Title: Attacking QPV with instantaneous non-local computation of low T-depth quantum circuits

Speakers: Florian Speelman

Collection: QPV 2023: Advances in quantum position verification

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Instantaneous non-local computation of low T-depth quantum circuits

Florian Speelman

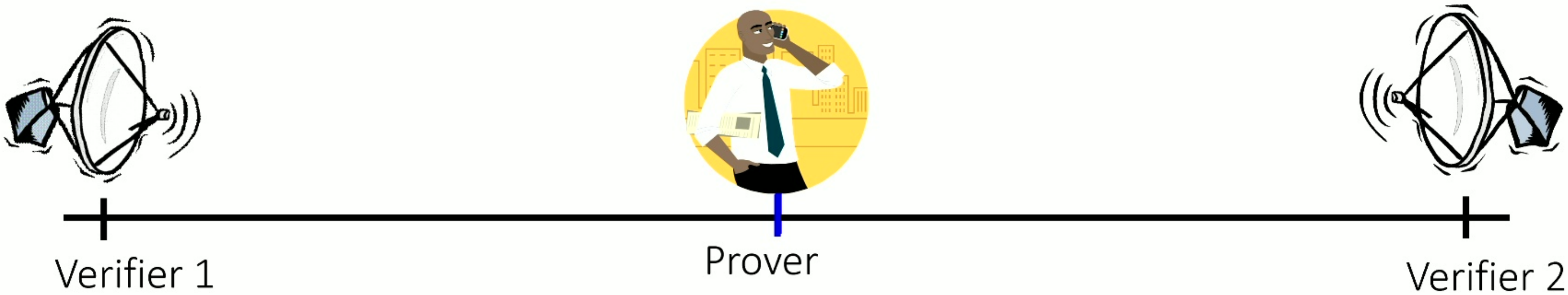
QPV 2023
September 21, 2023

QuSoft
Research Center for Quantum Software

Overview

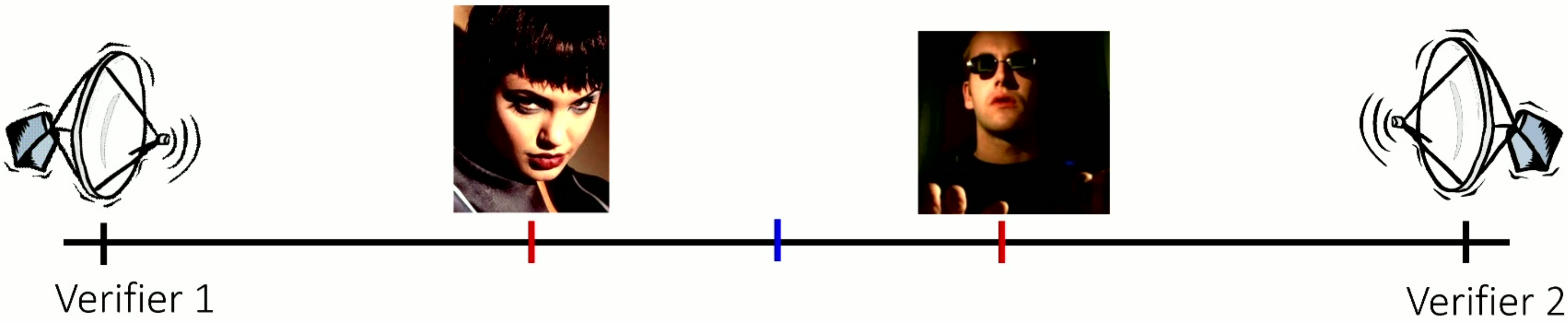
- Introducing QPV + INQC
- Warm-up: Clifford group protocol
- INQC for circuits with low T count
- The garden-hose model
- INQC for circuits with low T depth

Position Verification

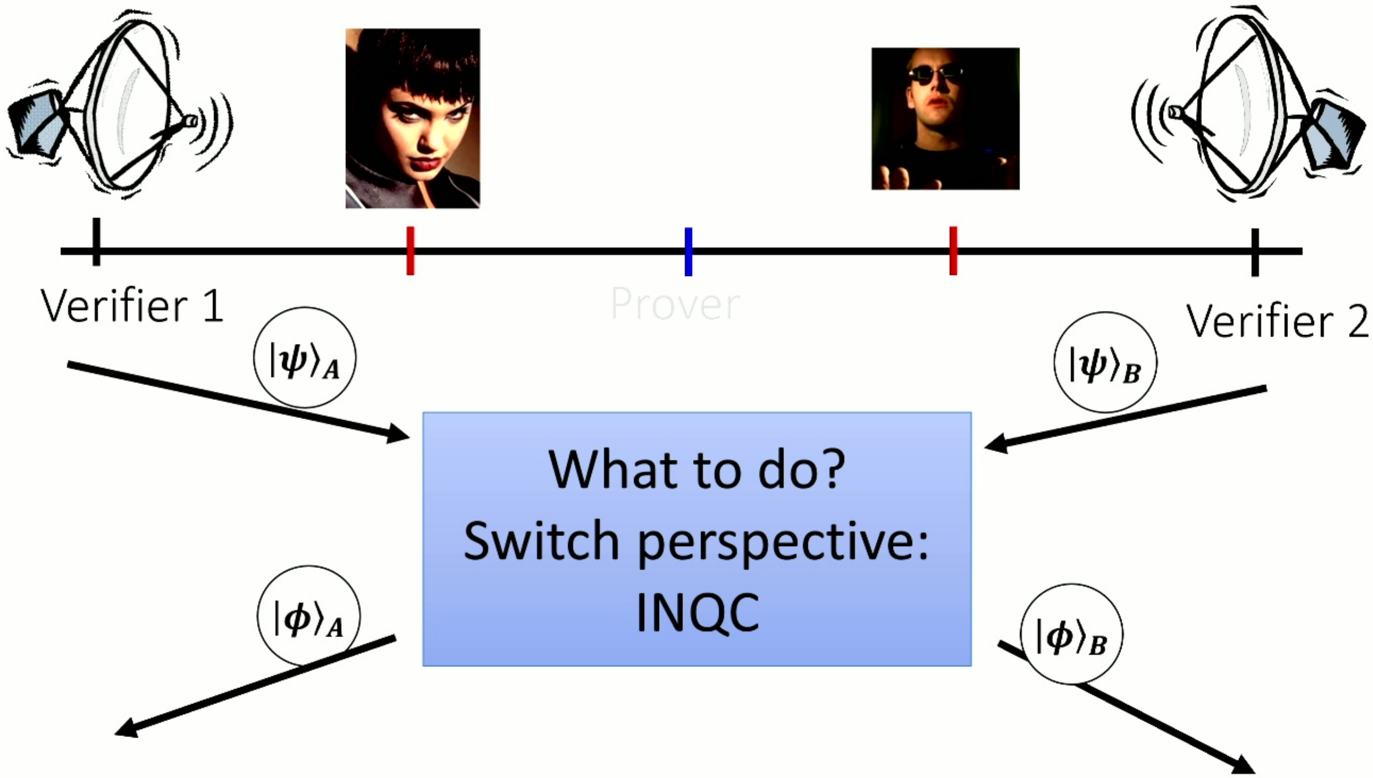


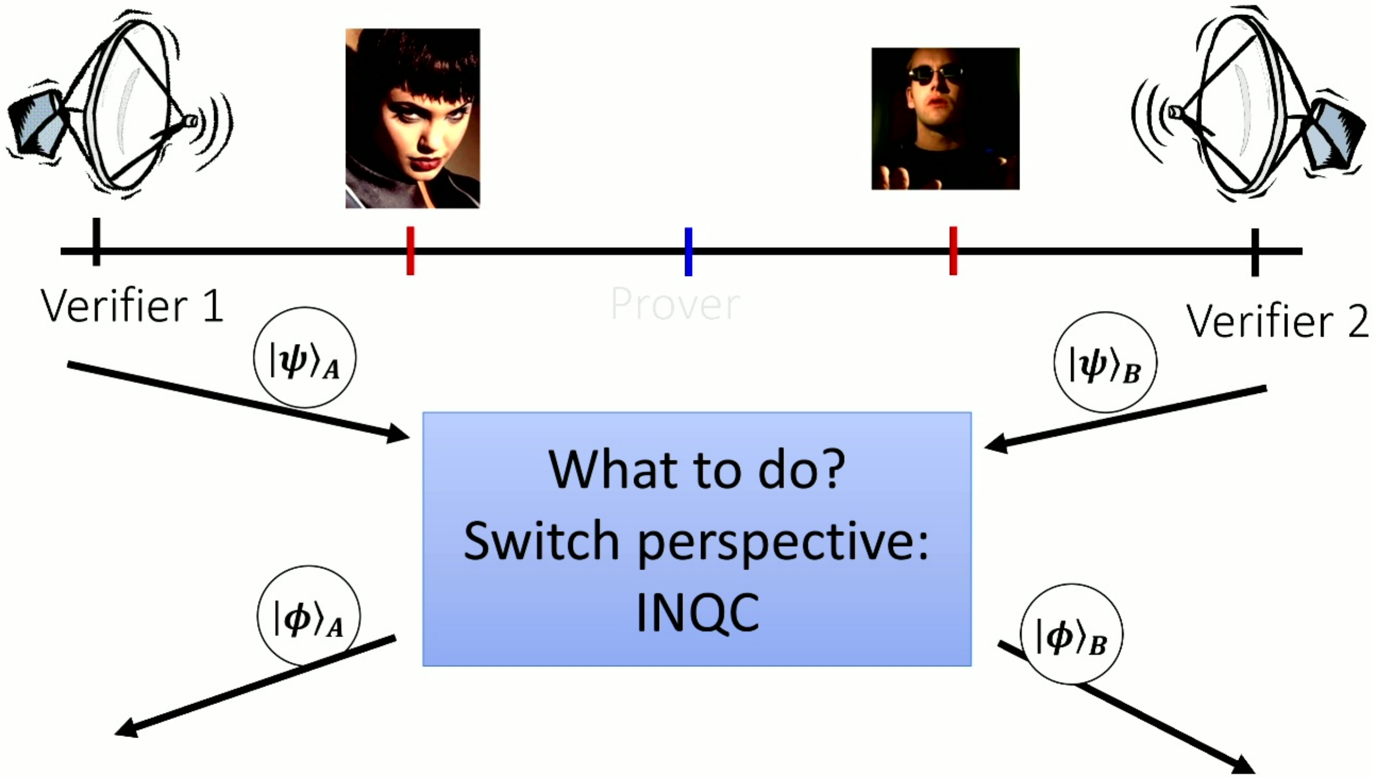
- Prover convince verifiers he is at a particular position
- Assumptions:
  - nothing faster than speed of light
  - **verifiers can coordinate**
  - disregard local computation time (for now)

Position Verification



- Prover convince verifiers he is at a particular position
- Assumptions:
  - nothing faster than speed of light
  - **verifiers can coordinate**
  - disregard local computation time (for now)
- attackers are a coalition of (fake) provers


Verifier 1
Prover
Verifier 2
$|\psi\rangle_A$
$|\psi\rangle_B$
What to do?
Switch perspective:
INQC
$|\phi\rangle_A$
$|\phi\rangle_B$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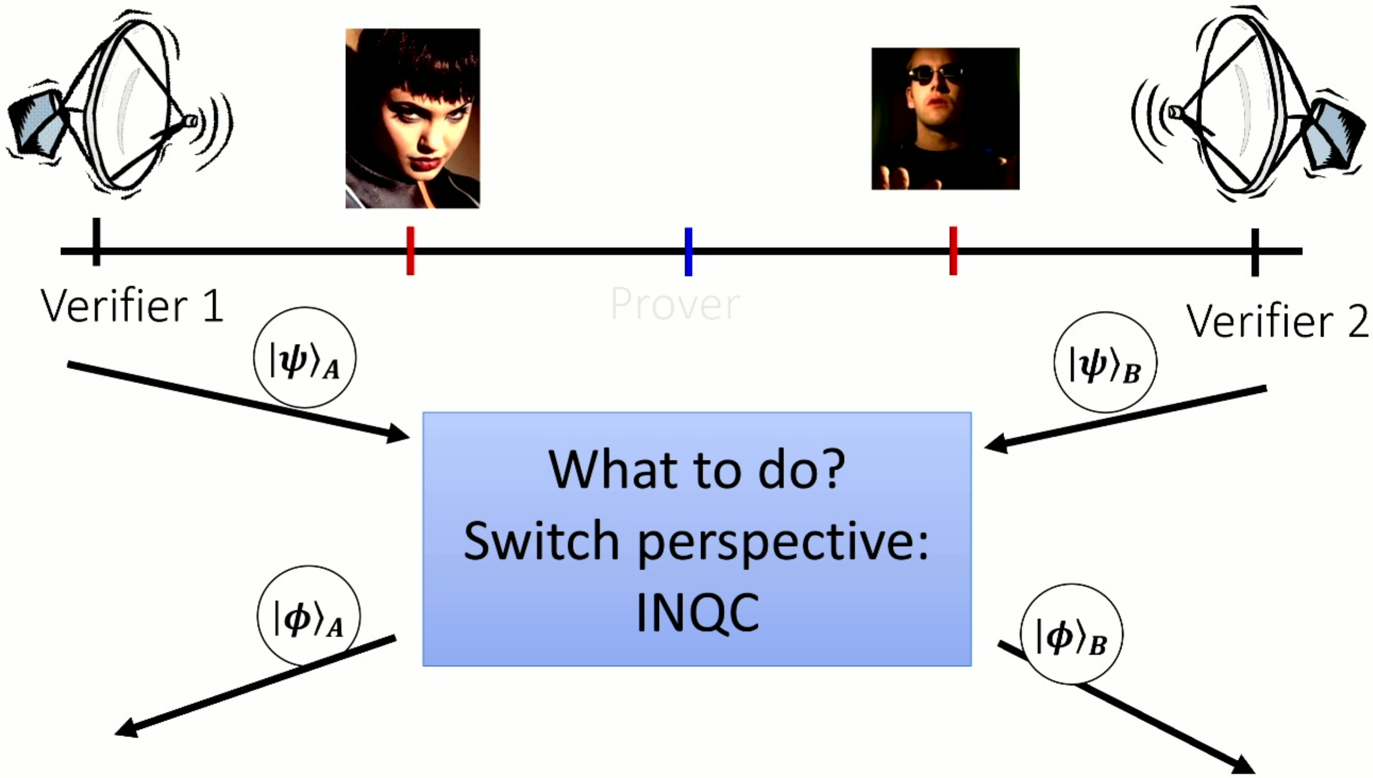
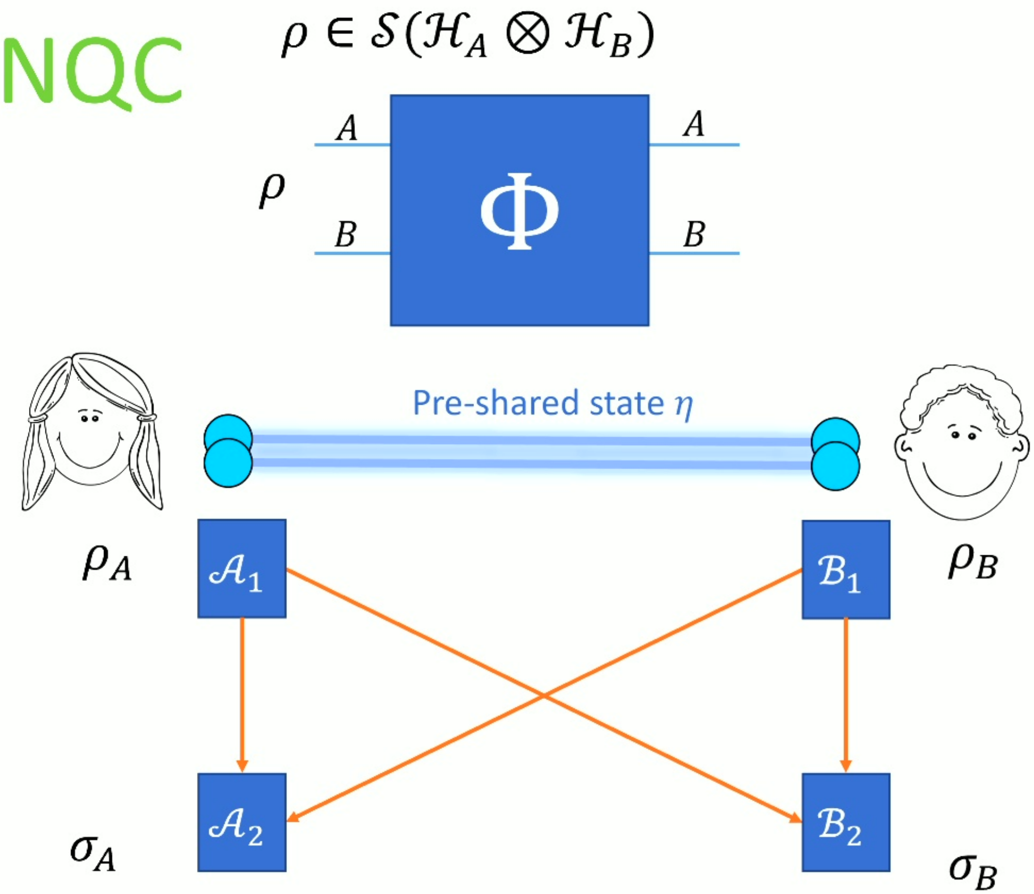


Verifiers check that they received
the correct quantum state in time

INQC



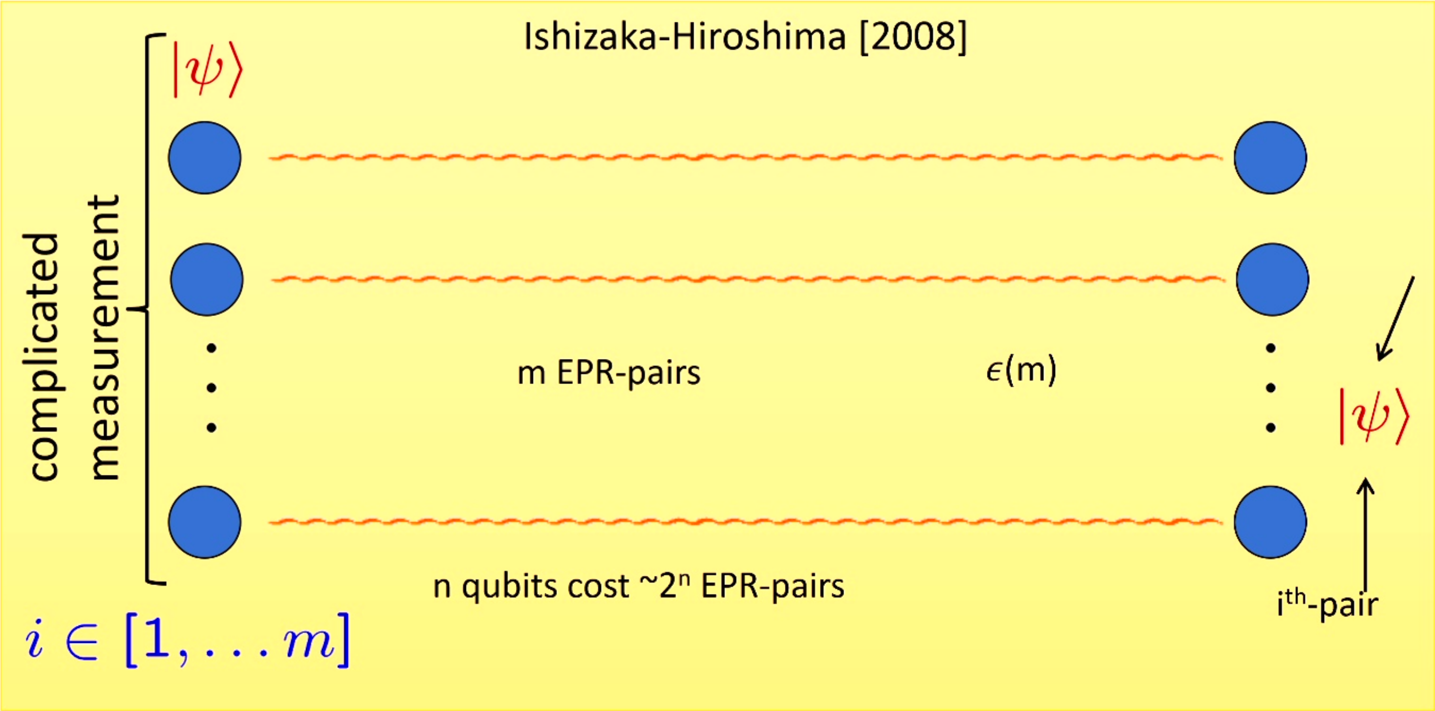
Alice and Bob perform an operation
with only a single round of simultaneous communication

$$\sigma = \Phi(\rho)$$

Good protocol means:
$$\|\Phi(\cdot) - (\mathcal{A}_2 \otimes \mathcal{B}_2)(\mathcal{A}_1 \otimes \mathcal{B}_1)(\cdot \otimes \eta)\|_\diamond \le \epsilon$$

General protocol: port-based teleportation

- Harry's Monday talk
- [Beigi König 2011] $O(n\frac{2^{8n}}{\epsilon^2})$ EPR pairs



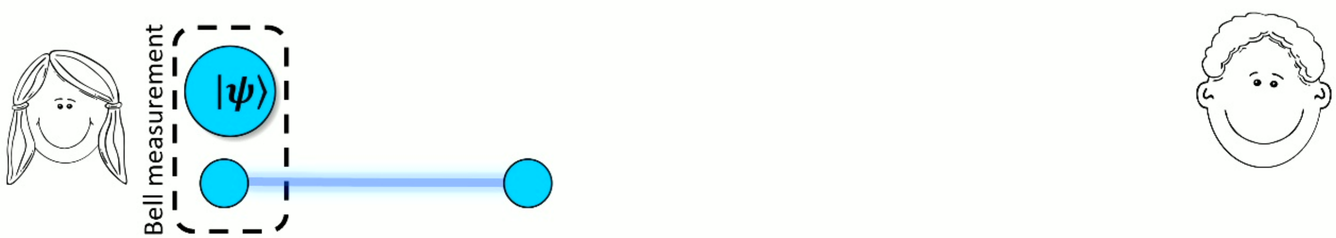
- But what if we want to do a simpler operation?

Teleportation

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

EPR pair: $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Teleportation transfers quantum information using classical bits + EPR pair



The Clifford group

- Generated by $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

- Commutation maps Pauli operators to Paulis (normalizer of Pauli group)
  e.g. $HX = ZH$, $PZ = ZP$, $PX = XZP$

Interaction with teleportation corrections:

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Interaction with teleportation corrections:

$$PX^aZ^b = X^aZ^{a\oplus b}P$$
$$HX^aZ^b = X^bZ^aH$$
$$CNOT X^{a_1}Z^{b_1}X^{a_2}Z^{b_2} = X^{a_1}Z^{b_1+b_2}X^{a_1+a_2}Z^{b_2}CNOT$$

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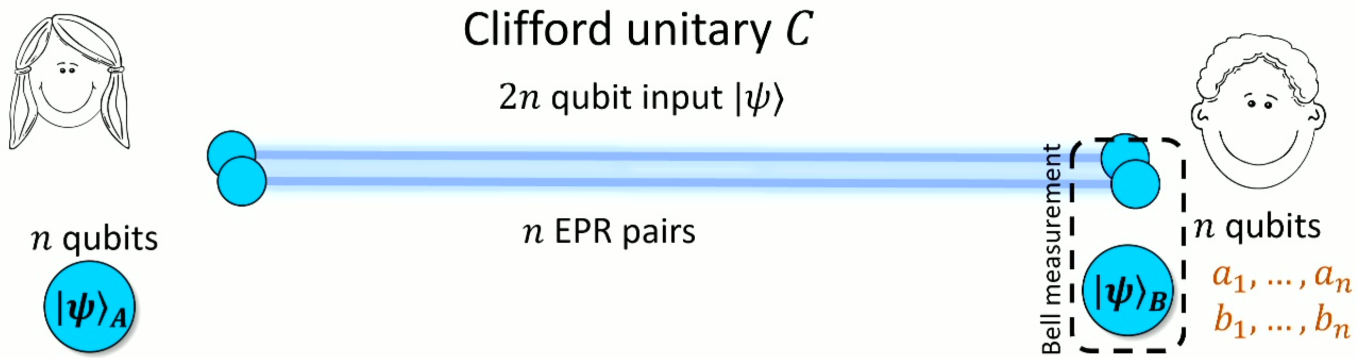
- Not a universal gate set
  Classical simulation possible

Warmup: clifford group protocol



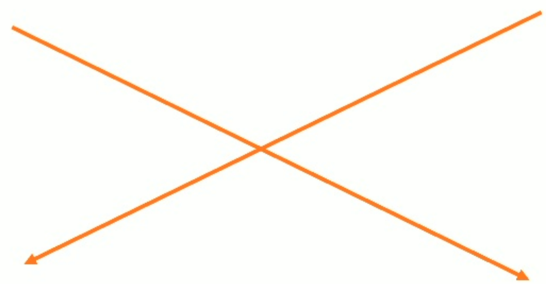
$I^{\otimes n} \otimes X^{a_1} Z^{b_1} \otimes \cdots \otimes X^{a_n} Z^{b_n} |\psi\rangle$

U

$$C\left(I^{\otimes n} \otimes X^{a_1} Z^{b_1} \otimes \cdots \otimes X^{a_n} Z^{b_n}\right)|\psi\rangle = \left(X^{c_1} Z^{d_1} \otimes \cdots \otimes X^{c_{2n}} Z^{d_{2n}}\right) C |\psi\rangle$$

Send Bob's n qubits back

Send $c_1, \ldots, c_n, d_1, \ldots, d_n$



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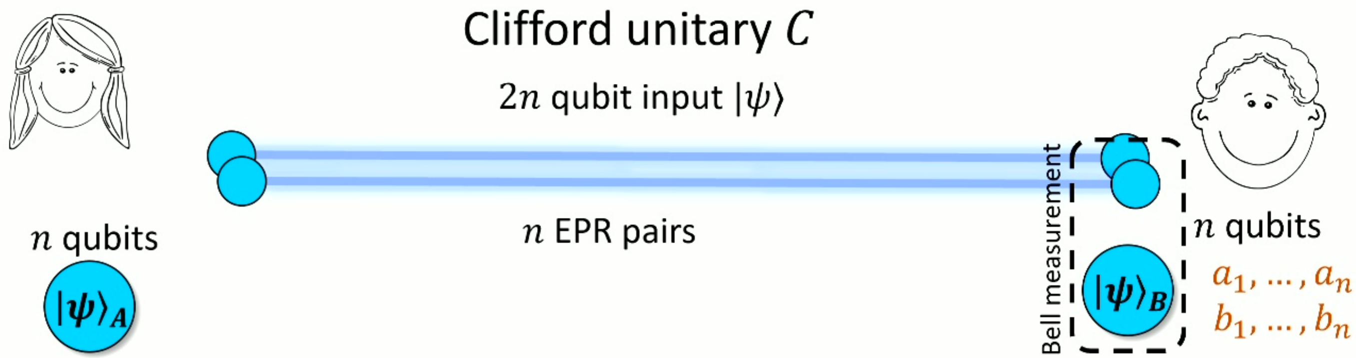

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Send Bob's n qubits back

Send $c_1, \dots, c_n, d_1, \dots, d_n$

Locally apply
$X^{c_1} Z^{d_1} \otimes \cdots X^{c_n} Z^{d_n}$

Locally apply
$X^{c_{n+1}} Z^{d_{n+1}} \otimes \cdots X^{c_{2n}} Z^{d_{2n}}$

Extending the gate set: T gate

T gate (also known as $\frac{\pi}{8}$ gate) is given by $\mathrm{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

Clifford+T can perform all quantum operations (universal set)

T gate on an uncorrected qubit:

$$TX = PXT$$
$$TZ = ZT$$

$$TX^aZ^b|\psi\rangle = P^aX^aZ^bT|\psi\rangle$$

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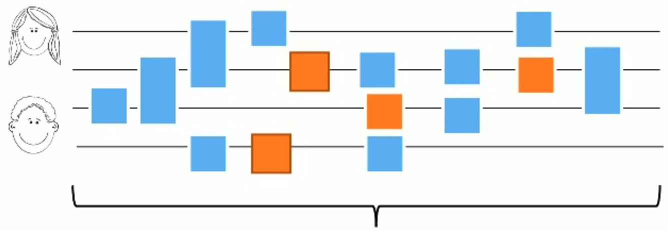
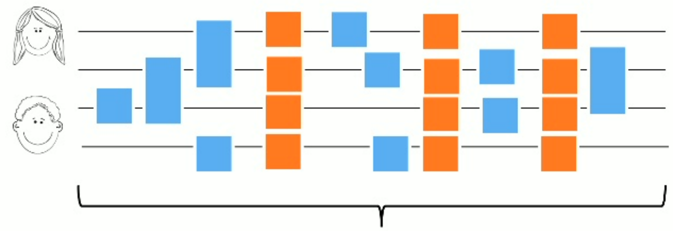
Handle the unwanted P gate in some way

Overview of results



T-count k

Entanglement $O(n2^k)$

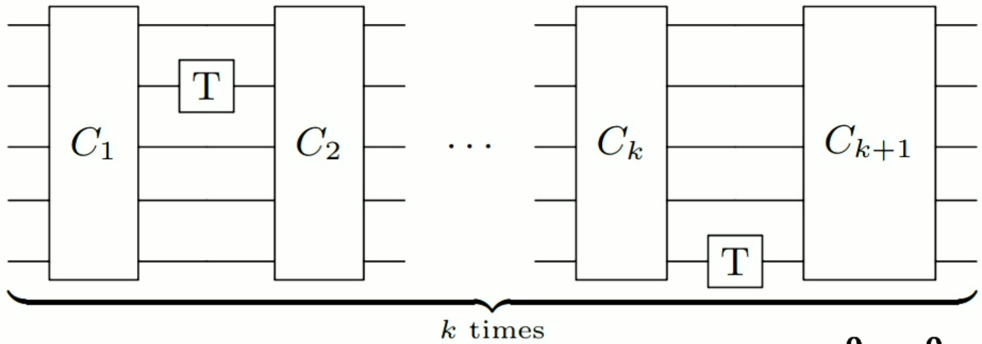


T-depth d

Entanglement $O(n^d)$

(No error, exactly simulates the circuit)

Also see: Monday's talk by Anne ➔ with PR-boxes we can do INQC of all poly-size circuits efficiently



Step 0: Bob teleports his $n/2$ qubits to Alice, holds $\mathrm{X}^{\boldsymbol{b_x^0}}\mathrm{Z}^{\boldsymbol{b_z^0}}|\psi_0\rangle$

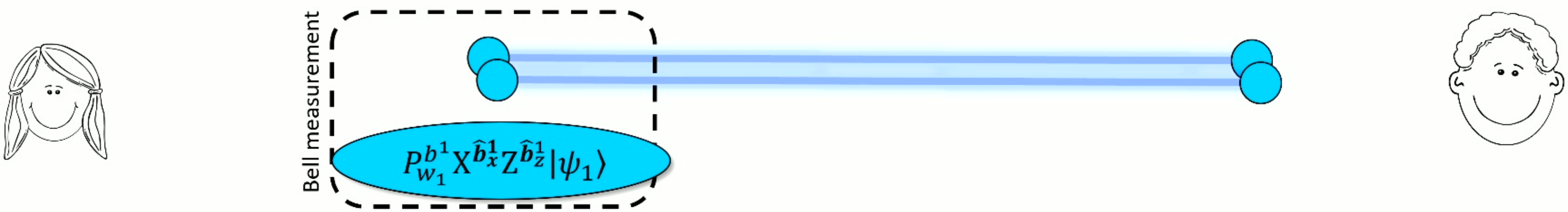
Step 1.a: Alice performs $C_1\mathrm{X}^{\boldsymbol{b_x^0}}\mathrm{Z}^{\boldsymbol{b_z^0}}|\psi_0\rangle = \mathrm{X}^{\widehat{\boldsymbol{b}}_x^{\boldsymbol{1}}}\mathrm{Z}^{\widehat{\boldsymbol{b}}_z^1}C_1|\psi_0\rangle$

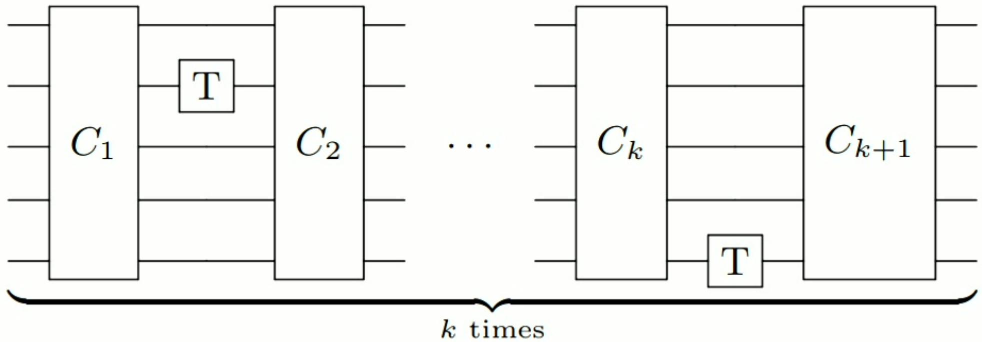
Step 1.b: Alice performs T on some wire w_1

$$\mathrm{T}_{w_1}\mathrm{X}^{\widehat{\boldsymbol{b}}_x^{\boldsymbol{1}}}\mathrm{Z}^{\widehat{\boldsymbol{b}}_z^1}C_1|\psi_0\rangle = P_{w_1}^{b^1}\mathrm{X}^{\widehat{\boldsymbol{b}}_x^{\boldsymbol{1}}}\mathrm{Z}^{\widehat{\boldsymbol{b}}_z^1}TC_1|\psi_0\rangle := P_{w_1}^{b^1}\mathrm{X}^{\widehat{\boldsymbol{b}}_x^{\boldsymbol{1}}}\mathrm{Z}^{\widehat{\boldsymbol{b}}_z^1}|\psi_1\rangle$$

with b^1 the w_1 entry of $\widehat{\boldsymbol{b}}_{\boldsymbol{x}}^{\boldsymbol{1}}$

Step 1.c: Alice teleports all qubits to Bob





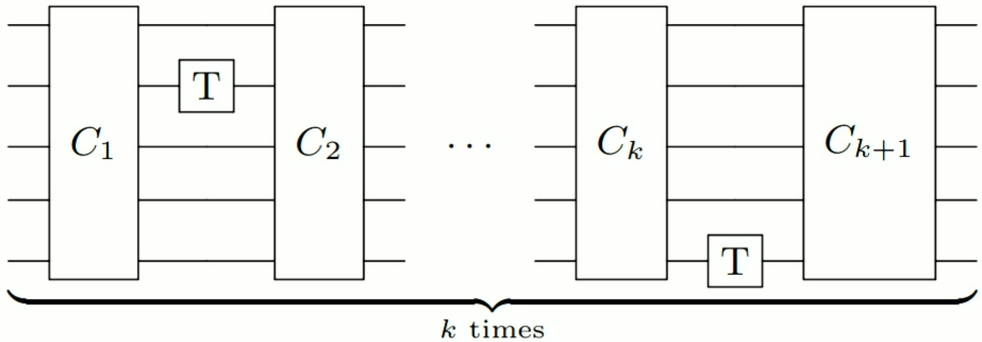
Step 1.d: Bob undoes old Paulis and applies $\left(P_{w_1}^{b^1}\right)^{-1}$

$$\left(P_{w_1}^{b^1}\right)^{-1} \mathrm{X}^{\boldsymbol{a_x^1}} \mathrm{Z}^{\boldsymbol{a_z^1}} P_{w_1}^{b^1} |\psi_1\rangle = \mathrm{Z}_{w_1}^{a^1 b^1} \mathrm{X}^{\boldsymbol{a_x^1}} \mathrm{Z}^{\boldsymbol{a_z^1}} |\psi_1\rangle$$

$\mathrm{Z}_{w_1}^{a^1}$

Step 1.e: Alice corrects extra Z if needed. Now one of the groups is back to starting invariant!

➔ Alice holds the qubits and Bob the teleportation corrections



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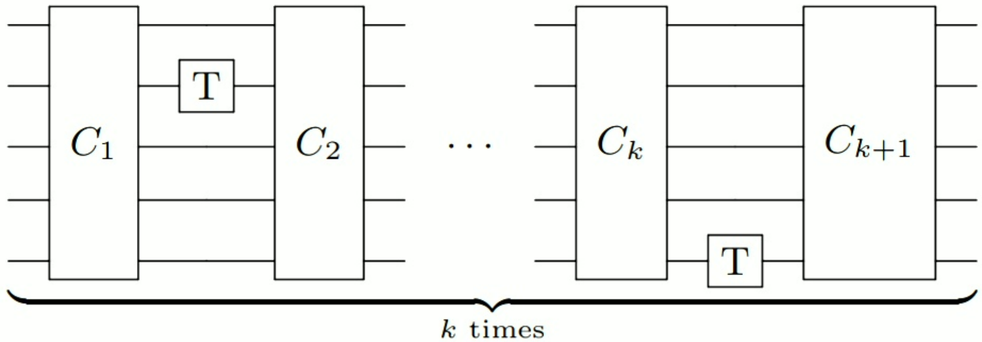

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$$\left(P_{w_1}^{b^1}\right)^{-1} X^{\boldsymbol{a}_x^1} Z^{\boldsymbol{a}_z^1} P_{w_1}^{b^1} |\psi_1\rangle = Z_{w_1}^{a^1 b^1} X^{\boldsymbol{a}_x^1} Z^{\boldsymbol{a}_z^1} |\psi_1\rangle$$

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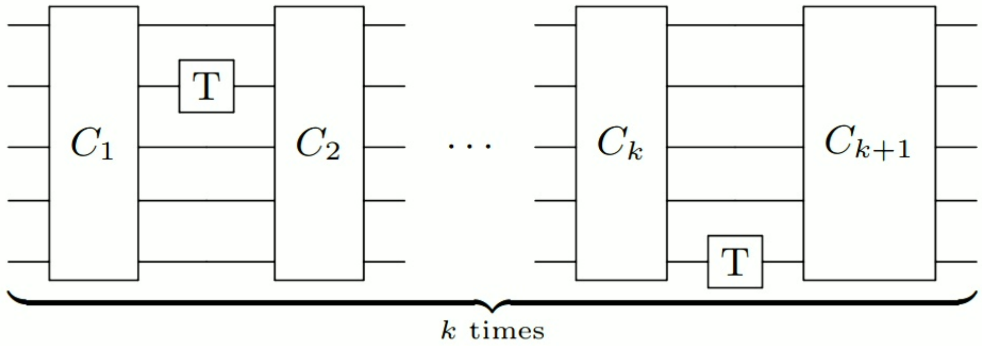
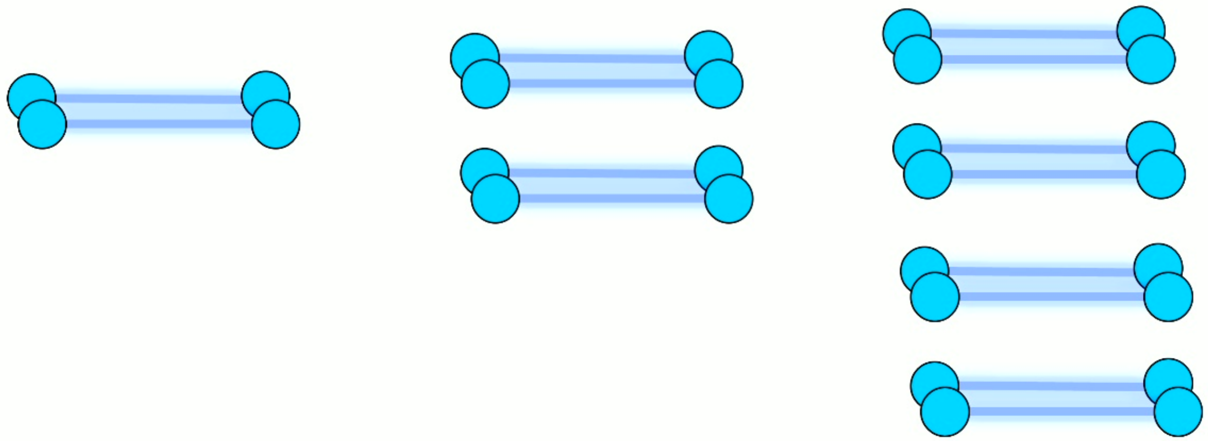
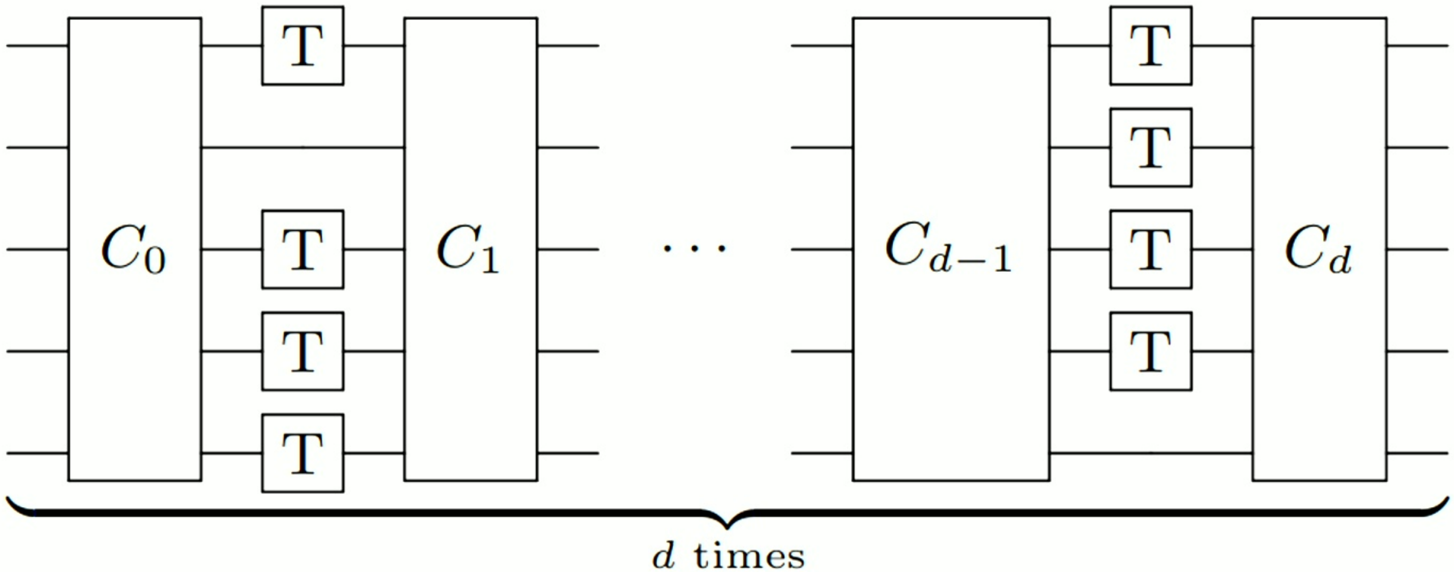


Step i: same as step 1, but Alice acts on all 2^{i-1} groups in parallel!





T-depth d

INQC with entanglement $O(n^d)$

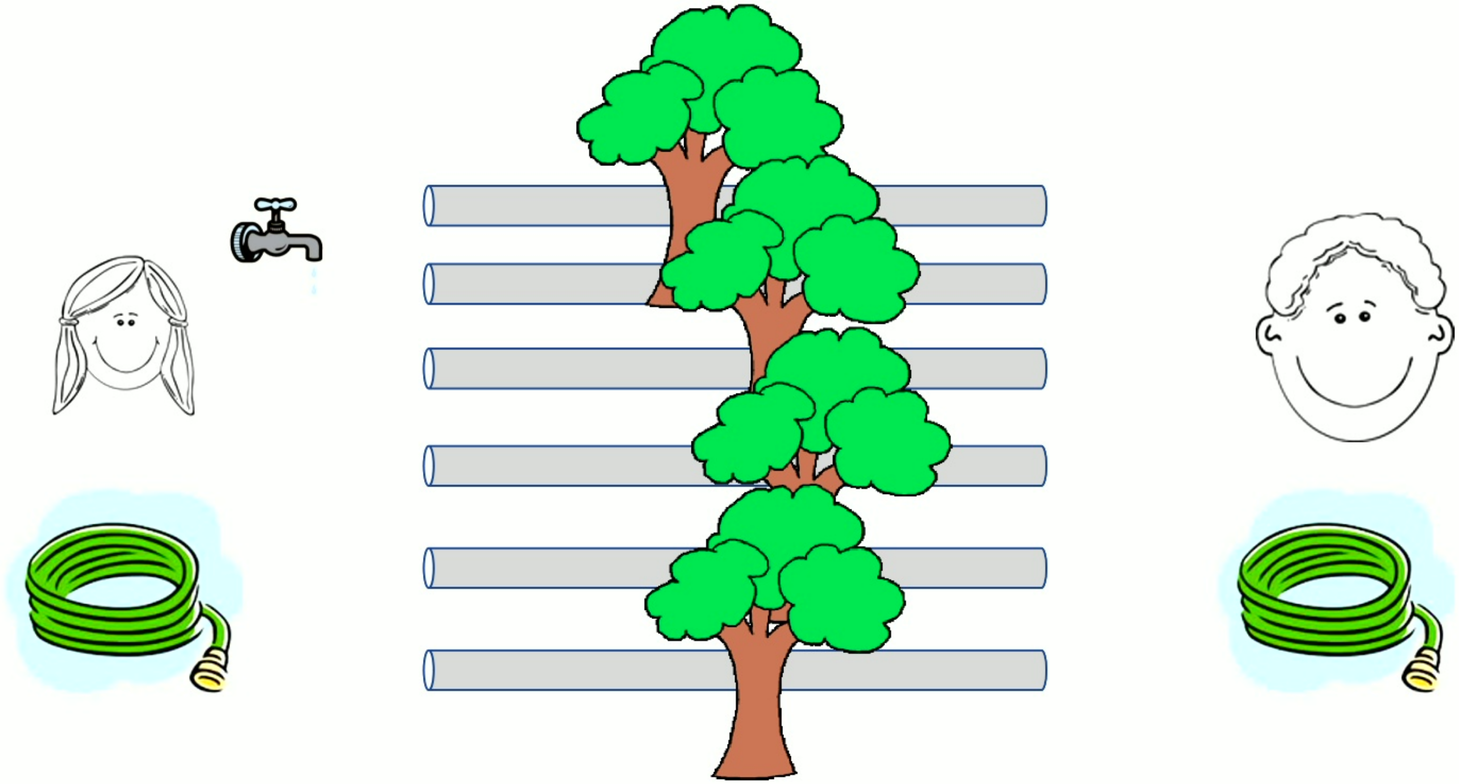
Main Lemma

$P^{f(x,y)}|\psi\rangle, x$ $\qquad\qquad$ y

$X^{g(x',y')}Z^{h(x',y')}|\psi\rangle, x'$ $\qquad\qquad$ y'

Consumes ebit dependent on garden-hose complexity of f
Garden-hose complexity of g, h is **linear** in garden-hose complexity of f

The Garden-Hose Model



share s waterpipes

The Garden-Hose Model

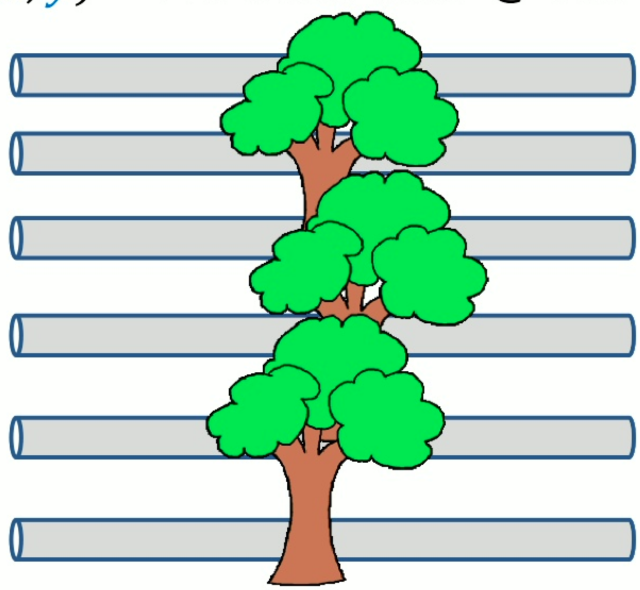
$$f: \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$$

$f(x, y) = 0$ if water exits @ Alice

$f(x, y) = 1$ if water exits @ Bob

$x \in \{0,1\}^n$

$y \in \{0,1\}^n$



- based on their inputs, players connect pipes with pieces of hose
- Alice also connects a water tap

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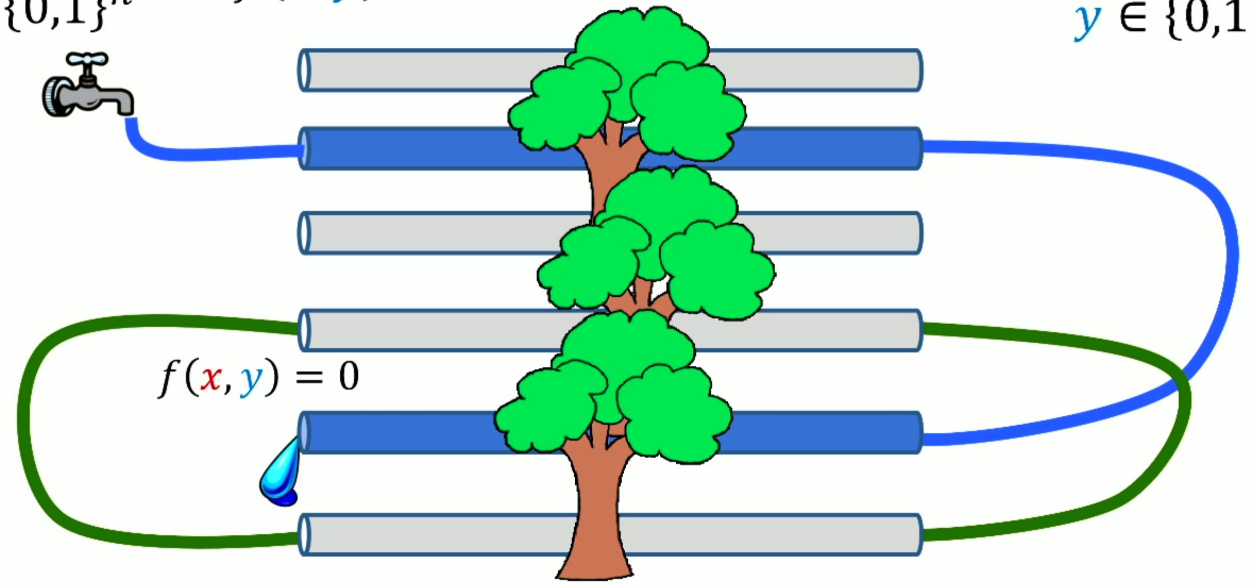

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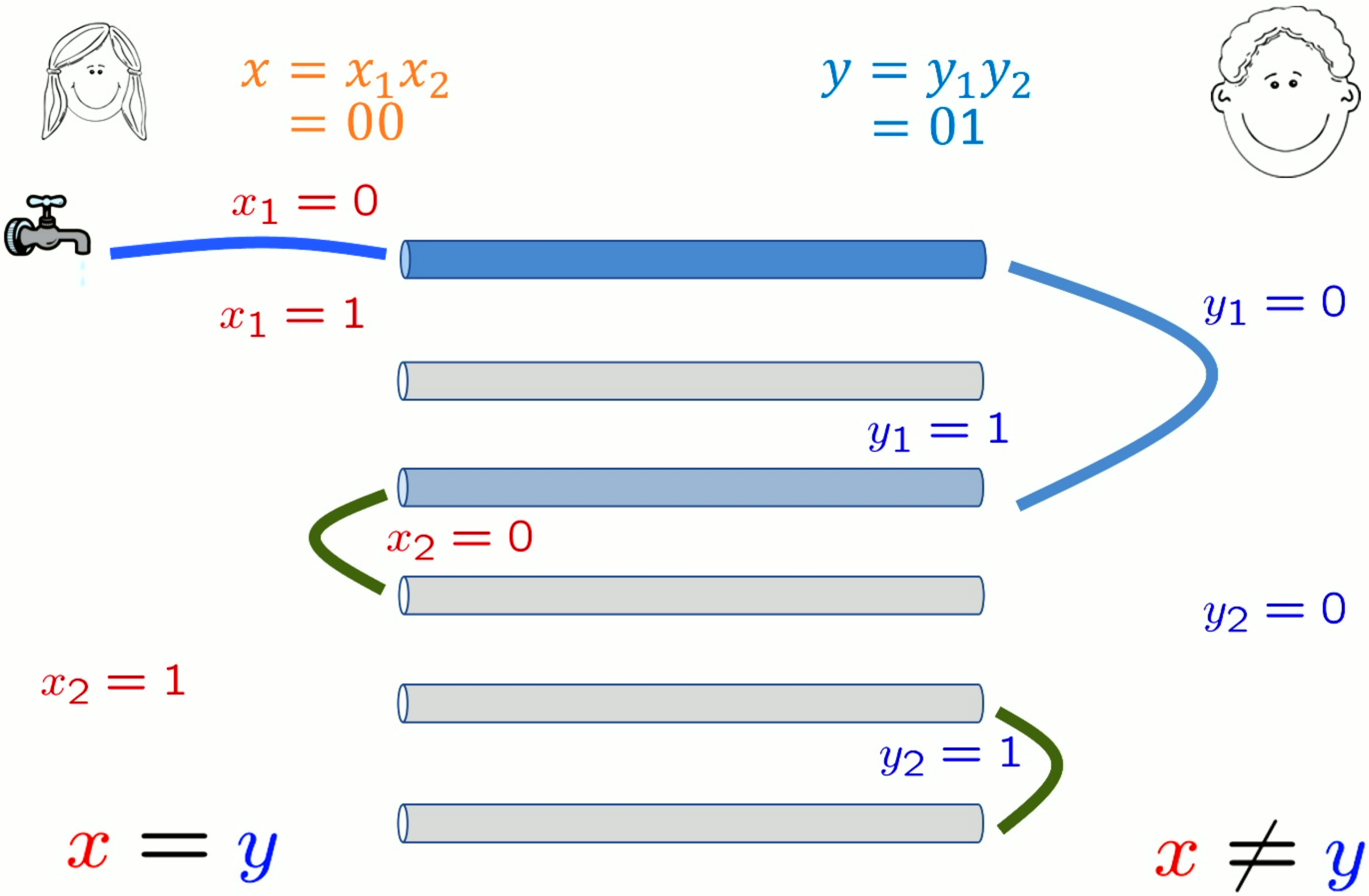
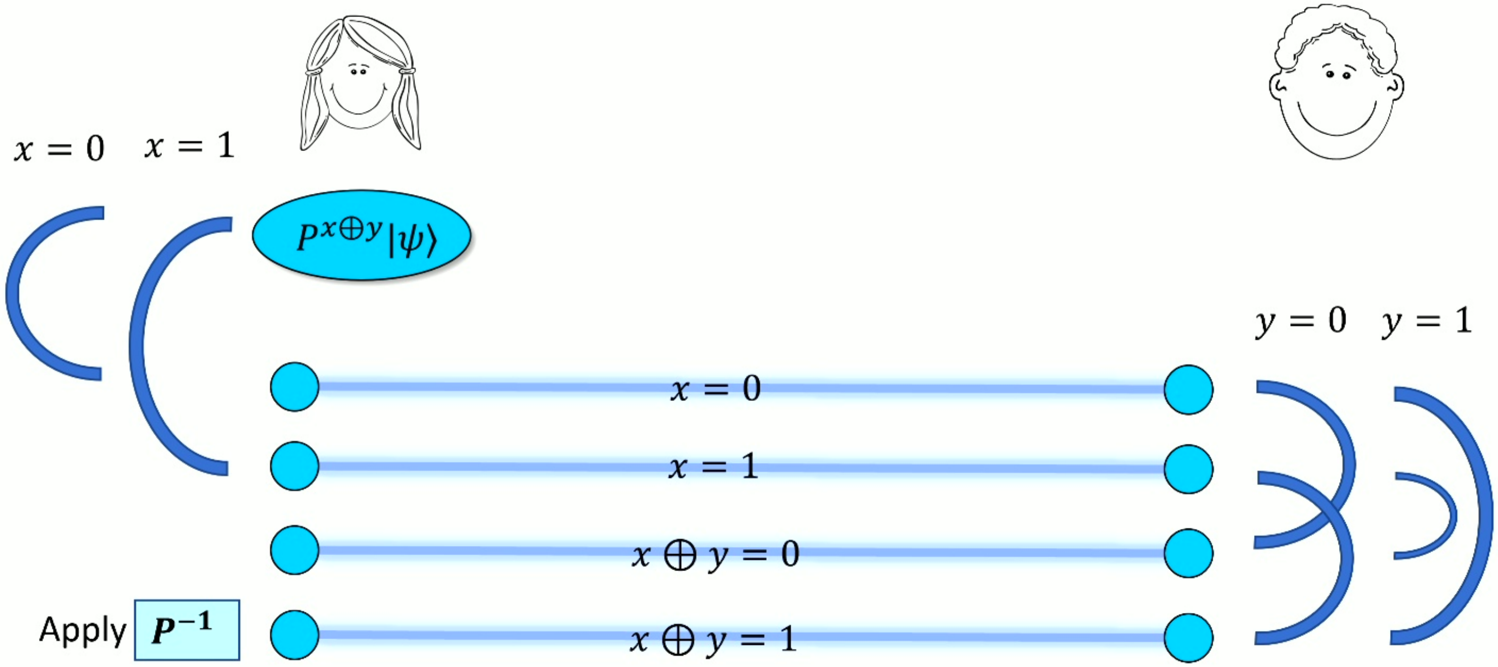
Example: Inequality on Two Bits



Some facts about Garden-hose complexity

- Inspired by attacks on routing QPV protocol
- Every f has $GH(f) \leq$ exponential
- f in logspace $\Rightarrow GH(f)$ is polynomial
  - Using Barrington's theorem (see Harry's talk)
- exists f with $GH(f)$ exponential (counting)
- for $g \in \{\text{equality, IP, majority}\}$:
$$GH(g) \geq n/\log n$$
  - using techniques from communication complexity

The garden-hose complexity describes how much entanglement we need to undo a correction. Example:



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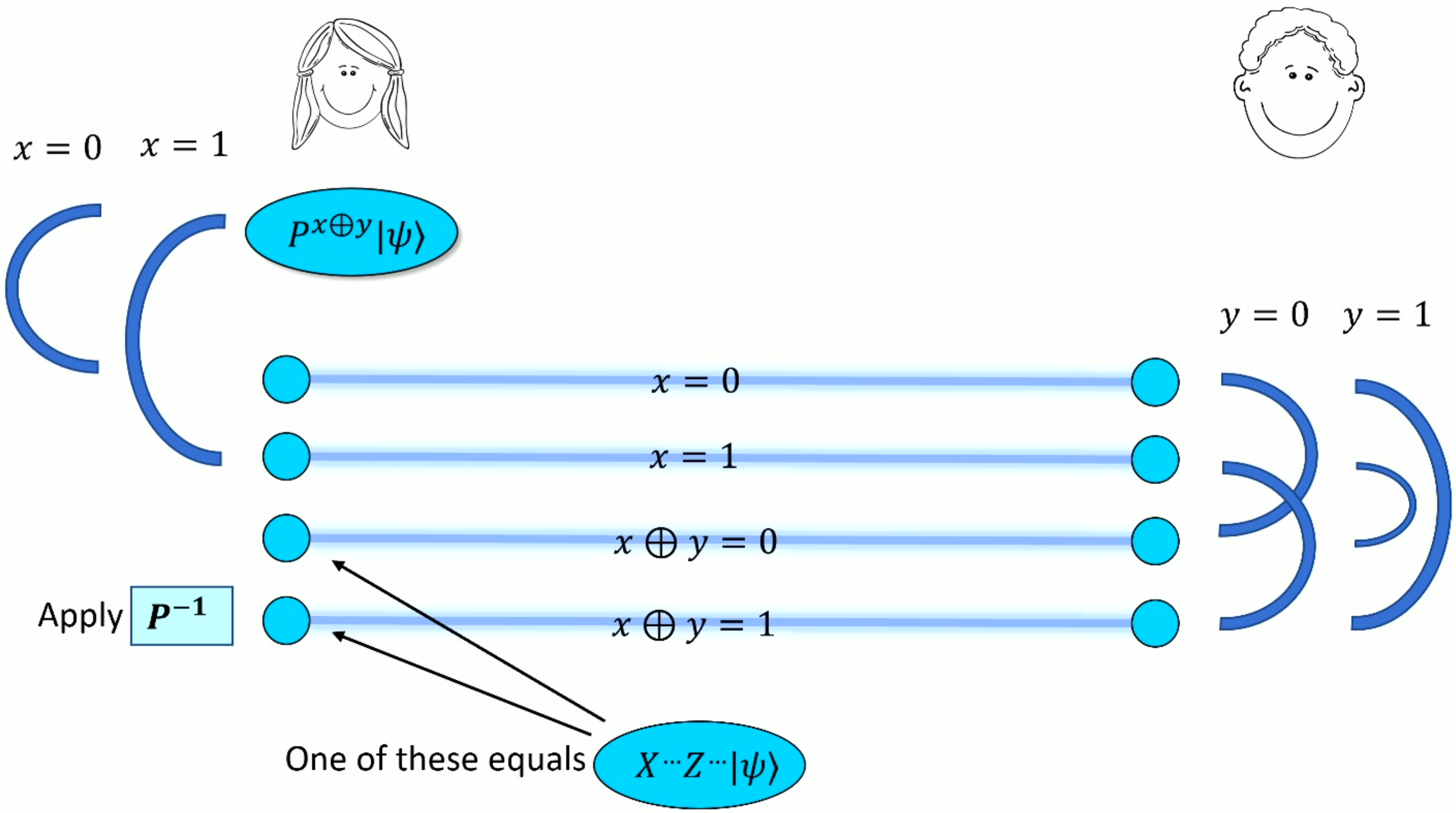
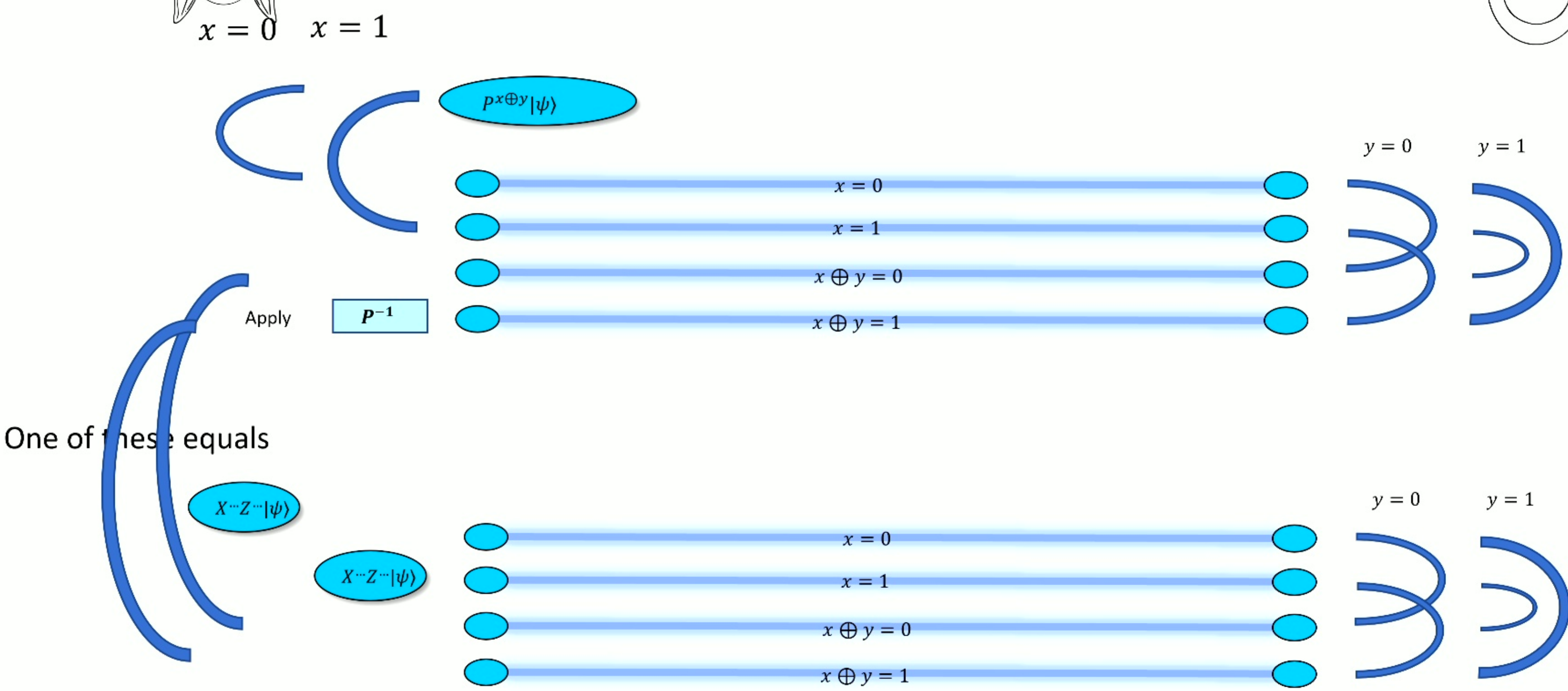
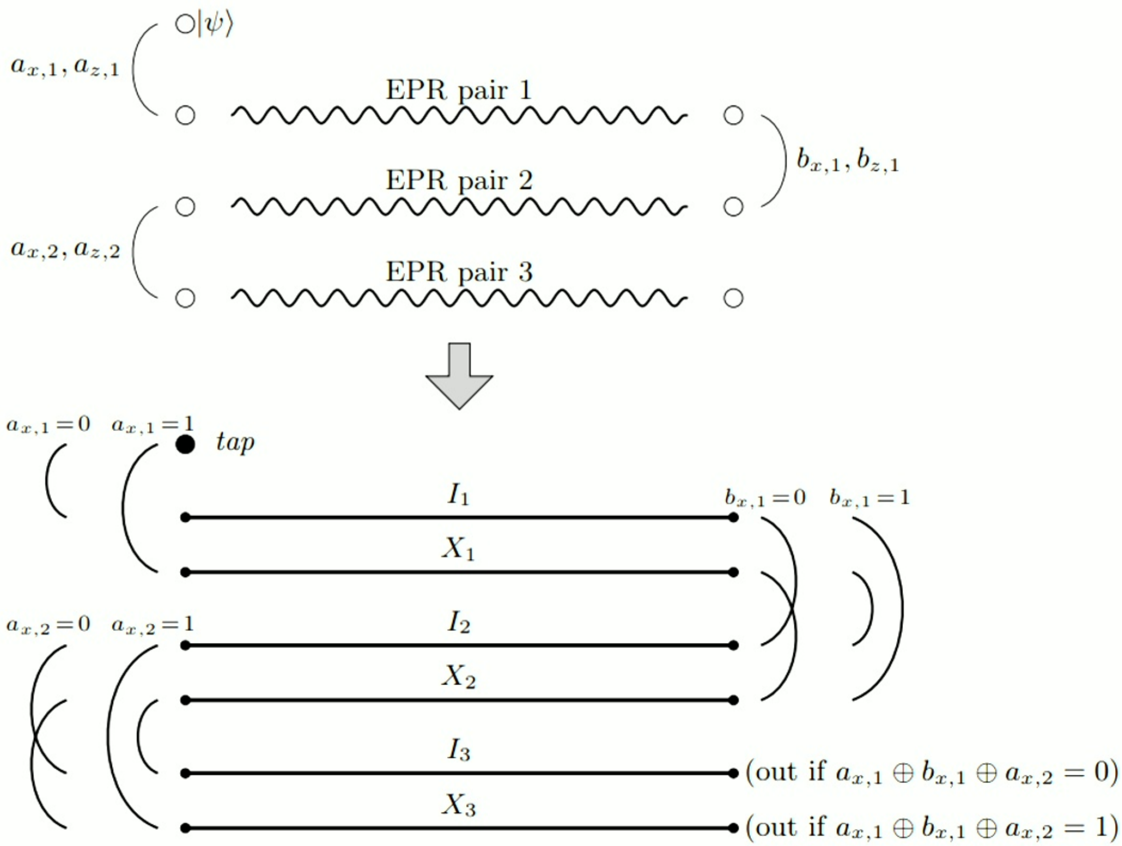


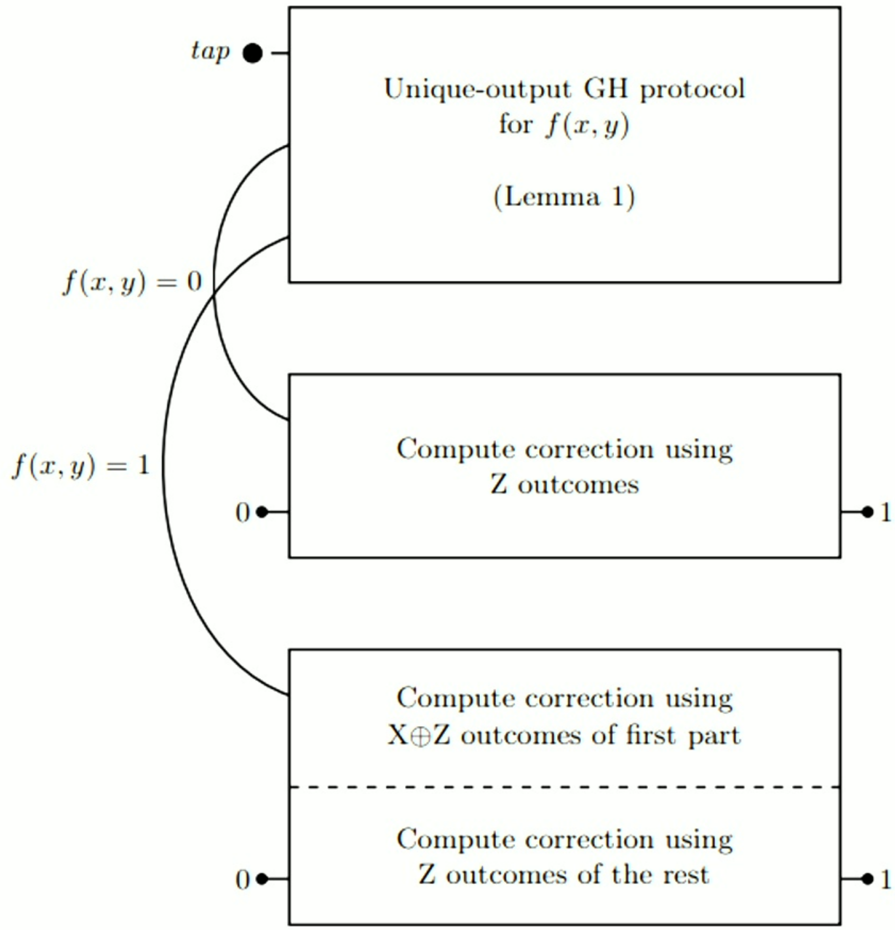
Observation 1: Previous attempt would lose track of qubit, but we can repeat the protocol in reverse to find it again



Observation 2: the garden-hose complexity of computing the Pauli corrections resulting from teleporting a qubit back-and-forth **k** times is linear in **k**.
"The garden-hose complexity of executing a garden-hose protocol of f linear in $\mathrm{GH}(f)$"



Correction to observation 2: The protocol is not just teleportations, but also involves some inverse phase gates if $f(x, y) = 1$ – what about the Z correction?



Lemma proof

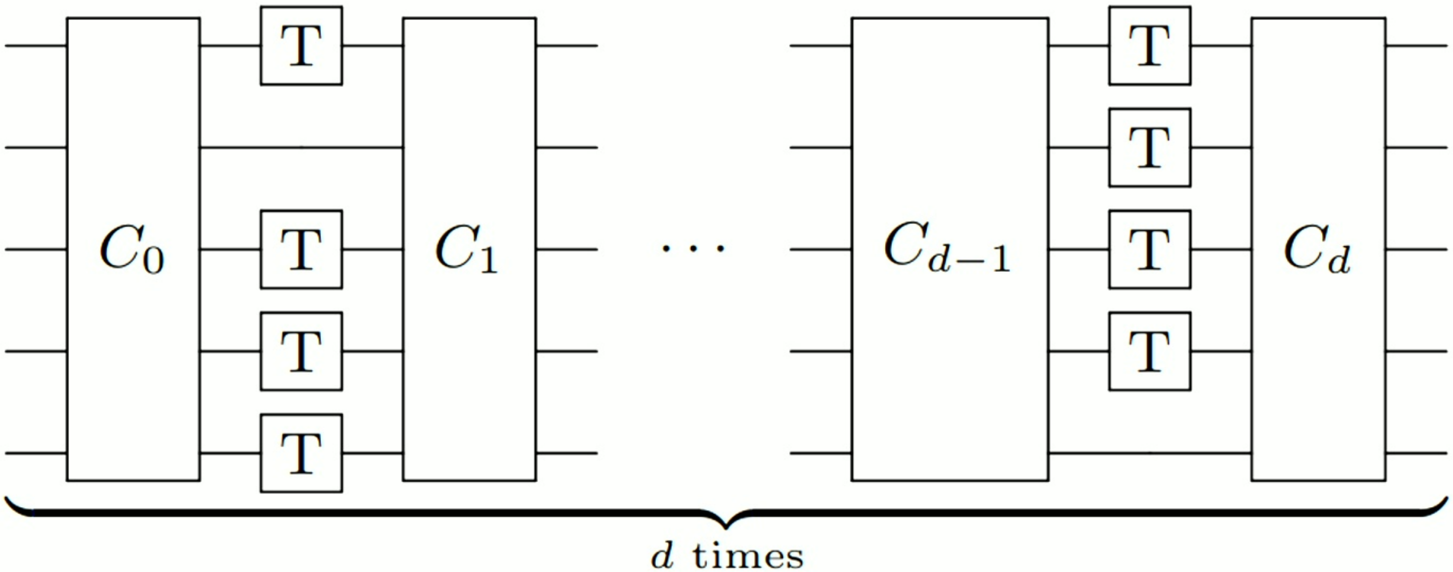
Alice starts with $P^{f(x,y)}|\psi\rangle$, x

Bob starts with y

- By **observation 1**: Alice and Bob perform the protocol to undo P, using $2\mathrm{GH}(f)$ EPR pairs

$X^{g(x',y')}Z^{h(x',y')}|\psi\rangle$, x' y'

- By **observation 2:** $\mathrm{GH(g)} \leq 4\mathrm{GH}(f) + 1$, $\mathrm{GH(h)} \leq 11\mathrm{GH}(f) + 2$



T-depth d

INQC with entanglement $O(n^d)$

We now know how handle T gates, put it all together

Proof sketch (INQC for T-depth)

- Every step, Alice holds qubits of the form $\bigotimes_i X^{g_i(x,y)} Z^{h_i(x,y)} |\psi\rangle$ with x, y teleportation corrections of Alice, Bob
- Clifford step: permute and sum functions
  - GH becomes approx $\leq \sum_i GH(g_i) + GH(h_i)$

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- Clifford step: permute and sum functions
  - GH becomes approx $\leq \sum_i GH(g_i) + GH(h_i)$
- T layer step:
  - For each qubit, $GH(g'_i) \leq 4GH(g_i') + 1$ and $GH(h'_i) \leq 11GH(h_i') + 2$

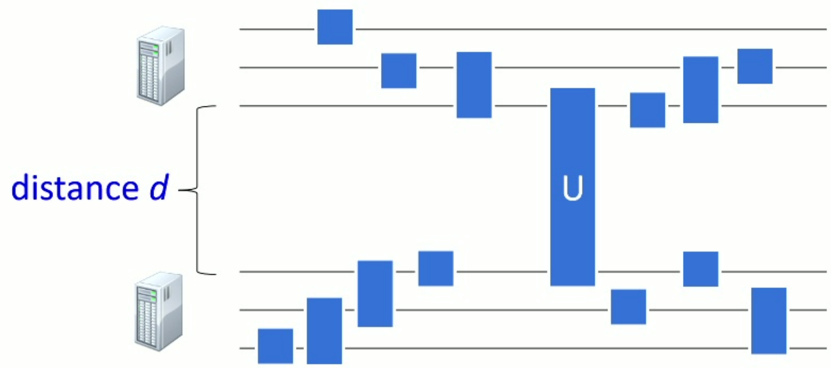
- Together (with some extras): complexity $(68n)^d$

INQC overview

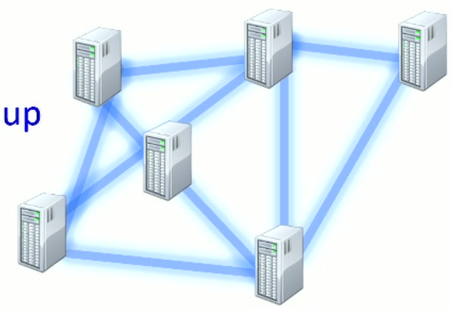
Circuit type	Result
General circuits	[Vaidman 2003, BCFGGOS 2011] $2^{\log(\frac{1}{\epsilon})}\ 2^{O(n)}$ EPR pairs [Beigi König 2011] $O(n\frac{2^{8n}}{\epsilon^2})$ EPR pairs
Two qubit circuits	[Gonzales Chitambar 2019] $8\log(\frac{1}{\epsilon}) + 22$
Clifford + T-count k	$O(n2^k)$ [Broadbent 2016] $O(n+k)$ with PR boxes *(Monday talk)*
Clifford + T-count d	$O((68n)^d)$
Small light-cone circuits	[Dolev Cree 2022]

Bonus application: distributed computing

- Quantum computation over spatially separated locations
- Normally executing U takes time 2*d*
  (send relevant qubit back and forth)
- Improved to time *d*, since we can make the communication simultaneous
- Trade entanglement for time
- Faster intelligent routing



More room for speed up with more parties / repeaters?



Bonus application: Homomorphic encryption

Classical case

Encrypt data so that another party can perform calculations on the encrypted data

Many applications

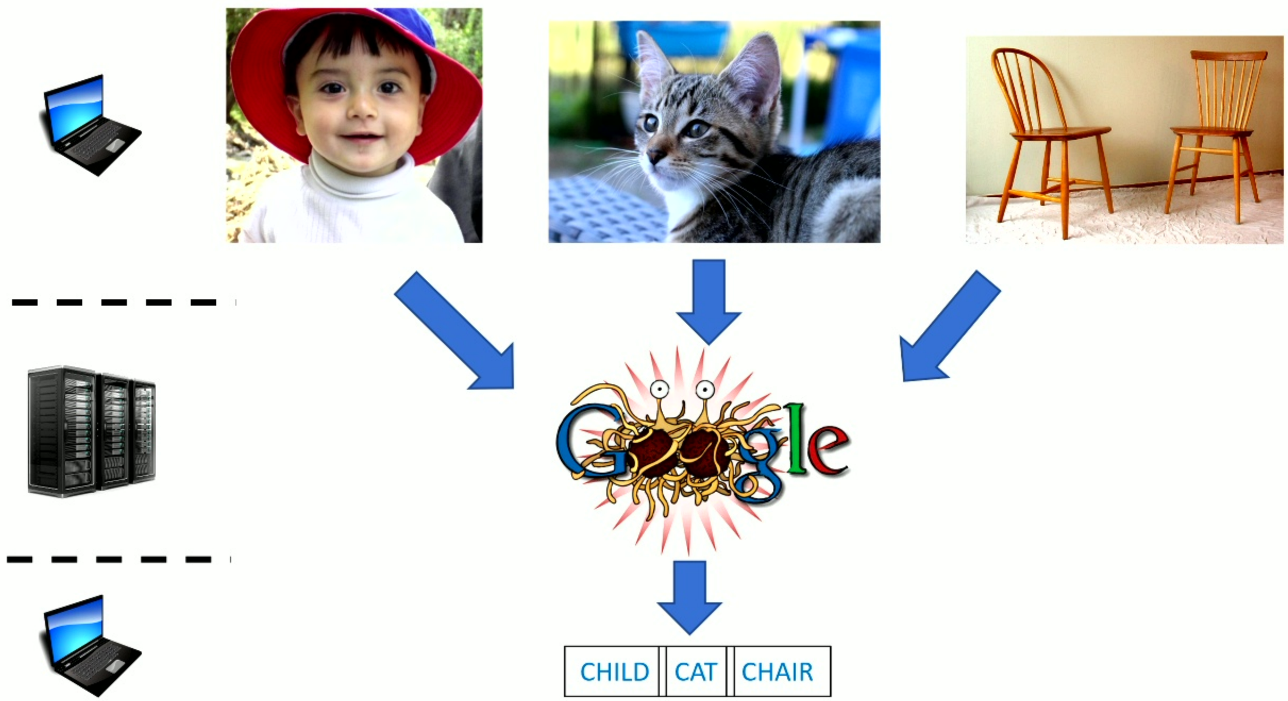
CHILD

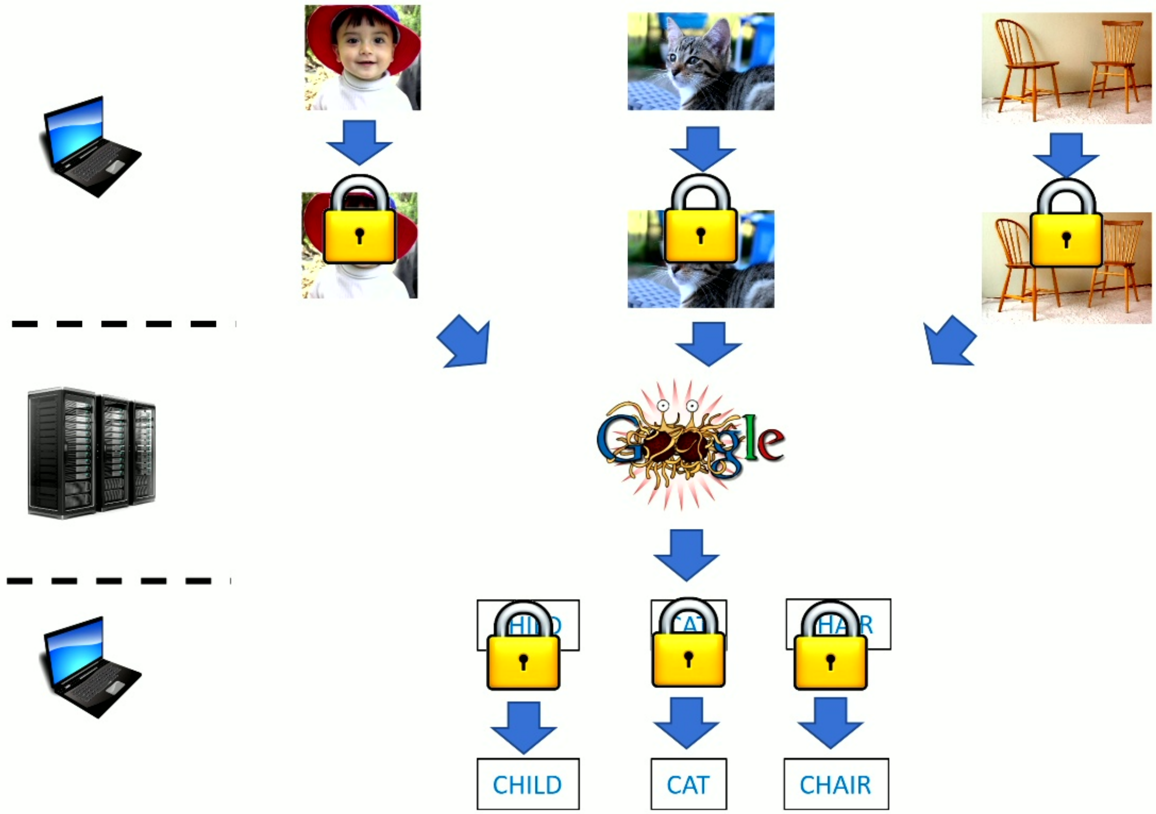
CAT

CHAIR

Tagging

Google DeepMind


CHILD
CAT
CHAIR


CHILD
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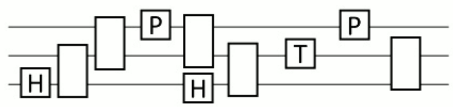
Quantum Homomorphic Encryption

Encrypt *quantum state*, instead of classical data

$$\rho \to \mathrm{QEnc}(\rho)$$

Execute *quantum circuit* on encrypted data



Quantum one-time pad ↔ uncorrected quantum teleportation
We can use the main lemma as a starting point
[DSS 2016]

Open questions (many)

- What about other circuit classes?
  - Fermions / match gates, CV, qudits…
- New tricks such as code-routing [Cree May 2023] better than garden-hose model?

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- (Details:) $(68n)^d$ is clearly not the right number. Proper gate teleportation easy way to reduce this.

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