Title: Classical Physics Lecture - 092723

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Collection: Classical Physics 2023/24

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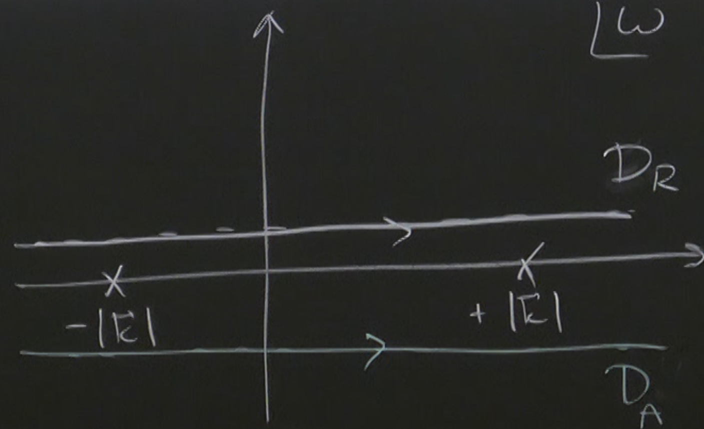
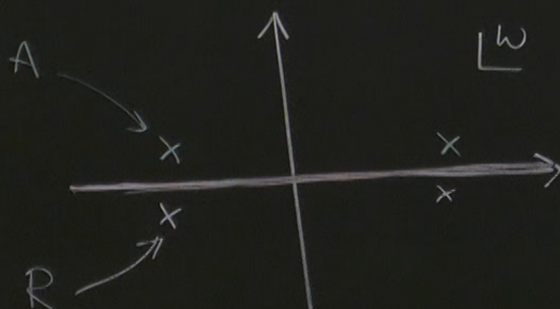
$k^\mu = (\omega, \vec{k})$



$$D_R(r) = -\frac{1}{2\pi}\Theta(t)\,\delta(r^\mu r_\mu)$$

$$D_A(r) = -\frac{1}{2\pi}\Theta(-t)\,\delta(r_\mu r^\mu)$$

$$|||$$



$$-k_\mu k^\mu = (\omega - |\vec{k}|)(\omega + |\vec{k}|)$$

$$\rightsquigarrow (\omega - |\vec{k}| + i\epsilon)(\omega + |\vec{k}| + i\epsilon)$$

$\ominus \quad \ominus$

k^μ

regularit.

In Fourier:

$$\hat{D}_R(k) = \frac{1}{(2\pi)^4} \frac{1}{(\omega + i\epsilon)^2 - |\vec{k}|^2}$$

$$= \frac{1}{(2\pi)^4} \frac{1}{\omega^2 - |\vec{k}|^2 + 2i\epsilon\omega}$$

"retarded propagator"

<u>Rmk</u> $\Box e^{ik_\mu x^\mu} = -k_\mu k^\mu e^{ik_\mu x^\mu}$

$$(\Box - 2\epsilon\partial_t) e^{ik_\mu x^\mu} = (\underbrace{-k_\mu k^\mu + 2i\epsilon\omega}_{\hat{D}_R(k)^{-1}}) e^{ik_\mu x^\mu}$$

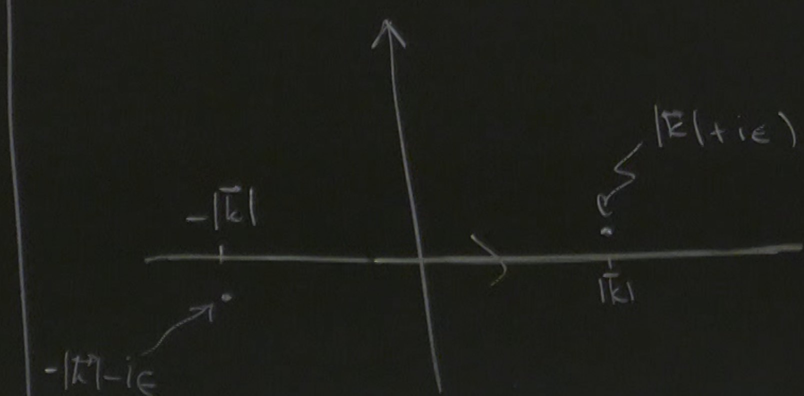
breaks time reversal sym.

A regularization that does <u>not</u> break time reversal sym?

$$\Box \to \Box + i\epsilon$$

$$\frac{-1}{(2\pi)^4}\frac{1}{k_\mu k^\mu} \to \frac{-1}{(2\pi)^4}\frac{1}{k_\mu k^\mu + i\epsilon} = \frac{1}{(2\pi)^4}\frac{1}{\omega^2 - |\vec{k}|^2 - i\epsilon} = \frac{1}{(2\pi)^4(\omega - |\vec{k}| - i\epsilon)(\omega + |\vec{k}| + i\epsilon)}$$

$\epsilon \equiv 2\epsilon|\vec{k}|$ (positive infinit.)



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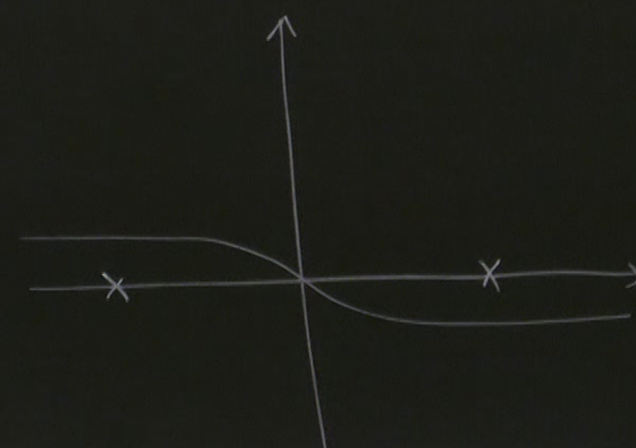
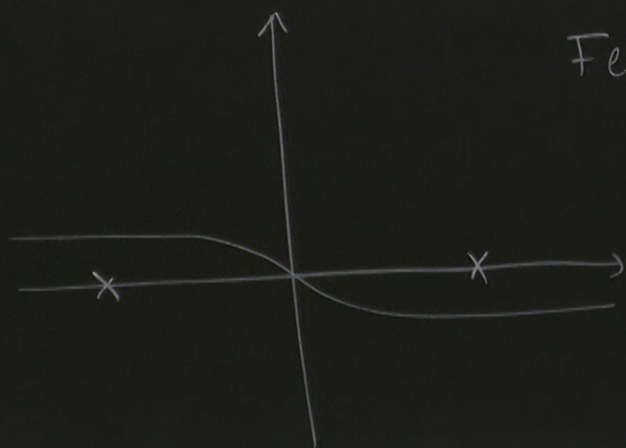


$\epsilon \equiv 2\epsilon|\vec{k}|$ (positive infinit.)

$$\frac{1}{(2\pi)^4}\frac{1}{\omega^2-|\vec{k}|^2-i\epsilon} = \frac{1}{(2\pi)^4(\omega-|\vec{k}|-i\epsilon)(\omega+|\vec{k}|+i\epsilon)}$$

$|\vec{k}|+i\epsilon)$

$\equiv$



Feynman prop.

$$D_F(r) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik_\mu r^\mu}}{k_\mu k^\mu + i\epsilon}$$

And:

$$D_R(r) = \Theta(t)\Big(D_+(r) \underbrace{- D_+(-r)}_{+D_-(r)}\Big)$$

$$D_A(r) = D_R(-r)$$

$$D_F(r) = \Theta(t)\, D_+(r) + \underbrace{\Theta(-t)\, D_+(-r)}_{-\Theta(-t)\, D_-(r)}$$

...n

...g the

pole.

$D_+(r)^*$

And:

- $D_R(r) = \Theta(t)\left(D_+(r) \underbrace{- D_+(-r)}_{+D_-(r)} \right)$

- $D_A(r) = D_R(-r)$

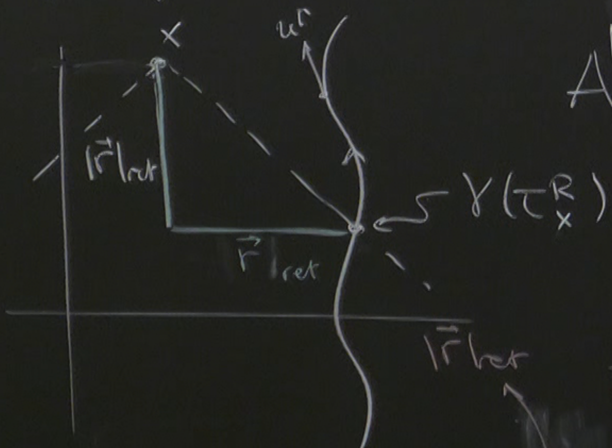
- $D_F(r) = \Theta(t) D_+(r) + \underbrace{\Theta(-t) D_+(-r)}_{-\Theta(-t) D_-(r)}$

these are all Green's function for different "boundary conditions"

$$\Box D_\circ(r) = \delta(r)$$

Lienard-Wiechert

Retarded sol. of $\Box A_\mu = -4\pi j_\mu$
for j^μ given by a point particle (in spacetime!)



$$A^\mu_R(x) = 2\int d\tau\; \Theta(x^0-\gamma^0(\tau))\,\delta\left((x-\gamma(\tau))^2\right) q u^\mu(\tau)$$

$$= \frac{q u^\mu(\tau)}{(x^\mu-\gamma^\mu(\tau))u_\mu(\tau)}\Bigg|_{\tau=\tau^R_x}$$

$$A^0_R \equiv \varphi^R(x) = \frac{-q}{\underbrace{\boxed{(x^0-\gamma^0(\tau^R_x))}}_{|\vec r|_{ret}} + \underbrace{\boxed{(x^i-\gamma^0(\tau^R_x))}}_{\vec r_{ret}} v_i}\Bigg|_{\tau_R} = \frac{q}{(1-\vec v\cdot\vec n)|\vec r|}\Bigg|_{ret}$$

$$\varphi^R = \frac{q}{\underbrace{(1-\vec{v}\cdot\vec{n})}_{\text{"redshift" due to relativistic change of frame}}|\vec{r}|}\Bigg|_{ret}$$

finite speed of light

$$\vec{A}^R = \frac{q\vec{v}}{(1-\vec{v}\cdot\vec{n})|\vec{r}|}\Bigg|_{ret}.$$

L.W. pot.

$$\vec{n} = \frac{\vec{r}}{|\vec{r}|}$$

In the presence of j^μ the most general EM field will have the form:

$$A_\mu(x) =$$

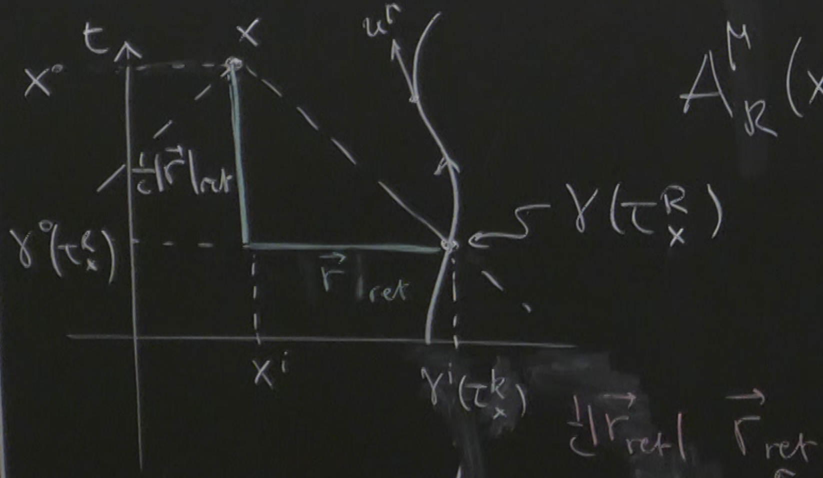
r)

δ(r)

Lienard-Wiechert

Retarded sol. of $\Box A_\mu = -4\pi j_\mu$

for j^μ given by a point particle (in spacetime!)



$$A^\mu_R(x) = 2\int d\tau\, \Theta\left(x^0-\gamma^0(\tau)\right)\,\delta\left(\left(x-\gamma(\tau)\right)^2\right) q u^\mu(\tau)$$

$$= \frac{q u^\mu(\tau)}{\left(x^\mu-\gamma^\mu(\tau)\right)u_\mu(\tau)}\Bigg|_{\tau=\tau^R_x}$$

$$A^0_R \equiv \varphi^R(x) = \frac{-q}{\overset{\frac{1}{c}|\vec r_{ret}|}{\boxed{\left(x^0-\gamma^0(\tau^R_x)\right)}} + \overset{\vec r_{ret}}{\boxed{\left(x^i-\gamma^0(\tau^R_x)\right)}}\, v_i}\Bigg|_{\tau_R} = \frac{q}{(1-\vec v\cdot\vec n)|\vec r|}\Bigg|_{ret}$$

...e speed
...light

L.W.
pot.

In the presence of j^μ
the most general EM field
will have the form:

$$A_\mu(x) \begin{cases} = A^\mu_{in}(x) - 4\pi \int d^4y\, j^\mu(y)\, G_R(y,x) \\ = A^\mu_{out}(x) - 4\pi \int d^4y\, j^\mu(x)\, G_A(y,x) \end{cases}$$

$A^\mu_R \equiv 0$ in the far past
(if j^μ is switched on/off)

a sol of the homogeneous eq.
$\Box A^\mu_{in} = 0$

Contribution to outgoing radiation from having switched on/off my source:

from two expression of same A_T

$$A^\mu_{rad} = A^\mu_{out} - A^\mu_{in} \overset{!}{=} -4\pi \int d^4y \left(G_R - G_A\right)(y,x)\, j^\mu(y)$$

Contribution to outgoing radiation from having switched on/off my source:

from two expressions of same A_T

$$A^\mu_{rad} = A^\mu_{out} - A^\mu_{in} \stackrel{\leftarrow}{=} -4\pi \int d^4y \, \underbrace{\left(G_R - G_A\right)}_{D_{PJ}(r)}(y,x) \, j^\mu(y)$$

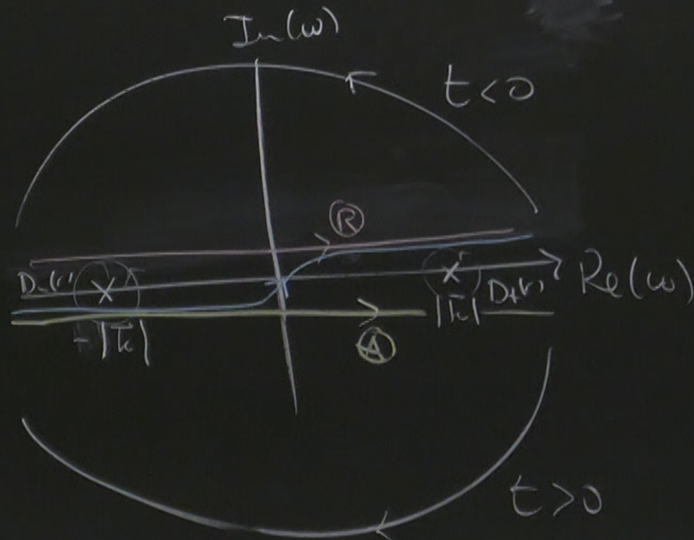
Pauli-Jordan

$= D_+(r) - D_-$

Rmk $\Box D_{PJ} = \Box(D_R - D_A) = \delta - \delta \equiv 0$

$\Rightarrow D_{PJ}$ is not a Green's function.

$$D(r) = \frac{1}{(2\pi)^4}\int d^3\vec{k}\; e^{i\vec{k}\cdot\vec{r}} \int d\omega\, \frac{e^{-i\omega t}}{(\omega - |\vec{k}|)(\omega + |\vec{k}|)}$$



$$D_R = \begin{cases} 0 & t<0 \\ -D_+(r) - D_-(r) & t>0 \end{cases}$$

$$D_A = \begin{cases} D_+(r) + D_-(r) & t<0 \\ 0 & t>0 \end{cases}$$

$$D_F = \begin{cases} D_-(r) & t<0 \\ -D_+(r) & t>0 \end{cases}$$