

Title: Protocols and Implementations of Quantum Position Verification

Speakers: Eric Chitambar, Paul Kwiat

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# Protocols and Implementations of Quantum Position Verification

**Eric Chitambar**

**Paul Kwiat**

**Ian George**

**QPV 2023: Advances in  
quantum position verification**

**Andrew Conrad**



# Protocols and Implementations of Quantum Position Verification *and Some Related Work on Relativistic QKD*

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# Outline

Principles and tools  
for QPV

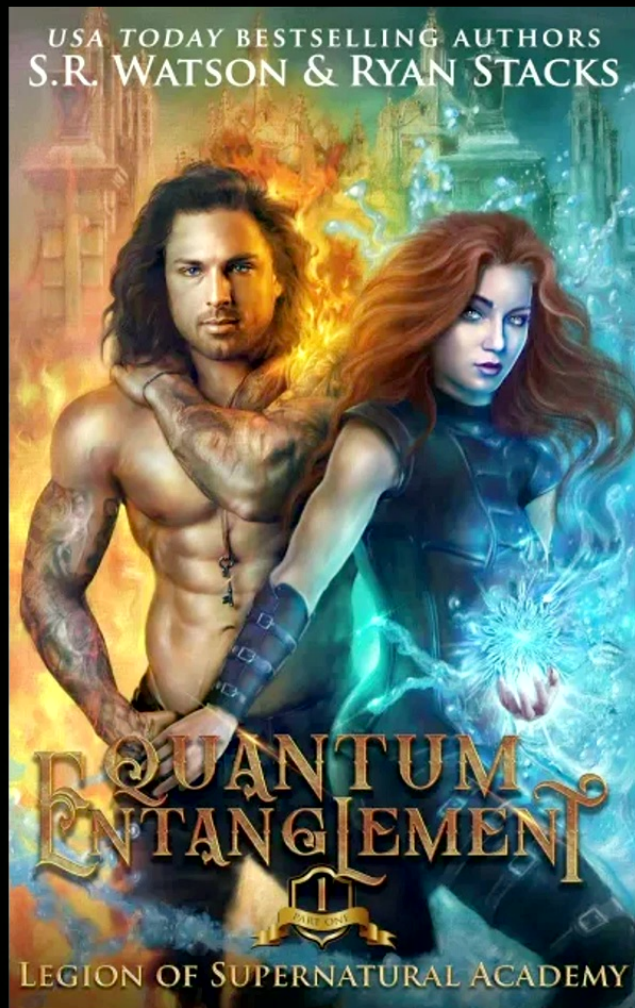
1. Relativistic QKD

2. QPV and state discrimination

Drone-based  
QPV protocol

3. An entanglement-distribution QPV protocol

4. Experimental implementation

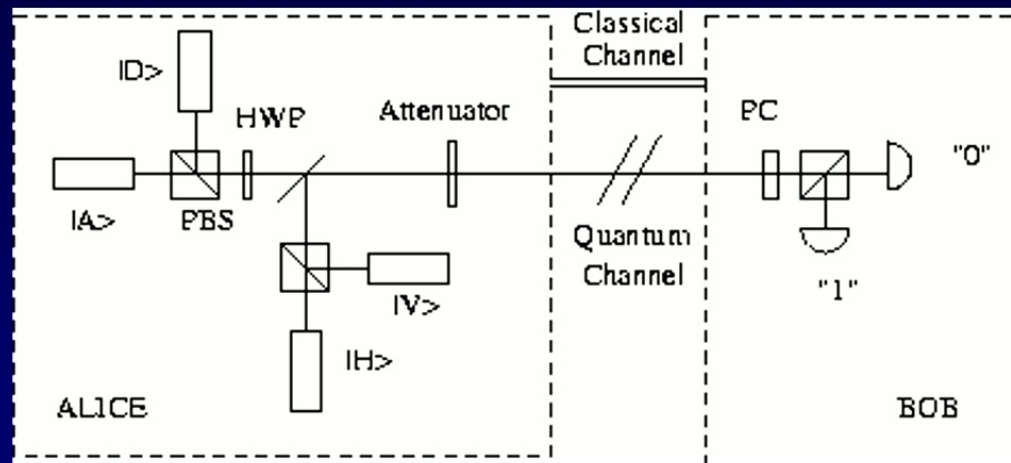


**Release Date: October 3, 2019**  
**Genre: Paranormal Romance**

*Genesis and Elysian are quantum entangled for a singular purpose. Their recruitment to the Legion of Supernatural Academy is unexpected but vital to the future of humanity. This unique series takes the world of supernatural academies to new heights with twisted tales, suspense-driven fantasy, and self-discovery.*

# Part I: Relativistic QKD

# BB84 (Six-state) Protocol



Alice transmits a photon in one of **four** (six) states.

Bob measures the photon in one of **two** (three) bases.

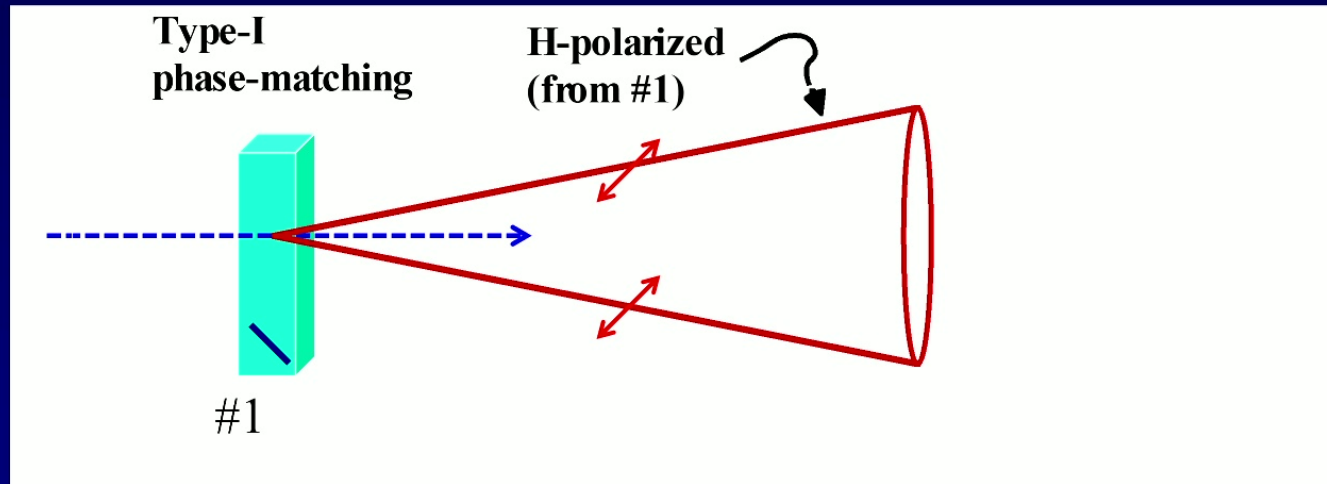
Alice and Bob sift out the trials -50% (33%) where they used same basis.

The sifted keys have "perfect" correlation.

An intrusive eavesdropper induces errors up to 25% (33%).

# How We Make Entanglement

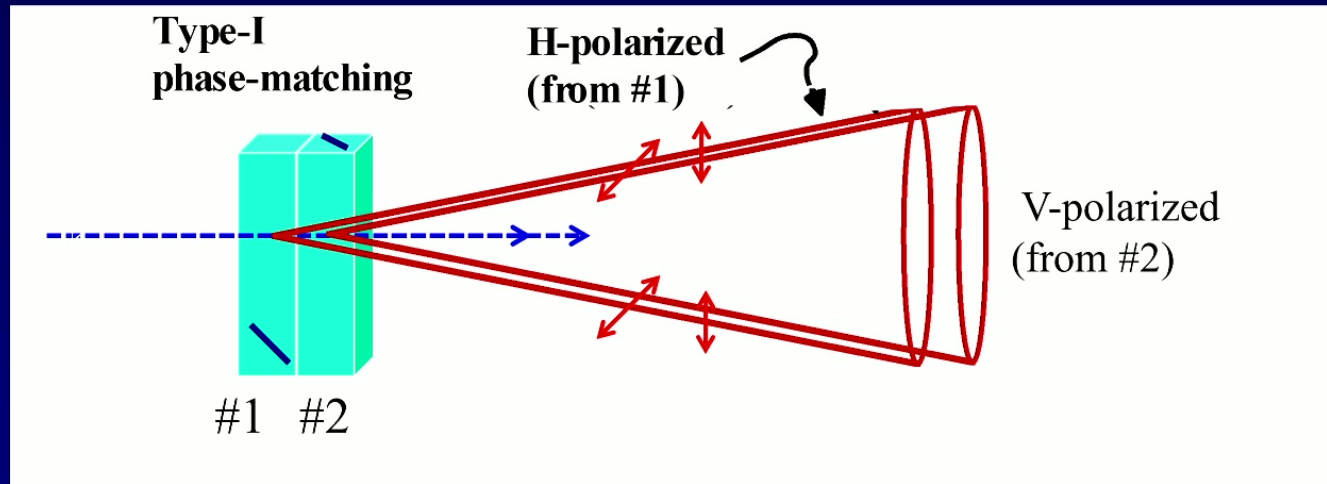
“Spontaneous DownConversion”: high-energy parent photon can split into two daughter photons (with same polarization)





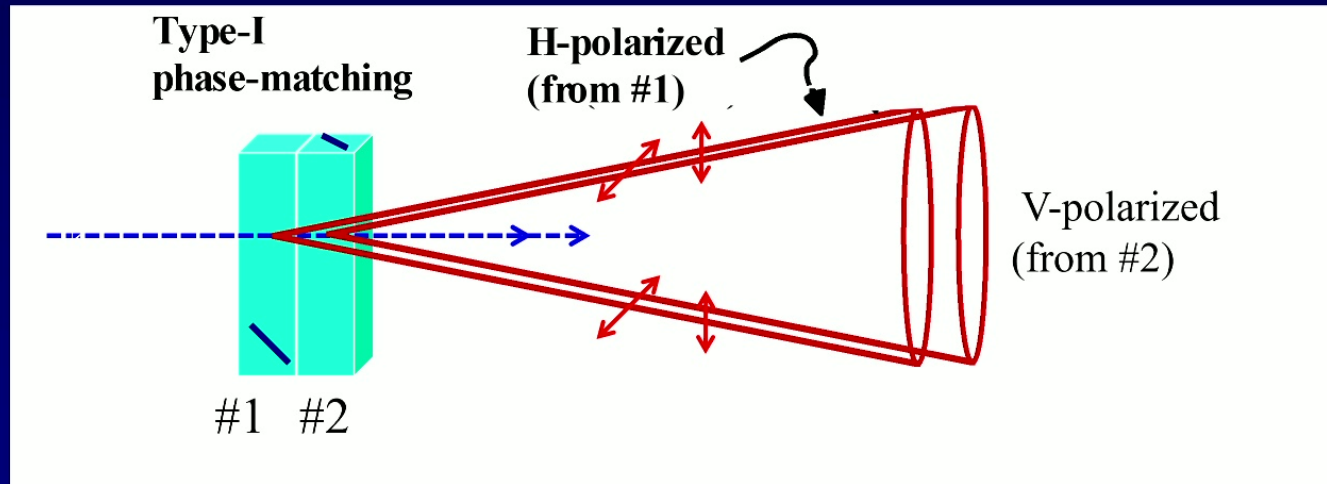
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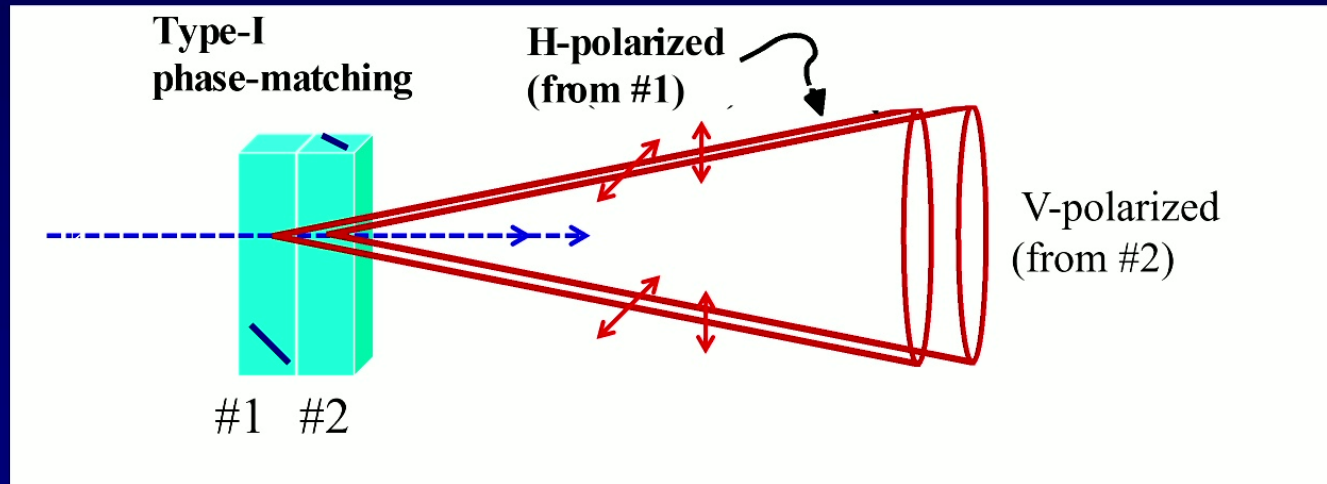


We don't know WHICH crystal created the pair of photons, but we know they both came from the same crystal

→ they MUST have the same polarization:  $|\psi\rangle = |H\rangle|H\rangle + |V\rangle|V\rangle$

# How We Make Entanglement

“Spontaneous DownConversion”: high-energy parent photon can split into two daughter photons (with same polarization)

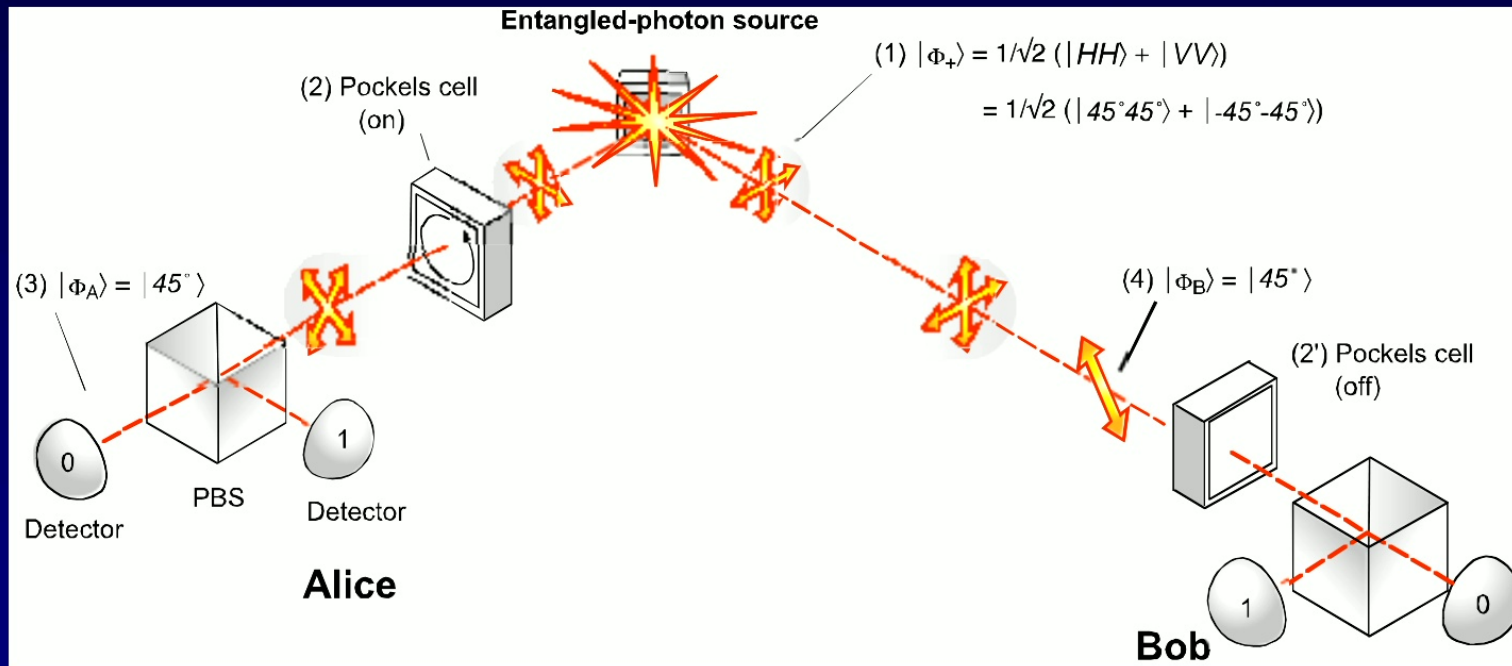


Foreshadowing:  
Changing the pump polarization → alters which/how much entanglement

We don't know WHICH crystal created the pair of photons, but we know they both came from the same crystal

→ they MUST have the same polarization:  $|\psi\rangle = |H\rangle|H\rangle + |V\rangle|V\rangle$

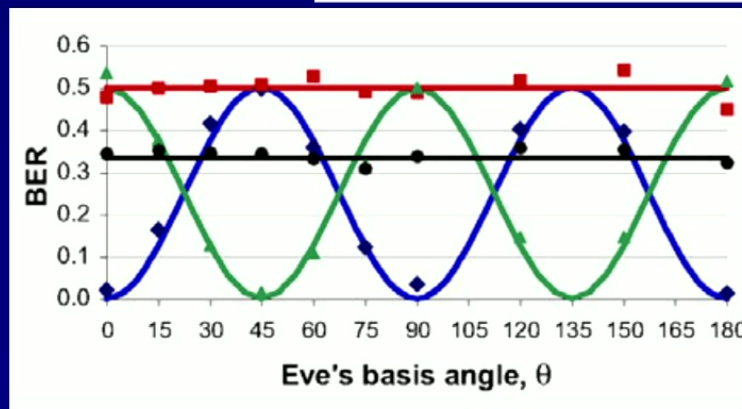
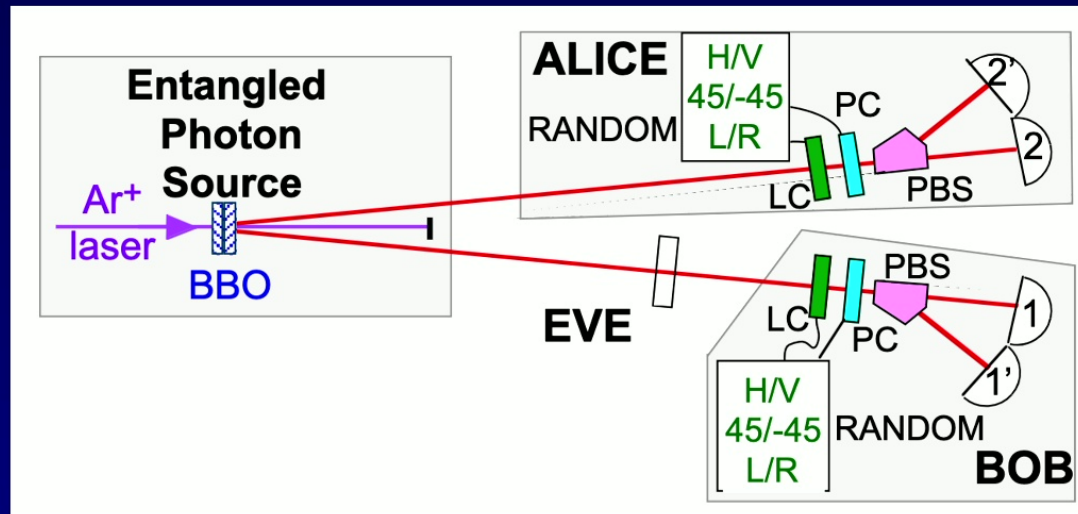
# Entangled-Photon Quantum Cryptography



- Alice & Bob randomly measure polarization in the (H/V) or the (+45/-45) basis.
- Discuss via a “public channel” which bases they used, *but not the results*.
- Discard cases (50%) where they used different bases → uncorrelated results.
- Keep cases where they used the same basis → *perfectly correlated results!*
- Define  $H \equiv “0” \equiv 45$ ,  $V \equiv “1” \equiv -45$ . **They now share a secret key.**

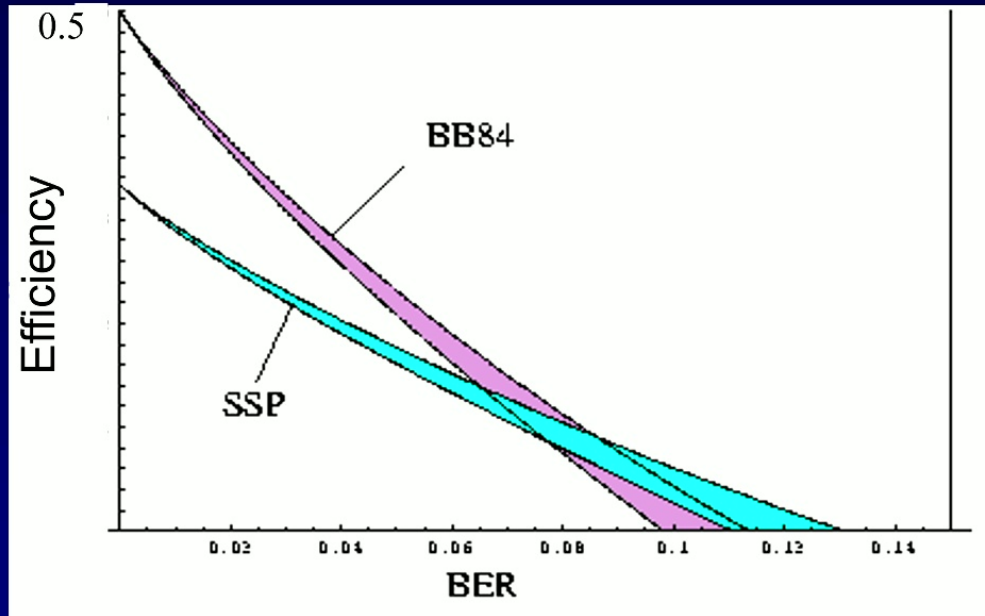
# Experimental Realization of Six-State QKD Protocol

{D. Enzer, PGK et al., New Journal Physics 4, 45.1 (2002)}



**Total BER is 33%, independent of attack strategy**  
**(cf. to 25% BER in BB84 4-state protocol)**

# The Trouble with Sifting



BB84: sifting  $\Rightarrow$  50% inefficiency

Six-State Protocol: sifting  $\Rightarrow$  66% inefficiency

## PGK Group, Circa 2006

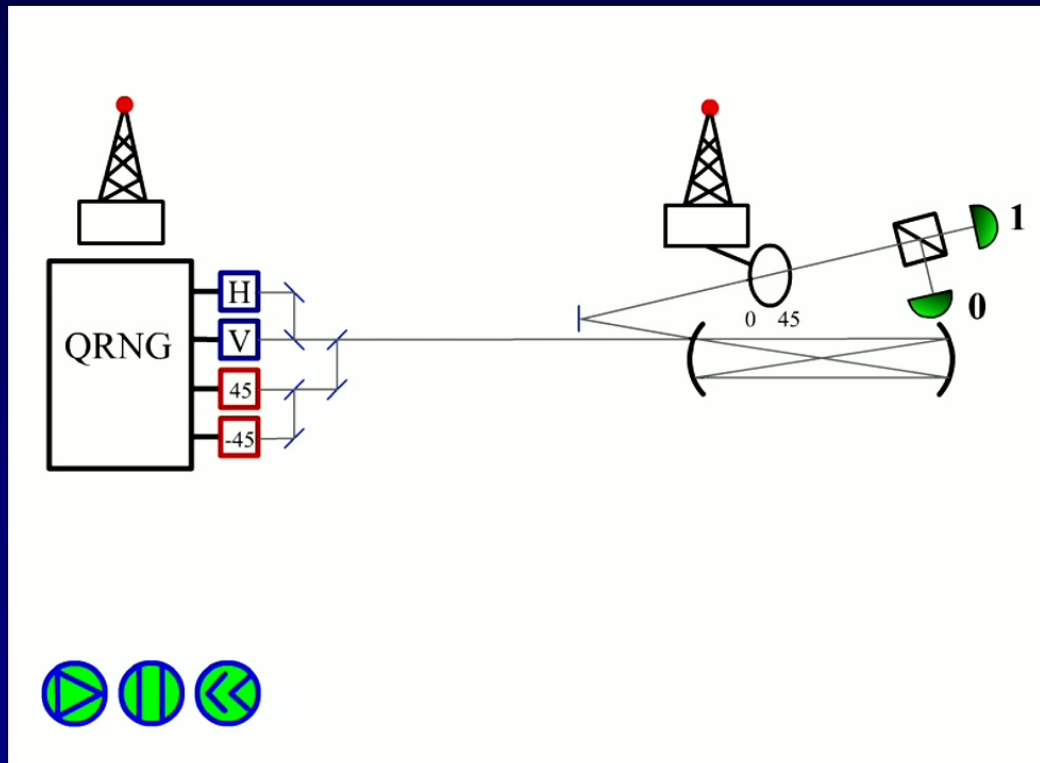
Graduate Students: Joe Altepeter, Julio Barreiro, Onur Hosten, Evan Jeffrey, Nicholas Peters, Radhika Rangarajan, Aaron VanDevender, Joseph Yasi

Undergraduates: Kyle Arnold, Gleb Akselrod, Rachel Hillmer, Kevin Uskali

Associated Theory Post-Doc: Tzu-Cheih Wei

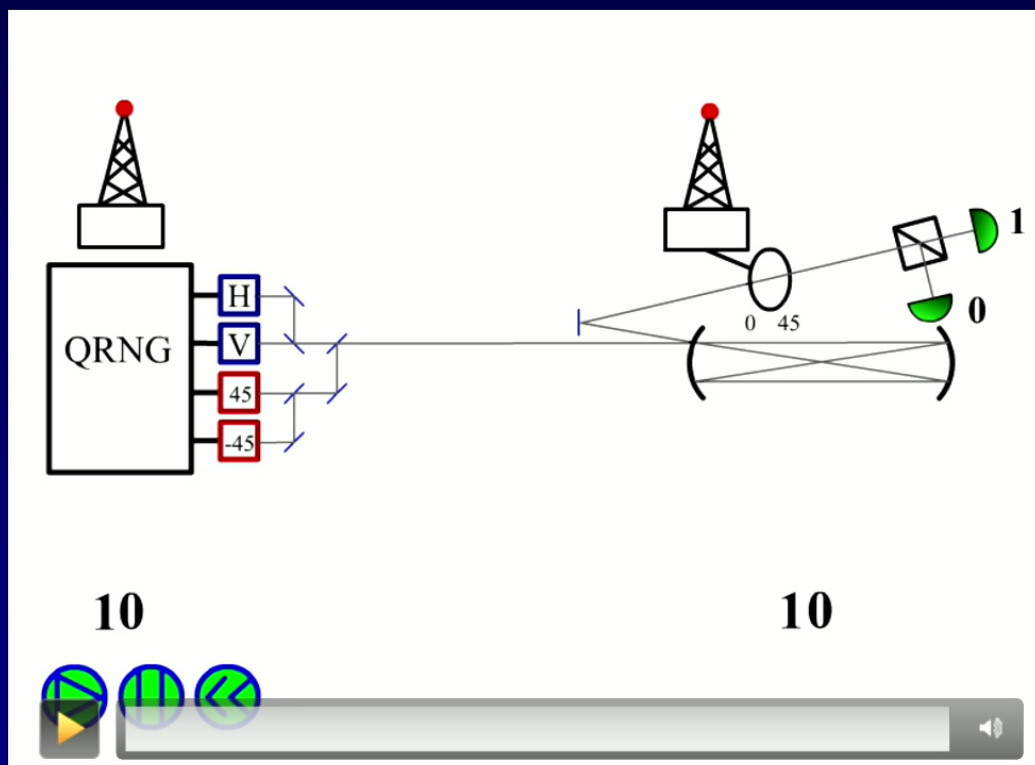


# “Relativistic” Quantum Cryptography



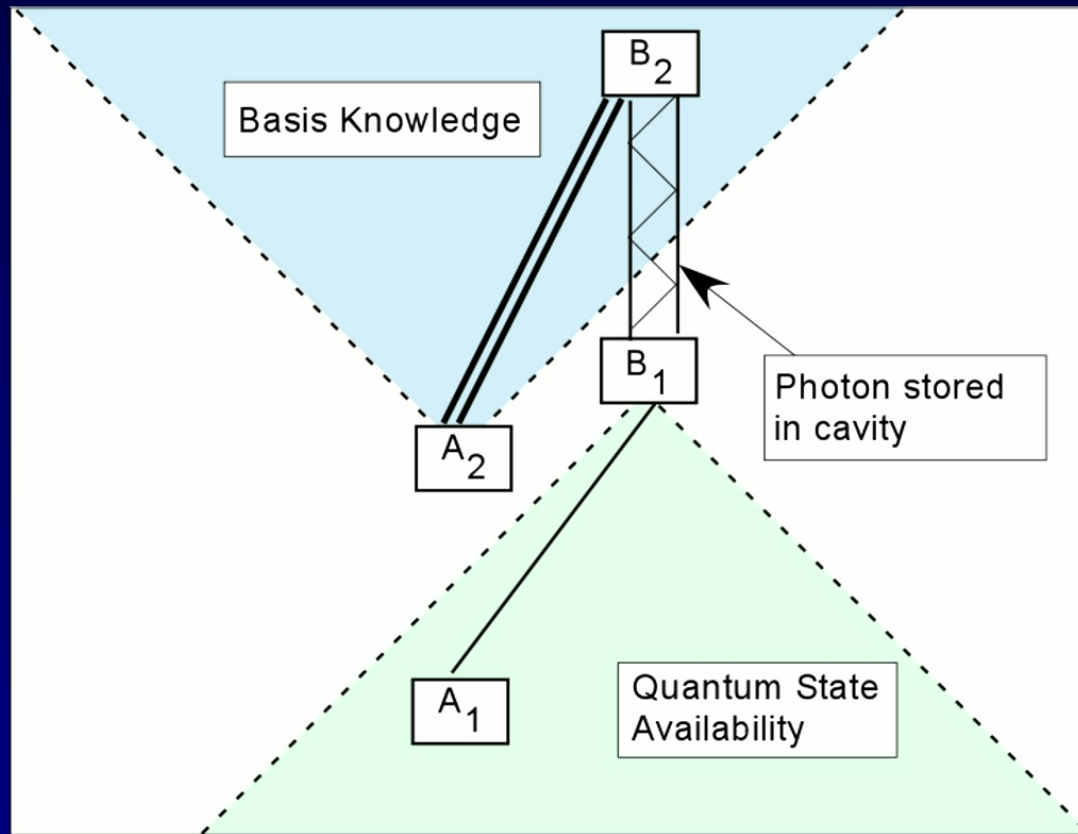


# “Relativistic” Quantum Cryptography



Bob stores each photon until Alice tells him which basis to use  
→ net efficiency is increased to 100% (in principle)  
→ same security as BB84 (Eve's  $\rho$  cannot depend on Bob)

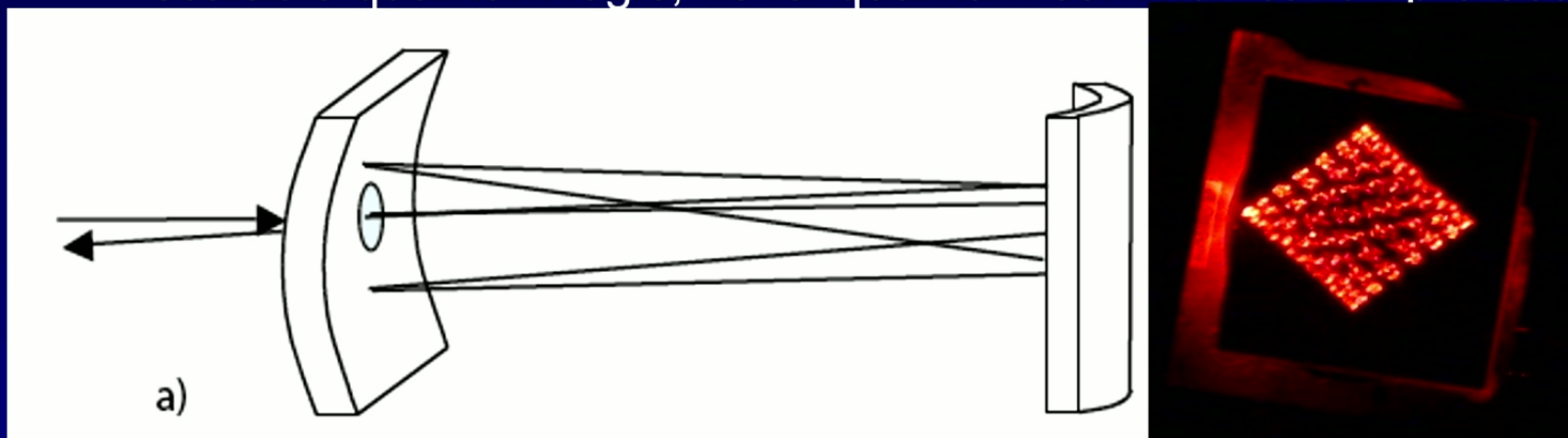
# QKD and Special Relativity



- These two light cones must not overlap
- $A_2$  may be before  $B_1$  in some reference frames
- *Alice and Bob must know their space-time coordinates*

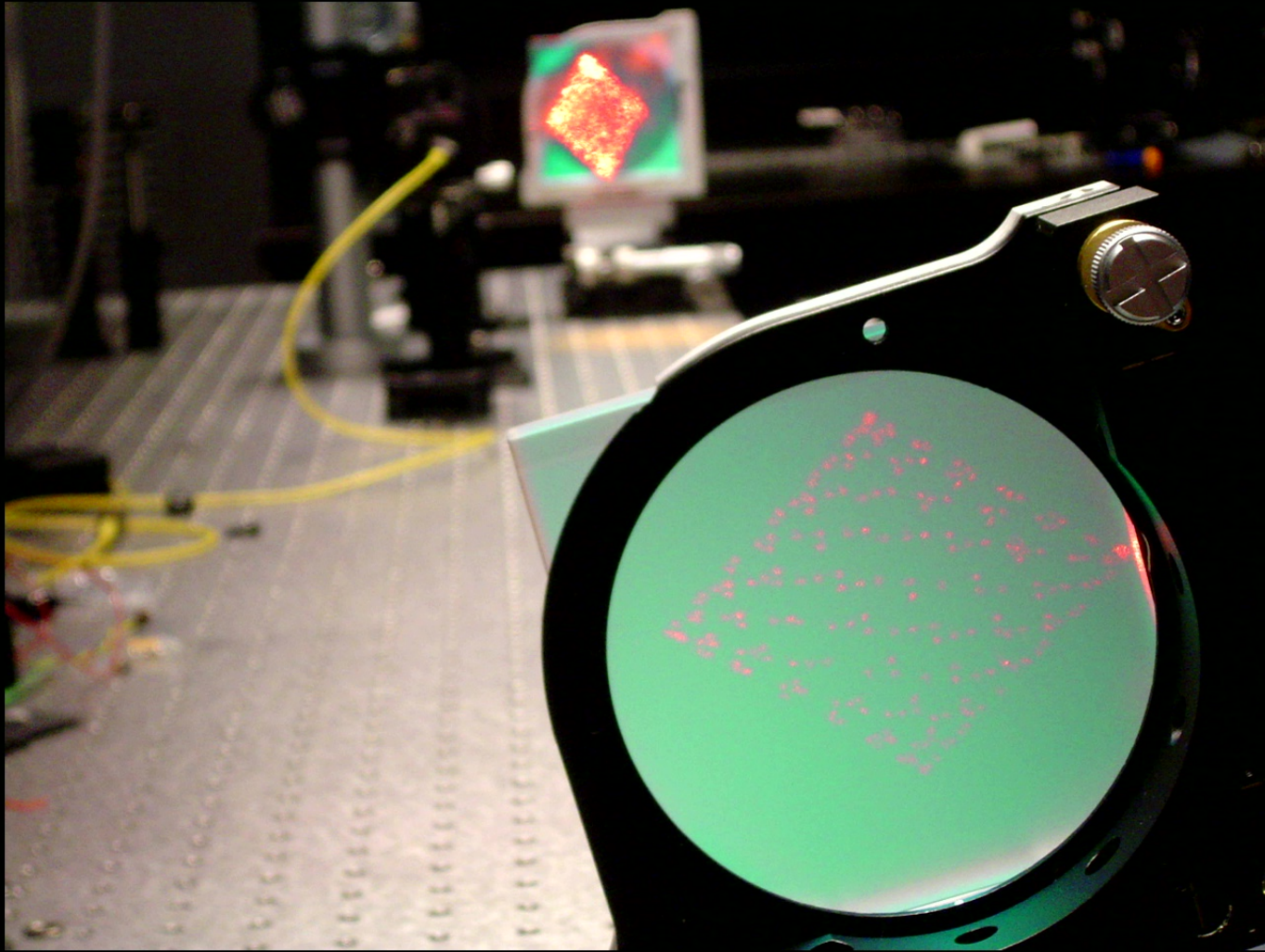
# Quantum Memory: low-loss optical delay line

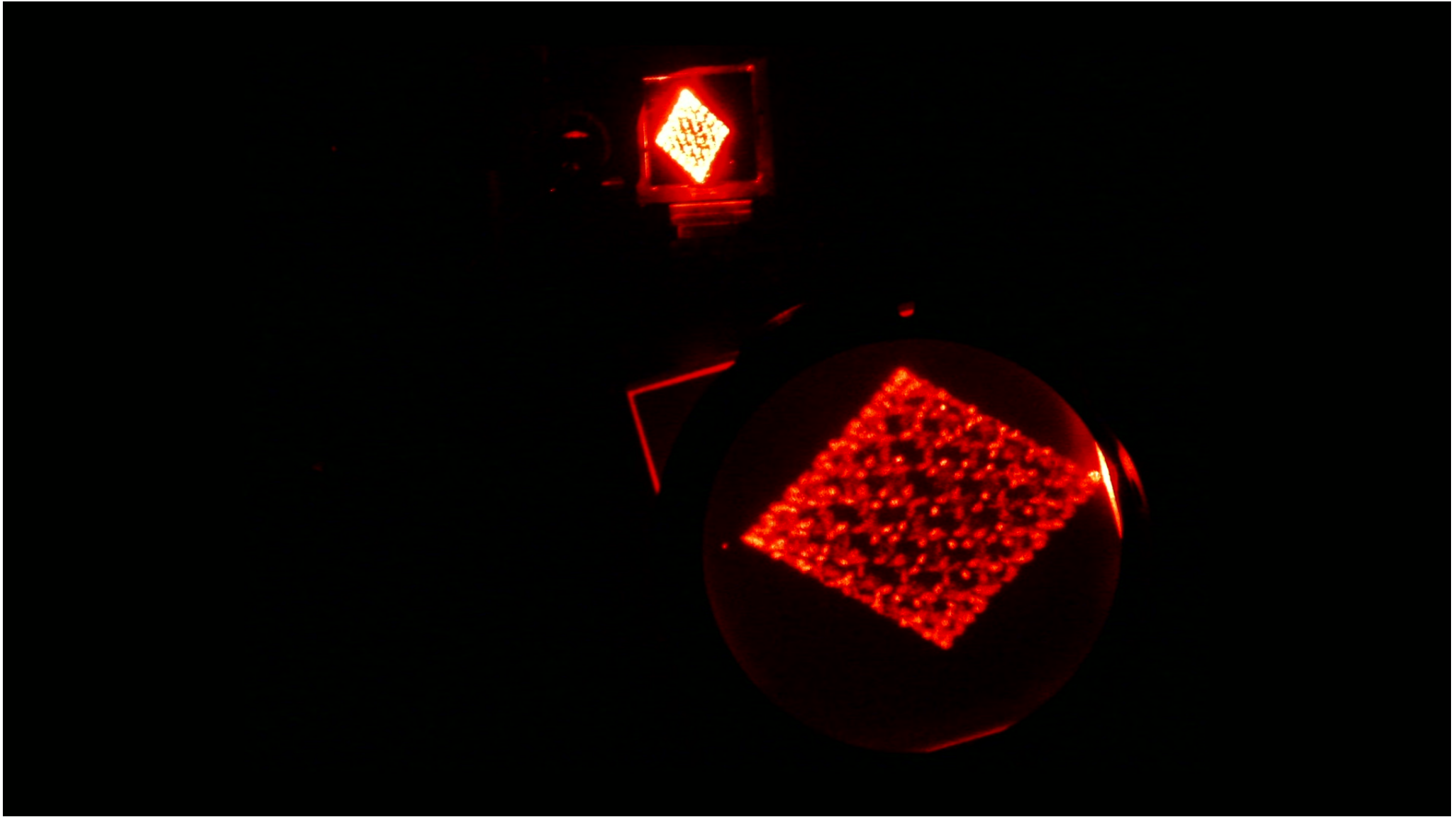
Applications to quantum cryptography, quantum “repeaters”, scalable quantum logic, novel quantum communication protocols



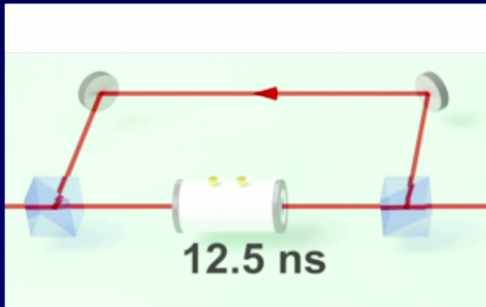
## Advantages

- High bandwidth ( $\sim 10$  nm)
- Polarization insensitive
- Adjustable time delay (10 ns – 10 $\mu$ s)
- Low loss (custom mirror coatings)
- Store multiple k-vectors, spatial modes

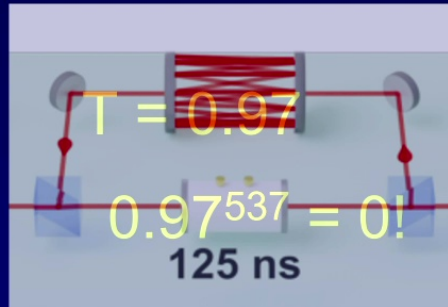




# FYI: Adjustable Quantum Memory

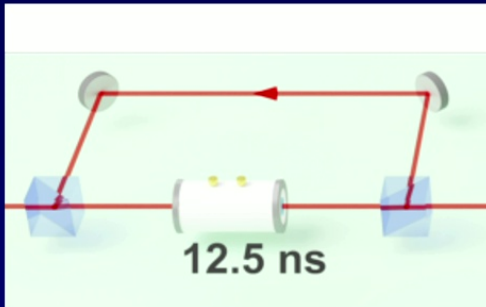


Flat-mirror cavity

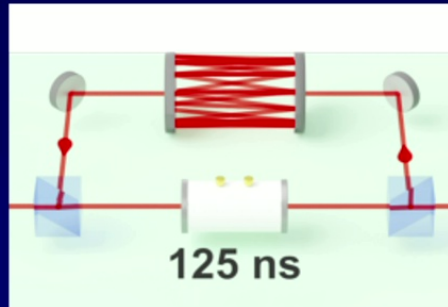


Herriott cell cavity

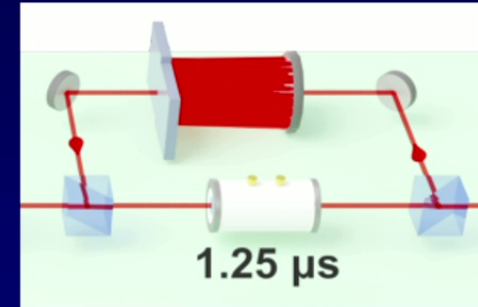
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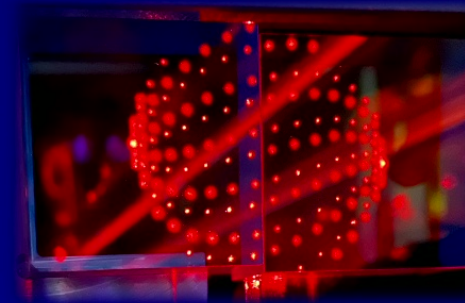
Modified Herriott cell cavity

Number of bounces  
limited by mirror area

E.g., 1.1-m spacing

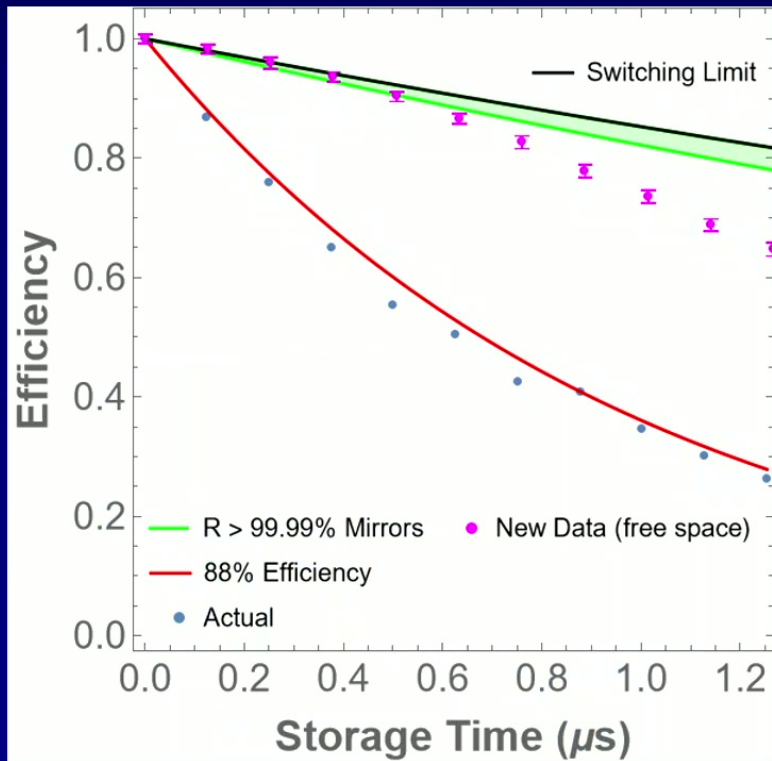
→ 339 reflections

→ 1.25- $\mu$ s delay

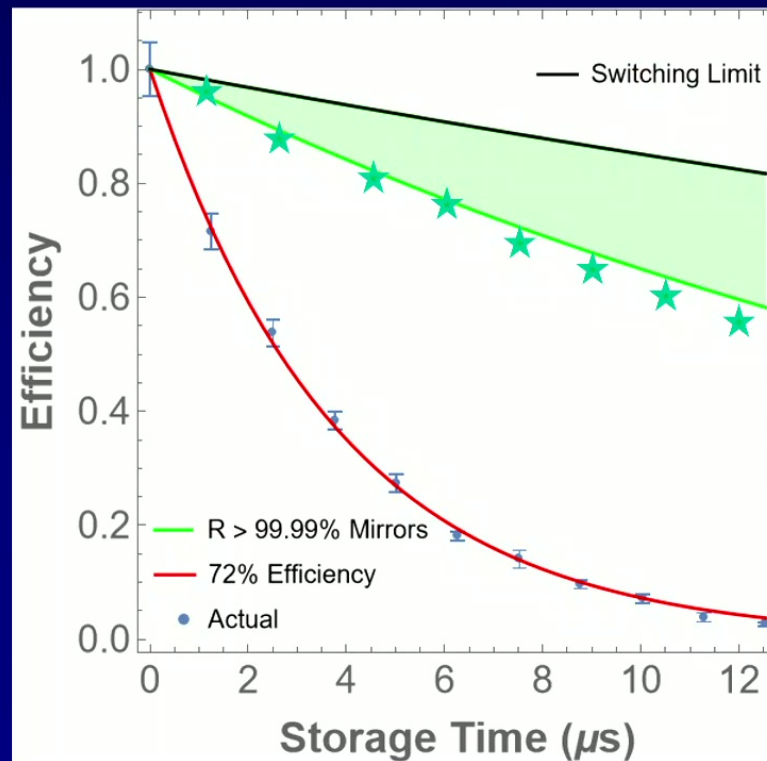


# Memory performance limited by mirror reflectivity and polarization-switching efficiency

While fiber-based memories must deal with fundamental dispersion and loss limitations, mirror coatings and active-switching technologies are continuously improving



125-ns loop  
end-to-end  
efficiency



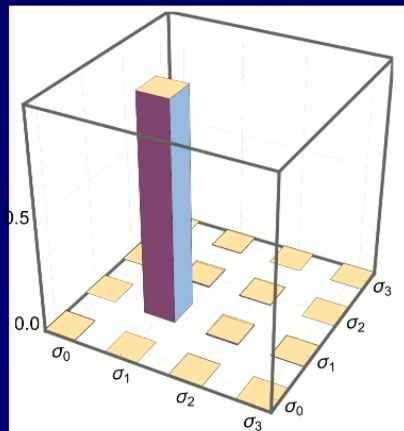
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# Memory preserves quantum state encoded onto photons

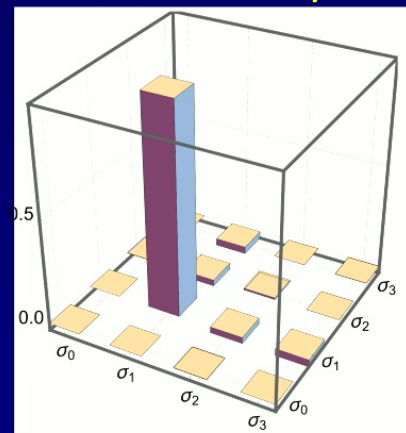
Propagating in free space and reflecting at mostly  $\sim 0^\circ$  angle of incidence prevents changes to the polarization state of the qubits being stored in the memory

12.5-ns loop



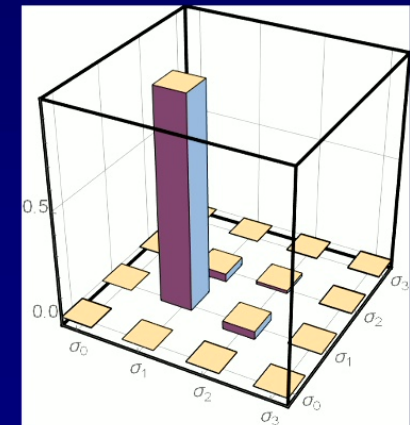
99.4(3)%  $\chi$ -fidelity

125-ns loop



99.0(1)%  $\chi$ -fidelity

1.25- $\mu$ s loop

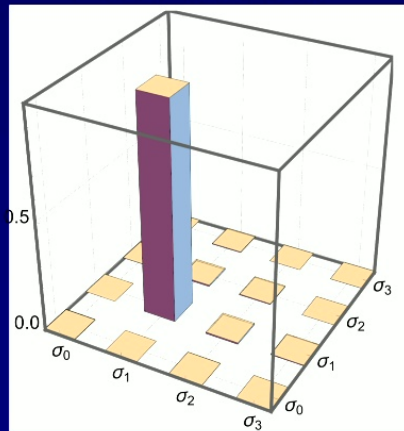


97.8(2)%  $\chi$ -fidelity

# Memory preserves quantum state encoded onto photons

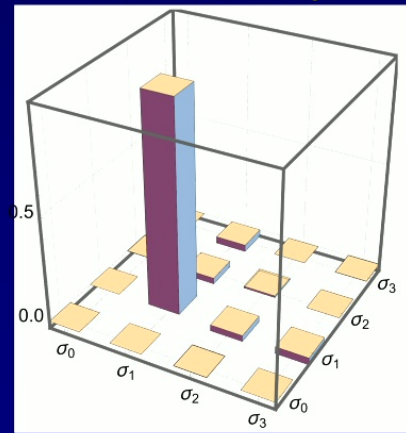
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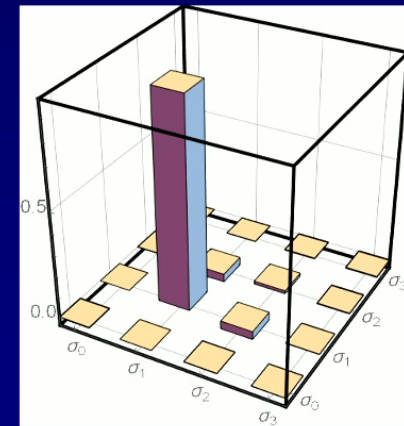
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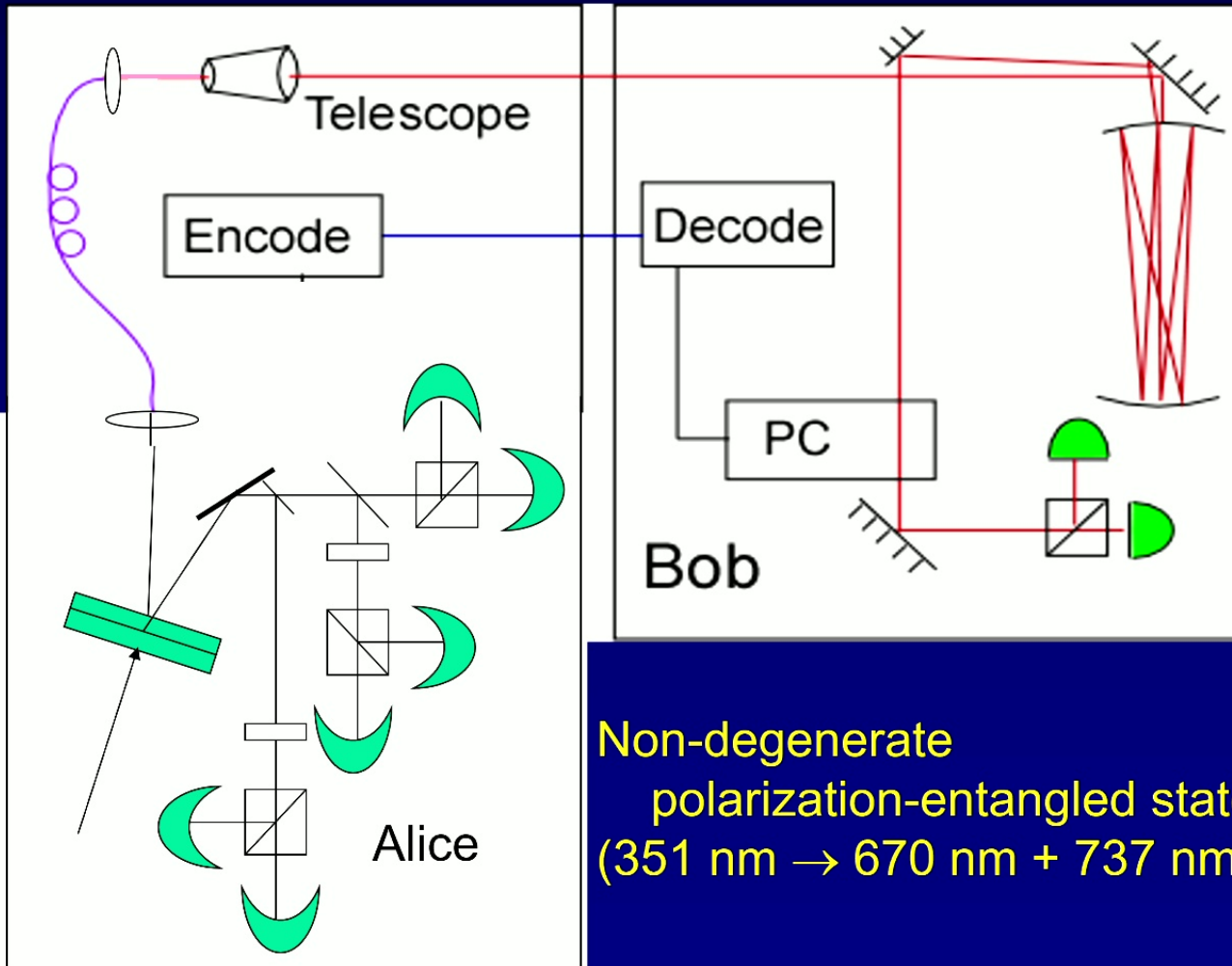


97.8(2)%  $\chi$ -fidelity

**Bandwidth: 1.5 THz**

**Time-Bandwidth:  $6 \times 10^6$**

# Incorporate *entangled* photon source

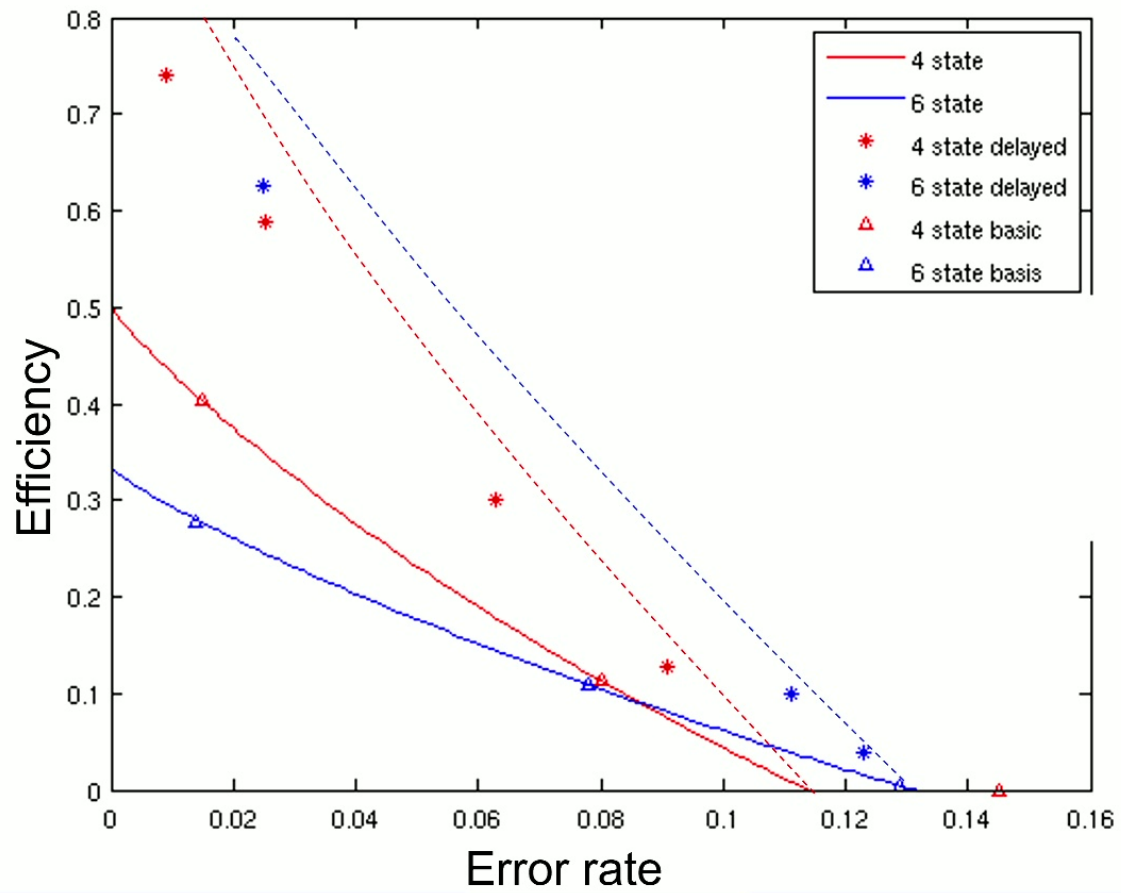


Non-degenerate  
polarization-entangled state  
(351 nm → 670 nm + 737 nm)

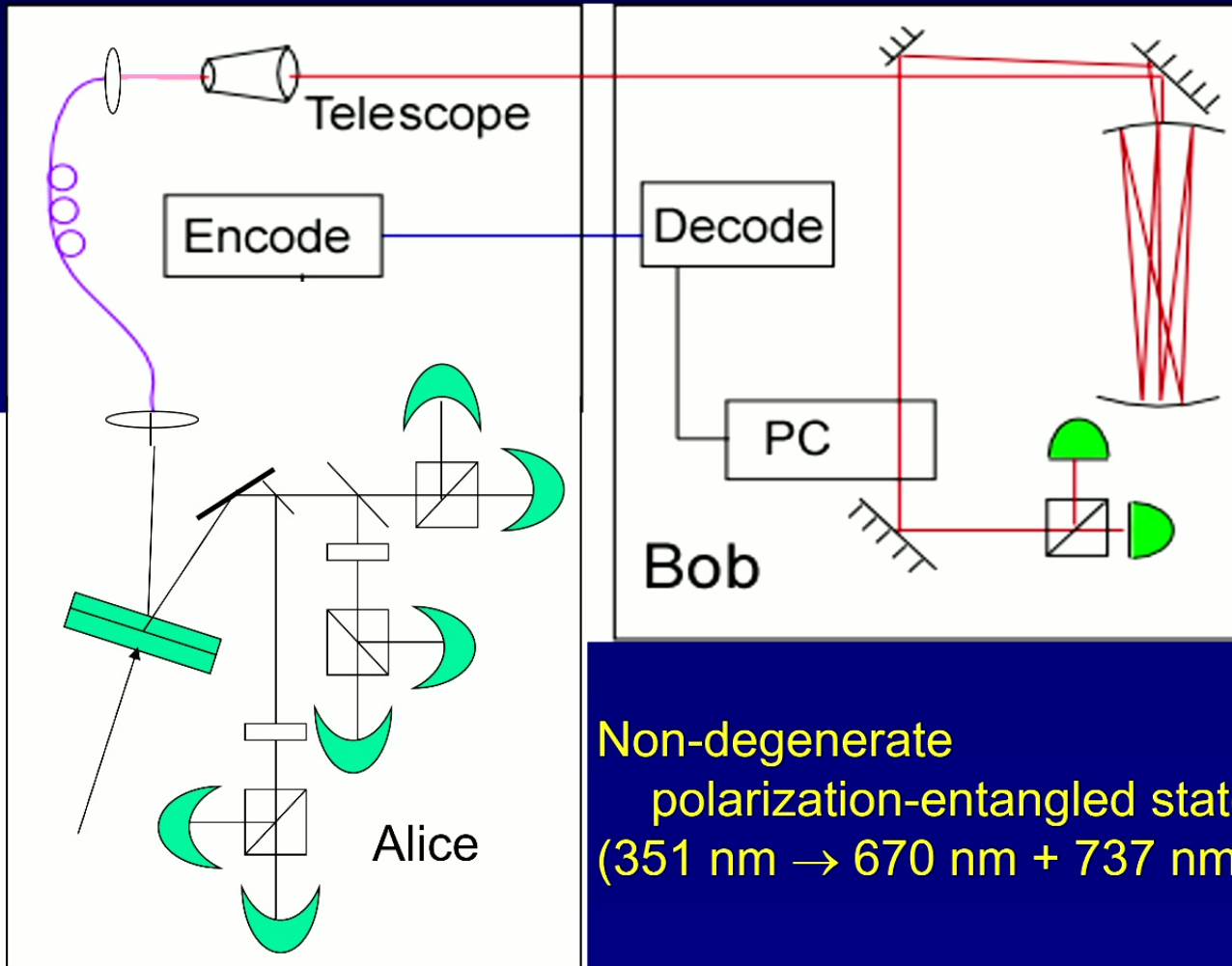
BB84, 30 mW pump power  
94 sifted bits/second  
2.5% error rate  
→ 65.5 secret bits/second

BB84, 90 mW pump power  
214 bits/second  
3.1% error rate  
→ 136 secret bits/second  
→ **yield enhancement = 1.3**

SSP, 90 mW pump power  
371 bits/second  
2.7% error rate  
→ 251 bits/second  
→ **yield enhancement = 2.1**



# Incorporate *entangled* photon source



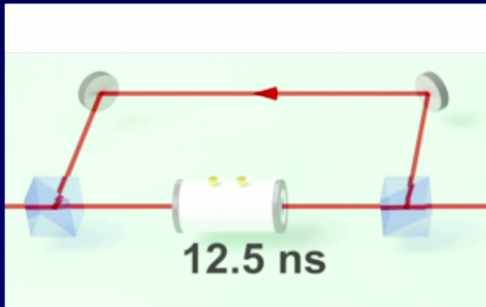
Non-degenerate  
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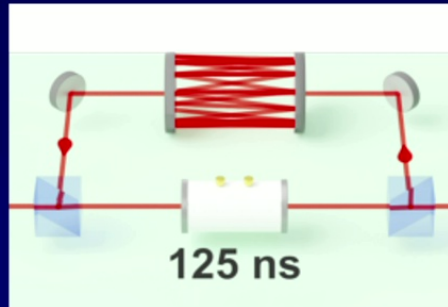
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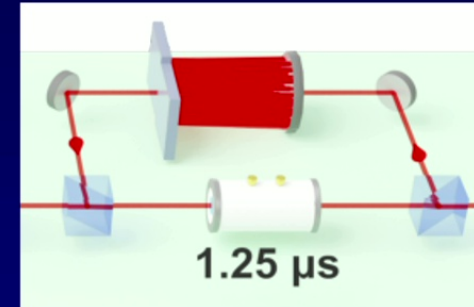
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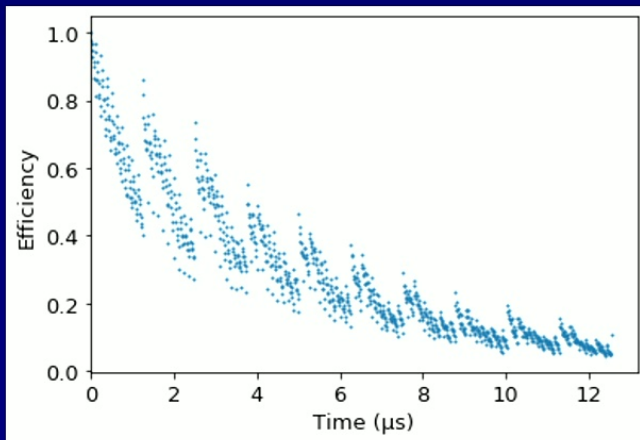
Flat-mirror cavity



Herriott cell cavity



Modified Herriott cell cavity

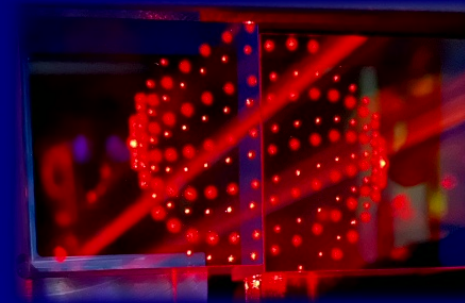


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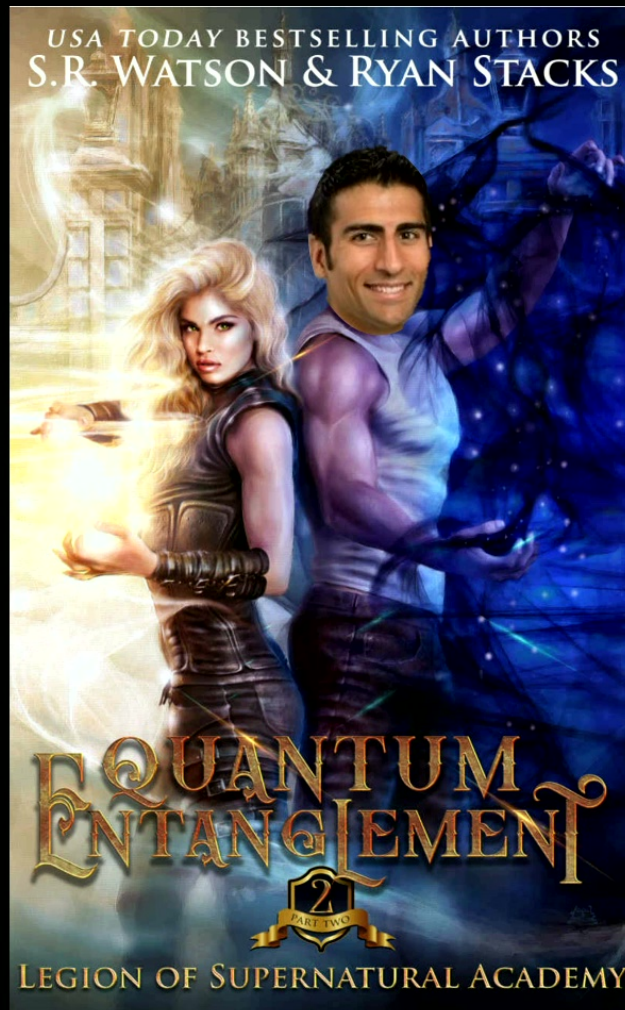
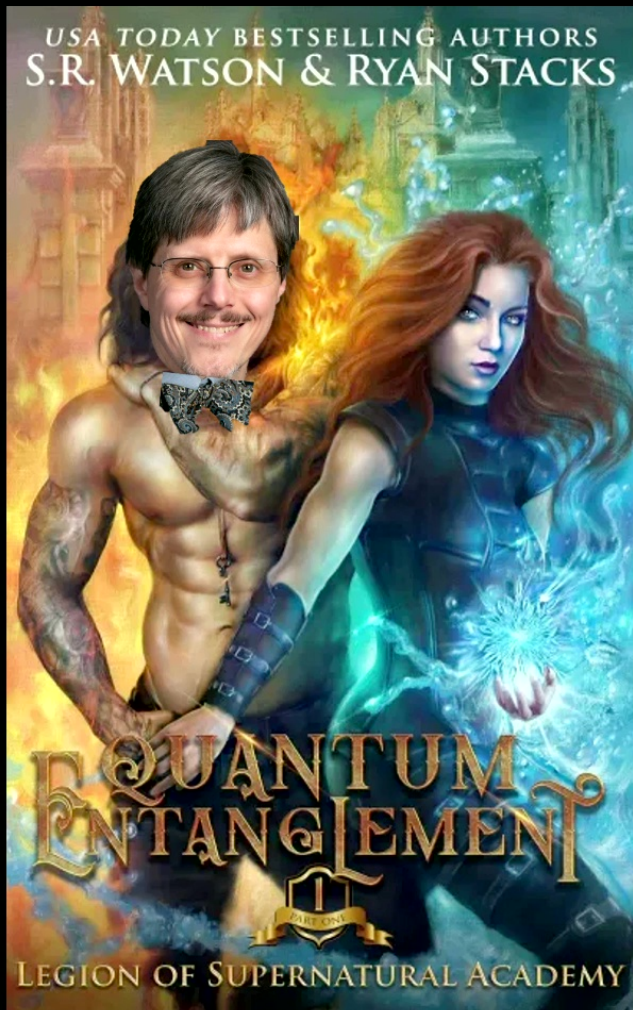
E.g., 1.1-m spacing

→ 339 reflections

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# Quantum Position Verification



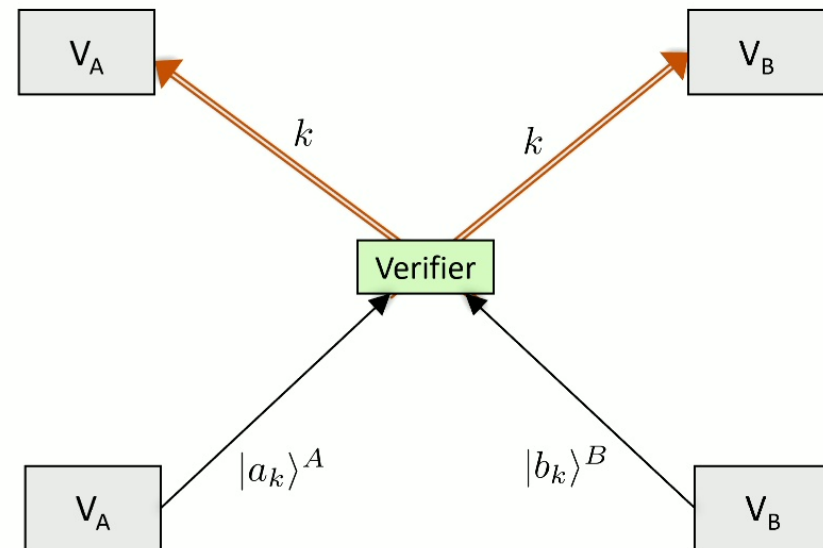
# QPV and State Discrimination

- Many QPV protocols can be understood as a state discrimination problem.

A family of  
orthogonal  
states

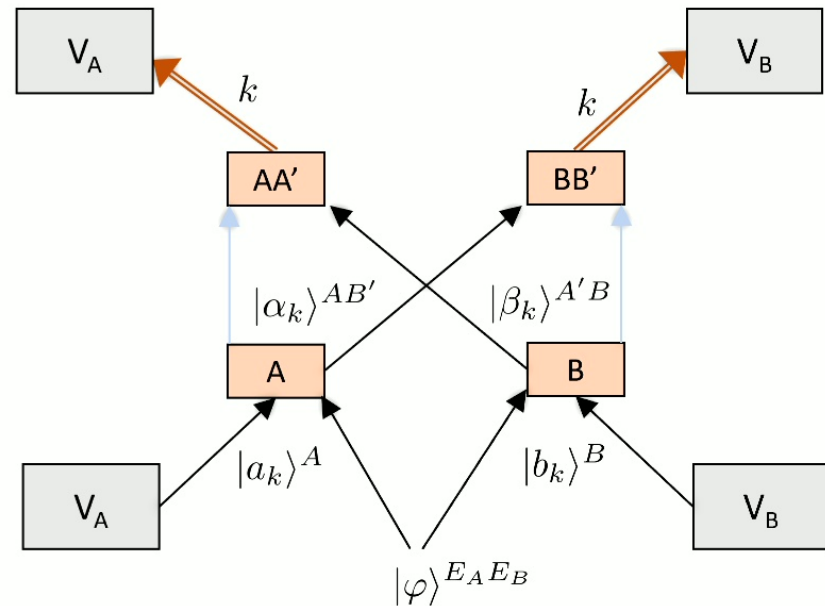
$$\left\{ \begin{array}{l} |\psi_1\rangle^{AB} = |0\rangle^A \otimes |0\rangle^B \\ |\psi_2\rangle^{AB} = |1\rangle^A \otimes |0\rangle^B \\ |\psi_3\rangle^{AB} = |+\rangle^A \otimes |1\rangle^B \\ |\psi_4\rangle^{AB} = |-\rangle^A \otimes |1\rangle^B \end{array} \right.$$

- The prover needs to identify which bipartite state  $|\psi_k\rangle^{AB}$  was sent by the verifiers.



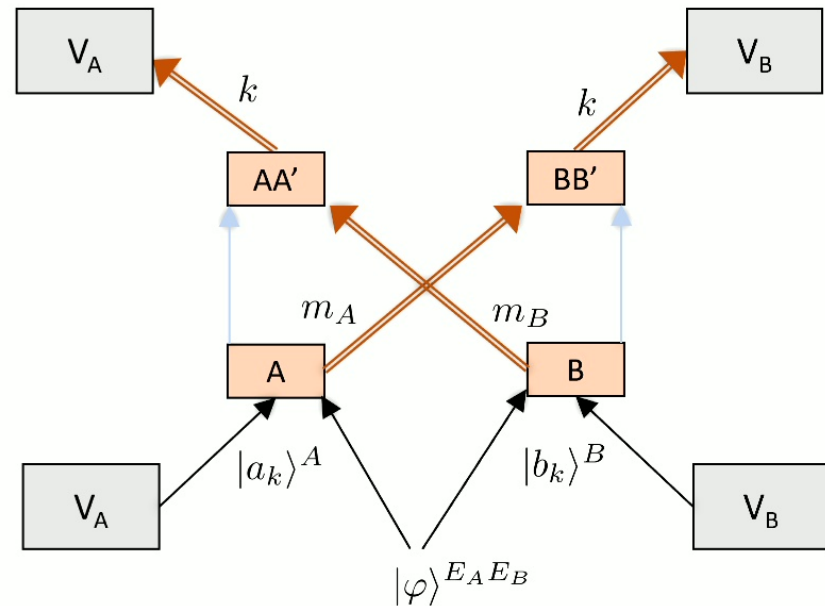
# QPV and State Discrimination

- In order to be secure, the orthogonality of the encoded states  $|\psi_k\rangle$  must be sufficiently nonlocal.
- They should not be distinguishable by *local operations* and *simultaneous communication*.
- Different adversarial models to consider:
  - Local operations and simultaneous quantum communication (**LOSQC**)
  - Entanglement-assisted local operations and simultaneous quantum communication (**eLOSQC**)



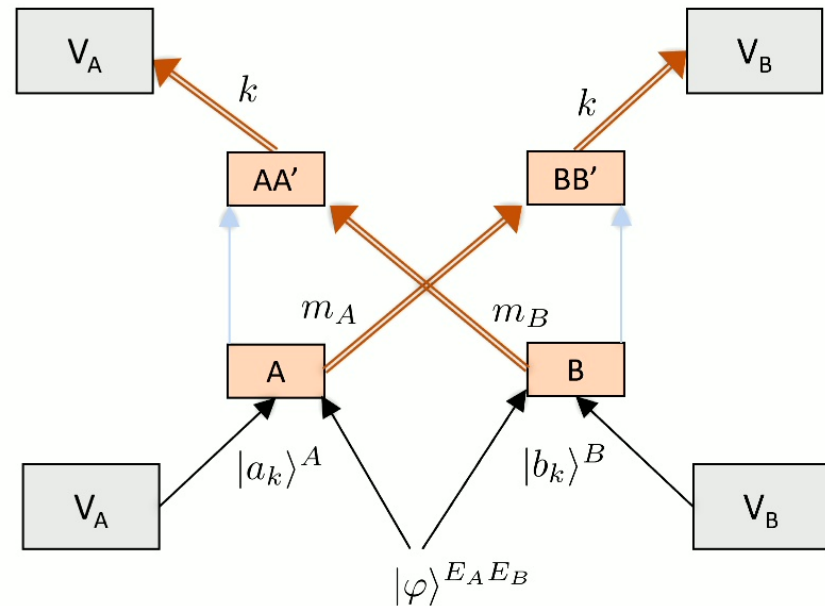
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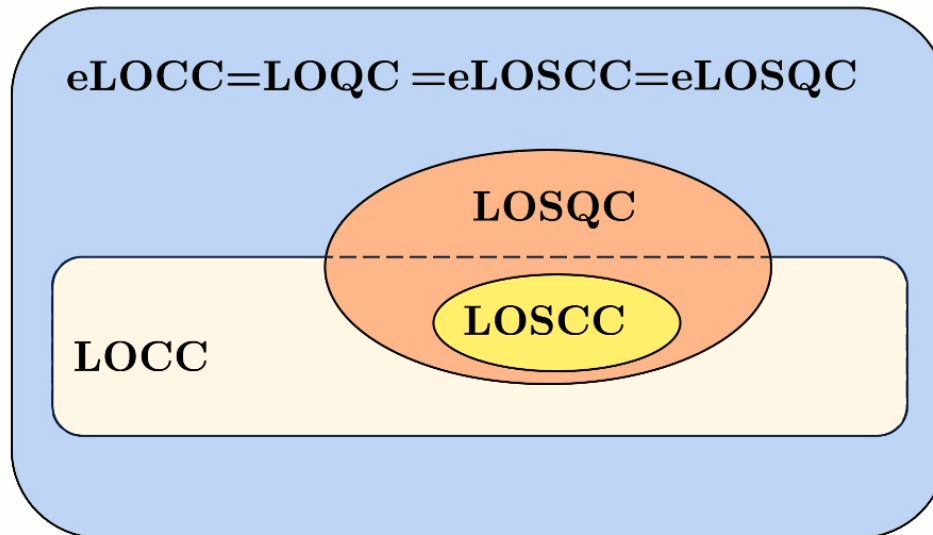


# Different Operational Classes

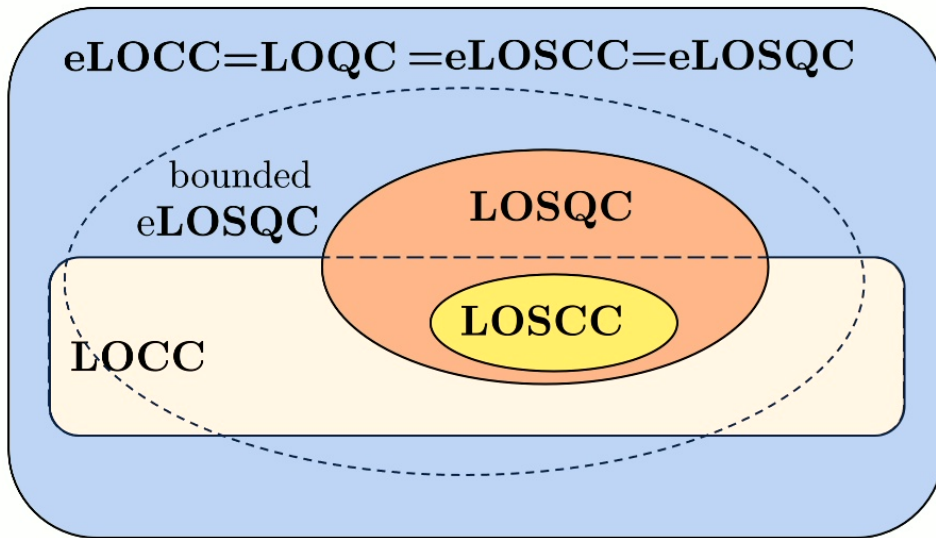
- These should be compared to standard:

- Local operations and classical communication (**LOCC**)
- Entanglement-assisted local operations and classical communication (**eLOCC**)
- Local operations and quantum communication (**LOQC**)

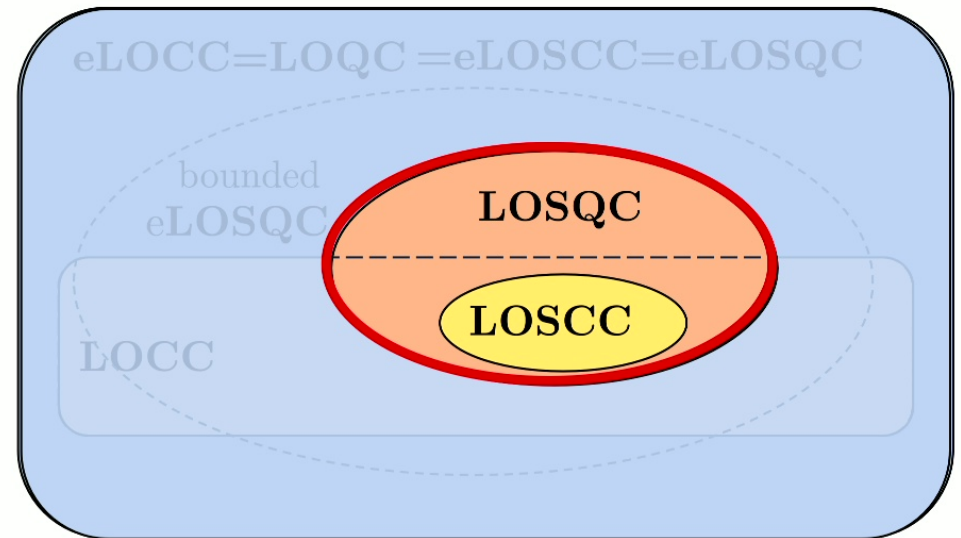
Unrestricted  
classical communication



# Different Operational Classes

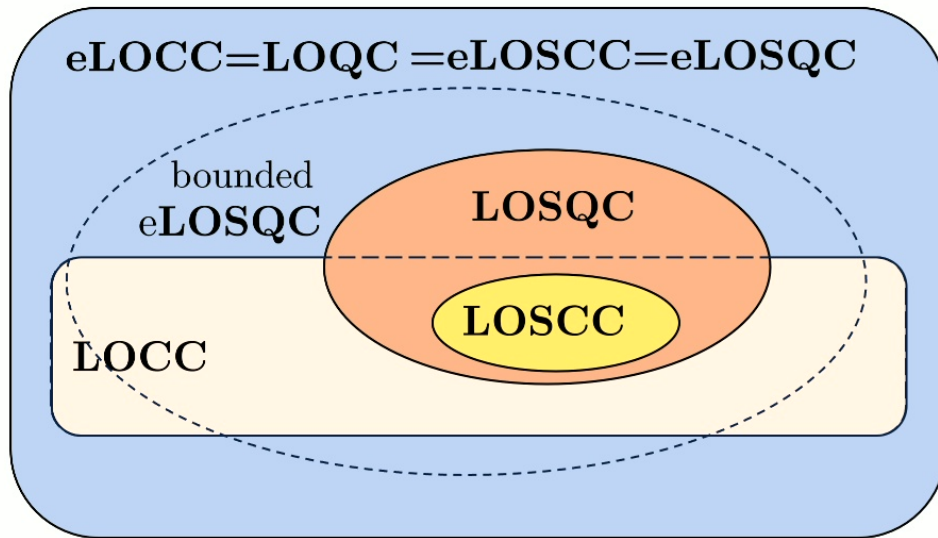


- The intermediate regime of **bounded entanglement** is where most QPV analysis sits.
- Every family of orthogonal  $\{|\psi_k\rangle\}_k$  that is difficult to discriminate using a class of operations constitutes a good QPV scheme under attacks from that class.

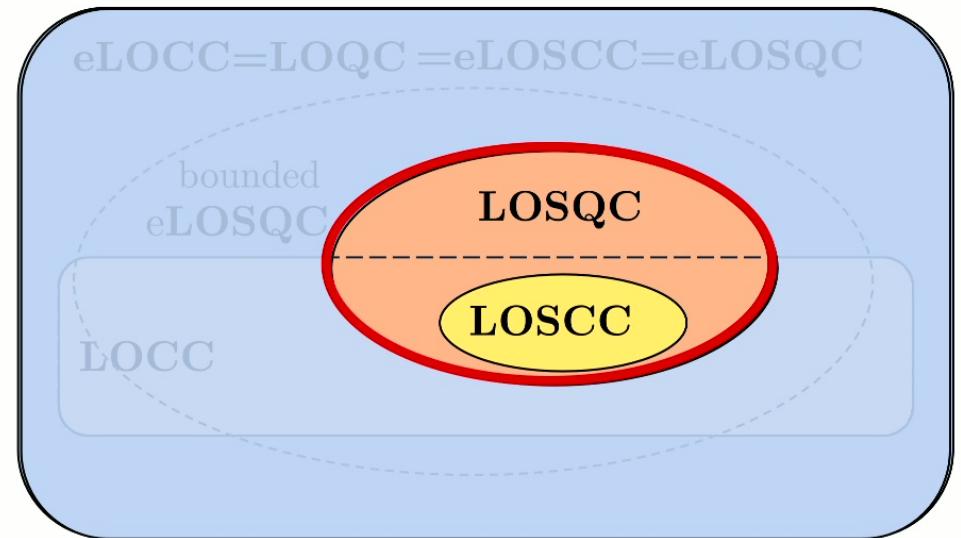


- The **no pre-shared entanglement** model is the simplest to analyze, but even in this scenario relatively little is known.

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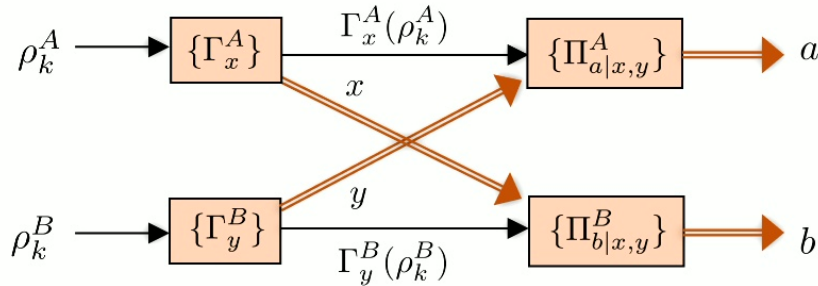


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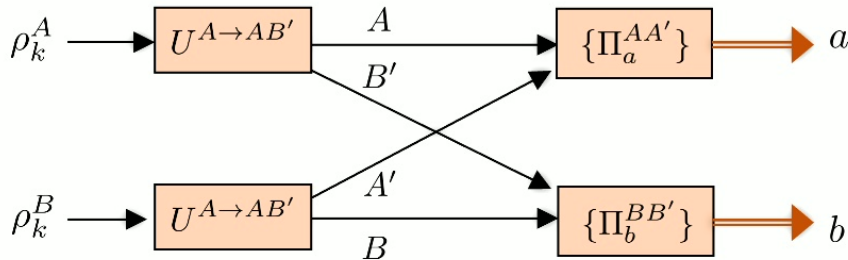
- Simplify the problem even further:  
How well can a family of orthogonal **product states**
- $$\{|\psi_k\rangle = |a_k\rangle^A |b_k\rangle^A\}_k$$
- be distinguished by LOSCC and LOSQC?

# The structure of LOSCC and LOSQC protocols

- The structure of LOSCC protocols:



- The structure of LOSQC protocols:



- Does the quantum communication help?

Yes!

Consider two copies of each Bell state

$$\begin{cases}
 |\psi_1\rangle = |\Phi^+\rangle^{AB'} \otimes |\Phi^+\rangle^{BA'} \\
 |\psi_2\rangle = |\Phi^-\rangle^{AB'} \otimes |\Phi^-\rangle^{BA'} \\
 |\psi_3\rangle = |\Psi^+\rangle^{AB'} \otimes |\Psi^+\rangle^{BA'} \\
 |\psi_4\rangle = |\Psi^-\rangle^{AB'} \otimes |\Psi^-\rangle^{BA'}
 \end{cases}$$

- Perfectly distinguishable by LOSQC but not LOSCC.

Yu, Duan, Ying, *PRL* 109, 020506 (2012).

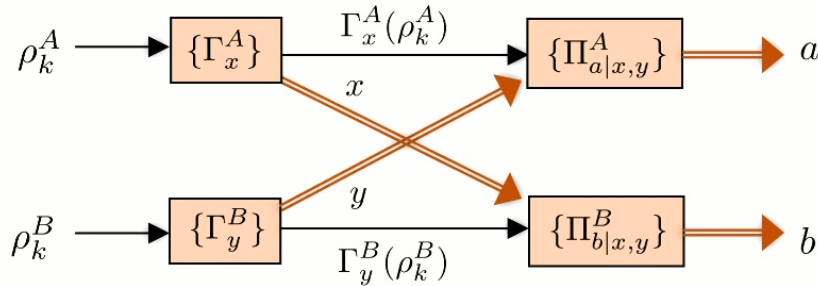
- Also true if coarse-grained.

Allerstorfer, Buhrman, Speelman, Lunel, arXiv:2208.04341.

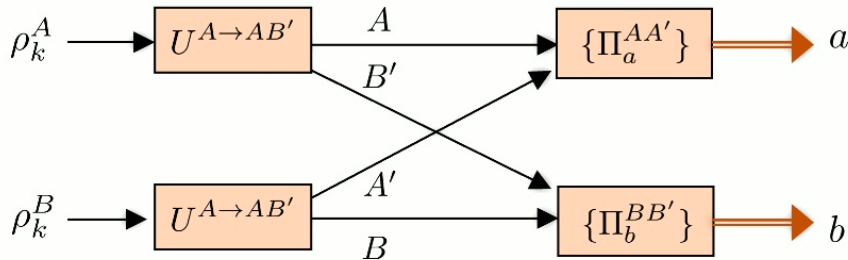


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- But these involve distinguishing entangled states.  
What about for product states?

# Distinguishing orthogonal product states

- This problem has a rich history in quantum information theory.
  - Any  $2 \otimes 2$  family of orthogonal product states can be perfectly distinguished by LOCC.

$$\left\{ \begin{array}{ll} |\psi_1\rangle = |0\rangle \otimes |\theta\rangle & |\psi_3\rangle = |1\rangle \otimes |\phi\rangle \\ |\psi_2\rangle = |0\rangle \otimes |\theta^\perp\rangle & |\psi_4\rangle = |1\rangle \otimes |\phi^\perp\rangle \end{array} \right.$$

Walgate and Hardy, **PRL** 89, 147901 (2002).

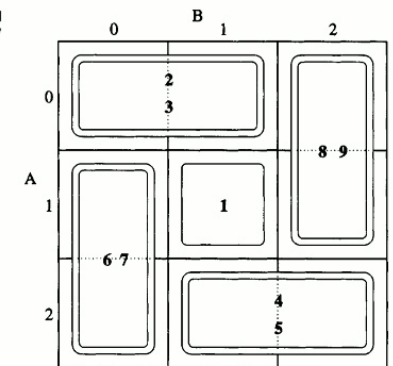
- Any  $2 \otimes n$  family of orthogonal product states can be perfectly distinguished by LOCC.

Bennett, DiVincenzo, Mor, Shor, Smolin, Terhal, **PRL** 82, 5385 (1999).

- There exists orthogonal product state that cannot be distinguished by LOCC

**“Nonlocality without entanglement”**

$$\left\{ \begin{array}{lll} |\psi_1\rangle = |1\rangle \otimes |1\rangle & |\psi_4\rangle = |2\rangle \otimes |1+2\rangle & |\psi_7\rangle = |1-2\rangle \otimes |0\rangle \\ |\psi_2\rangle = |0\rangle \otimes |0+1\rangle & |\psi_5\rangle = |2\rangle \otimes |1-2\rangle & |\psi_8\rangle = |0+1\rangle \otimes |2\rangle \\ |\psi_3\rangle = |0\rangle \otimes |0-1\rangle & |\psi_6\rangle = |1+2\rangle \otimes |0\rangle & |\psi_8\rangle = |0-1\rangle \otimes |2\rangle \end{array} \right.$$



Bennett, DiVincenzo, Fuchs, Mor, Rains, Shor, Smolin, Wootters, **PRA** 59, 1070 (1999).

# Distinguishing orthogonal product states

**Proposition** [I.George, R. Allerstorfer, P. Lunel, E.C.]:

- For perfect discrimination of  $2 \otimes 2$  orthogonal product states, LOSQC=LOSCC and the states must have the form:

$$\left\{ \begin{array}{ll} |\psi_1\rangle = |0\rangle \otimes |0\rangle & |\psi_3\rangle = |1\rangle \otimes |0\rangle \\ |\psi_2\rangle = |0\rangle \otimes |1\rangle & |\psi_4\rangle = |1\rangle \otimes |1\rangle \end{array} \right.$$

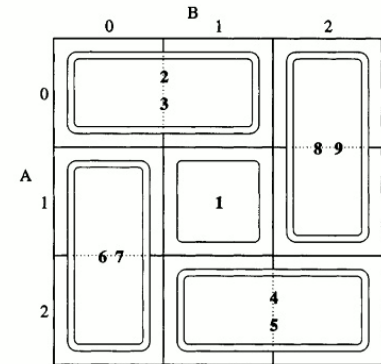
- A  $2 \otimes n$  family of orthogonal product states can be perfectly distinguished by LOSC iff it has the form:

$$\left\{ \begin{array}{l} |0\rangle^A \otimes |j\rangle^B \\ |1\rangle^A \otimes (x_j|j\rangle + y_j|j+1\rangle)^B \\ |g_i\rangle^A \otimes |i\rangle^B \quad \text{for } i > 2m+1 \end{array} \right\} \quad \text{for } j \in \{0, 2, 4, \dots, 2m\}$$

# Distinguishing orthogonal product states

- But what about the sausage states?

$$\left\{ \begin{array}{lll}
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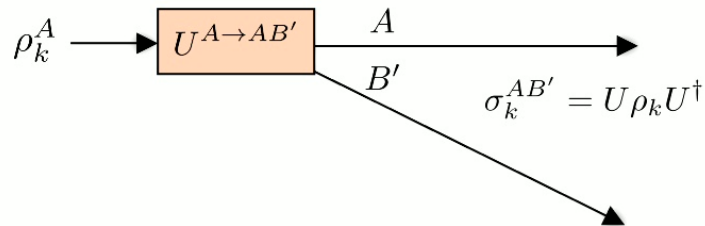
- These states cannot be distinguished by LOSCC.
- They also cannot be distinguished by LOSQC (see theorem below).
- What about two copies of the states:  $\{|\psi_k\rangle^{\otimes 2} = |a_k\rangle^{\otimes 2} \otimes |b_k\rangle^{\otimes 2}\}$ ?  $\Rightarrow$

- Distinguishable by LOSQC
- Distinguishable by LOSCC

## Conjecture:

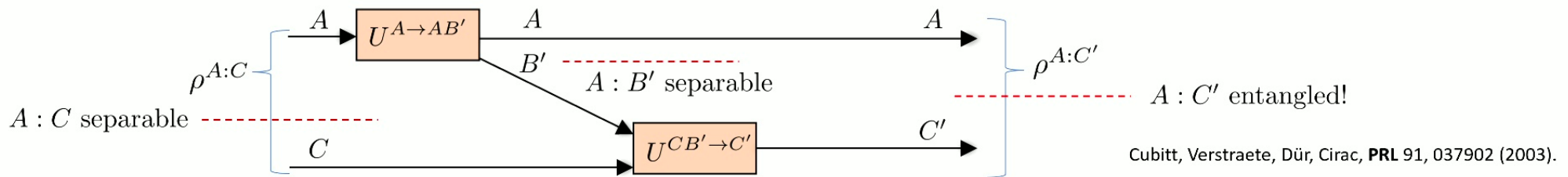
Two copies of any set of orthogonal product states is sufficient for LOSCC discrimination (or at least the ensemble must have a large number of states).

# LOSQC is more powerful than LOSCC



- Distinguish between two types of quantum communication:
  - **Separable communication**, i.e.  $\sigma_k^{AB'}$  is separable for all  $k$ .
  - **Entangled communication**, i.e.  $\sigma_k^{AB'}$  is entangled for some  $k$ .

- Separable communication can be used to perform non-classical tasks, like entanglement distribution.



**Theorem** [I.George, R. Allerstorfer, P. Lunel, E.C.]:

The four states can be perfectly distinguished by LOSQC only if entangled communication is used:

$$\left\{ \begin{array}{ll} |\psi_1\rangle = |0\rangle \otimes |0+1\rangle & |\psi_3\rangle = |1\rangle \otimes |0+2\rangle \\ |\psi_2\rangle = |0\rangle \otimes |0-1\rangle & |\psi_4\rangle = |1\rangle \otimes |0-2\rangle \end{array} \right.$$

# LOSQC state discrimination with error

- Perfect state discrimination is interesting from a fundamental perspective, but not for practical QPV.
- **QPV question:**

Given an ensemble  $\{|\psi_k\rangle\}_k$ , what is the smallest error probability in state discrimination using LOSQC?

**Theorem** [I.George, R. Allerstorfer, P. Lunel, E.C.]:

Let  $\{|\psi_k\rangle^{AB} = |a_k\rangle^A |b_k\rangle^B\}_k$  be an ensemble of product states that contains four states of the form

$$\begin{aligned} |\psi_0\rangle^{AB} &= |a_0\rangle^A |b_0\rangle^B, \\ |\psi_1\rangle^{AB} &= |a_1\rangle^A |b_1\rangle^B, \\ |\psi_2\rangle^{AB} &= |a_2\rangle^A (\cos \theta |b_0\rangle + e^{i\phi} \sin \theta |b_1\rangle)^B \\ |\psi_3\rangle^{AB} &= |a_3\rangle^A (\cos \theta |b_0\rangle - e^{i\phi} \sin \theta |b_1\rangle)^B, \end{aligned}$$

with  $\langle a_0 | a_1 \rangle \neq 0$ . Suppose Alice and Bob can identify each state with at least probability  $1 - \epsilon$  using some LOBQC protocol. Then

$$2\epsilon + \frac{4\sqrt{\epsilon(1-\epsilon)}}{|\langle a_0 | a_1 \rangle|^2} + \sqrt{1 - |\langle a_2 | a_3 \rangle|^2} > 1.$$

# LOSQC state discrimination with error

**Example:** Generalized BB84 states:

$$\begin{aligned} |\psi_0\rangle^{AB} &= |0\rangle^A \otimes |0\rangle^B, \\ |\psi_1\rangle^{AB} &= |0\rangle^A \otimes |1\rangle^B, \\ |\psi_2\rangle^{AB} &= |1\rangle^A \otimes (\cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle)^B \\ |\psi_3\rangle^{AB} &= |1\rangle^A \otimes (\cos \theta |0\rangle - e^{i\phi} \sin \theta |1\rangle)^B \end{aligned}$$

The LOSQC error probability  $P_{err}$  is lower bounded as:  $P_{err} > \frac{1}{4} \left( \frac{1}{2} - \frac{1}{\sqrt{5}} \right) \approx 1.3\%$ .

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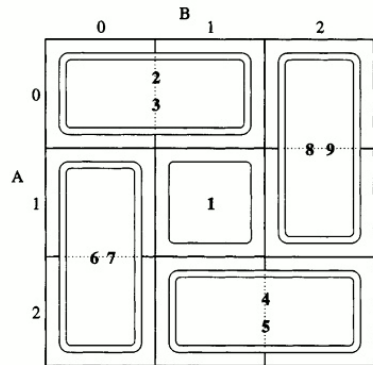
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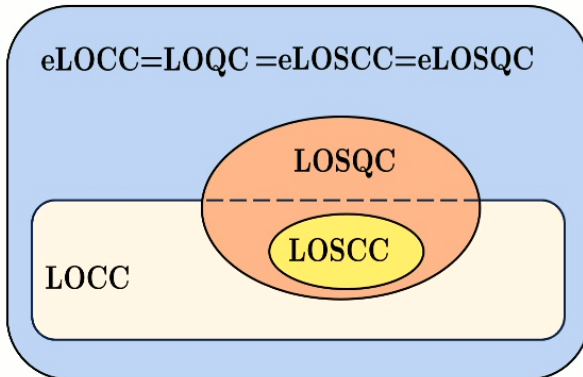
$$P_{err} > \frac{1}{9} \left( \frac{1}{2} - \frac{2}{\sqrt{17}} \right) \approx .16\%.$$



# Open problems and future directions

- What are the necessary and sufficient conditions for product state discrimination under LOCC and LOSQC?
- **Copy complexity:** How many copies of an ensemble state do Alice and Bob need before they can perfectly discriminate by LOCC?

$$\{|\psi_k\rangle^{\otimes n} = |a_k\rangle^{\otimes n} \otimes |b_k\rangle^{\otimes n}\}$$

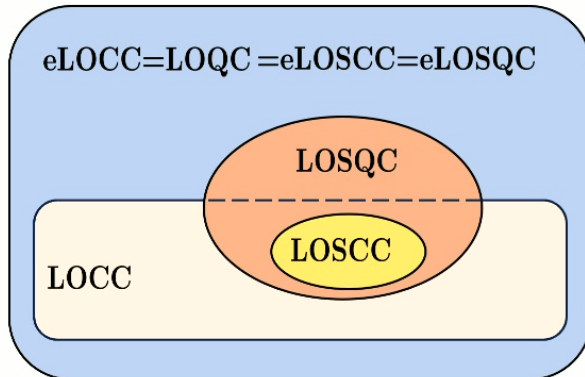


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What are the entanglement costs for state discrimination under eLOSCC and eLOSQC?

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One ebit suffices for perfect discrimination

Lo and Lau **PRA** 83, 012322 (2011).

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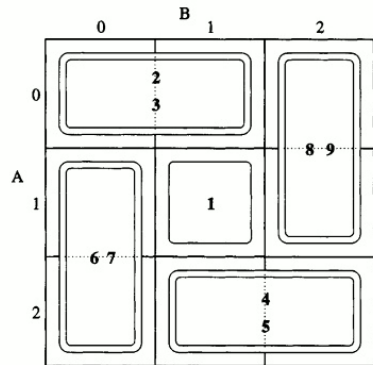
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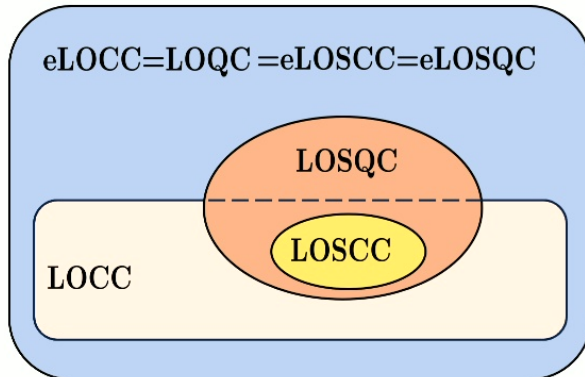
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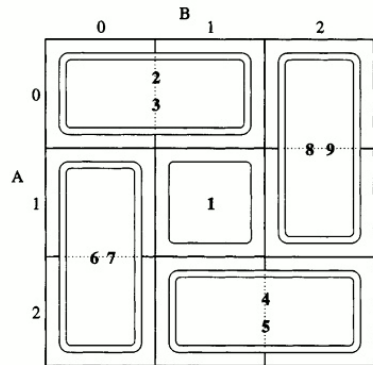
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# Near-term realization of QPV

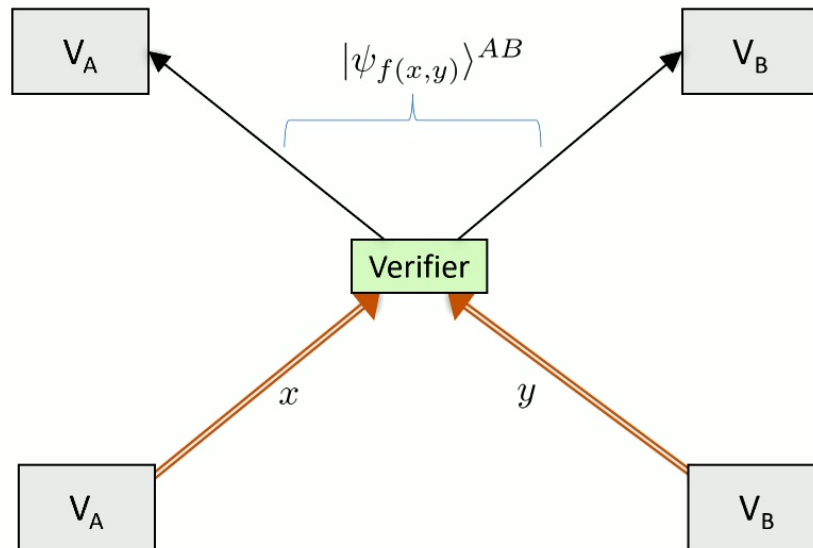
- How to implement QPV using today's (or tomorrow's) technology?
- There will generally be a trade-off between the feasibility of implementation and the security guarantees.
- Suggested heuristic benchmark for first-generation QPV implementations:

The scheme should be secure assuming the adversaries have the same capabilities as the honest prover (in terms of quantum memory, measurements, gates, **channel loss** etc.).

- This allows for greater flexibility in protocol designs.



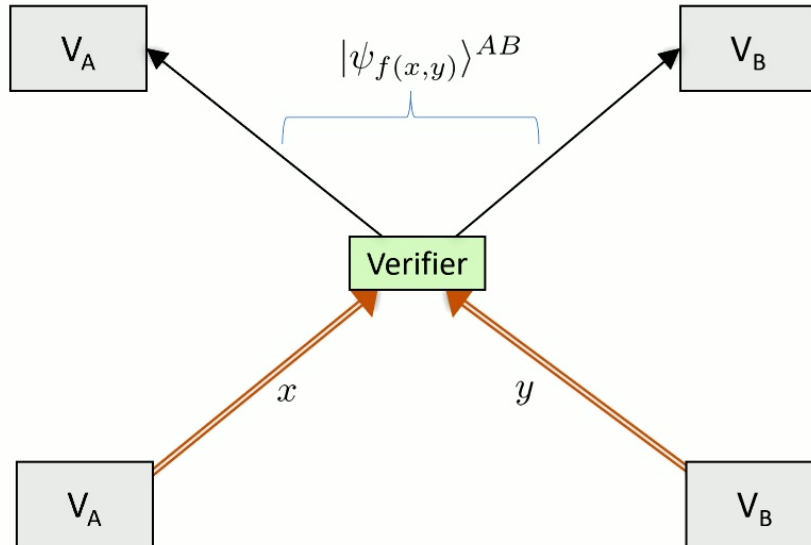
# Entanglement preparation as a QPV task



- **Idea:** Force the honest prover to prepare different entangled states.



# Entanglement preparation as a QPV task

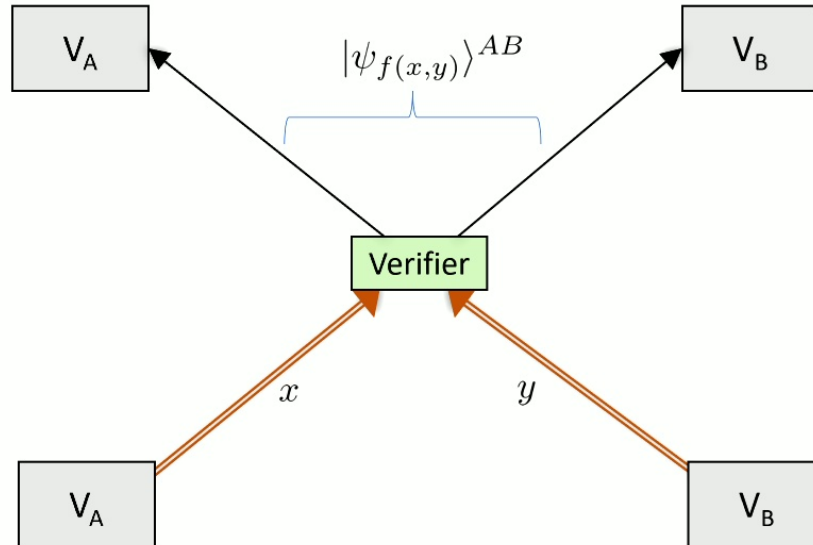


- **Idea:** Force the honest prover to prepare different entangled states.
- **Advantage:** No quantum measurement required for the prover; only an entanglement source.

Suitable for deployment on a drone!

- **Intuition for why this works:**
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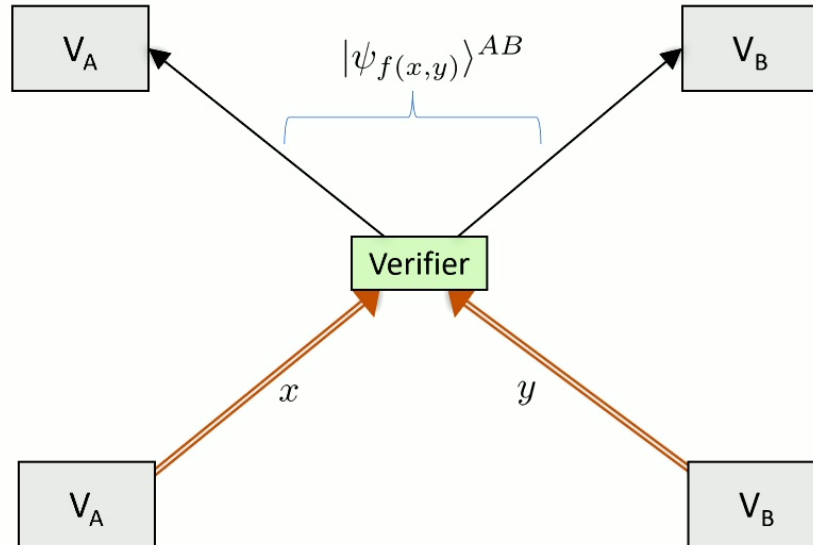
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**Theorem** [I. George, A. Conrad, E.C., P.K.]:

If the adversaries are not allowed quantum memory, then there is a secure entanglement distribution QPV protocol that tolerates **any rate of loss** and error rate  $\delta \leq 3.34\%$ .

# Entanglement preparation as a QPV task



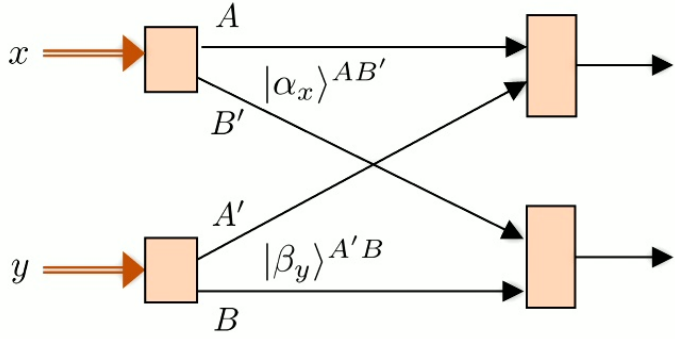
- Choose  $|\psi_{f(x,y)}\rangle = \cos[f(x,y)]|00\rangle + \sin[f(x,y)]|11\rangle$

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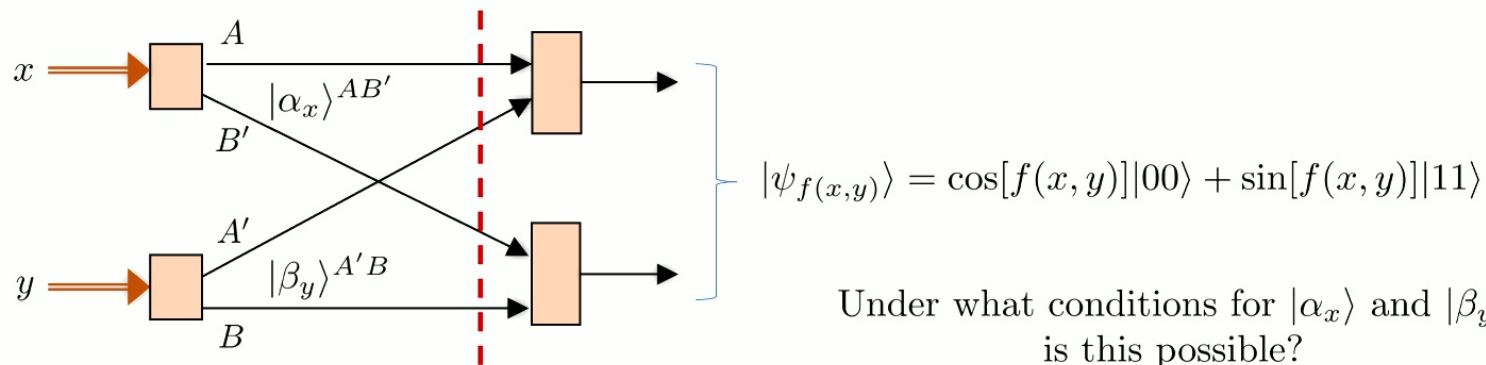
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- **Intuition for why this works:**
  - Entanglement *manipulation* is difficult in the **LOSQC** model;  
i.e. transforming  $|\psi_{f(x,y)}\rangle \mapsto |\psi_{f(x',y')}\rangle$

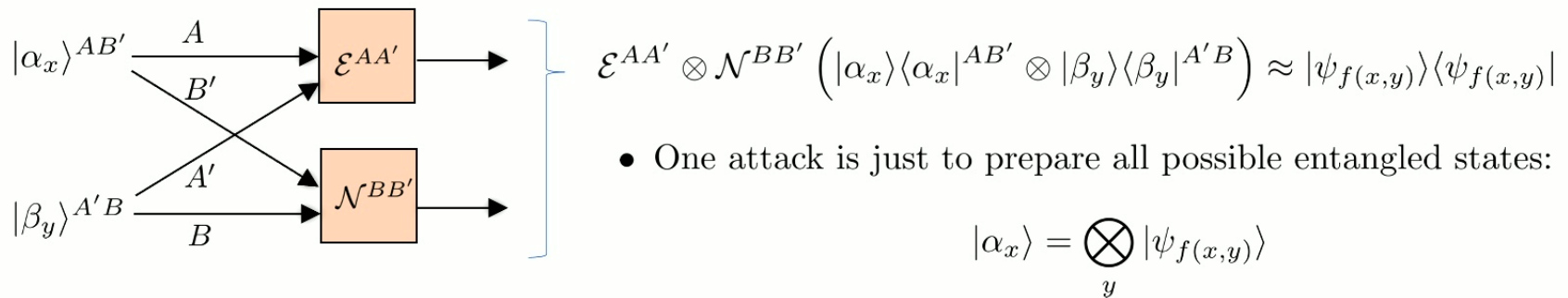
# LOQC entanglement distribution



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At this point in time no more communication is allowed.

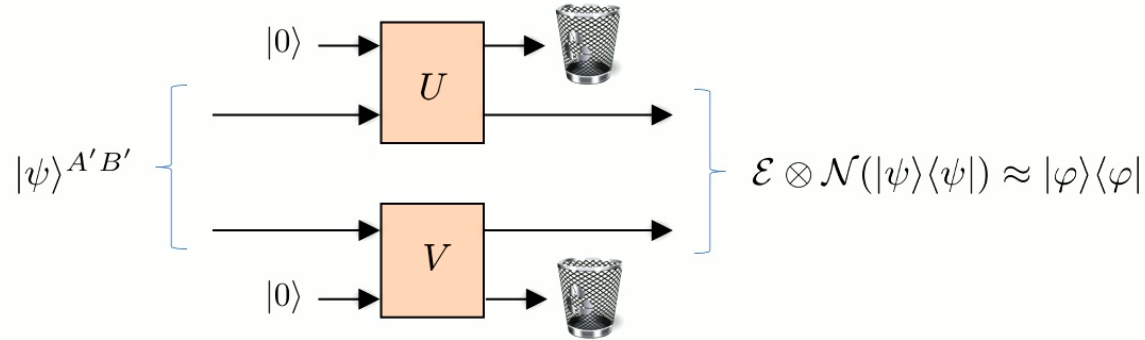


# Entanglement manipulation with no communication

General problem statement:

Given two bipartite entangled states  $|\psi\rangle^{A'B'}$  and  $|\varphi\rangle^{AB}$  how well can Alice and Bob transform  $|\psi\rangle \mapsto |\varphi\rangle$  by local operations (and shared randomness)?

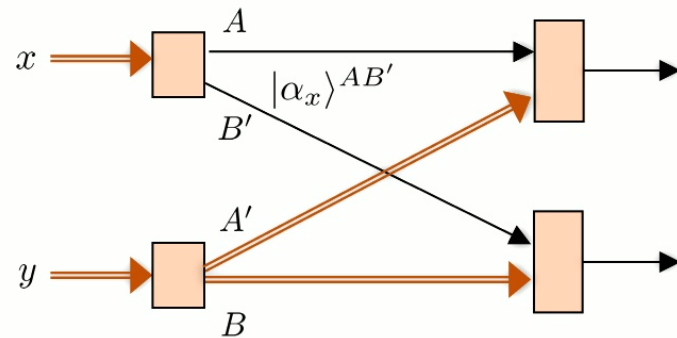
$$F_{\text{LO}}(|\psi\rangle \rightarrow |\varphi\rangle) := \max_{\mathcal{E}, \mathcal{N}} \langle \varphi | \mathcal{E} \otimes \mathcal{N}(|\psi\rangle\langle\psi|) | \varphi \rangle$$



**Theorem** [I. George, E.C.]:

$$F_{\text{LO}}(|\psi\rangle \rightarrow |\varphi\rangle) = \max_{P'} F((P \otimes P')^\downarrow, Q^\downarrow) \quad \text{where } P^\downarrow \text{ and } Q^\downarrow \text{ are the ordered squared-Schmidt coefficients of } |\psi\rangle \text{ and } |\varphi\rangle, \text{ and } |P'| \leq |P||Q|.$$

# Security against single ebit attacks



Adversarial power is equal to the honest prover's power.

- Consider a class of attacks in which the adversaries can exchange just a single ebit.

**Theorem** [I. George, A. Conrad, E.C., P.K.]:

There is an entanglement distribution QPV protocol with transmission rate  $\eta$  and loss rate  $\delta$  that is secure against one-ebit attacks provided

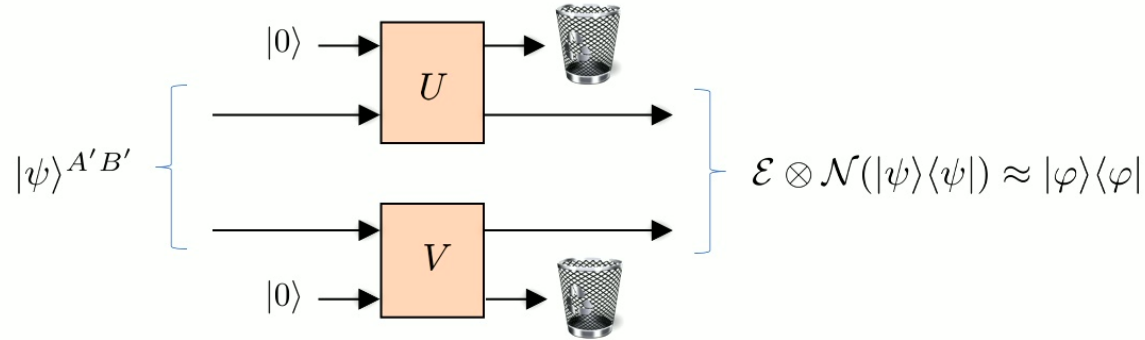
$$\delta(\eta) \leq \min_{p \in [\frac{1}{2}, 1]} \max_{\theta_{x,y}} \frac{1}{4} (1 - \sin(2\theta_{x,y})) \left( \eta - \cos(\theta_{x,y})\sqrt{p} + \sin(\theta_{x,y})\sqrt{1-p} \right)^2.$$

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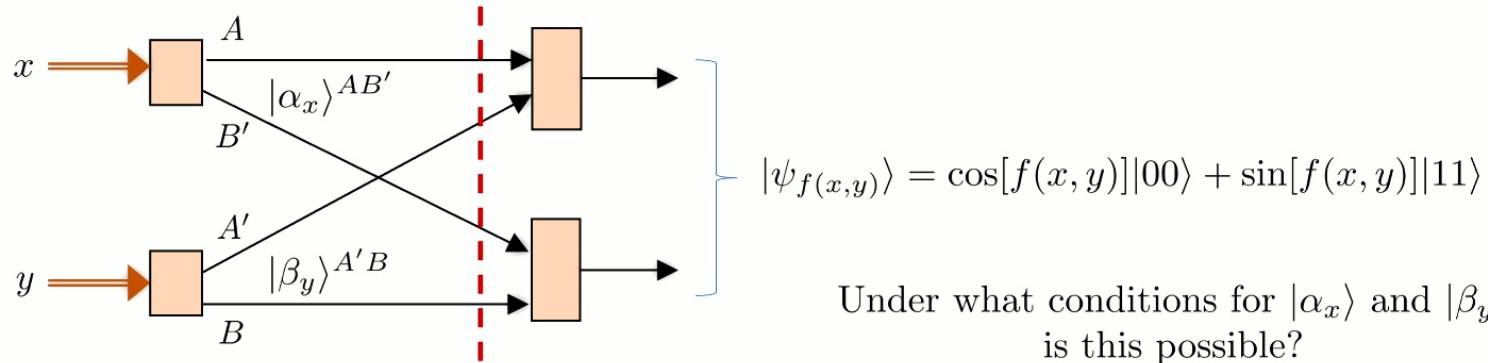


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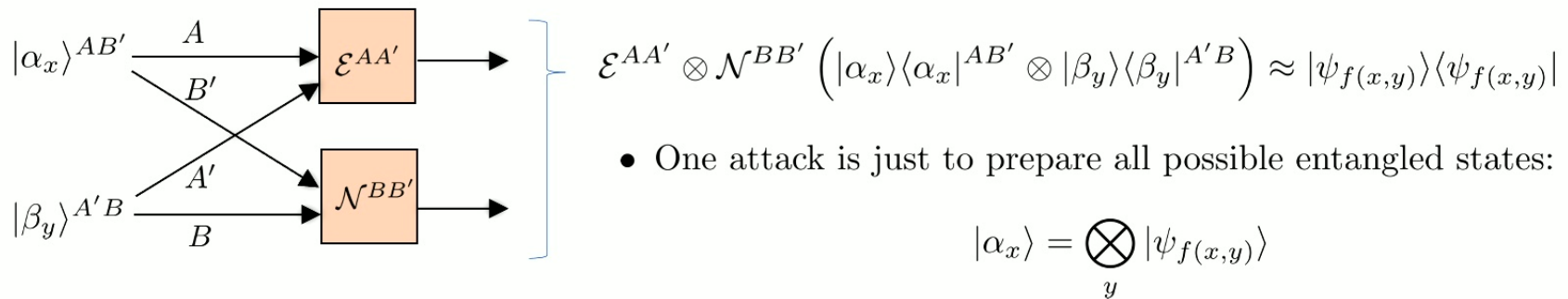
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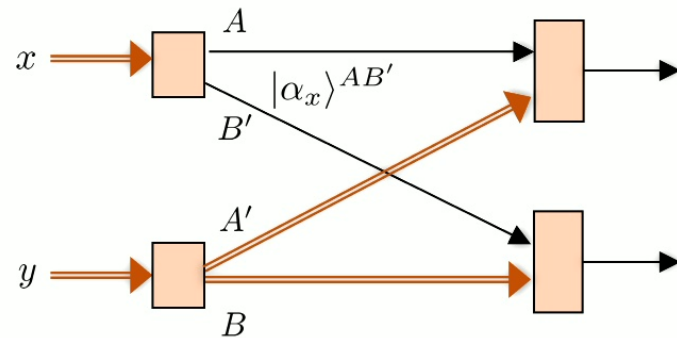
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- One attack is just to prepare all possible entangled states:

- But this requires large entanglement. Is it optimal?

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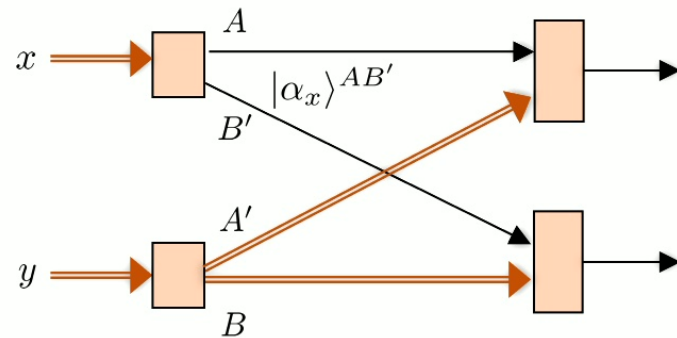
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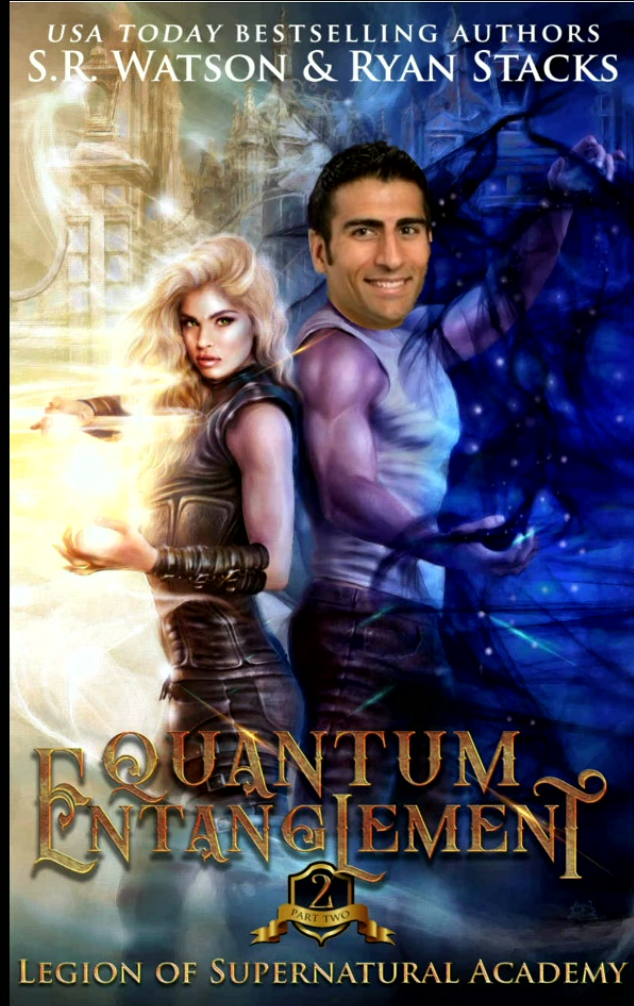
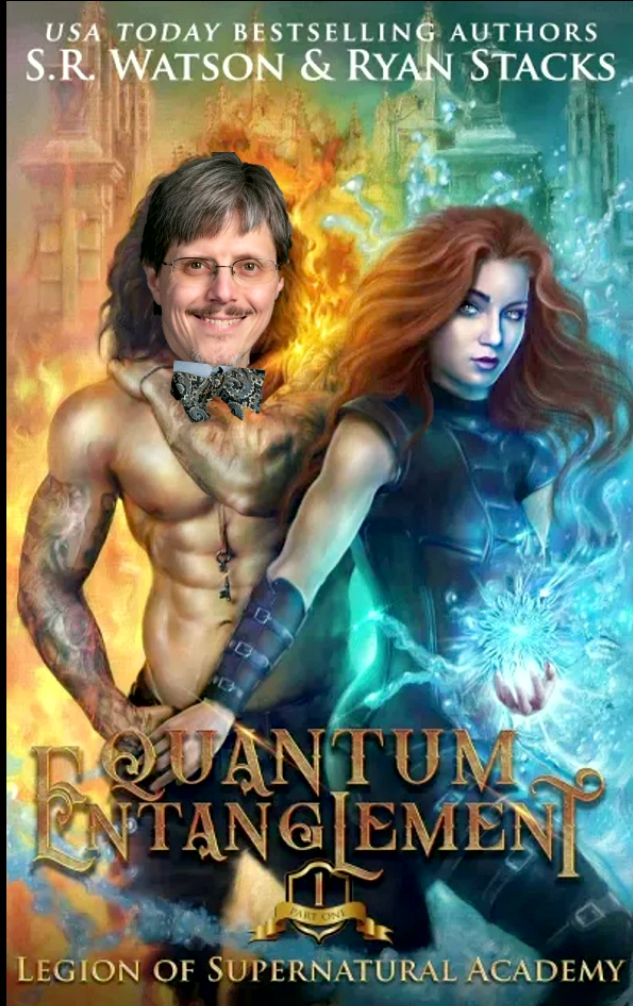
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In particular, we can tolerate an error rate of 0.2% and loss of 3%.

- This is stronger than the original BB84 protocol, which is completely insecure under single ebit attacks.

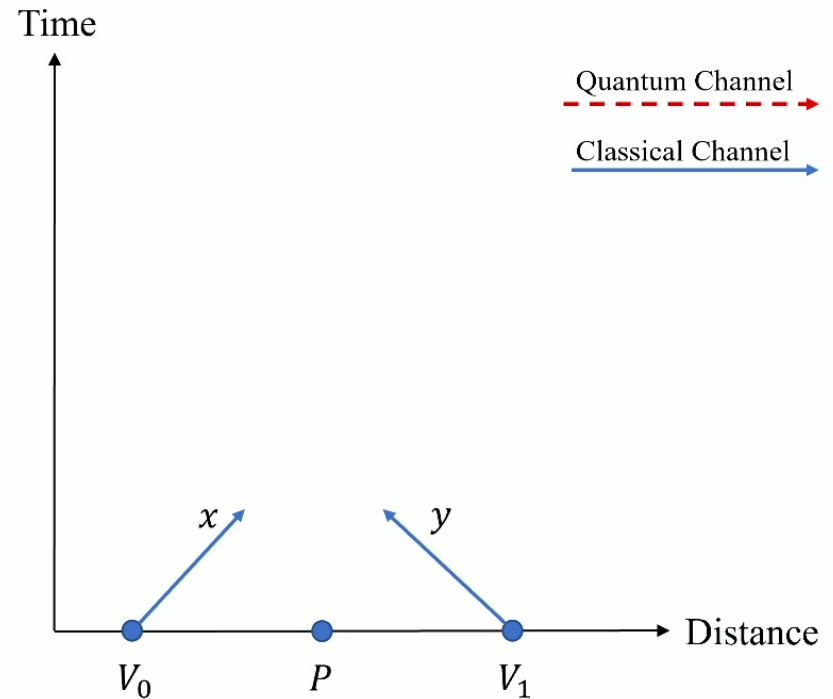


# Part IV: Experimental Implementation

# Our Approach

## Our Protocol

1. Verifiers  $V_0, V_1$  send classical random bit strings  $x, y$ , respectively
  - Where  $x \cdot y = \theta \in (0, \frac{\pi}{4})$



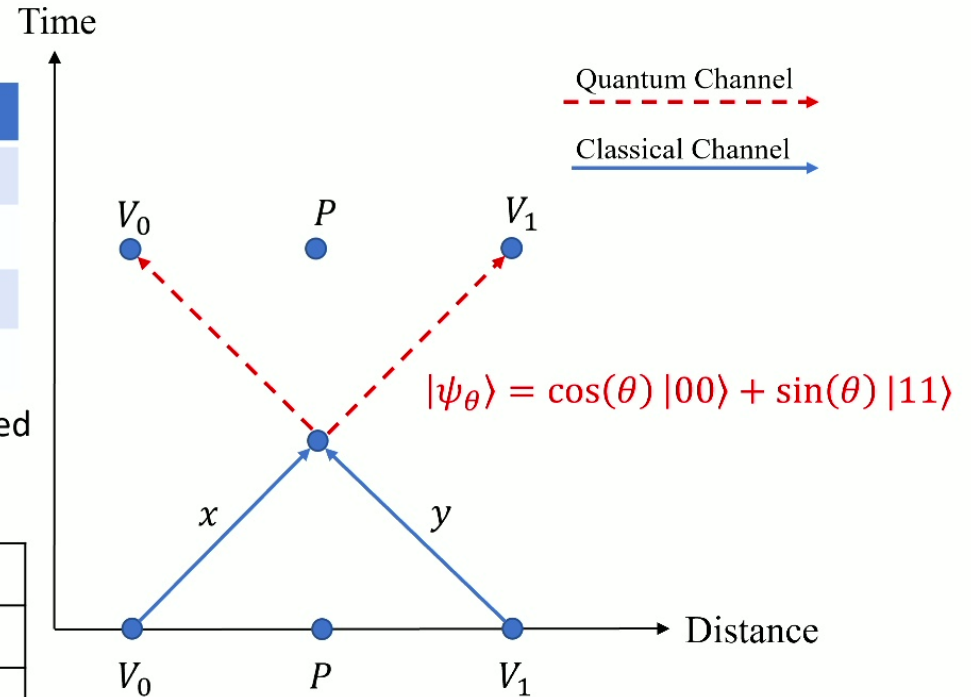
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Verifiers select random measurement basis:

Case	$V_0$	$V_1$
1	$ +\rangle\langle+ ,  -\rangle\langle- $	$ \theta\rangle\langle\theta ,  \theta^\perp\rangle\langle\theta^\perp $
2	$ +\rangle\langle+ ,  -\rangle\langle- $	$ -\theta\rangle\langle-\theta ,  -\theta^\perp\rangle\langle-\theta^\perp $
3	$ \theta\rangle\langle\theta ,  \theta^\perp\rangle\langle\theta^\perp $	$ +\rangle\langle+ ,  -\rangle\langle- $
4	$ -\theta\rangle\langle-\theta ,  -\theta^\perp\rangle\langle-\theta^\perp $	$ +\rangle\langle+ ,  -\rangle\langle- $

**Note:** If the target modulated entanglement state is produced  $|\psi_\theta\rangle$  by an honest prover, then the following measurement outcomes are not possible

Bad Outcome	$V_0$	$V_1$
1	$ +\rangle\langle+ $	$ \theta^\perp\rangle\langle\theta^\perp $
2	$ -\rangle\langle- $	$ -\theta^\perp\rangle\langle-\theta^\perp $
3	$ \theta^\perp\rangle\langle\theta^\perp $	$ +\rangle\langle+ $
4	$ -\theta^\perp\rangle\langle-\theta^\perp $	$ -\rangle\langle- $

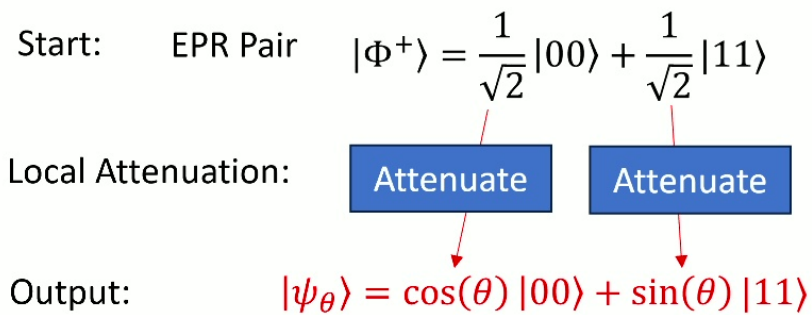


If a Bad Outcome is measured  $\rightarrow$  Cheating is detected

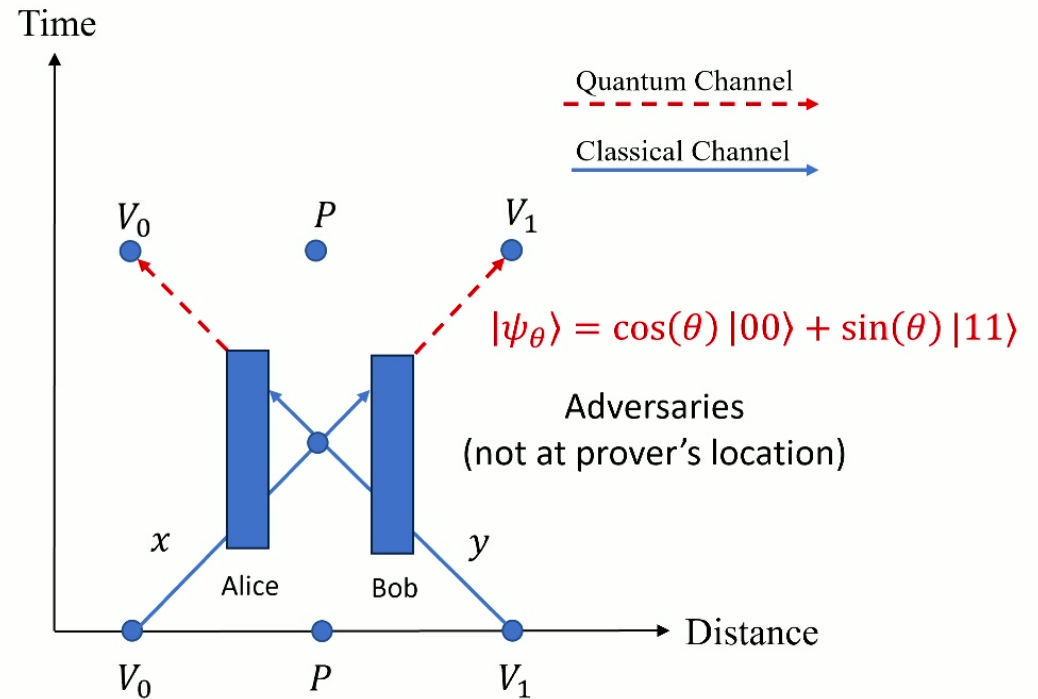
# Attacks: 1 e-bit + Quantum Memory

## Attackers

- If attackers have **1 entangled bit** (e-bit) and a **quantum memory**:
  - Attackers can attenuate an EPR state to the target state



Attackers produce  $|\psi_\theta\rangle$  with 50% success probability



If channel loss > 50%, then attackers win

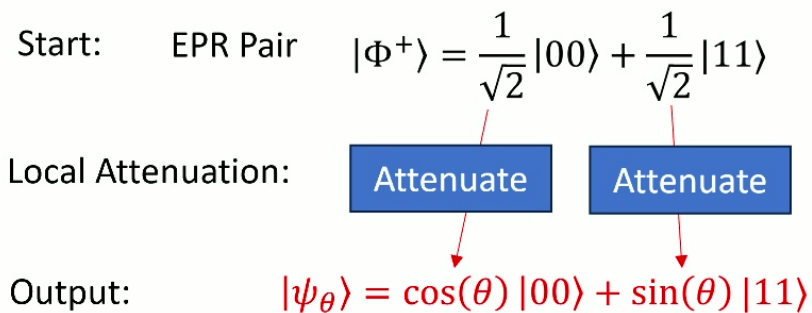
→ Loss intolerance



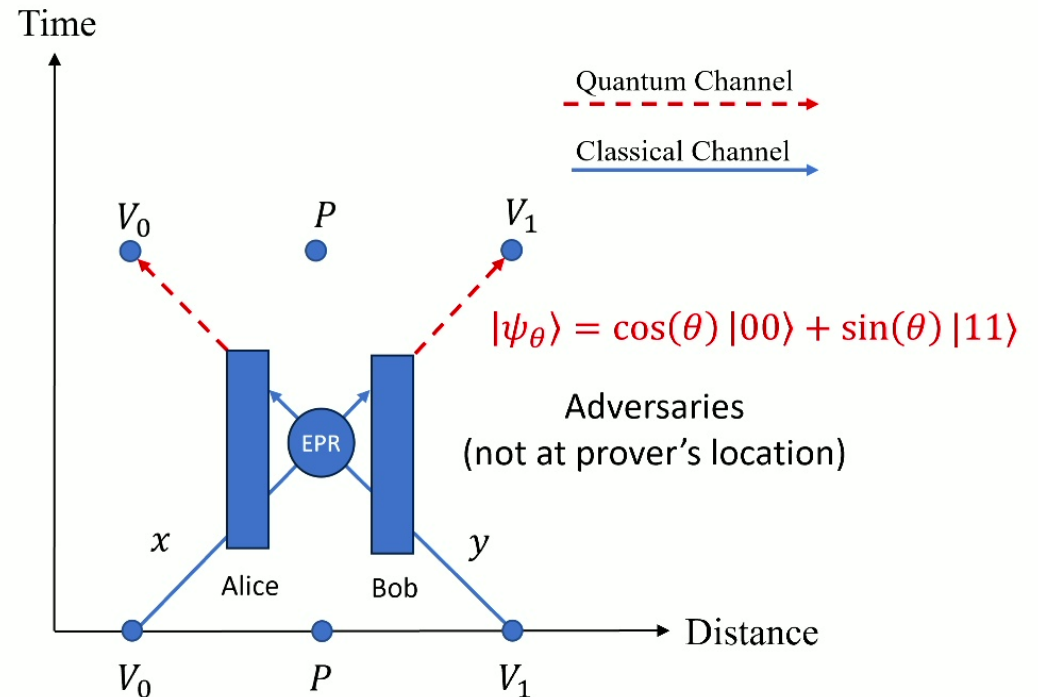
# Attacks: No Quantum Memory

## Attackers

- If attackers have **1 entangled bit (e-bit)** and **no quantum memory**:
  - Attackers can attenuate an EPR state to the target state, but the EPR pair must originate at the prover's location, thus the verifiers win



Attackers produce  $|\psi_\theta\rangle$  with 50% success probability

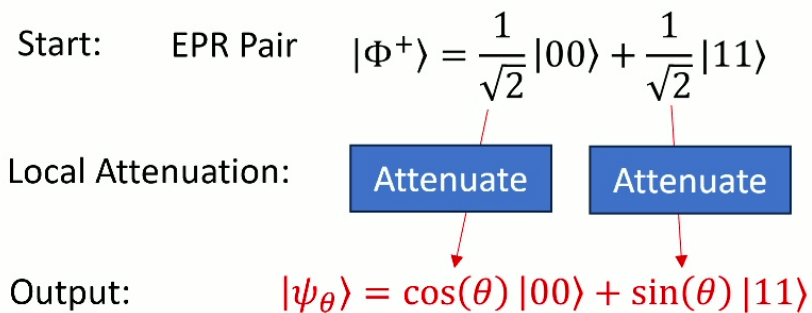


If the attackers lack a quantum memory, then our modulated entanglement protocol achieves complete loss tolerance

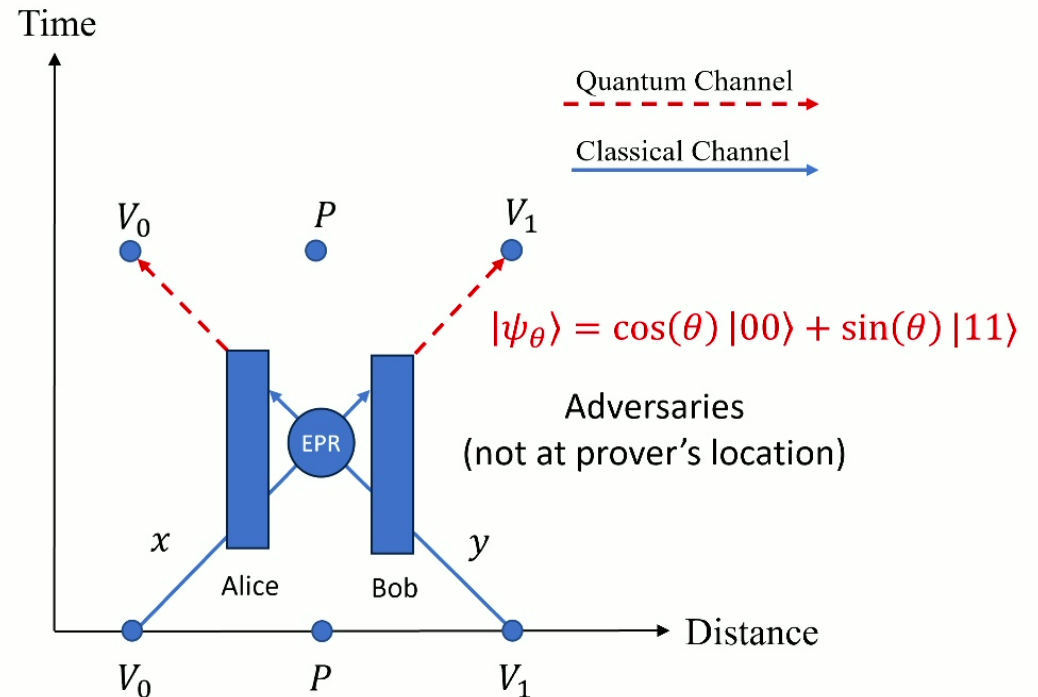
# Attacks: No Quantum Memory

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- If attackers have **1 entangled bit (e-bit)** and **no quantum memory**:
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If the attackers lack a quantum memory, then our modulated entanglement protocol achieves complete loss tolerance

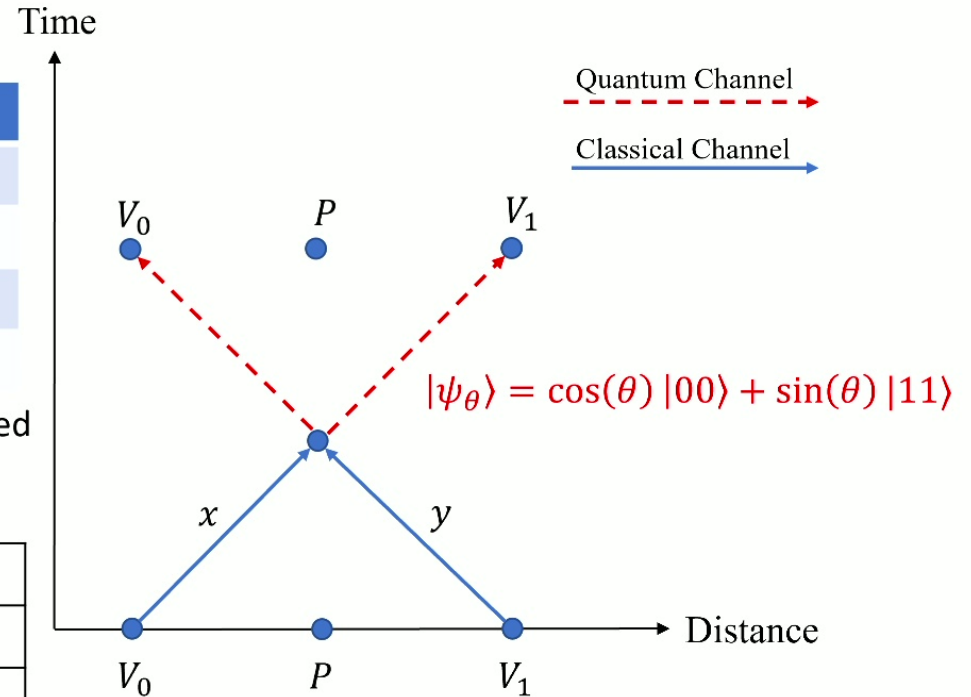
# Our Approach

Verifiers select random measurement basis:

Case	$V_0$	$V_1$
1	$ +\rangle\langle+ ,  -\rangle\langle- $	$ \theta\rangle\langle\theta ,  \theta^\perp\rangle\langle\theta^\perp $
2	$ +\rangle\langle+ ,  -\rangle\langle- $	$ -\theta\rangle\langle-\theta ,  -\theta^\perp\rangle\langle-\theta^\perp $
3	$ \theta\rangle\langle\theta ,  \theta^\perp\rangle\langle\theta^\perp $	$ +\rangle\langle+ ,  -\rangle\langle- $
4	$ -\theta\rangle\langle-\theta ,  -\theta^\perp\rangle\langle-\theta^\perp $	$ +\rangle\langle+ ,  -\rangle\langle- $

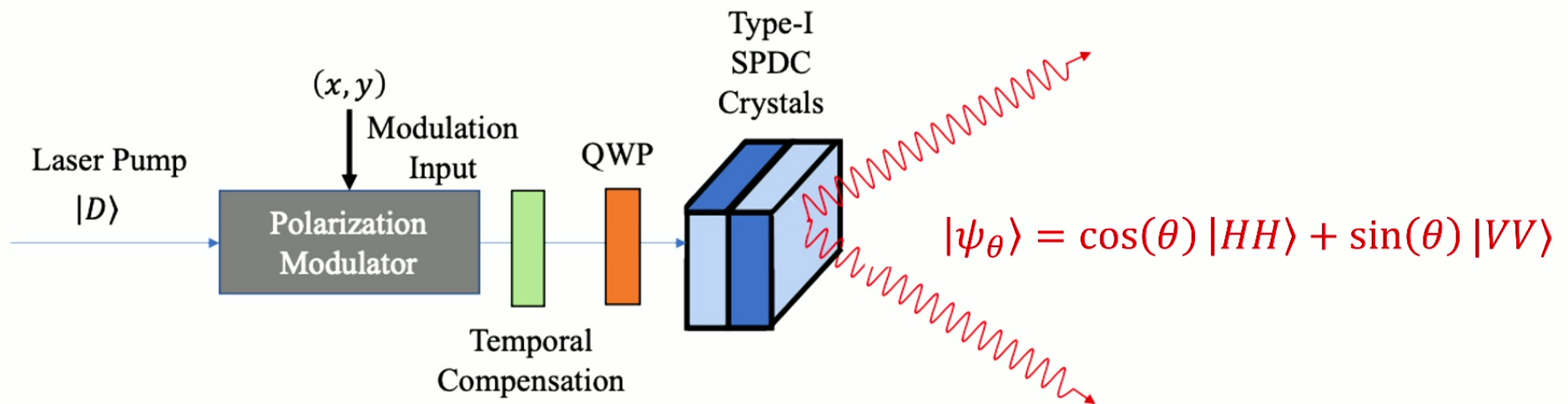
**Note:** If the target modulated entanglement state is produced  $|\psi_\theta\rangle$  by an honest prover, then the following measurement outcomes are not possible

Bad Outcome	$V_0$	$V_1$
1	$ +\rangle\langle+ $	$ \theta^\perp\rangle\langle\theta^\perp $
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3	$ \theta^\perp\rangle\langle\theta^\perp $	$ +\rangle\langle+ $
4	$ -\theta^\perp\rangle\langle-\theta^\perp $	$ -\rangle\langle- $



If a Bad Outcome is measured  $\rightarrow$  Cheating is detected

# Hardware Implementation



Changing the pump polarization  
→ alters how much entanglement

# Building upon previous work

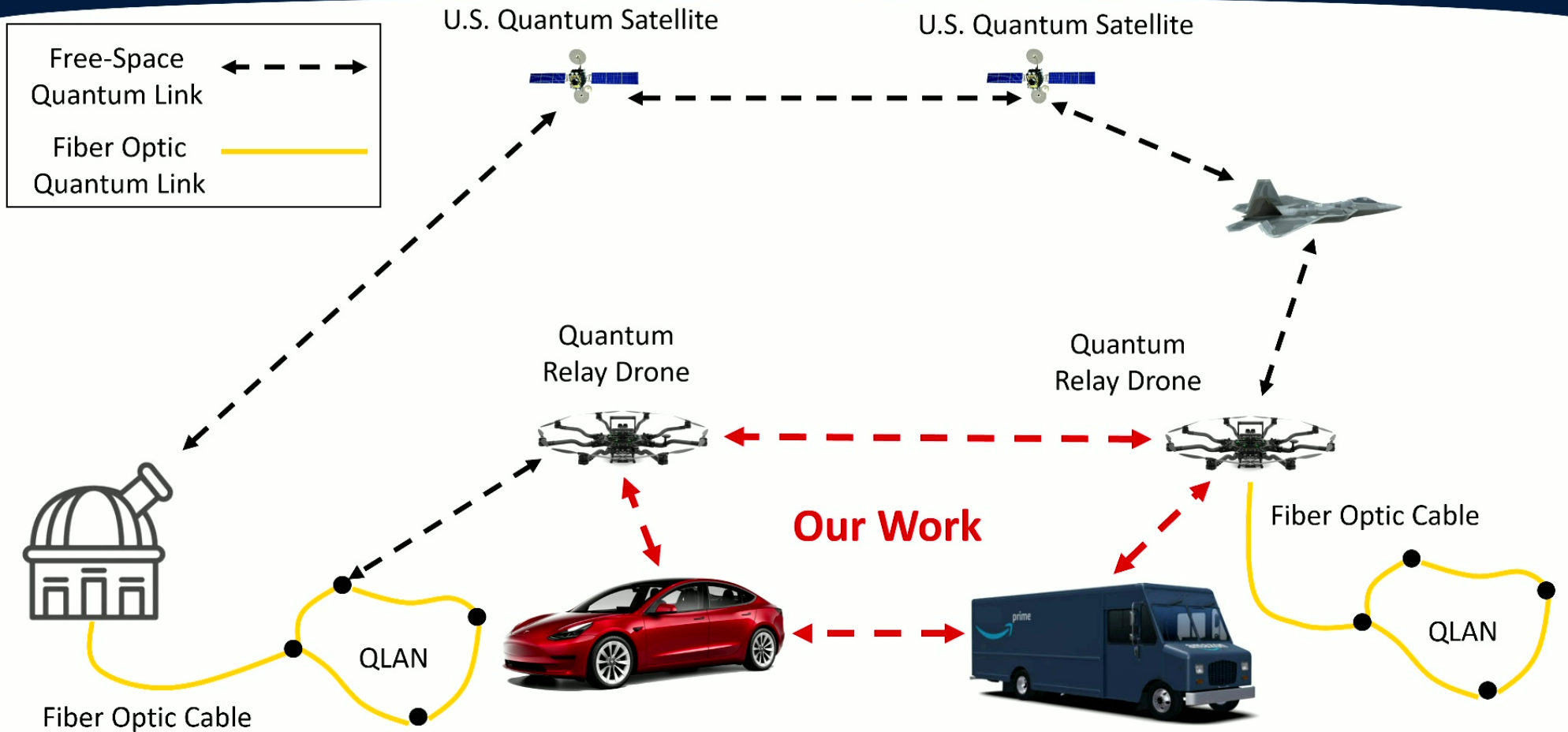
Andrew Conrad<sup>1</sup>, Samantha Isaac<sup>2</sup>, Roderick Cochran<sup>3</sup>,  
Daniel Sanchez-Rosales<sup>3</sup>, Timur Javid<sup>1</sup>, Shuen Wu<sup>1,2</sup>,  
Prof. Daniel Gauthier<sup>3</sup>, Prof. Paul Kwiat<sup>1,2</sup>

<sup>1</sup>Department of Electrical Engineering, University of Illinois Urbana-Champaign (UIUC), Urbana, IL

<sup>2</sup>Department of Physics, Illinois Quantum Information Science & Technology Center (IQUIST)  
University of Illinois Urbana-Champaign (UIUC), Urbana, IL

<sup>3</sup>Department of Physics, The Ohio State University, Columbus, OH

# Free-Space Quantum Network



# System Design

## System Overview:

- Quantum Transmitter (Alice)
  - Quantum Key Distribution (QKD) source:
    - Resonant cavity LED
    - Decoy state
    - Polarization encoded
  - Custom optics benches
- Quantum Receiver (Bob)
  - Single-Photon Detectors (SPCM-AQ4C)
  - FPGA-based Time-Tagger
  - Qubit-based Time Synchronization (Post-processing)
- Pointing, Acquisition, and Tracking (PAT) system
- Mobile Platforms:
  - Drone
  - Car

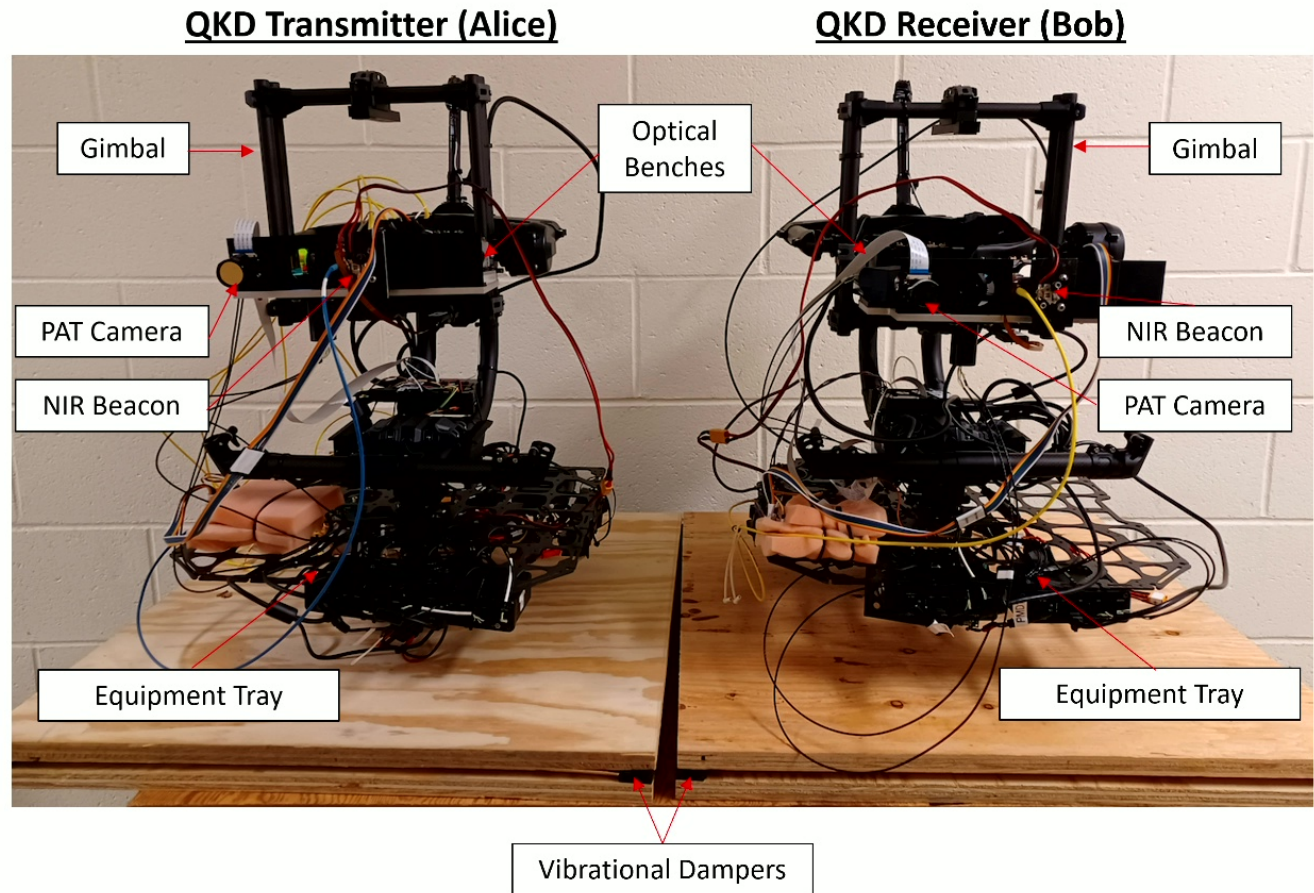


Image Courtesy Timur Javid

# Modular Design

## Modular Design:

- Our QKD system shares no resources with host mobile platform
  - Power
  - Control
  - Communication
- Single quick-release connection with drone
  - Place QKD transmitter (receiver) on other platforms (e.g., vehicle) with no required hardware changes

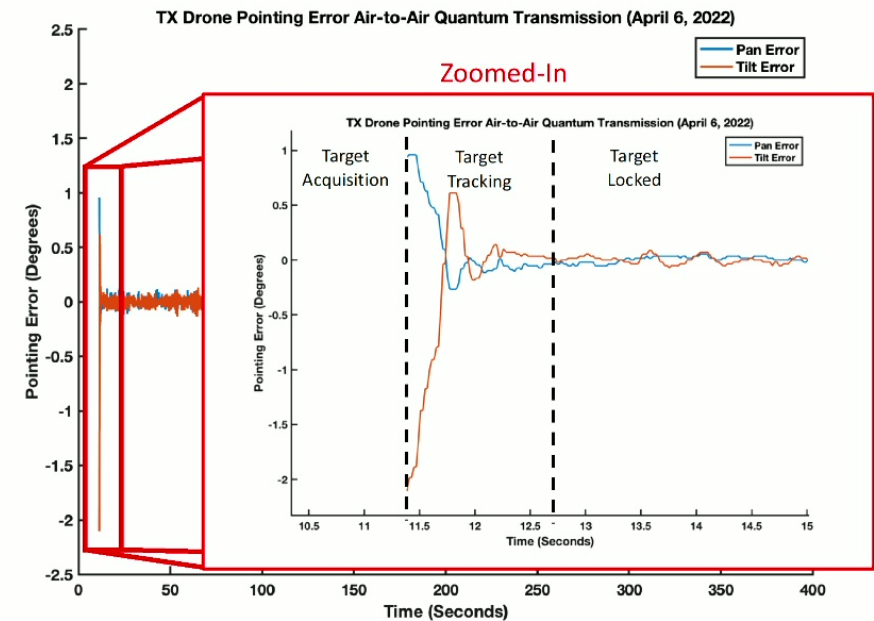
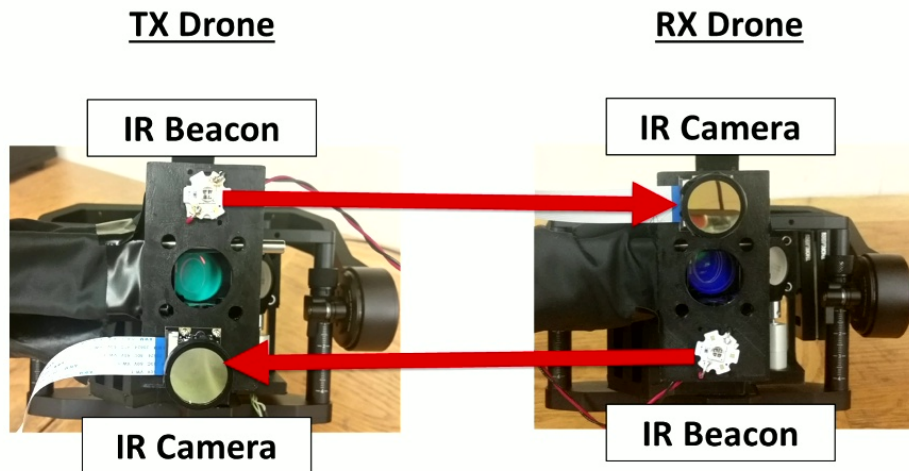




# PAT Subsystem (Course Adjustment)

## Outer-Control Loop Calibration

- Initial Pointing, acquisition, and course pointing
- IR Beacon/IR Camera
- Image processing to identify location in camera's reference frame
- Feedback Control



## Gimbal (Movi Pro)



- Tracking Performance:
  - Pan RMS Error =  $0.0230^\circ$
  - Tilt RMS Error =  $0.0263^\circ$

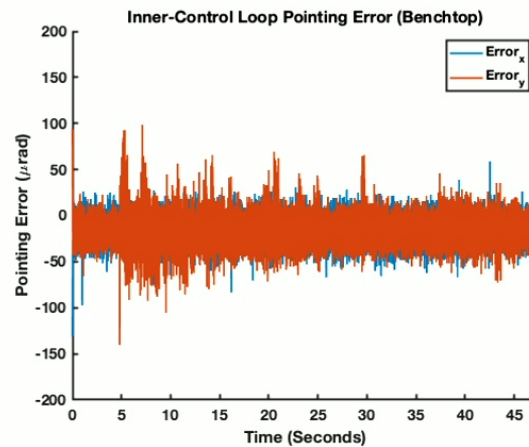
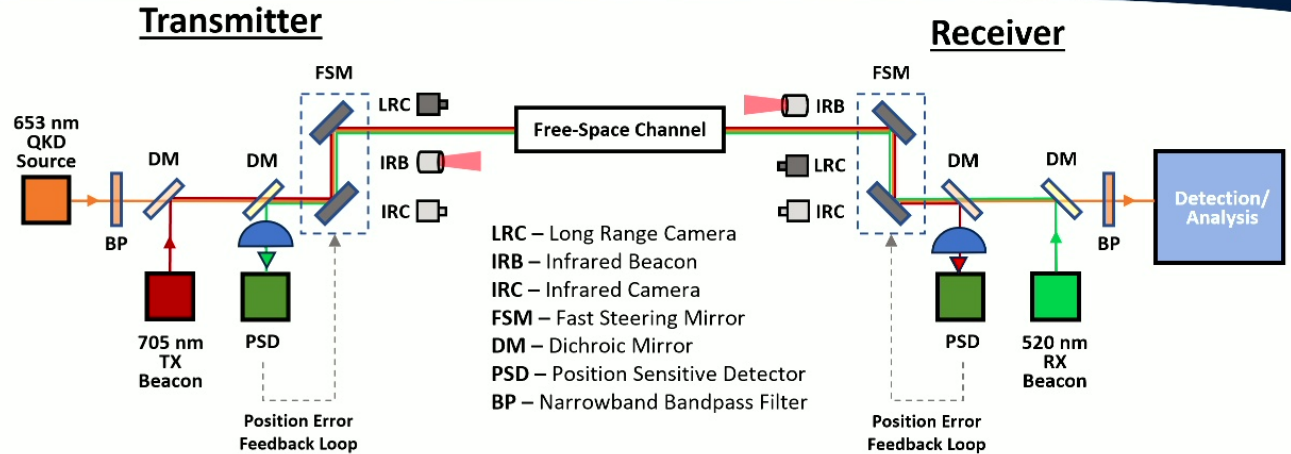
Gimbal Jitter Specification =  $0.02^\circ$

# PAT Subsystem (Fine Adjustment)

## PAT Subsystem (Fine Adjustment)

- Co-propagating laser beacons
  - Transmitter: 705-nm beacon
  - Receiver: 520-nm beacon
- Fast Steering Mirrors + Position Sensitive Diode (PSD)
- Senses incoming beacon beam Angle of Arrival (AoA)
- Raspberry Pi single-board computer
- Local (no PAT communication between drones)

### Fast Steering Mirrors (Model LR-17)



Pointing Error<sub>x</sub> = 21.1  $\mu\text{rad}$   
 Pointing Error<sub>y</sub> = 22.9  $\mu\text{rad}$

# Air-to-Air Classical Locking

## Drone Platform

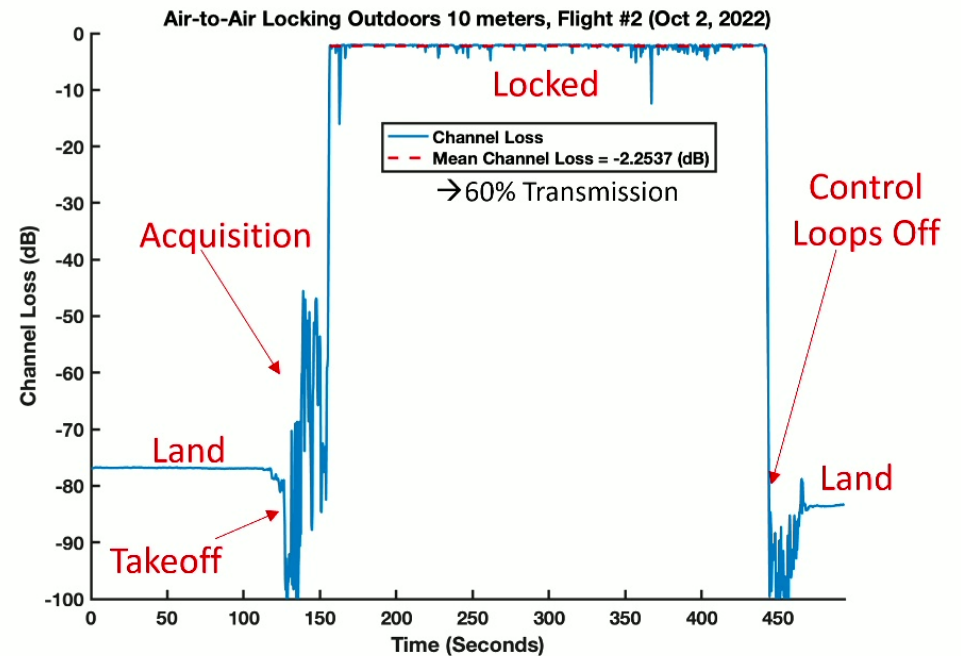
- Alta 8 Pro Drone
- 20 lbs payload capacity
- Two 10,000 mA-hr Lithium Polymer Batteries



Image Courtesy Timur Javid

## System Characterization

- Classical Air-to-Air Locking into multimode fiber
- Average 2.25 dB Channel Loss (60% transmission)
- 10-meter distance



# Drone Air-to-Air QKD Flights (Nov 2<sup>nd</sup>, 2022)

## Air-to-Air QKD Setup

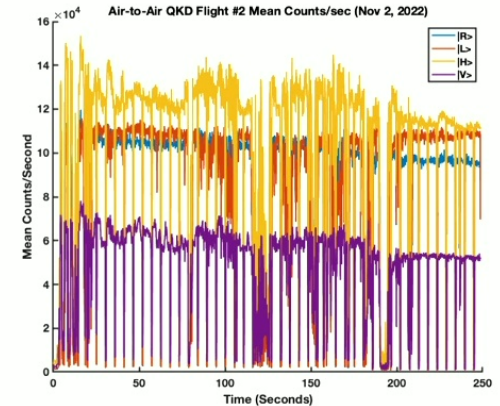
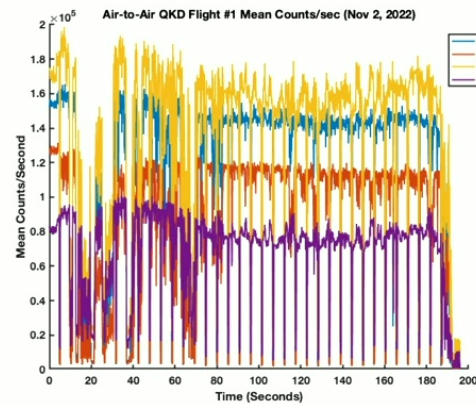
- Both drones hovering
- 10-meter distance between drones
- Altitude  $\sim$ 5 meters above ground



Image Courtesy Timur Javid

## Quantum Transmission

- Average QBER = 2.9% (R/L Basis), 3.0% (H/V Basis)
- 1<sup>st</sup> demonstration of drone-to-drone QKD
- Collaborating with Lütkenhaus group to develop tailor-made finite key analysis



	Flight #1	Flight #2	Flight #3
QBER (R/L)	2.0%	1.8%	5.0%
QBER (H/V)	3.6%	3.1%	2.4%
Mean Photon Number $\mu$	0.78	0.78	0.73

# 70 mph Vehicle-to-Vehicle Quantum Transmission

## Car-to-Car Quantum Setup

- 70 mph
- Interstate Highway (I-57)
- Outer-Control Loop only (Near-IR Beacon)
- No alignment lasers
- Attenuated laser quantum source
- Coupled into multi-mode and single-mode fiber
- Achieved 70 mph 28.6 dB SNR into multimode fiber and 17.4 dB SNR into single-mode fiber
- **We believe this is the first demonstration of a car-to-car quantum link on public highway**

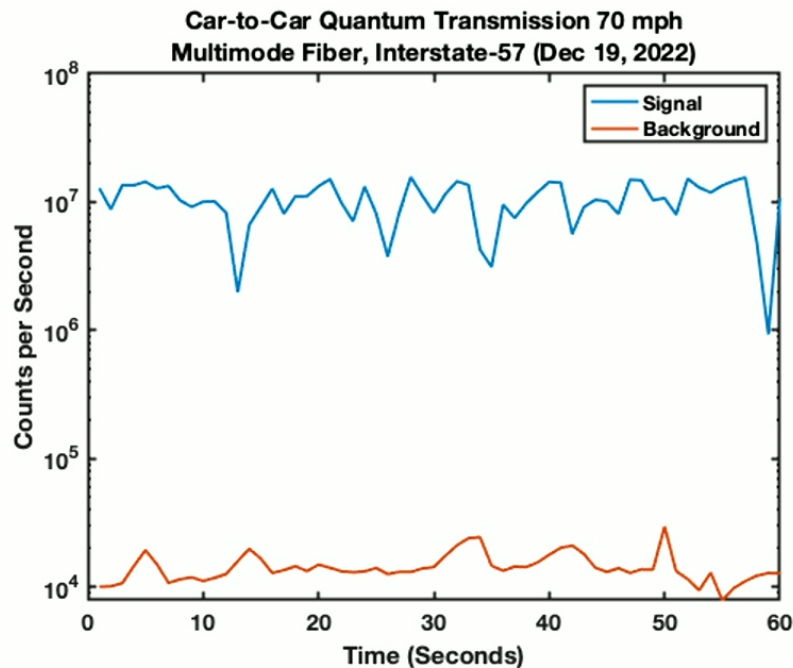


Image Courtesy Google Earth

# 70 mph Car-to-Car into Multimode Fiber

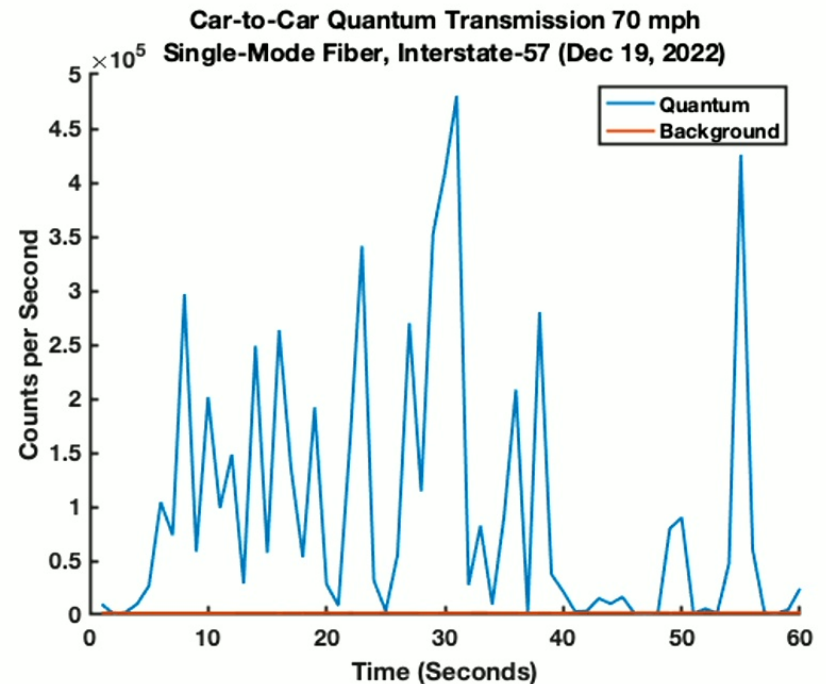
## Multimode Fiber

- Mean Signal = 10,465,380 counts/sec
- Mean Background = 14,440 counts/sec
- Mean Signal-to-Noise (SNR) = 28.6 dB



## Single-Mode Fiber (SMF)

- SMF needed for quantum teleportation, entanglement swapping, etc.
- Mean Signal = 97,080 counts/sec
- Mean Background = 1,730 counts/sec
- Mean Signal-to-Noise (SNR) = 17.4 dB



# SEAQUE: Space Entanglement Annealing QUantum Experiment

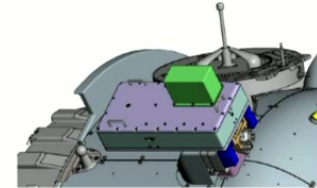
## **I** ILLINOIS

Project Lead  
Optical Payload  
Control Board

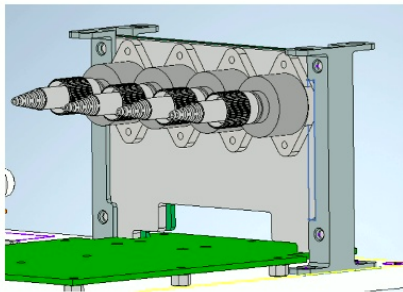
JPL  
Funding and Program  
Management



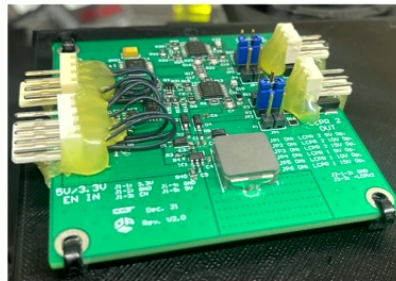
Laboratory for Advanced Space  
Systems at Illinois  
Electrical Platform and Interface  
with Nanoracks



University of Waterloo  
Detector Module



National University of Singapore  
Liquid Crystal Electronics



AdVR  
SPDC waveguide

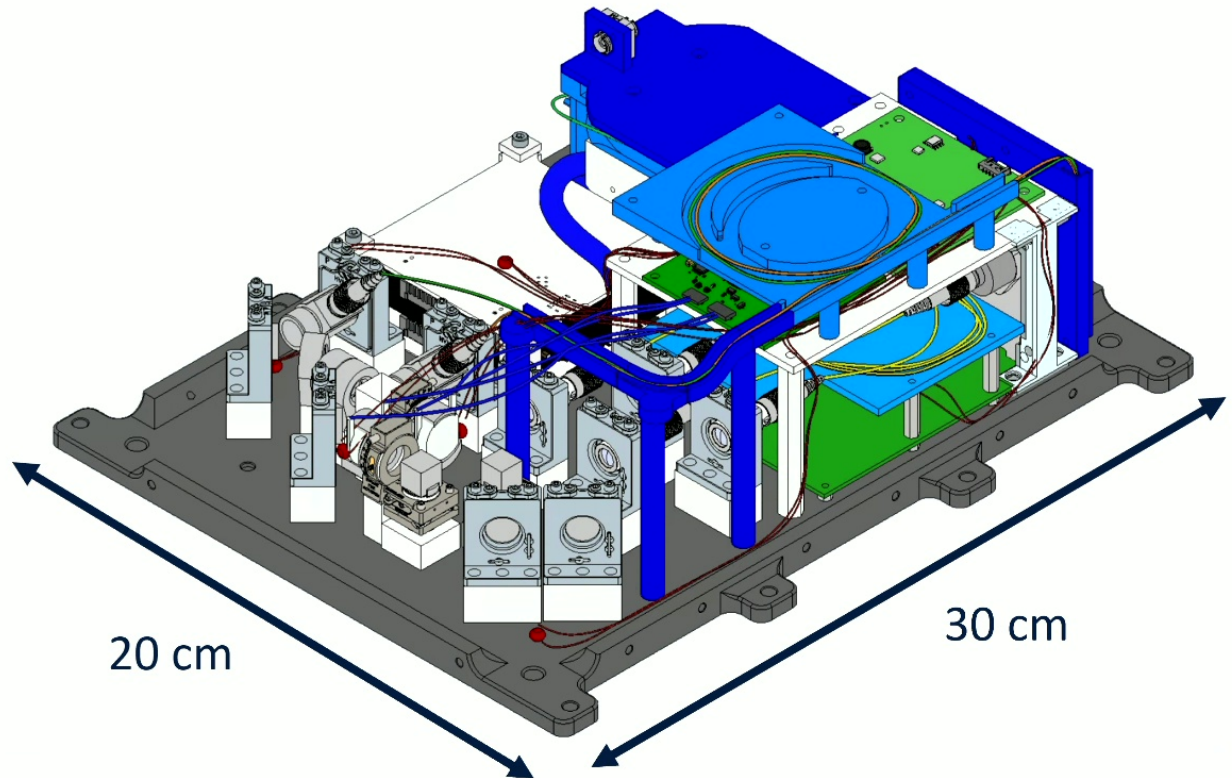


# Goals

## Goal 1

Demonstrate capabilities of quantum light systems in space

- Create and verify entanglement
- Integrated optics





# Goals

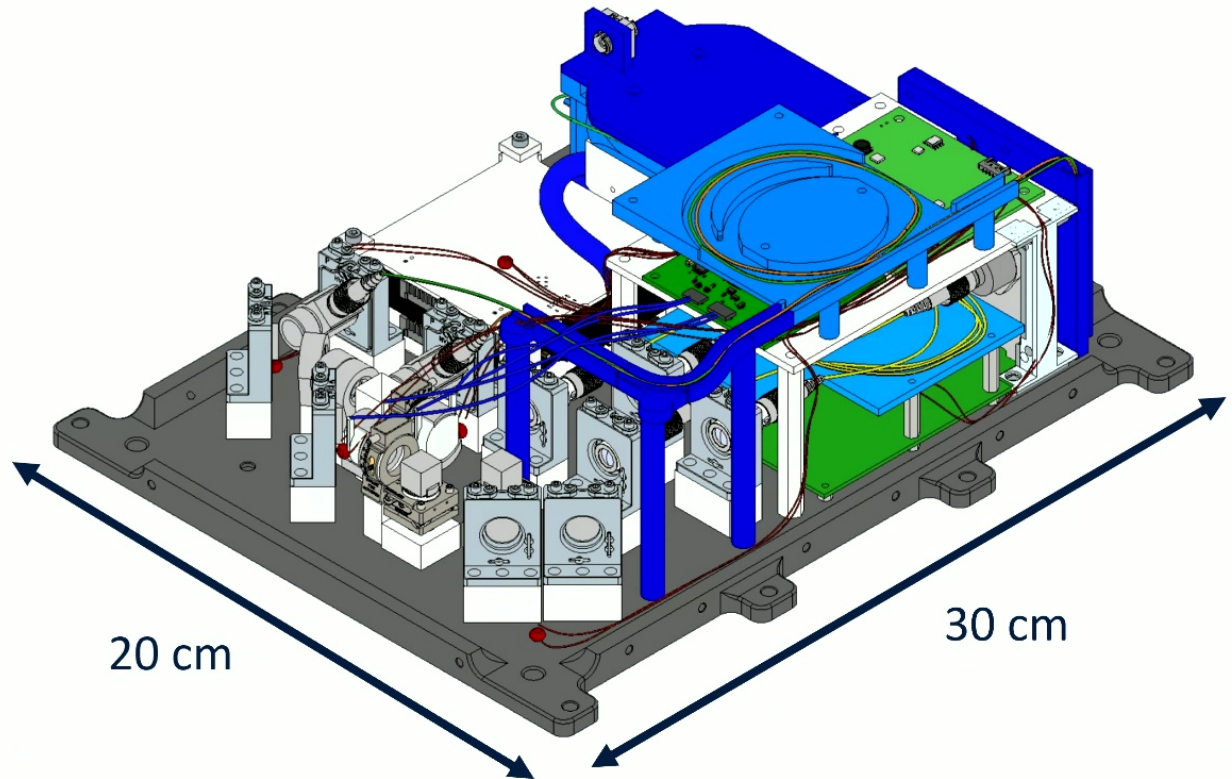
## Goal 1

Demonstrate capabilities of quantum light systems in space

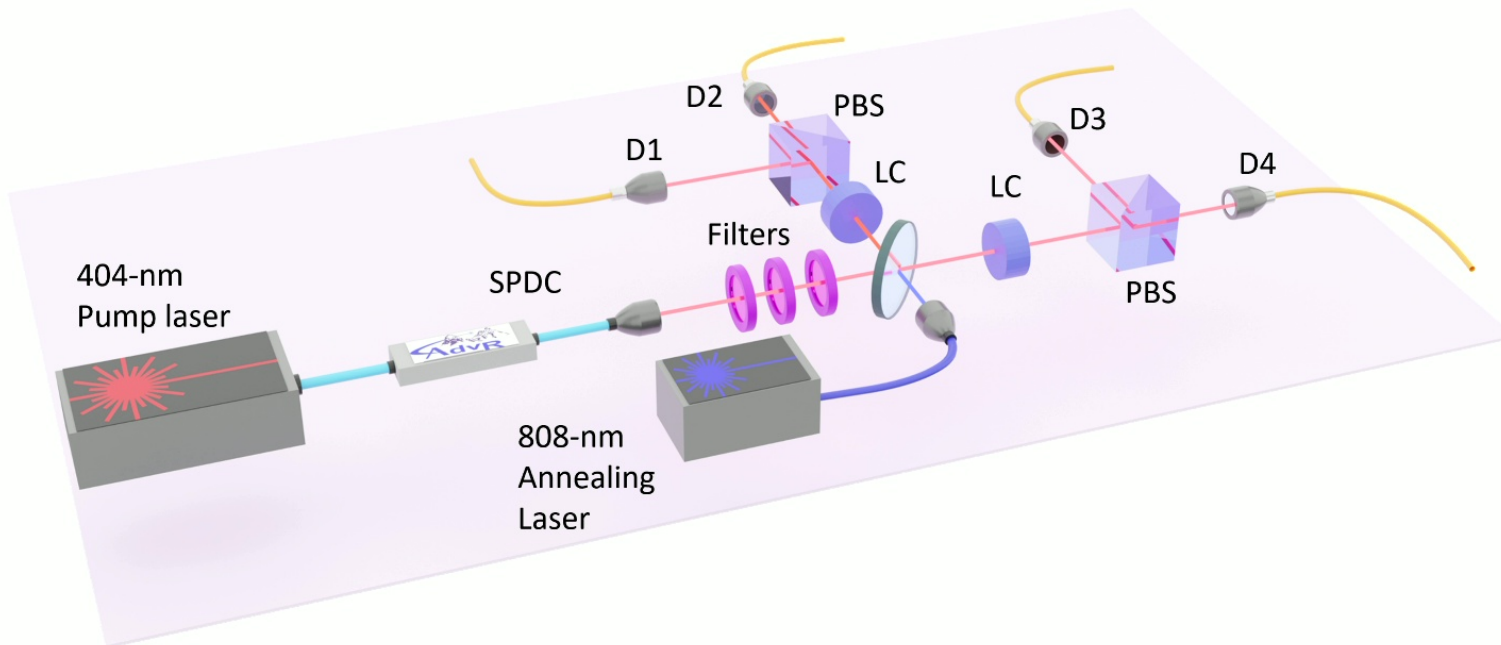
- Create and verify entanglement
- Integrated optics

## Goal 2

Perform detector “self-healing” through laser annealing



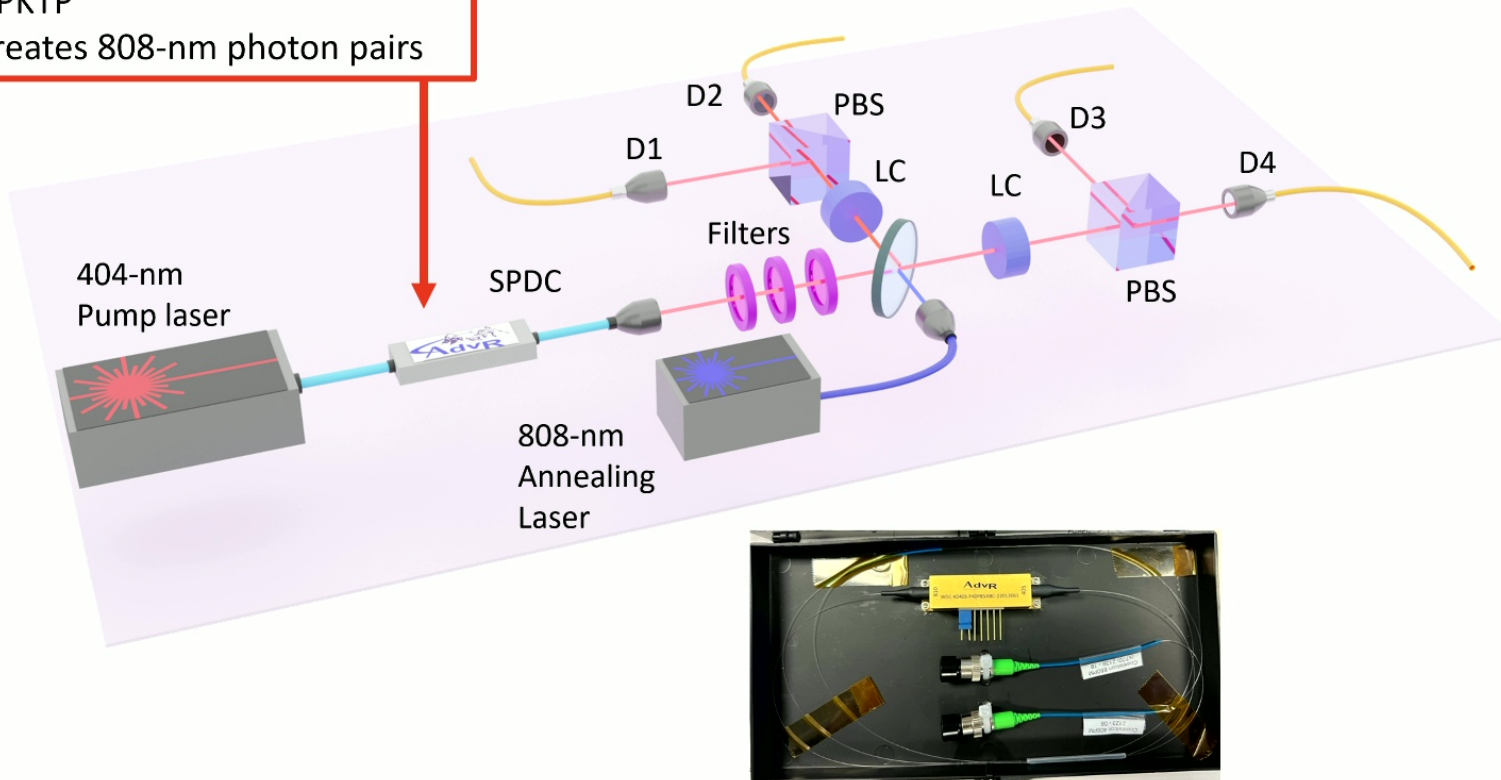
# Entanglement Source



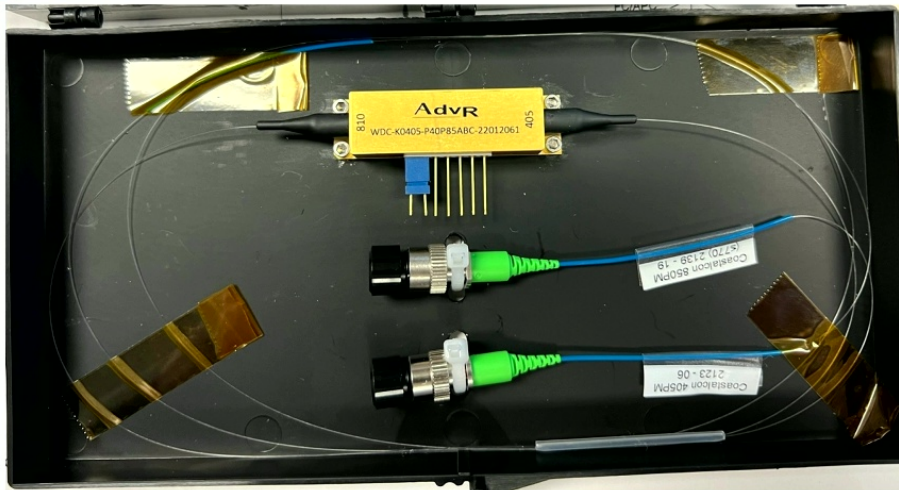
# Entanglement Source

## Waveguide:

- PPKTP
- creates 808-nm photon pairs



# SEAQUE Entanglement Source



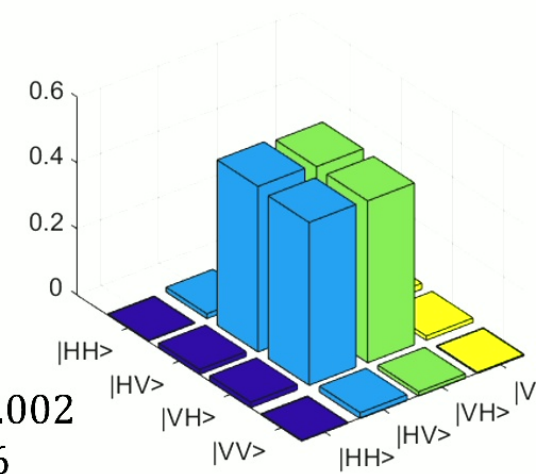
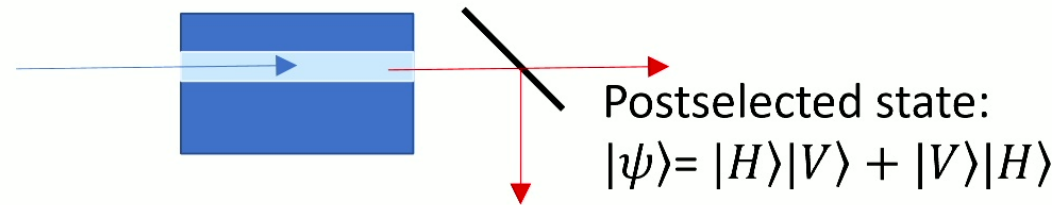
Fiber-In/Fiber-Out Timing Compensated SPDC Module  
 WDC-K0405-P40P85ABC  
 SN: 22012061

## Optical Characterization

Pump Wavelength	404.88 nm
Pair Rate	25 MHz/mW (power in input fiber)
2-Photon Visibility	95% with 3 nm filter
Module Degeneracy Temperature	45.1°C



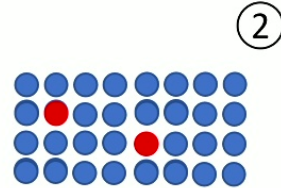
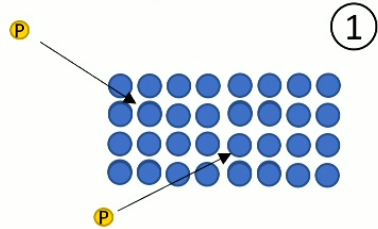
PPKTP waveguide, Type II  
 405nm  $\rightarrow$  810 nm (H) + 810 nm (V)



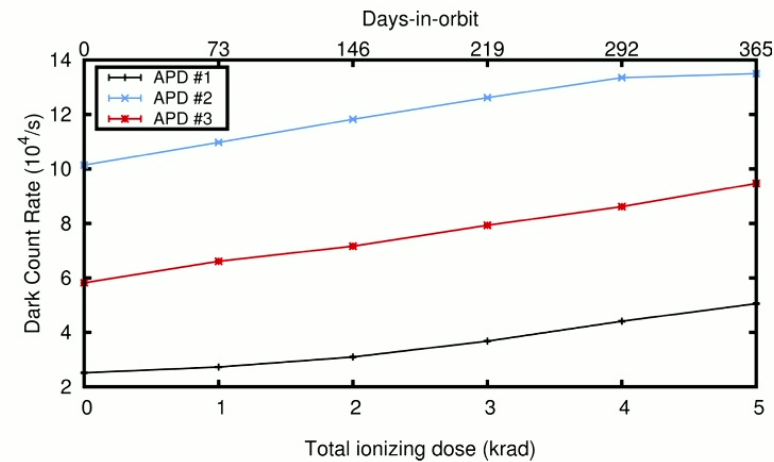
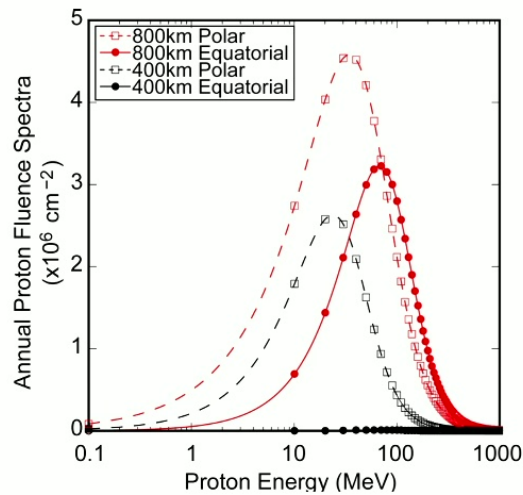
Fidelity:  $0.991 \pm 0.001$   
 Concurrence:  $0.984 \pm 0.002$   
 Bell Test:  $2.758 \pm 0.006$

# Radiation & Single-Photon Detectors

Si Avalanche PhotoDiode single-photon detectors accumulate damage while exposed to high energy protons in low-earth orbit.



Protons ionize and displace atoms in the semiconductor crystal, raising the number of detector **dark counts** (erroneous detection events).



Tan, Chandrasekara, Cheng, & Ling, "Silicon avalanche photodiode operation and lifetime analysis for small satellites," Opt. Expr. **21**, 16946 (2013)

# Healing through Annealing

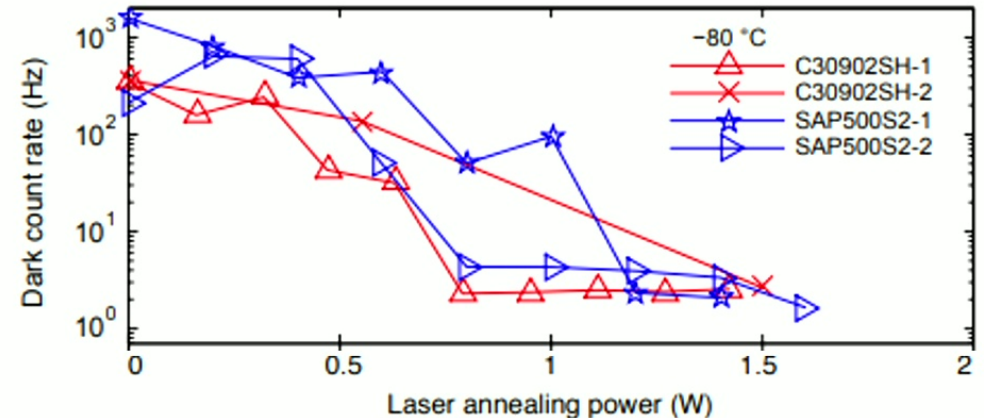
Radiation damage on single photon detectors can be **reduced** through annealing.

## Thermal Annealing:

- Entire detector is heated.
- Found to reduce dark count rate by  $\sim 6.6$  times

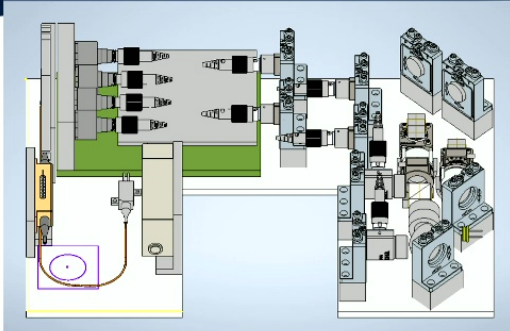
## Laser Annealing:

- A high power ( $\sim 0.5$ -2 Watts) laser sined onto the detector (provides a focused heating)
- Found to reduce dark count rate by 5.5-758 times (near  $-80^\circ\text{C}$ )

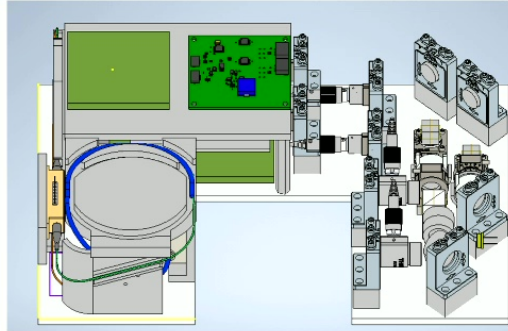
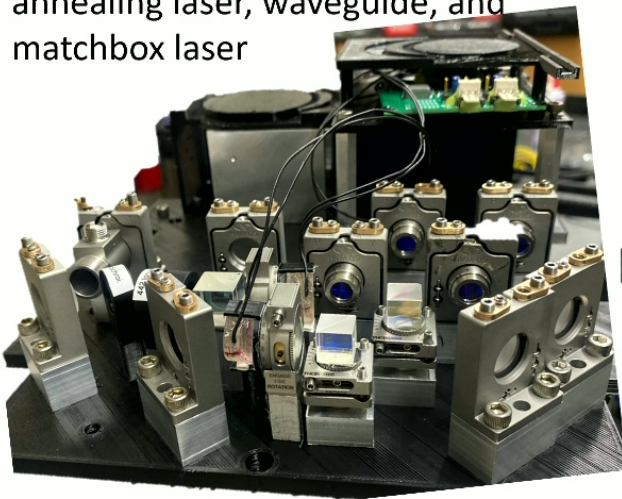


Lim, *et al.* "Laser annealing heals radiation damage in avalanche photodiodes", *EPJ Quantum Technol.* **4**, 11 (2017)

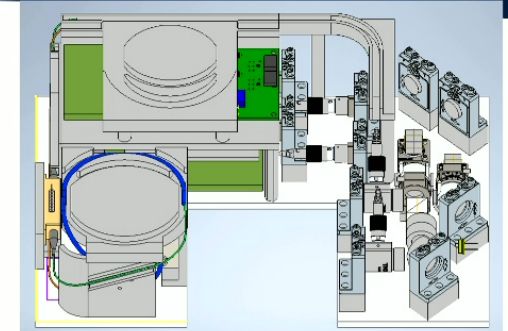
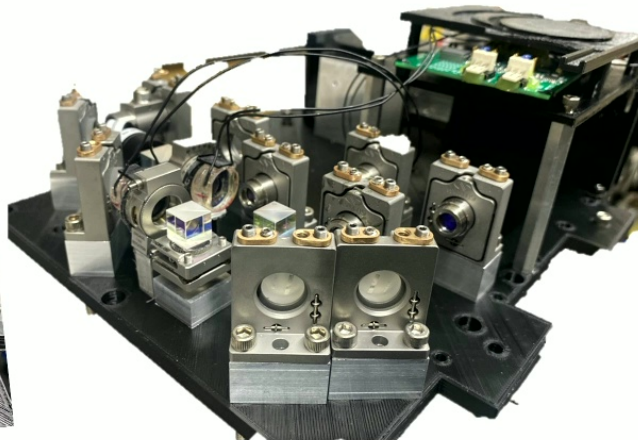
# Layout design and Assembly Order



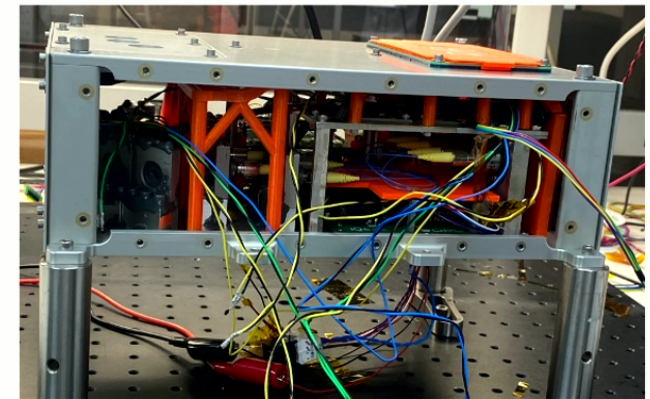
**Step 1:** mount optics, detector board, annealing laser, waveguide, and matchbox laser



**Step 2:** mount electronics and fiber supports



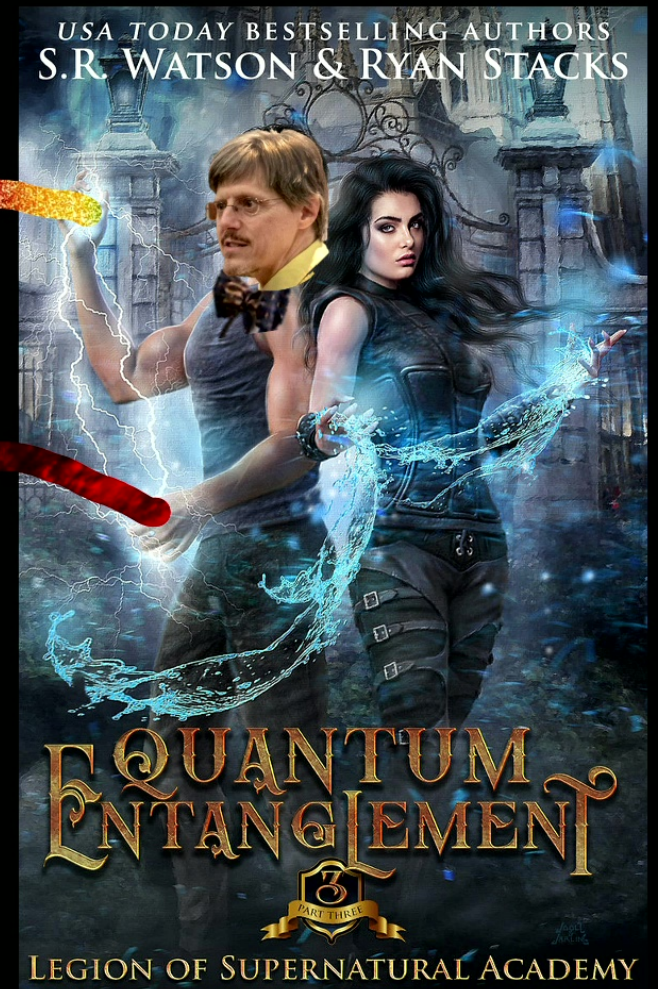
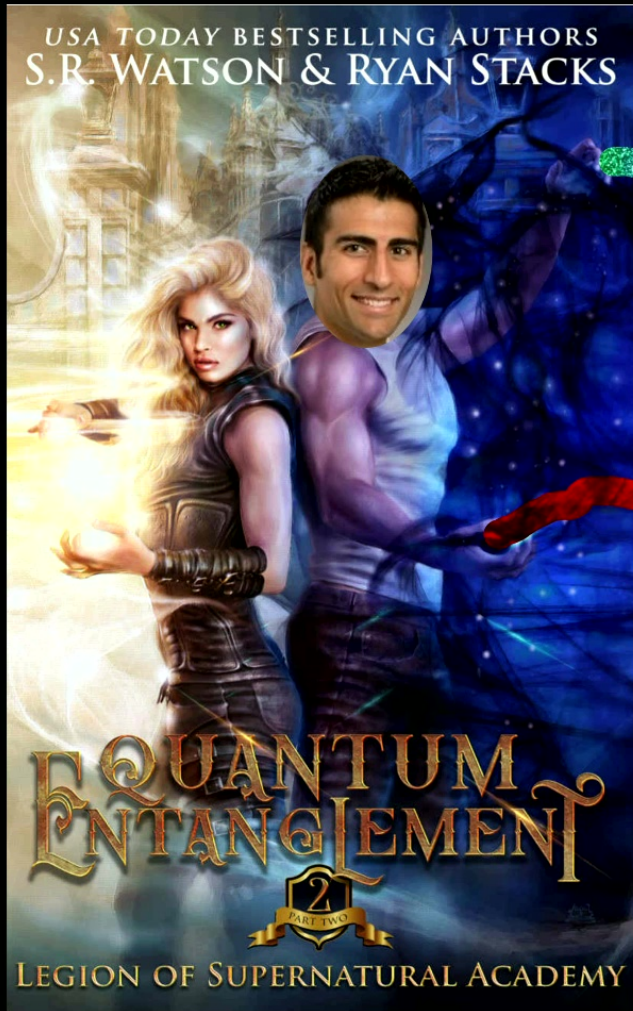
**Step 3:** mount final fiber supports over electronics board



KQC: Kwiat's  
Quantum  
Consortium  
(Cohort, Clan,  
Collective,  
Comrades, ...)







Questions?