Title: Quantum influences and event relativity

Speakers: Nicholas Ormrod

Series: Quantum Foundations

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Abstract: A number of no-go theorems suggest that theories upholding both unitarity and relativity must deny that events are absolute. I'll show how quantum causal influences allow us to articulate an attractive conception of relational events. This will lead us towards a precise, observer-independent reformulation of quantum theory, in which relational events emerge from causation.

Zoom link: https://pitp.zoom.us/j/98649944363?pwd=Y25NOUROelY2ZmFpVWhTZFErV2MwUT09

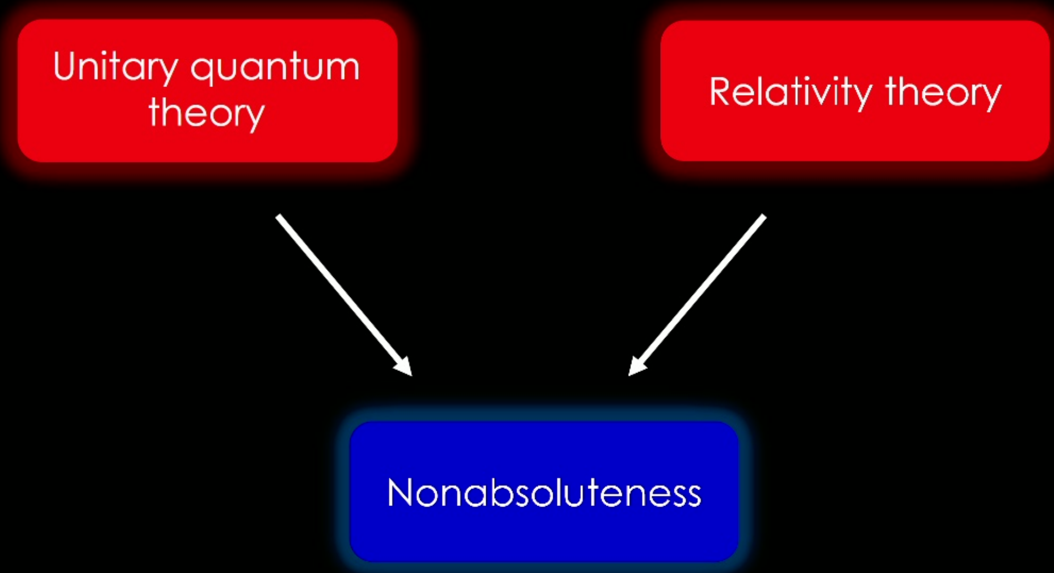
quantum influences and event relativity

towards a precise reformulation of quantum theory based on causation

Nick Ormrod
University of Oxford

the absoluteness of observed events

$$(O_1, \ldots, O_n)$$



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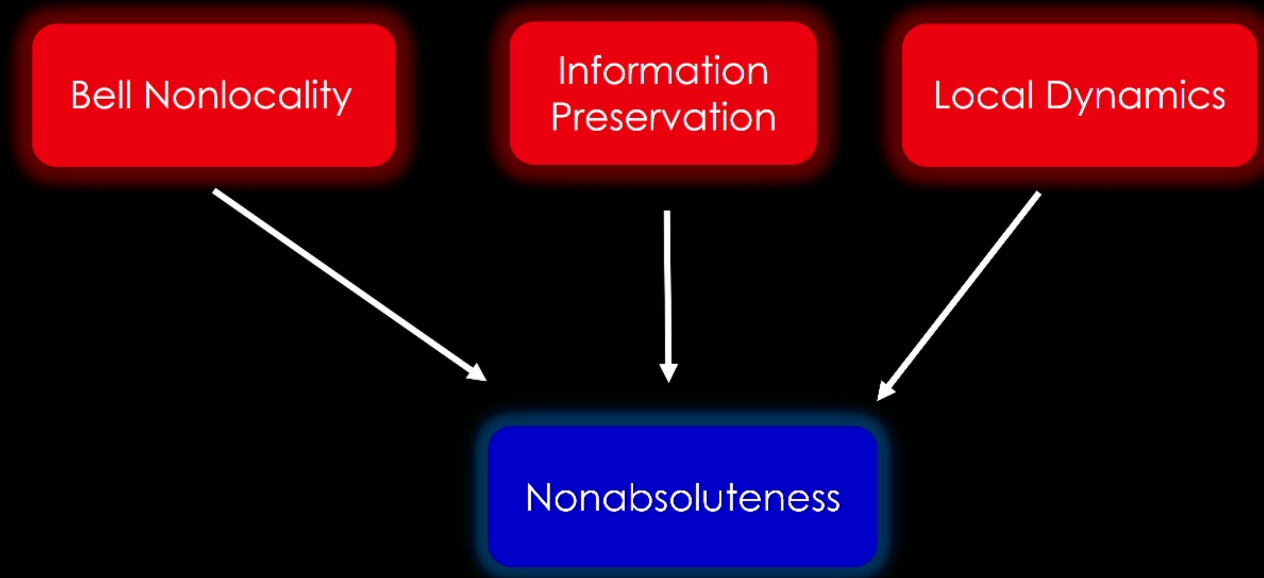


conceptions of event relativity

A bomb explodes relative X, but not relative to Y

- Everett: X and Y are branches
- Consistent histories: X and Y are consistent sets
- “Relational” approaches: X and Y are observers
  - events obtain relative to the system that `observes', as a result of an interaction with the observed system

quantum influences and event relativity

- quantum influences are just what we need to articulate a better understanding of event relativity

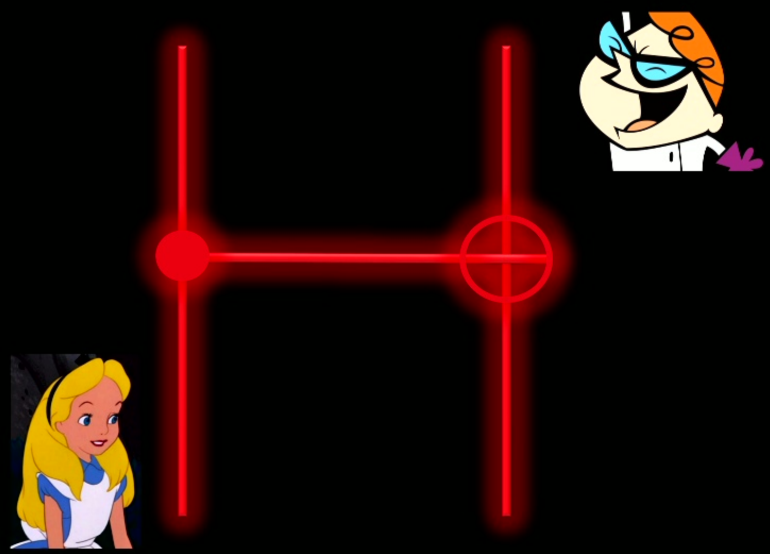
- it is a conception of event relativity that avoids the problems with consistent histories and Everett, which formalises some (though not all) of the ideas from relational approach, but which does so in a way that is conceptually clear and mathematically precise

- **ultimately, we will be led to a precise, observer-independent reformulation of quantum theory, in which causation is fundamental, and relational events emerge from it**

what is causation?

- Signalling => causation
- But causation =/> signalling:

CNOT(i, j) = (i, j+i)

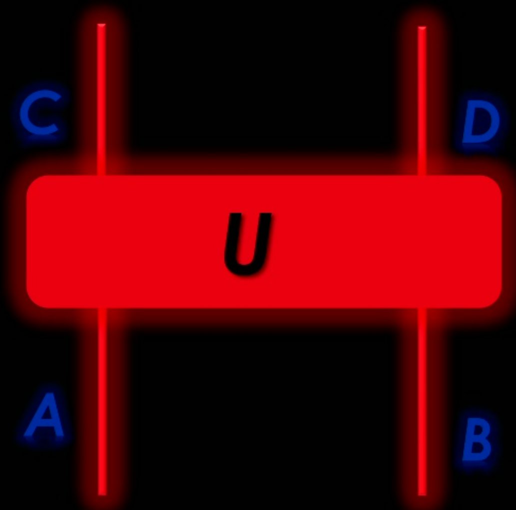


what is causation?

classically, there is causal influence from the variable A to the variable B if and only if B depends nontrivially on A in the function that fully describes the dynamics

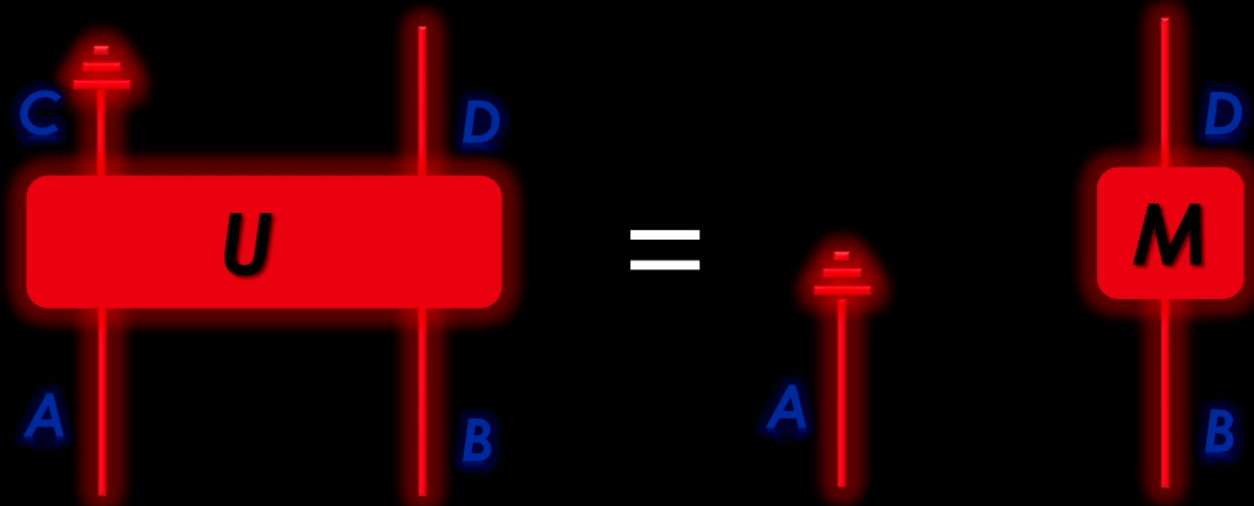
the full function tells you whole the dynamical structure, including **how** one system depends on another; the causal structure just tells you **whether** one system depends on the other

quantum influence as unitary dependence

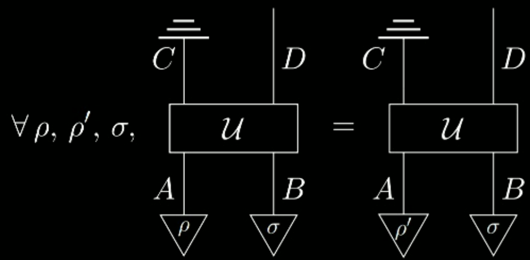


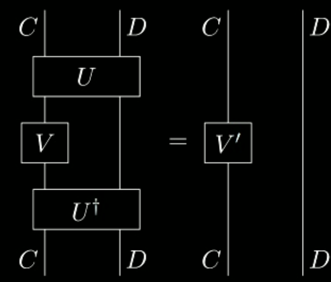
quantum influence as unitary dependence

no causal influence from A to D if and only if:

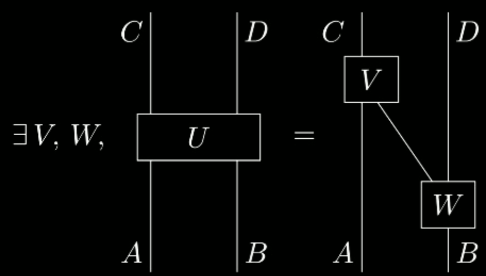


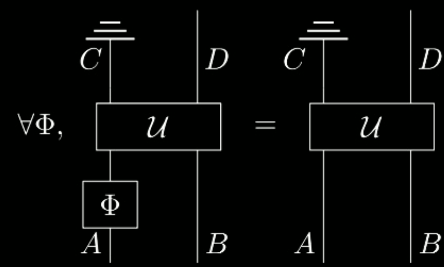
equivalent notions





$$[U(\mathcal{A} \otimes I_B)U^\dagger, I_C \otimes \mathcal{D}] = 0$$





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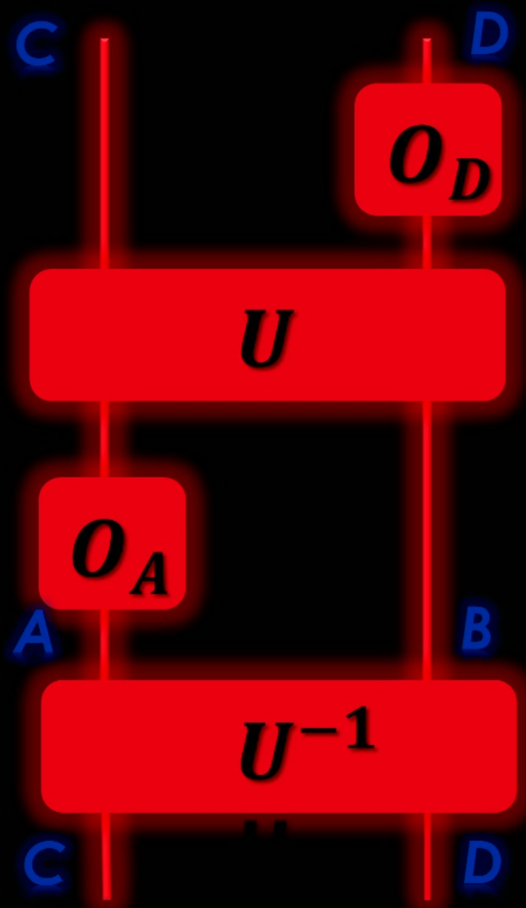

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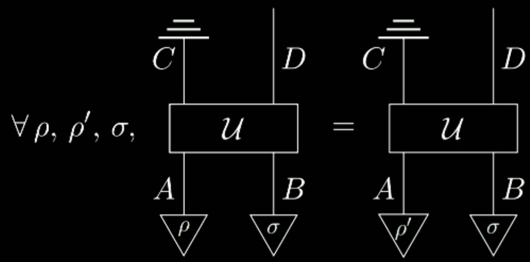
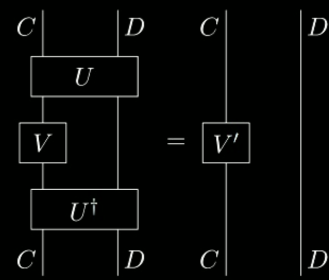
nice properties

$$[U(\mathcal{A} \otimes I_B)U^\dagger, I_C \otimes \mathcal{D}] = 0$$

- **time symmetry:** A influences D through U if and only if D influences A through U^{-1}
- **causal supervenience**: influences between composite systems are uniquely fixed by influences between subsystems
  - e.g. A influences $D_1 \otimes D_2$ if and only if either A influences D_1 or A influences D_2

equivalent notions





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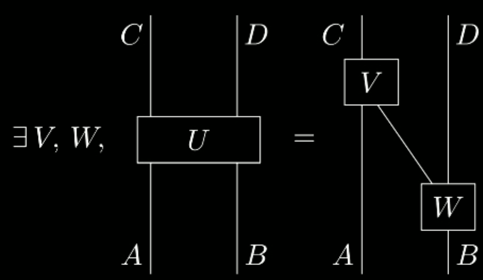

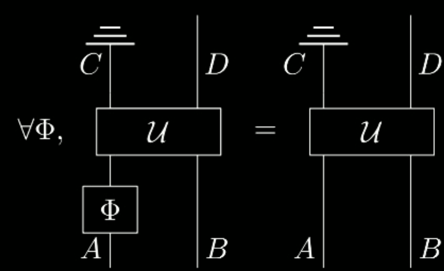

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quantum causal models

- define causal structure of an arbitrary unitary circuit or supermap
- determine whether a nonunitary supermap is compatible with a given causal structure
- show that a nonunitary supermap is compatible with a DAG if and only if it satisfies a markov condition for that dag
- quantum analogue of Reichenbach's common cause principle and d-separation theorem
- do-calculus
- extension to systems affected by sectorial constraints

projective decompositions and projective algebras

$$\mathcal{H}_A = \bigoplus_i \mathcal{H}_A^i$$

- projective decomposition $\{P_A^i\}_i$: the set of projectors onto each subspace
- projective algebra

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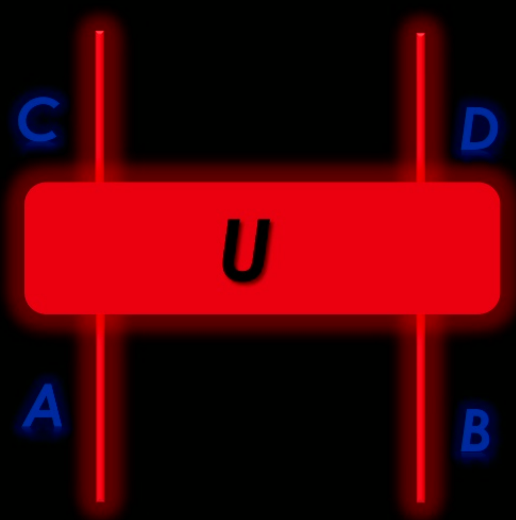
- projective decomposition $\{P_A^i\}_i$: the set of projectors onto each subspace
- projective algebra $\mathcal{P}_A := \mathtt{span}(\{P_A^i\}_i)$

interference influences



equivalently: an agent who chooses a channel with Kraus operators from $\mathcal{P}_A$ can signal to an agent who chooses a measurement with operator elements from $\mathcal{P}_D$

no interference influence from $\mathcal{P}_A$ to $\mathcal{P}_D$ if and only if:

$$[U(\mathcal{P}_A \otimes I_B)U^\dagger, I_C \otimes \mathcal{P}_D] = 0$$

nice properties

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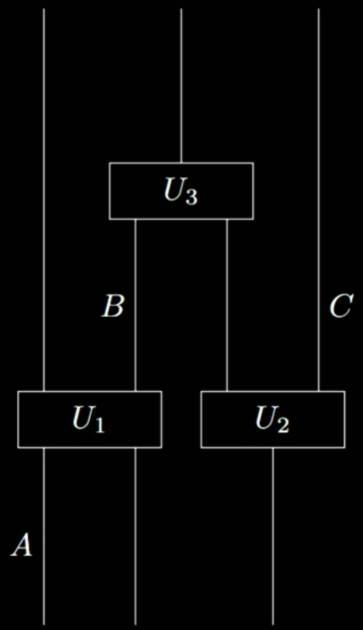
- **time symmetry:** $\mathcal{P}_A \xrightarrow{U} \mathcal{P}_D \Leftrightarrow \mathcal{P}_D \xrightarrow{U^{-1}} \mathcal{P}_A$
- **causal supervenience:** $A \rightarrow D \Leftrightarrow \exists \mathcal{P}_A, \mathcal{P}_D : \mathcal{P}_A \rightarrow \mathcal{P}_D$
  - so the interference causal structure is strictly a fine-graining of the previous sort of causal structure

events and projective decompositions



$$\{P_A^i\}_i,\ \{P_B^j\}_j,\ \{P_C^k\}_k$$

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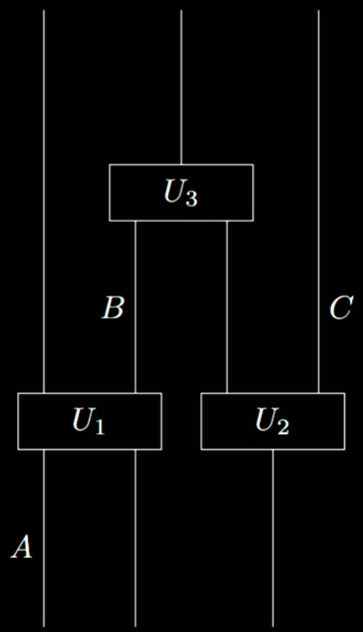
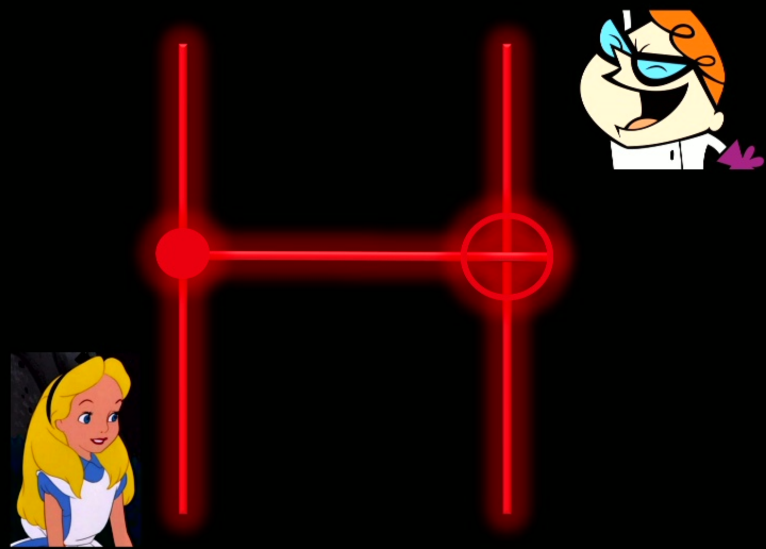


$$\{P_A^i\}_i,\ \{P_B^j\}_j,\ \{P_C^k\}_k$$

where do they come from???

- event := the selection of a single projector from a certain projective decomposition

events from influences



events from influences: preference

- definition: "D prefers $\mathcal{P}_A$" if and only if:

$$\mathcal{P}_A \not\to \mathcal{P}_D \forall \mathcal{P}_D$$

$$[\mathcal{P}'_A, \mathcal{P}_A] \neq 0 \implies \exists \mathcal{P}_D : \mathcal{P}'_A \to \mathcal{P}_D$$

essential uniqueness: if the above two conditions still hold when we substitute $\mathcal{P}''_A \neq \mathcal{P}_A$ for $\mathcal{P}_A$, then $\mathcal{P}''_A$ is a coarse-graining of $\mathcal{P}_A$

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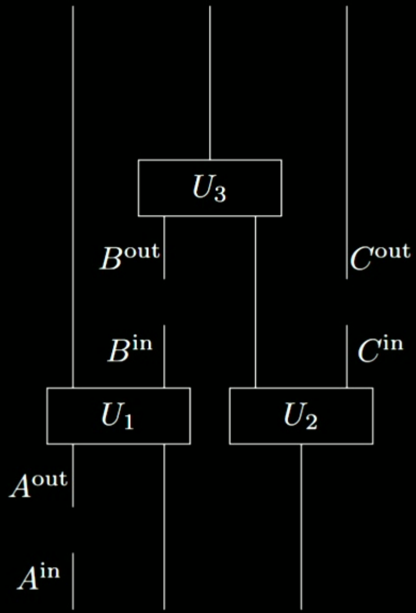
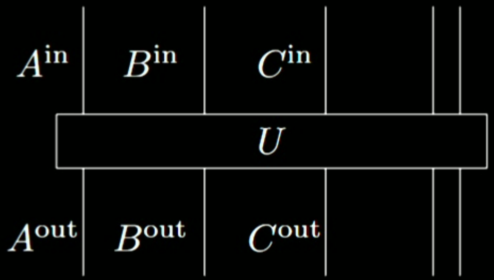
- theorem: D prefers $\mathcal{P}_A$ if and only if:

$$\begin{aligned}\mathcal{P}_A &= \texttt{centre}((\mathcal{A} \otimes I_B) \cap \texttt{comm}(U^{-1}(I_C \otimes \mathcal{D})U)) \\ &= \texttt{centre}(\tilde{\mathcal{A}} \cap \texttt{comm}(\tilde{\mathcal{D}}))\end{aligned}$$

events from influences: frames

frame F ={A, B, C}





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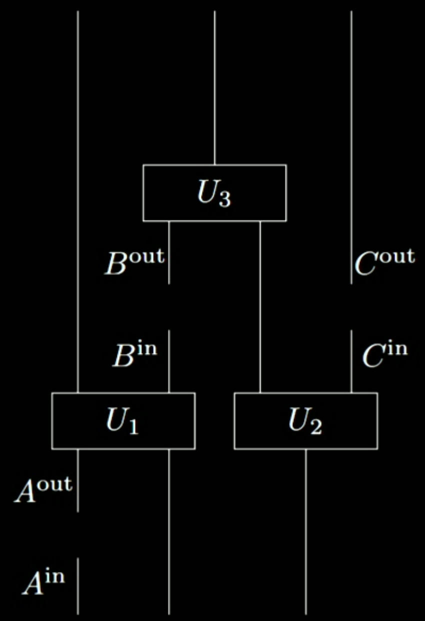

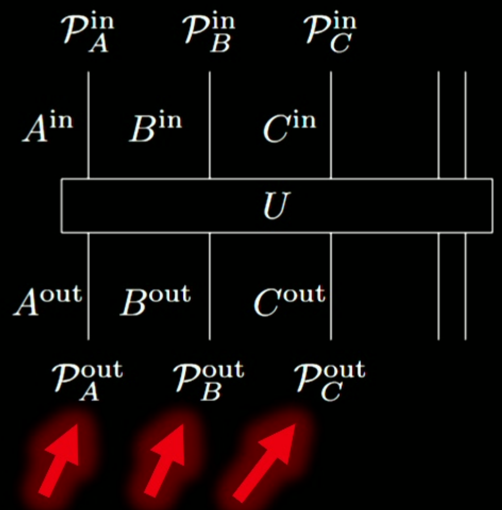

consistent histories from frames

- theorem: the decompositions derived using a single frame form a *consistent set of histories*, so we can “consistently” use the probability formula:

$$\mathrm{Prob}(e_1, e_1', \ldots e_n, e_n') = \mathrm{Tr}(P^{e_n'}_{A_n^{\mathrm{out}}} P^{e_n}_{A_n^{\mathrm{in}}} \ldots P^{e_1'}_{A_1^{\mathrm{out}}} P^{e_1}_{A_1^{\mathrm{in}}} (I/d) P^{e_1}_{A_1^{\mathrm{in}}} P^{e_1'}_{A_1^{\mathrm{out}}} \ldots P^{e_n}_{A_n^{\mathrm{in}}} P^{e_n'}_{A_n^{\mathrm{out}}})$$

… where “consistently” means that the probability function is linear

a causal explanation of consistency

- when decompositions, the probability formula simplifies to

$$
\begin{aligned}
\text{Prob}(e_1, e_1', \dots e_n, e_n') &= \text{Tr}(P^{e_n'}_{A_n^{\text{out}}} P^{e_n}_{A_n^{\text{in}}} \dots P^{e_1'}_{A_1^{\text{out}}} P^{e_1}_{A_1^{\text{in}}} (I/d) P^{e_1}_{A_1^{\text{in}}} P^{e_1'}_{A_1^{\text{out}}} \dots P^{e_n}_{A_n^{\text{in}}} P^{e_n'}_{A_n^{\text{out}}}) \\
&= \frac{1}{d} \text{Tr}(P^{e_1}_{A_1^{\text{in}}} P^{e_1'}_{A_1^{\text{out}}} \dots P^{e_n}_{A_n^{\text{in}}} P^{e_n'}_{A_n^{\text{out}}})
\end{aligned}
$$

which is manifestly linear.

- this simplification is due to the causal structure…

a causal explanation of consistency

$$\mathcal{P}_X^{\text{out}} \not\to \mathcal{P}_Y^{\text{out}}$$

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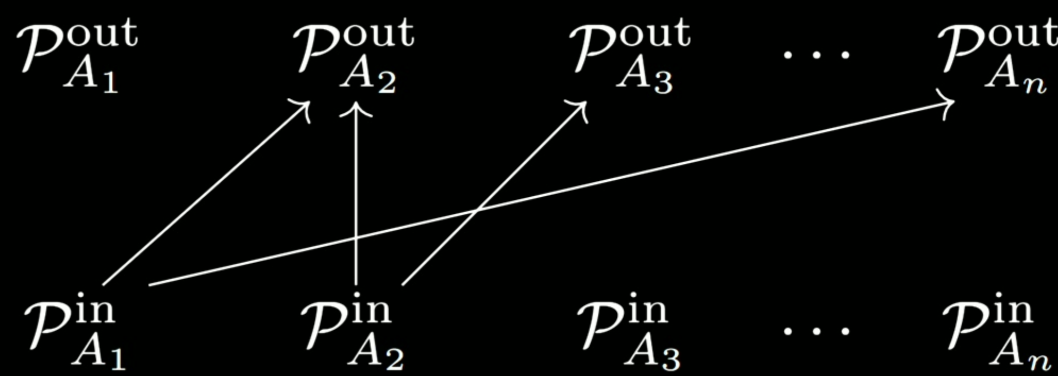


$$\begin{aligned}\text{Prob}(e_1, e_1', \ldots e_n, e_n') &= \text{Tr}(P^{e_n'}_{A_n^{\text{out}}} P^{e_n}_{A_n^{\text{in}}} \ldots P^{e_1'}_{A_1^{\text{out}}} P^{e_1}_{A_1^{\text{in}}} (I/d) P^{e_1}_{A_1^{\text{in}}} P^{e_1'}_{A_1^{\text{out}}} \ldots P^{e_n}_{A_n^{\text{in}}} P^{e_n'}_{A_n^{\text{out}}}) \\ &= \frac{1}{d} \text{Tr}(P^{e_1}_{A_1^{\text{in}}} P^{e_1'}_{A_1^{\text{out}}} \ldots P^{e_n}_{A_n^{\text{in}}} P^{e_n'}_{A_n^{\text{out}}})\end{aligned}$$

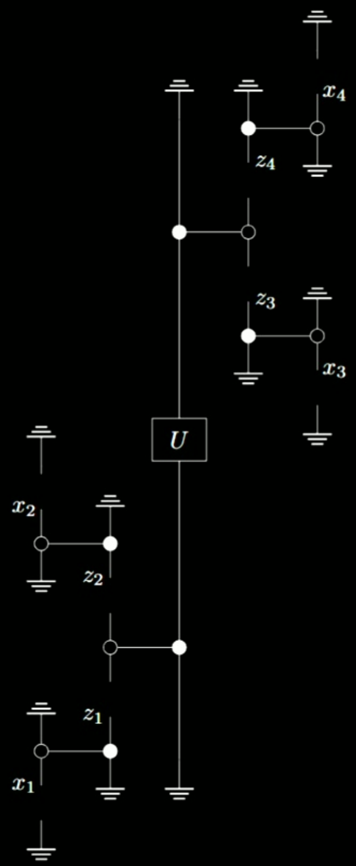
a causal conception of event relativity

- an event takes place relative to the frame in which its corresponding projective algebra was derived
- when the same projective algebra is derived in two different frames, there is no requirement that the events match on a given run

a precise, observer-independent reformulation of quantum theory

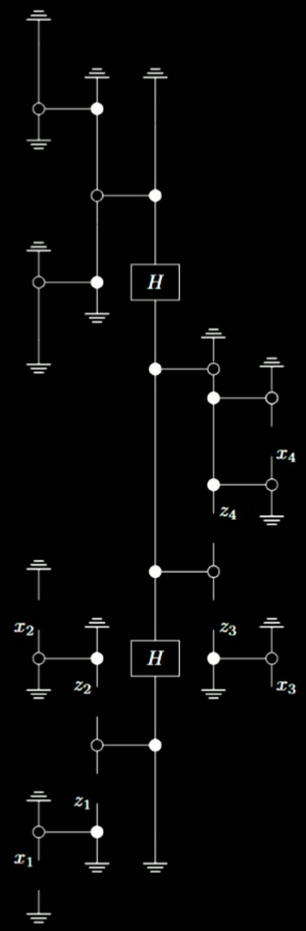
1. DYNAMICS are given by unitary circuits. That a circuit takes place is taken as a primitive and observer-independent fact about reality.
2. An EVENT is the selection of a projector from a projective decomposition. Events take place relative to a frame, and correspond to projectors that are preferred by that frame.
3. Events are RELATIONAL, in the sense that if the same projective decomposition features in two different frames, it needn't be the case that the same projector is selected on each run. However, the fact e_F that an event e took place relative to the frame F is absolute.
4. In a given frame, the PROBABILITY of a set of events are given by taking the matrix product of all the corresponding projectors, then tracing and dividing by the dimension of the Hilbert space.

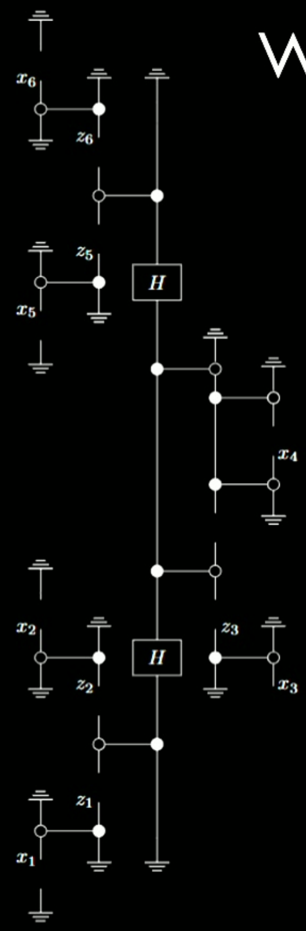
prepare, measure



$$\mathrm{Prob}(z_3 + z_4 | z_1 + z_2) = |\langle z_3 + z_4 | U | z_1 + z_2 \rangle|^2$$

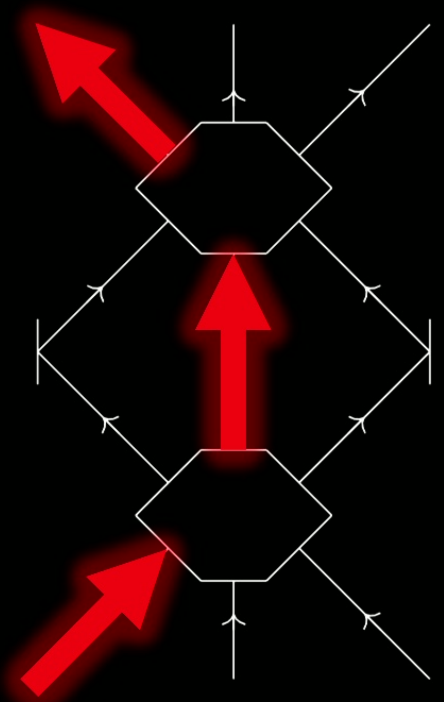
wigner's friend





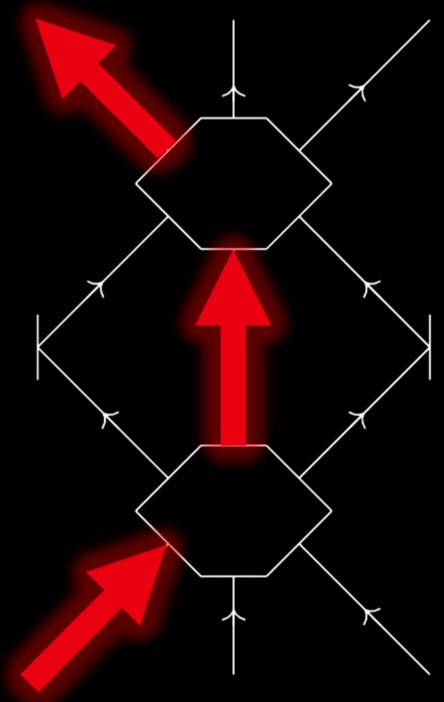
three box paradox

- consistent set S: in on the left and out on right => left path between beamsplitters
- consistent set S': in on the left and out on right => middle path between beamsplitters

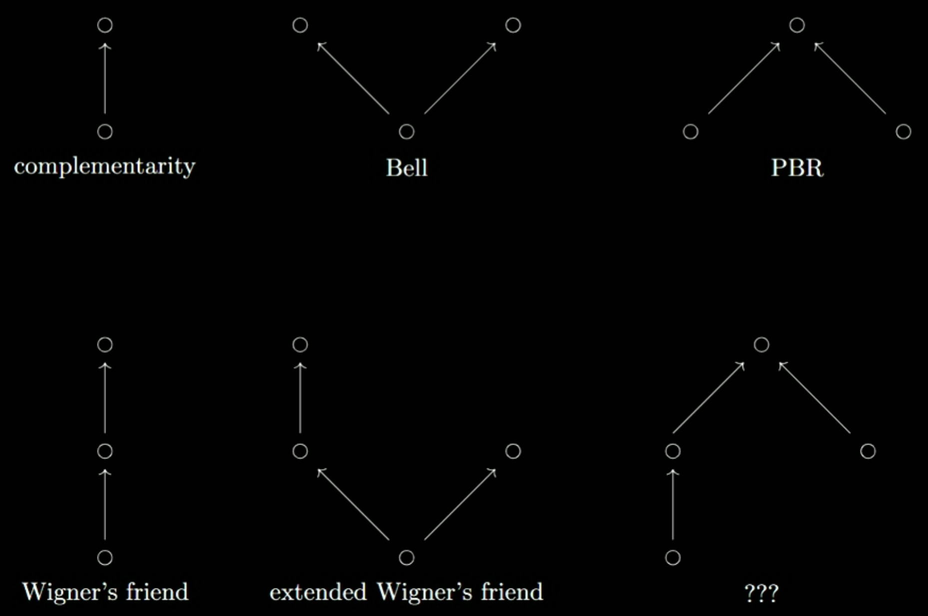


three box paradox

- consistent set S: in on the left and out on right => left path between beamsplitters
- consistent set S': in on the left and out on right => middle path between beamsplitters
- but both S and S' form chains of interference influences, and so the decompositions cannot be derived in a single frame



classifying quantum phenomena



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frames → maximal frames?

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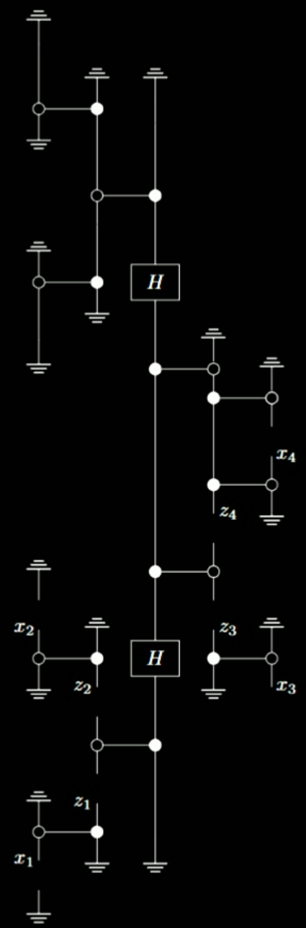
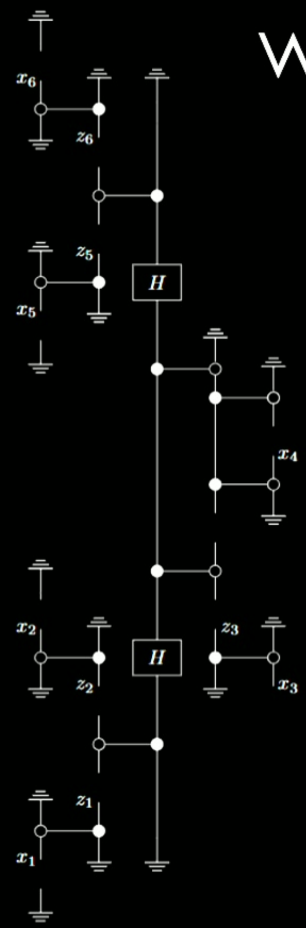
dynamics → causation?

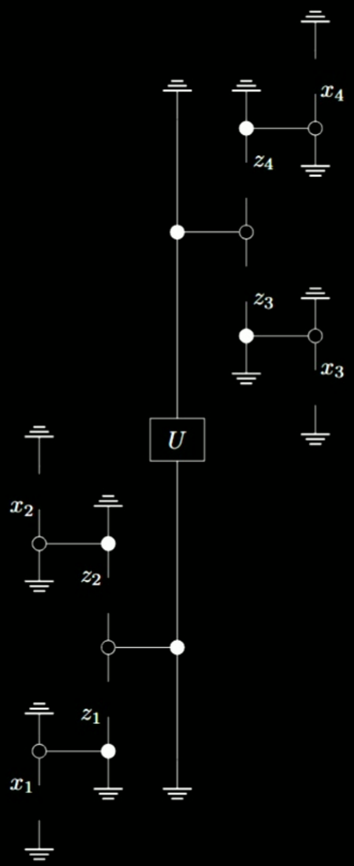
frames → maximal frames?

wigner's friend





prepare, measure



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