sharing https://wall.sli.do to 10.30.14.191:6443 Stop sharing Share this tab instead



"We can't solve problems by using the same kind of thinking we used when we created them"

Albert Einstein

Image Source : Albert Einstein Drawing, favpng.com/png_view/transparent-albert-einstein-albert-einstein-drawing-png/PQyWim5D

Sharing https://wall.sli.do to 10.30.14.191:6443 Stop sharing Share this tab instead

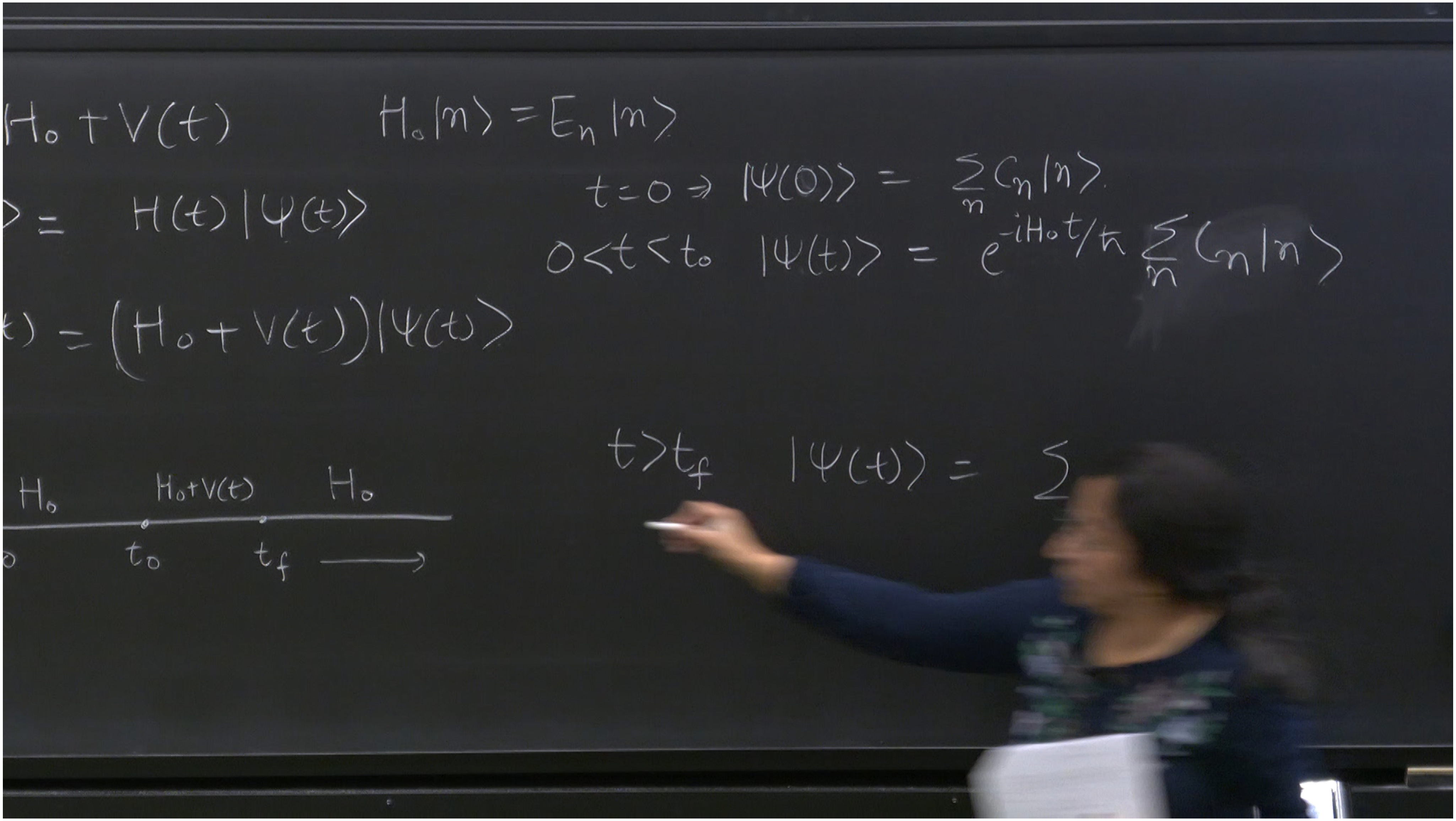
Time Dependent Perturbation

H^0 — $H^0 + V(t)$ — H^0

t_0 t_f

Schrödinger Equation $$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \underbrace{(H^0 + V(t))}_{H(t)} |\psi(t)\rangle$$

What are the probabilities of finding the system in different eigen states after the perturbation is turned off?


H₀ + V(t)
H₀|n⟩ = Eₙ|n⟩
t=0 ⇒ |Ψ(0)⟩ = Σₙ cₙ|n⟩
⟩ = H(t)|Ψ(t)⟩
0<t<t₀ |Ψ(t)⟩ = e^{-iH₀t/ħ} Σₙ cₙ|n⟩
t) = (H₀ + V(t))|Ψ(t)⟩
t>t_f |Ψ(t)⟩ = Σ
H₀ H₀+V(t) H₀
t₀ t_f

$H_0 + V(t)$

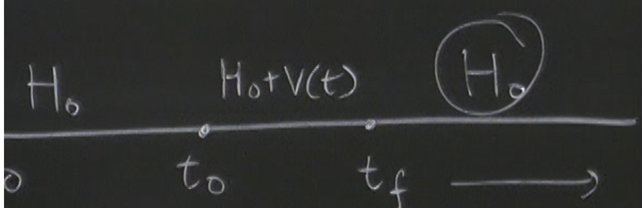
$H_0|n\rangle = E_n|n\rangle$

$\rangle = H(t)|\psi(t)\rangle$

$t=0 \Rightarrow |\Psi(0)\rangle = \sum_n c_n|n\rangle$

$0<t<t_0 \quad |\Psi(t)\rangle = e^{-iH_0 t/\hbar}\sum_n c_n|n\rangle$

$t) = \left(H_0 + V(t)\right)|\Psi(t)\rangle$



$t>t_f \quad |\Psi(t)\rangle = \sum c_n(t)|n\rangle$

(t)

$(t)|\Psi(t)\rangle$

$H_0 + V(t))|\Psi(t)\rangle$

$H_0|n\rangle = E_n|n\rangle$

$t=0 \Rightarrow |\Psi(0)\rangle = \sum_n c_n|n\rangle$

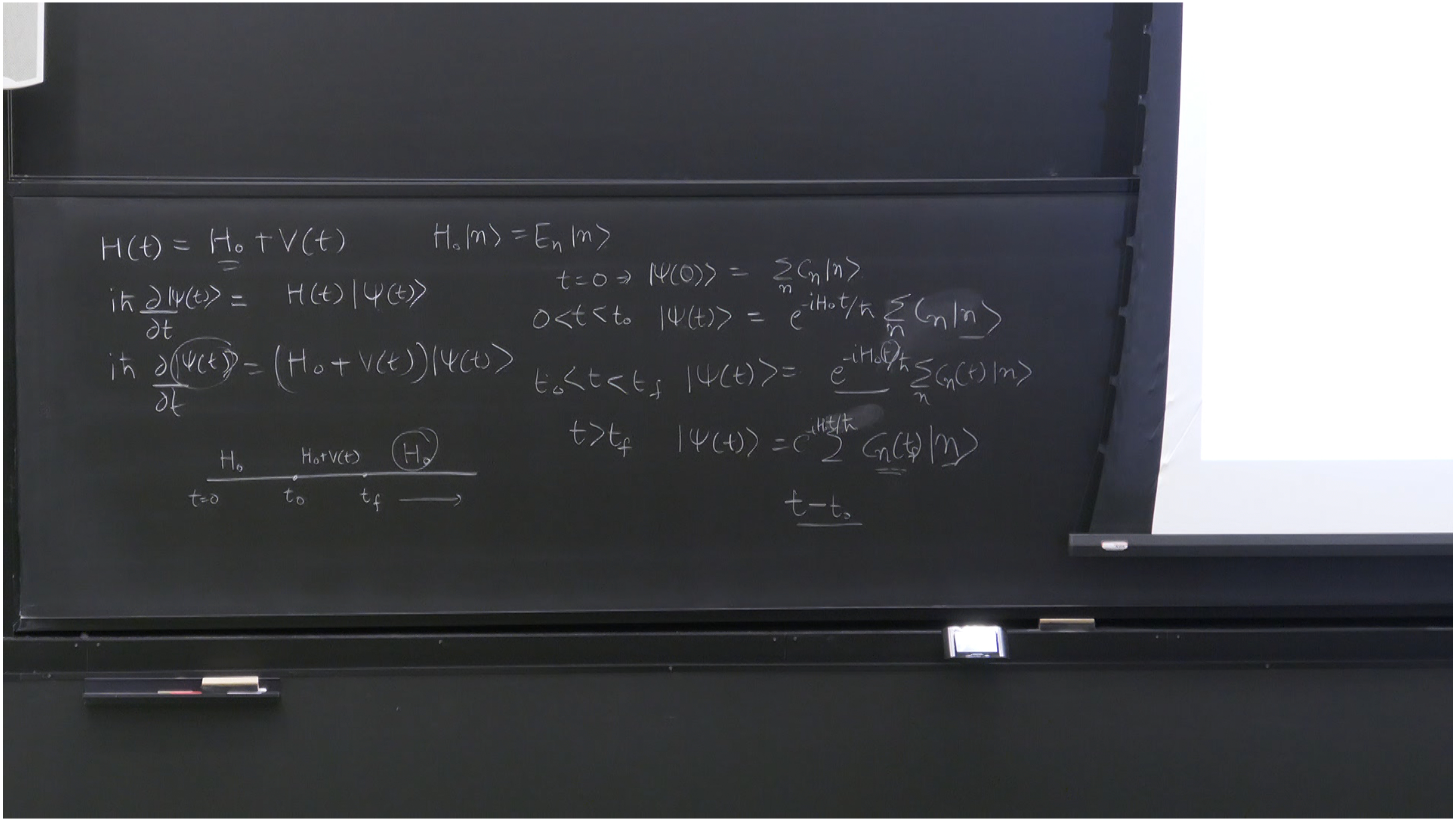
$0<t<t_0 \quad |\Psi(t)\rangle = e^{-iH_0 t/\hbar}\sum_n c_n|n\rangle$

$t_0<t<t_f \quad |\Psi(t)\rangle = e^{-iH_0 t/\hbar}\sum_n c_n(t)|n\rangle$

$t>t_f \quad |\Psi(t)\rangle = e^{-iH_0 t/\hbar}\sum c_n(t_f)|n\rangle$

$H_0+V(t)$ $\quad$ H_0

t_f ⟶

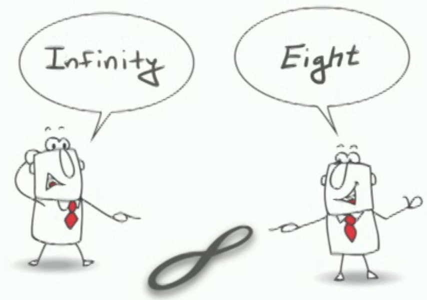

H(t) = H₀ + V(t)
H₀|n⟩ = Eₙ|n⟩
iħ ∂|Ψ(t)⟩/∂t = H(t)|Ψ(t)⟩
iħ ∂|Ψ(t)⟩/∂t = (H₀ + V(t))|Ψ(t)⟩
t=0 ⇒ |Ψ(0)⟩ = Σₙ cₙ|n⟩
0<t<t₀ |Ψ(t)⟩ = e^{-iH₀t/ħ} Σₙ cₙ|n⟩
t₀<t<t_f |Ψ(t)⟩ = e^{-iH₀(t)/ħ} Σₙ cₙ(t)|n⟩
t>t_f |Ψ(t)⟩ = e^{-iH₀t/ħ} Σ cₙ(t_f)|n⟩
t−t₀
H₀
H₀+V(t)
H₀
t=0
t₀
t_f

Interaction Picture

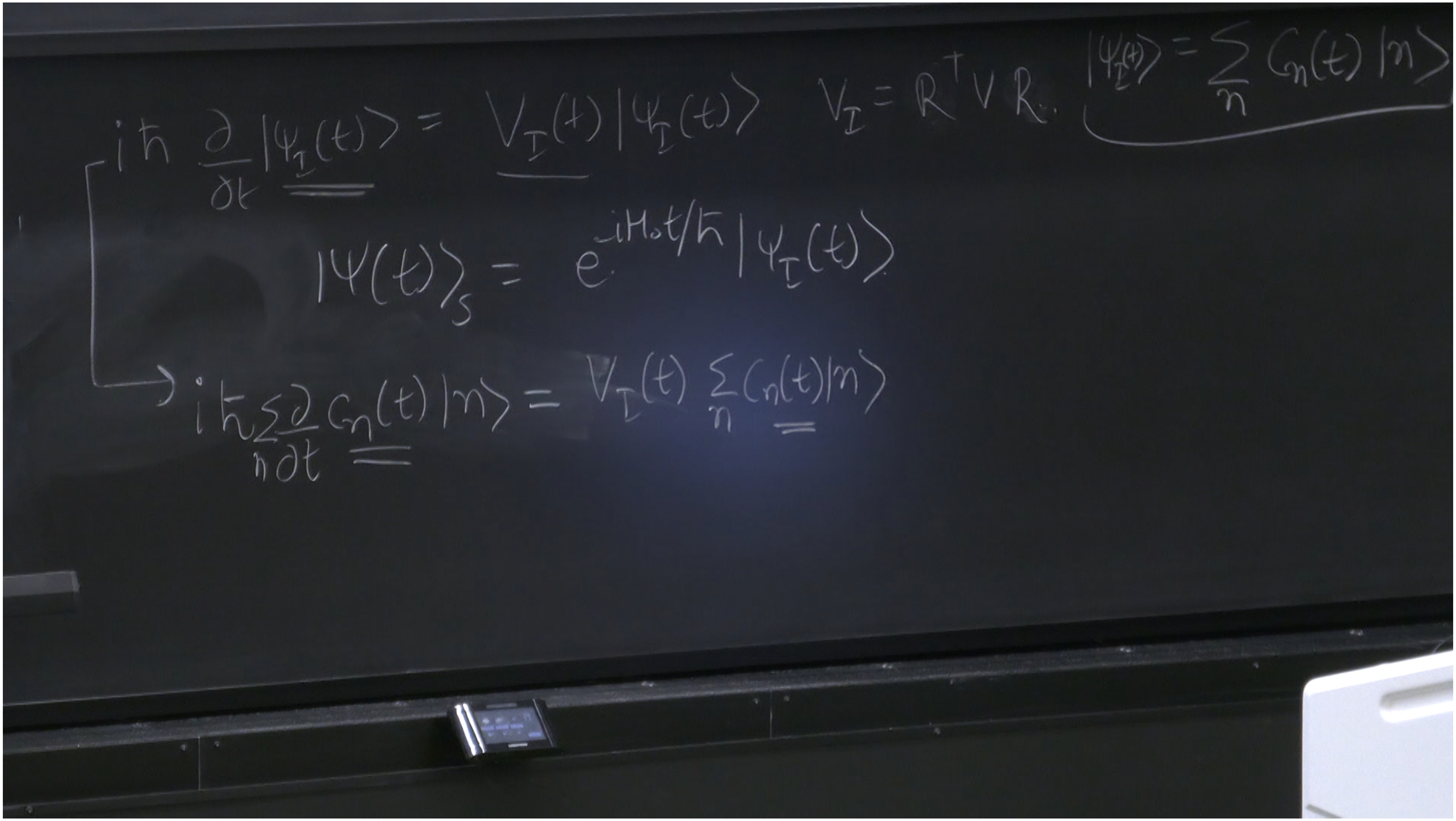
The both **states** and operators (**observables**) evolve with time

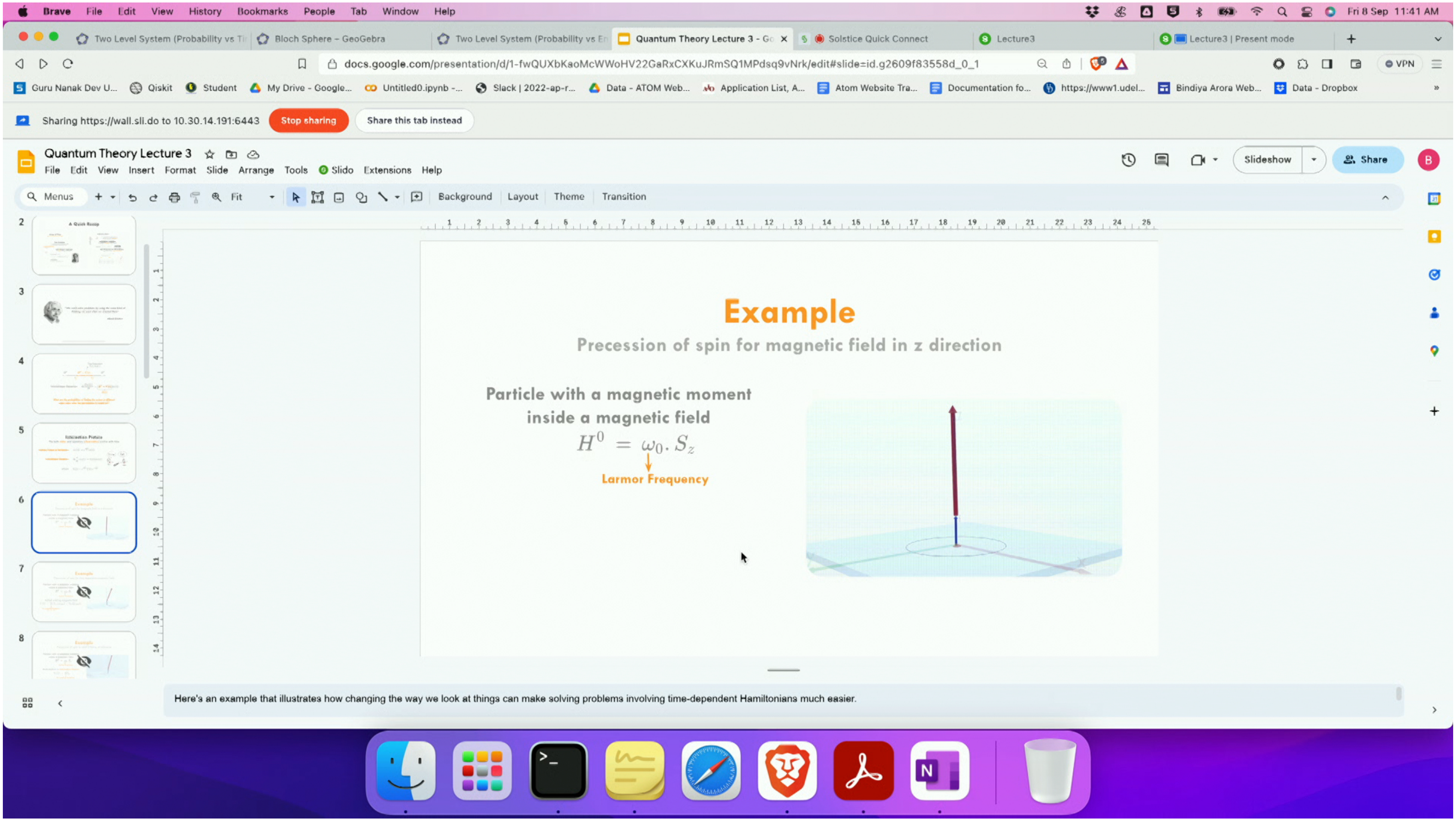
Shifting Frame of Reference $|\psi_I(t)\rangle = e^{\frac{iH^0 t}{\hbar}}|\psi(t)\rangle$

Schrödinger Equation $i\hbar\frac{d}{dt}|\psi_I(t)\rangle = V_I(t)|\psi_I(t)\rangle$



where $V_I(t) = e^{\frac{iH^0 t}{\hbar}}\, V(t)\, e^{-\frac{iH^0 t}{\hbar}}$


$i\hbar \frac{\partial}{\partial t}|\psi_I(t)\rangle = V_I(t)|\psi_I(t)\rangle$
$V_I = R^\dagger V R$
$|\psi_I(t)\rangle = \sum_n c_n(t)|n\rangle$
$|\psi(t)\rangle_S = e^{-iH_0 t/\hbar}|\psi_I(t)\rangle$
$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t)|n\rangle = V_I(t) \sum_n c_n(t)|n\rangle$


Quantum Theory Lecture 3
Example
Precession of spin for magnetic field in z direction
Particle with a magnetic moment
inside a magnetic field
$H^0 = \omega_0 . S_z$
Larmor Frequency
Here's an example that illustrates how changing the way we look at things can make solving problems involving time-dependent Hamiltonians much easier.

Example

Precession of spin in rotated frame of reference

Particle with a magnetic moment inside a magnetic field

$$H^0 = \omega_0 . S_z$$

Larmor Frequency

Perturbation in Interaction Picture

$$V_I(t) = \Omega S_x$$

For small perturbation $\Omega << \omega_0$

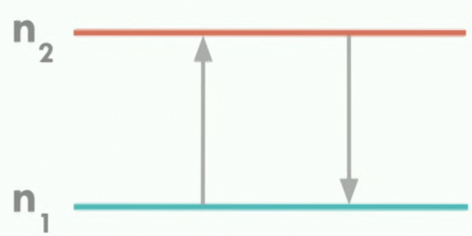
This slide is skipped in slideshow mode

Here's an example that illustrates how changing the way we look at things can make solving problems involving time-dependent Hamiltonians much easier.

Sharing https://wall.sli.do to 10.30.14.191:6443 Stop sharing Share this tab instead

Time Evolution

Analyzing the effect of time-dependent potential on transition between eigenstates



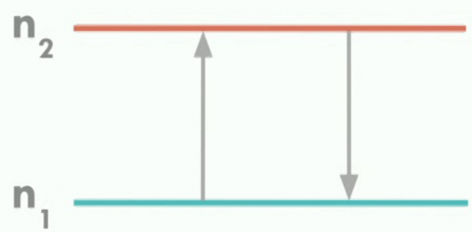
Expanding Wave Function in Basis of H^0 $|\psi_I(t)\rangle = \sum_n c_n(t)|n\rangle$

Equation for Coefficients

$$i\hbar \frac{d}{dt} \sum_m c_m(t)|m\rangle = V_I(t) \sum_n c_n(t)|n\rangle$$

Time Evolution

Analyzing the effect of time-dependent potential on transition between eigenstates



Expanding Wave Function in Basis of H^0 $\quad |\psi_I(t)\rangle = \sum_n c_n(t)|n\rangle$

Equation for Coefficients $\quad i\hbar \frac{d}{dt} \sum_m c_m(t)|m\rangle = V_I(t) \sum_n c_n(t)|n\rangle$

Coupled Differential Equations

Using Orthogonality of eigen states, $V_{mn}(t) = \langle m|V(t)|n\rangle$ **and** $\omega_{mn} = \dfrac{E_m - E_n}{\hbar}$

$$i\hbar \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12}e^{i\omega_{12}t} & \cdots \\ V_{21}e^{i\omega_{21}t} & V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}$$

Exactly | **Perturbatively**

$$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t) \langle m|n\rangle = \sum_n \underbrace{\langle m|V_I(t)|n\rangle} c_n(t)$$

$$i\hbar\, \dot{c}_m(t) = \sum_n c_n(t) \underbrace{\langle m|V(t)|n\rangle}_{V_{mn}.} e^{i(E_m - E_n)t/\hbar}$$

Sharing https://wall.sli.do to 10.30.14.191:6443 Stop sharing Share this tab instead

n Level System

Consider a **sudden violent perturbation** at t=0

$t = 0$

Perturbation $$V(t) = \begin{bmatrix} 0 & \alpha \\ \alpha^\star & 0 \end{bmatrix} \delta(t)$$

Coupled Differential Equation $$i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn}(t) c_n(t)$$

$$i\hbar \sum_n \frac{\partial}{\partial t} c_n(t)\langle m|n\rangle = \sum_n \underbrace{\langle m|V_I(t)|n\rangle}\, c_n(t)$$

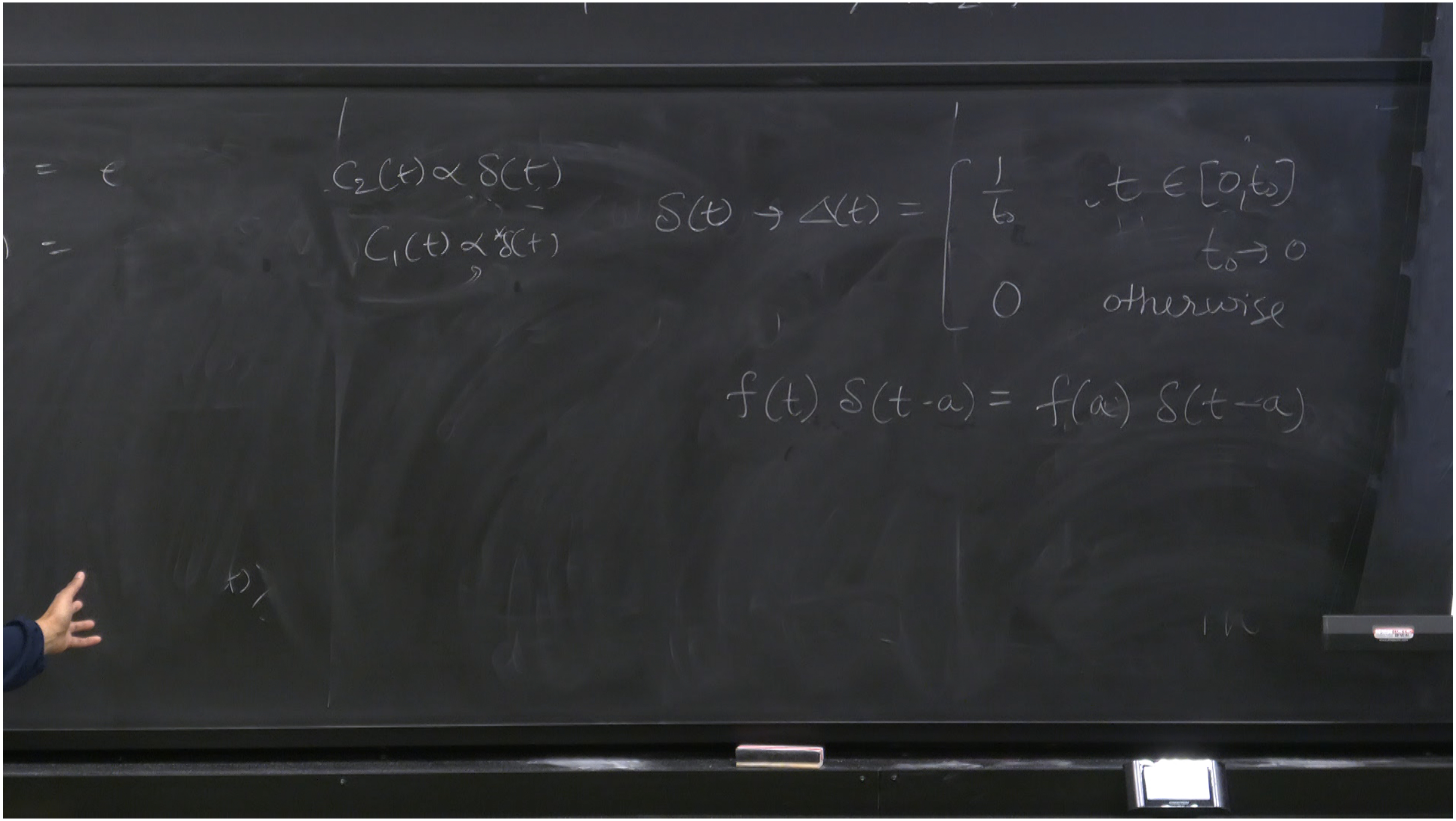
$$i\hbar\, \dot{c}_m(t) = \sum_n c_n(t)\, \underbrace{\langle m|V(t)|n\rangle}_{V_{mn}}\, e^{i(E_m-E_n)t/\hbar}$$

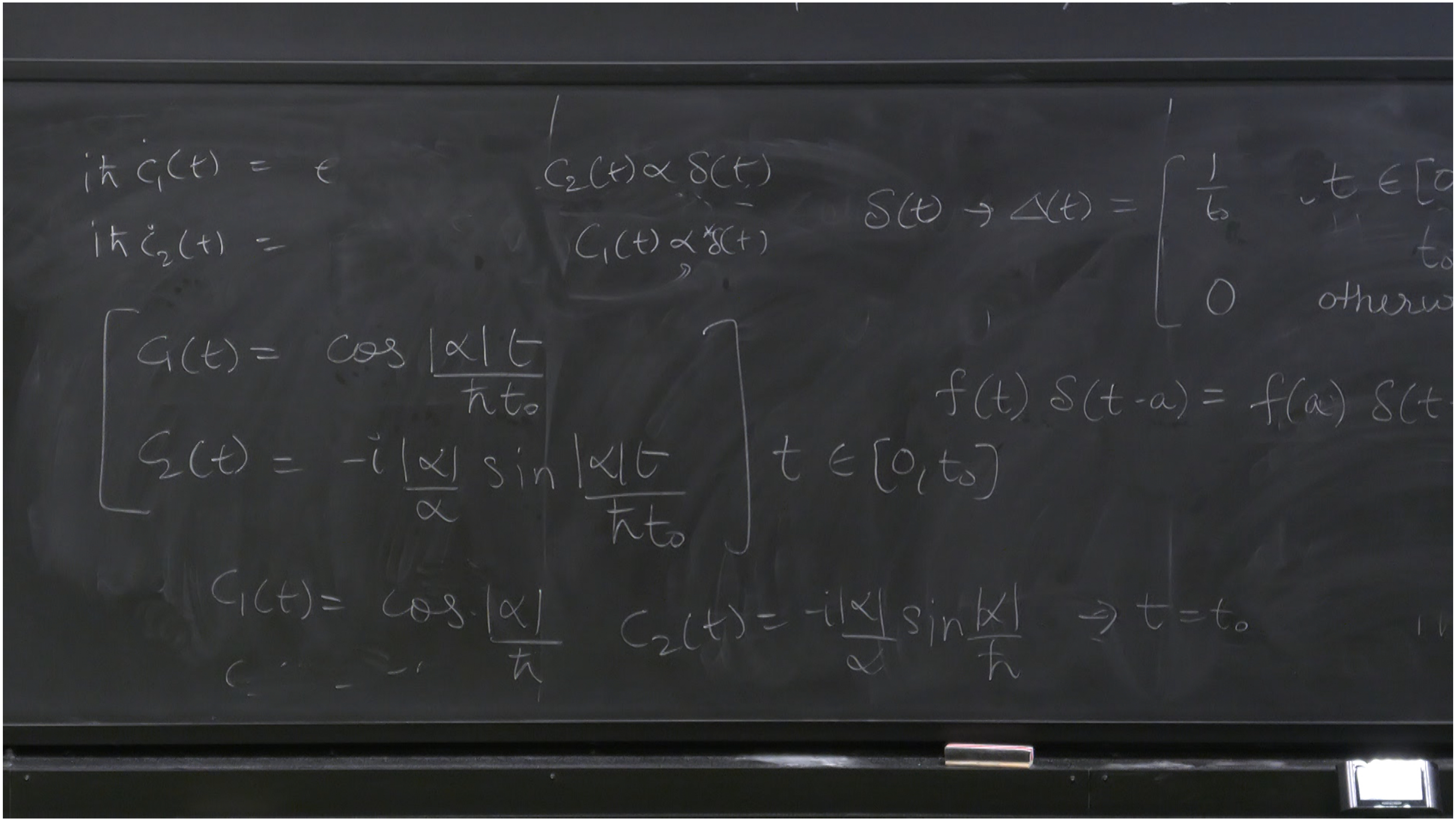
<u>Exactly</u>

at $t = 0_-$ $\quad c_1(0_-) = 1 \quad c_2(0_-) = 0$

$$i\hbar\, \dot{c}_1(t) = e^{i\omega_{12}t} V_{12} c_2(t) + \cancel{e^{i\omega_{11}t} V_{11}} c_1(t)$$

$$i\hbar\, \dot{c}_2(t) = e^{-i\omega_{12}t} V_{21} c_1(t) + e^{i\omega_{22}t} \cancel{V_{22}} c_2(t)$$


C2(t) ∝ δ(t)
C1(t) ∝ δ(t)
δ(t) → Δ(t) = 1/t0 , t ∈ [0,t0]
t0 → 0
0 otherwise
f(t) δ(t-a) = f(a) δ(t-a)


iħ ċ1(t) = ε
iħ ċ2(t) =
C2(t) ∝ δ(t)
C1(t) ∝ δ(t)
δ(t) → Δ(t) = 1/t0 , t ∈ [0
t0
0 otherw
C1(t) = cos |α| t / ħ t0
C2(t) = −i |α|/α sin |α| t / ħ t0
t ∈ [0, t0]
f(t) δ(t − a) = f(a) δ(t
C1(t) = cos |α|/ħ
C2(t) = −i |α|/α sin |α|/ħ
⇒ t = t0

Sharing https://wall.sli.do to 10.30.14.191:6443 Stop sharing Share this tab instead

Perturbative Solution

No Transitions — Single Transitions

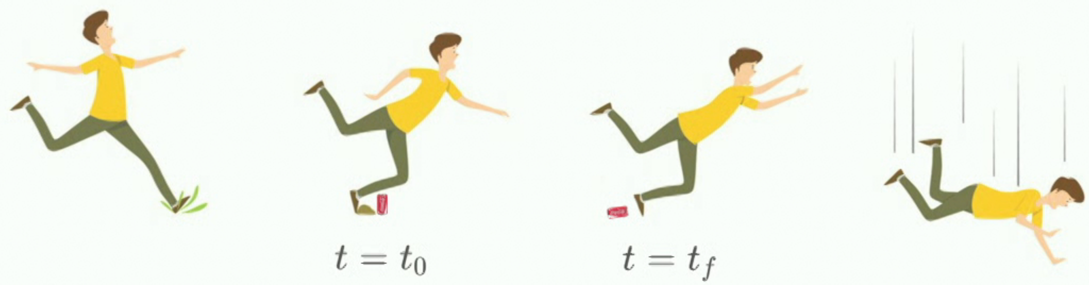
$$c_m(t) = \delta_{mi} + \frac{1}{i\hbar}\sum_n \int_{t_0}^{t_f} dt' c_n(t_0) V_{mn}(t') e^{i\omega_{mn}t'} +$$

$$\left(\frac{1}{i\hbar}\right)^2 \sum_n \sum_l \int_{t_0}^{t_f} \int_{t_0}^{t'} dt' dt'' c_l(t'') V_{nl}(t'') e^{i\omega_{nl}t''} V_{mn}(t') e^{i\omega_{mn}t'} + \cdots$$

Double Transitions

n Level System

Consider a **constant perturbation** turned on from $t=t_0$ to $t=t_f$



Perturbation $V(t) = \begin{cases} 0 & otherwise \\ V & t_0 < t < t_f \end{cases}$

Coupled Differential Equation $i\hbar \dot{c}_m(t) = \sum_n e^{i\omega_{mn}t} V_{mn}(t) c_n(t)$

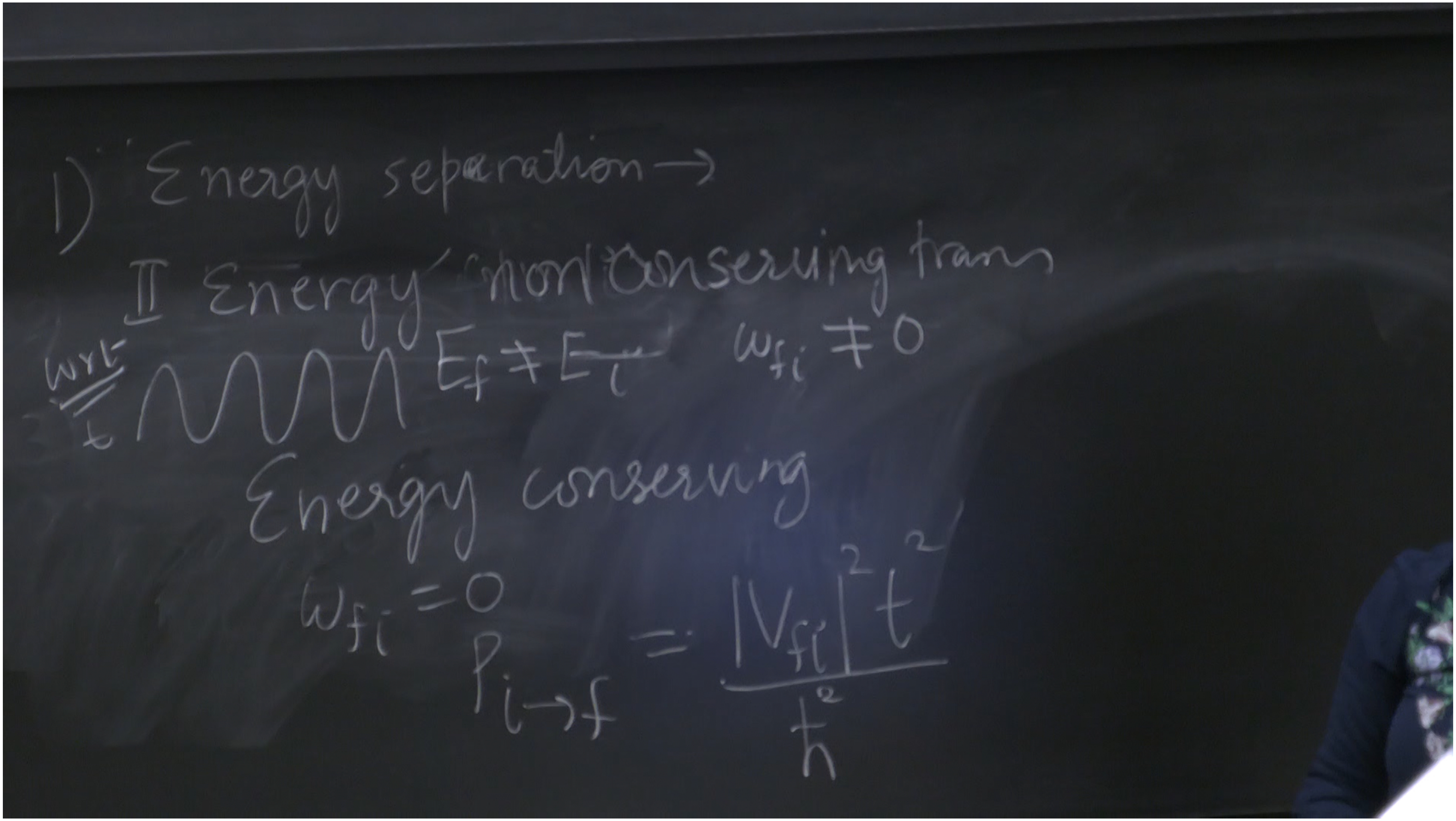
Upto First Order $c_m(t) = \delta_{mi} + \frac{1}{i\hbar} \sum_n \int_{t_0}^{t_f} dt' c_n(t_0) V_{mn}(t') e^{i\omega_{mn}t'}$

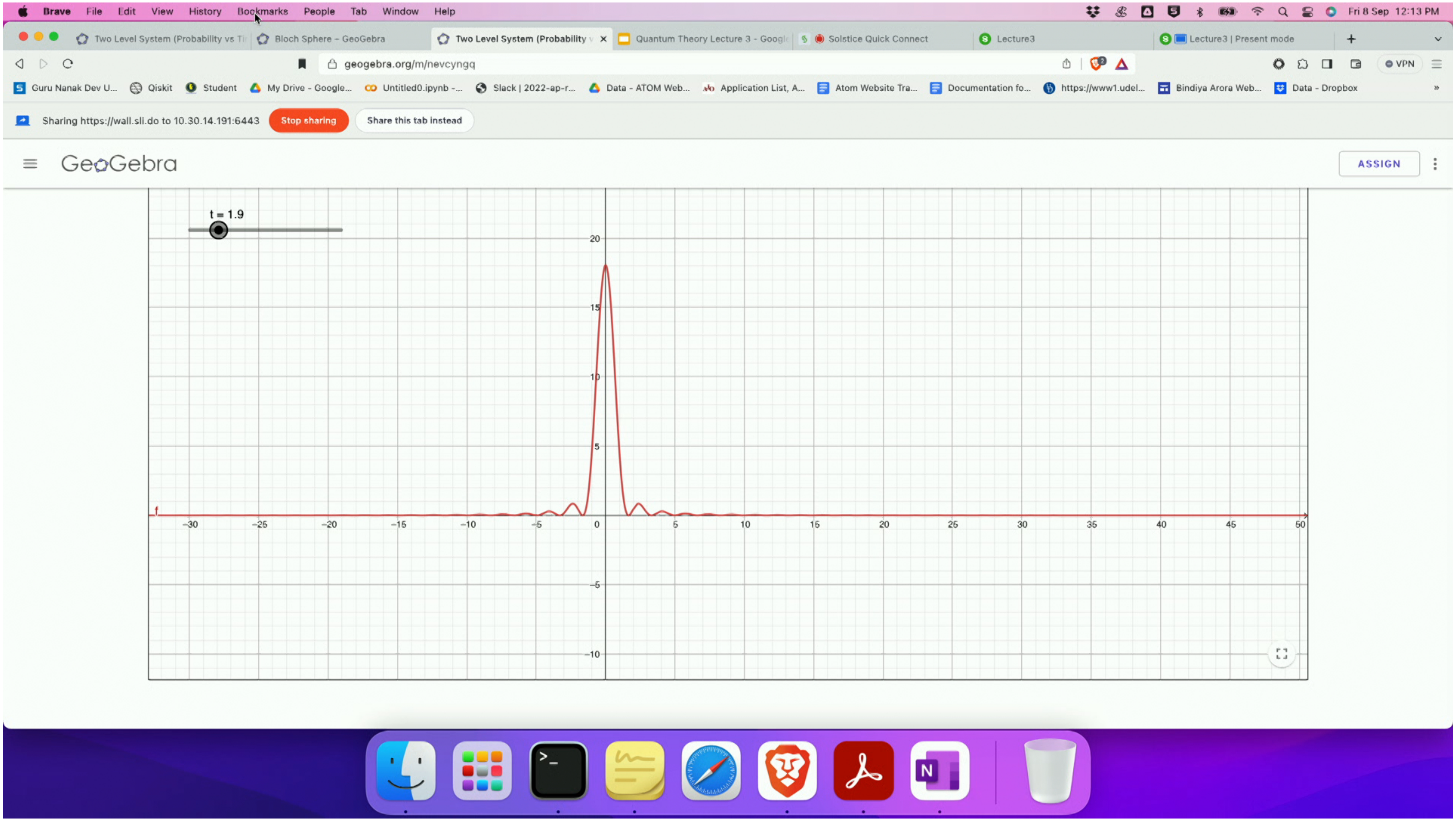
$C_i(0) = \delta_{mi}$ Constant

$$P_{i\to f}(t) = |C_f^{(1)}(t)|^2$$

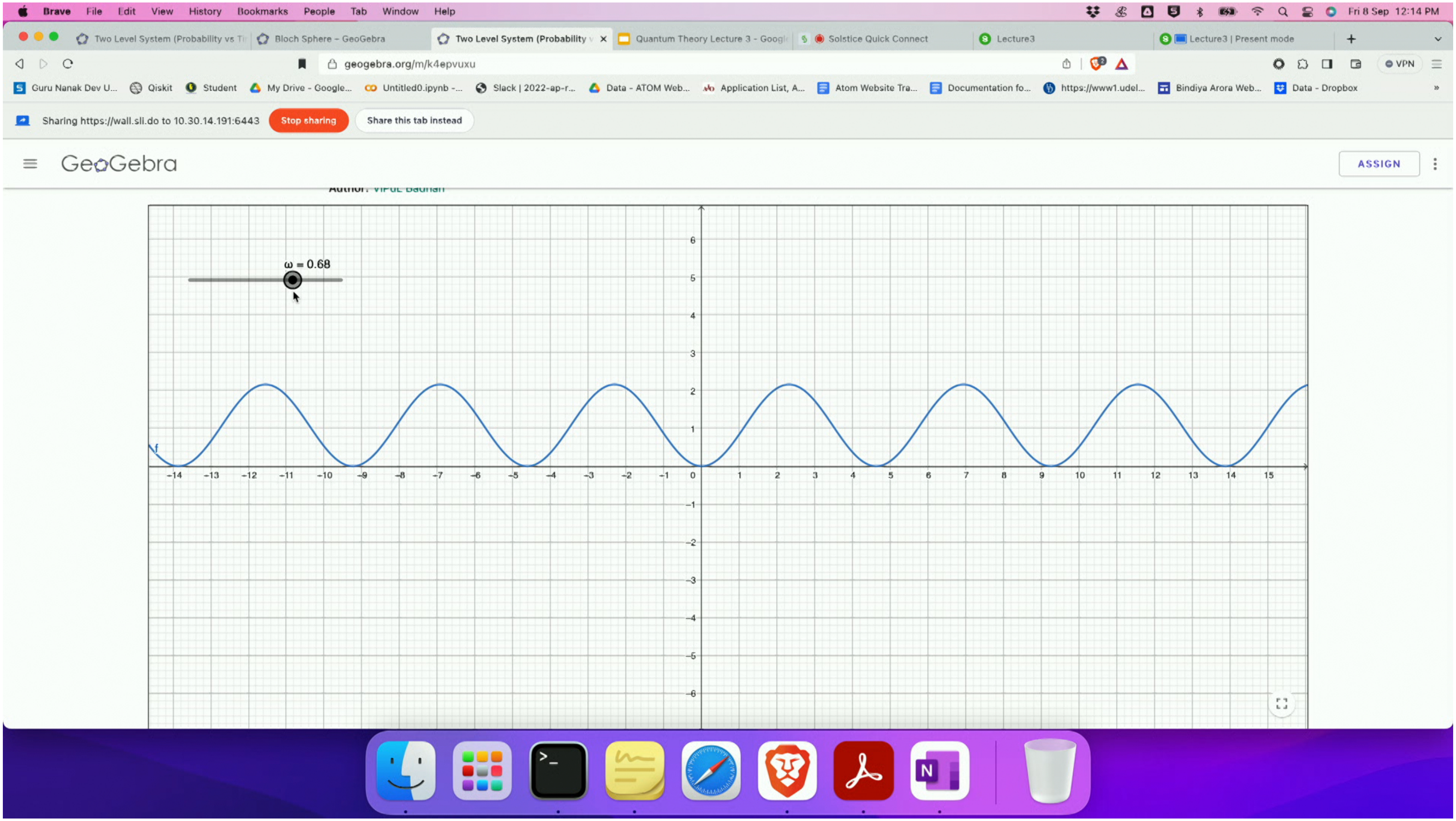
$$C_f^{(1)}(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t')\, e^{i\omega_{fi}t'}\, dt'$$

$$\boxed{P_{i\to f}(t) = \frac{4 V_{fi}^2}{\hbar^2 \omega_{fi}^2} \sin^2\left(\frac{\omega_{fi} t}{2}\right)}$$


1) Energy separation →
II Energy non conserving trans
wrt t
$E_f \neq E_i$ $\omega_{fi} \neq 0$
Energy conserving
$\omega_{fi} = 0$
$P_{i\to f} = \frac{|V_{fi}|^2 t^2}{\hbar^2}$



Two Level System (Probability
geogebra.org/m/nevcyngq
Sharing https://wall.sli.do to 10.30.14.191:6443
Stop sharing
Share this tab instead
GeoGebra
ASSIGN
t = 1.9
f



geogebra.org/m/k4epvuxu
Sharing https://wall.sli.do to 10.30.14.191:6443
Stop sharing
Share this tab instead
GeoGebra
ASSIGN
ω = 0.68
f

Fermi's Golden Rule

Energy levels are not discrete but has a finite width

No. of states per unit energy around E_f

$$\sum_f P_{i\to f}(t) = \int P_{i\to f}(t)\,\overbrace{\rho(E_f)}\,dE_f$$

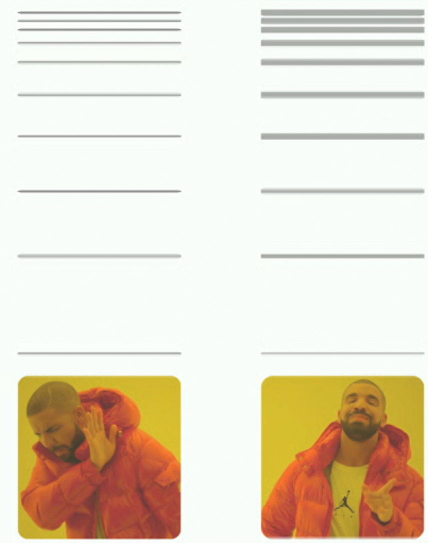
Over Entire Spectrum

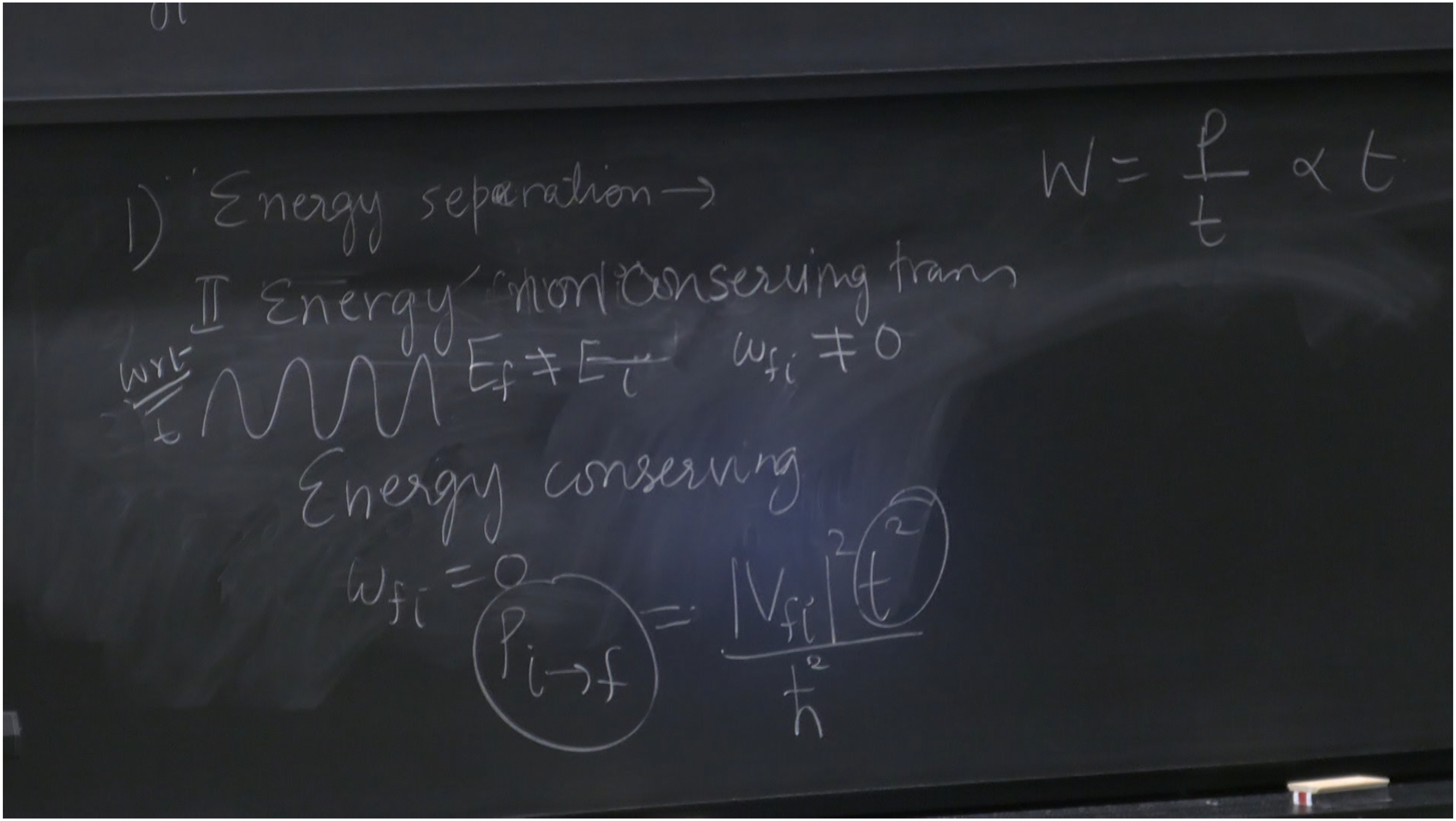
$$= \frac{2\pi |V_{fi}|^2 \rho(E_f)\, t}{\hbar}$$

Rate of transition Independent of time

Fermi's Golden Rule

Discrete Width | Finite Width




1) Energy separation →
W = ℏ/t ∝ t
II Energy non conserving trans
wrt t
E_f ≠ E_i
ω_fi ≠ 0
Energy conserving
ω_fi = 0
P_i→f = |V_fi|² t² / ℏ²