Title: Quantum Error-Correction and Holographic Task

Speakers: Beni Yoshida

Collection: QPV 2023: Advances in quantum position verification

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Quantum error-correction in holographic scattering

Beni Yoshida (Perimeter Institute)

Plan of the talk

✤ (Recap) AdS/CFT correspondence as quantum error-correcting code.

(Harlow-Pastawski-Preskill-BY 2015)

✤ (Recap) Revisiting the holographic task in AdS/CFT.

(May 2019)

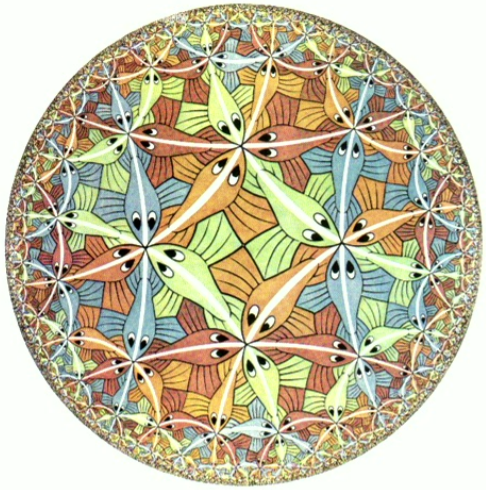
✤ Multiple-input protocol based on entanglement-assisted QECCs.

(May-Sorce-BY 2022)

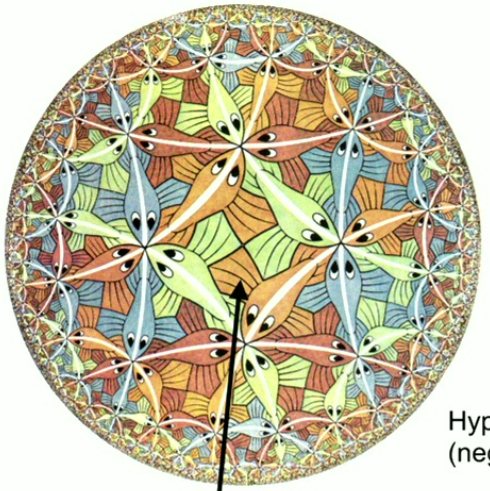
✤ Remarks on transversal logical gates.

AdS/CFT correspondence

✤ Quantum gravity in bulk = CFT on boundary ?



Boundary D-dimensional conformal field theory (without gravity)



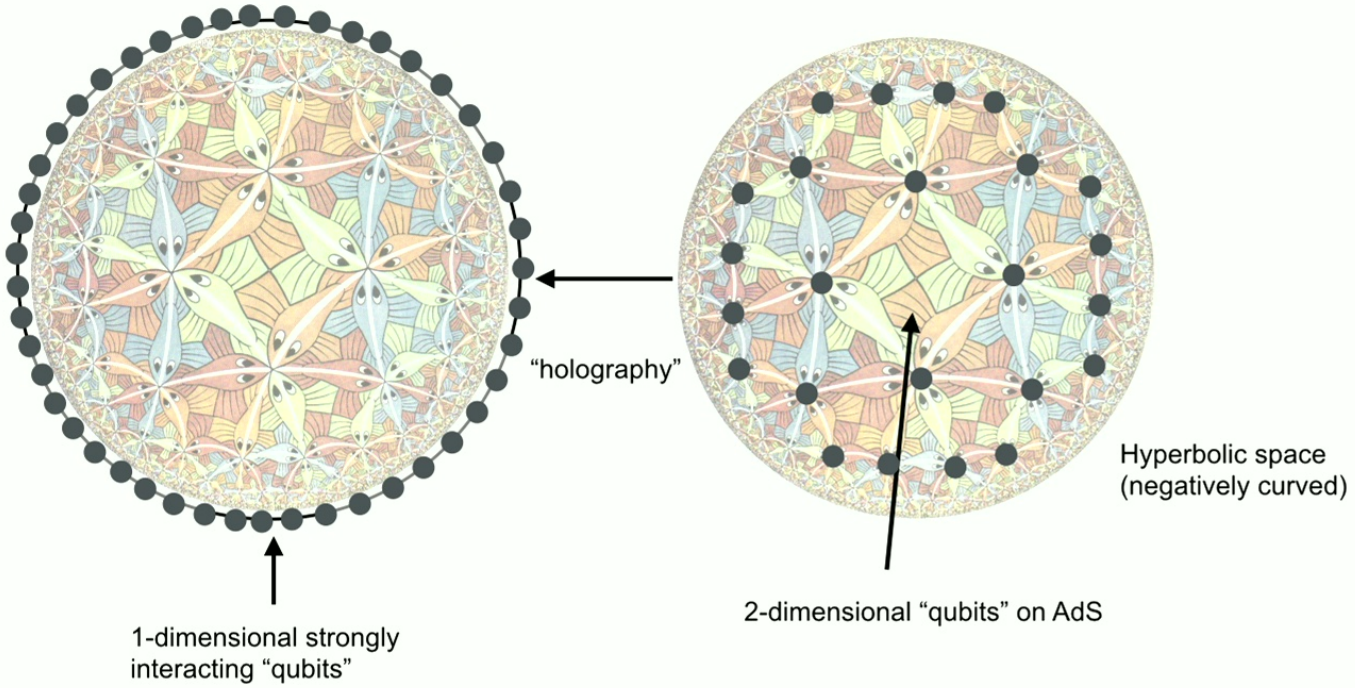
Hyperbolic space (negatively curved)

Bulk (D+1)-dimensional theory with gravity on AdS space

AdS/CFT correspondence

✤ Quantum gravity in bulk = CFT on boundary ?

✤ Bulk info is encoded in boundary, like a hologram.



Quantum entanglement in AdS/CFT

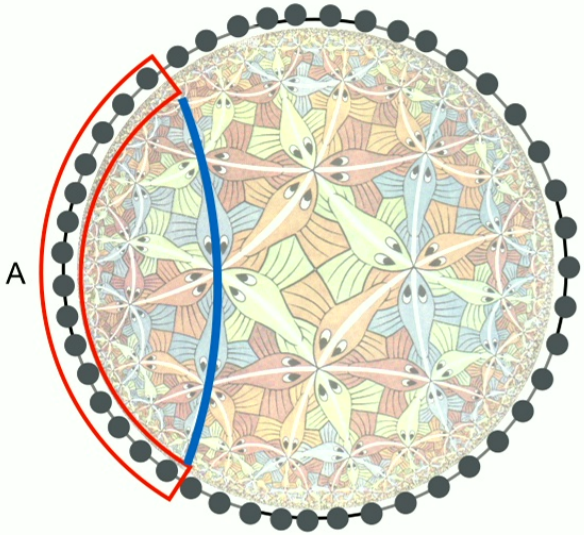
- [Ryu-Takayanagi formula 06]

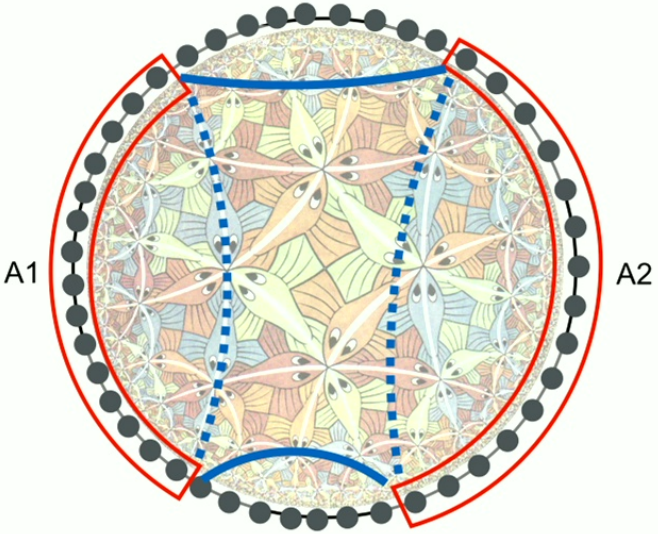
Quantum

$$S(A) = \frac{1}{4G_N} \min_{\gamma_A}(\text{area}(\gamma_A))$$

Gravity

Minimize over spatial bulk surfaces homologous to A

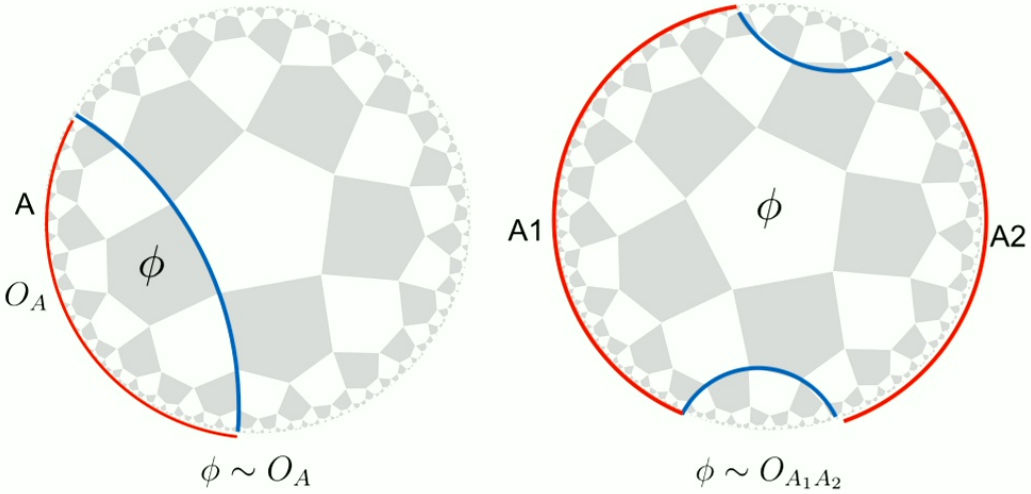




Bulk operator vs boundary operator

✤ Entanglement wedge reconstruction

A bulk operator ϕ can be represented by some boundary operators supported on A if ϕ is contained inside the entanglement wedge of A.

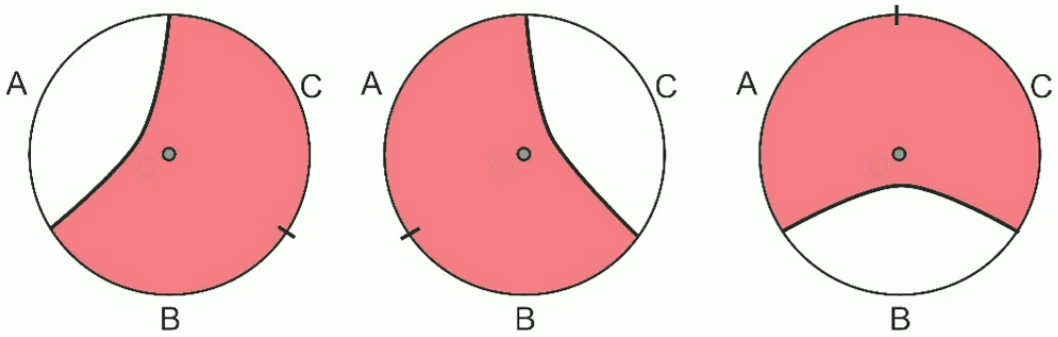


- "Proven" by using a generalized RT formula (Jafferis et al)
- No explicit recipe is known for more than one intervals

Bulk locality puzzle

✤ The reconstruction recipe leads to a paradox

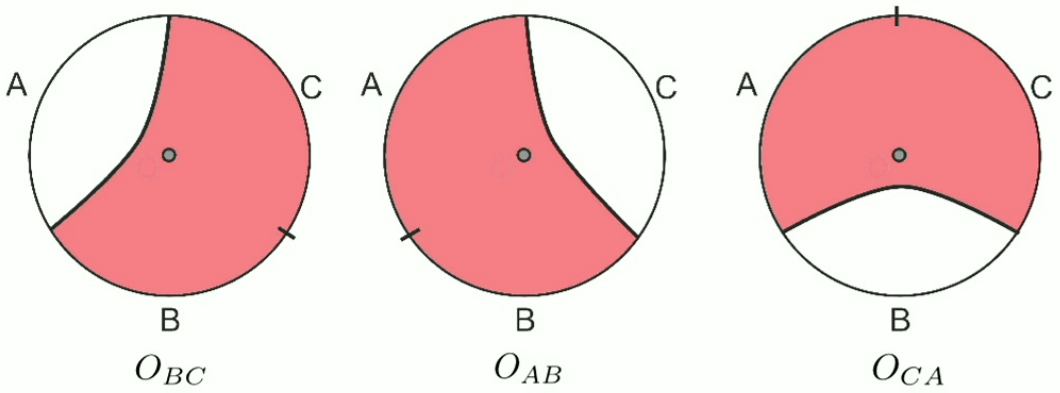
- All the bulk operators must correspond to identity operators on the boundary ?



(Almheiri-Dong-Harlow)

Quantum error-correction in AdS/CFT ?

✤ The AdS/CFT correspondence can be viewed as a quantum error-correcting code.

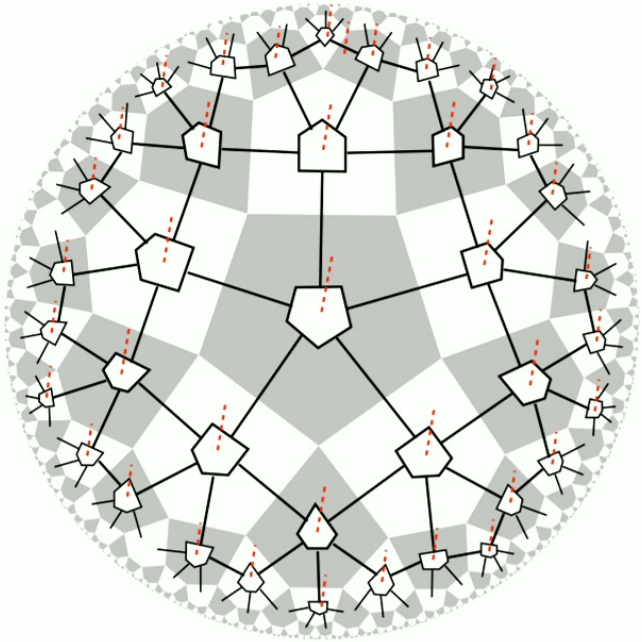


They are different operators, but act in the same manner in a low energy subspace.

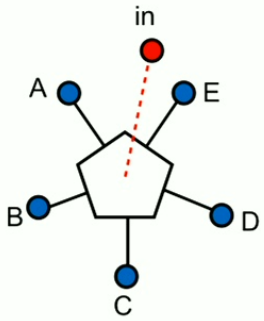
cf. Quantum secret-sharing code

Holographic quantum error-correcting code

✤ A tiling of random tensor network works.



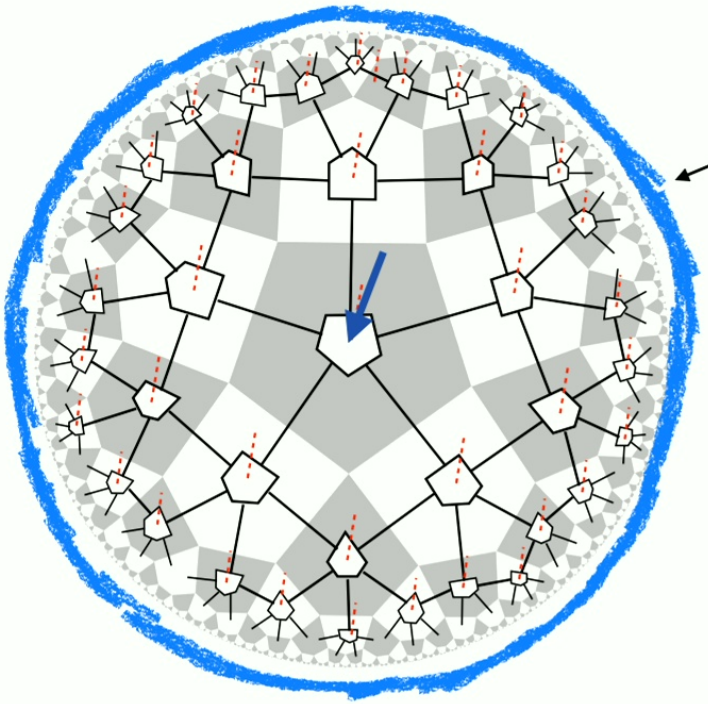
Ryu-Takayanagi formula, entanglement wedge reconstruction etc



Haar random tensor

Bulk center is very complex

✤ One can “push” the bulk operator to the boundary.



Very complicated expression

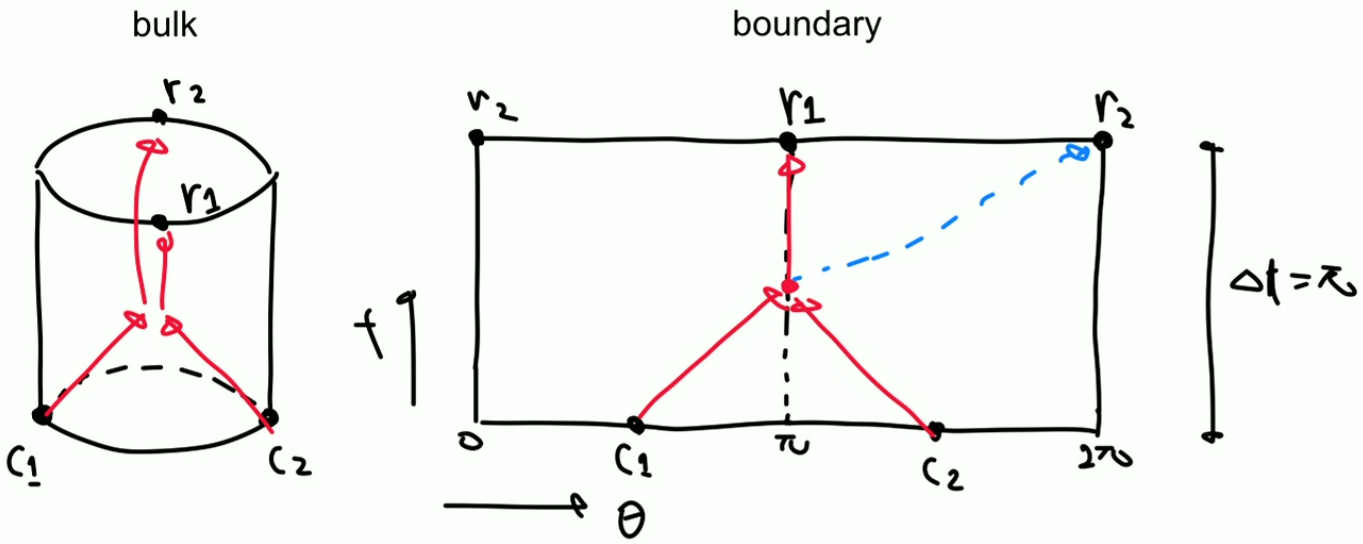
(Cannot be implemented by short-depth quantum circuit)

Holographic scattering and QECC

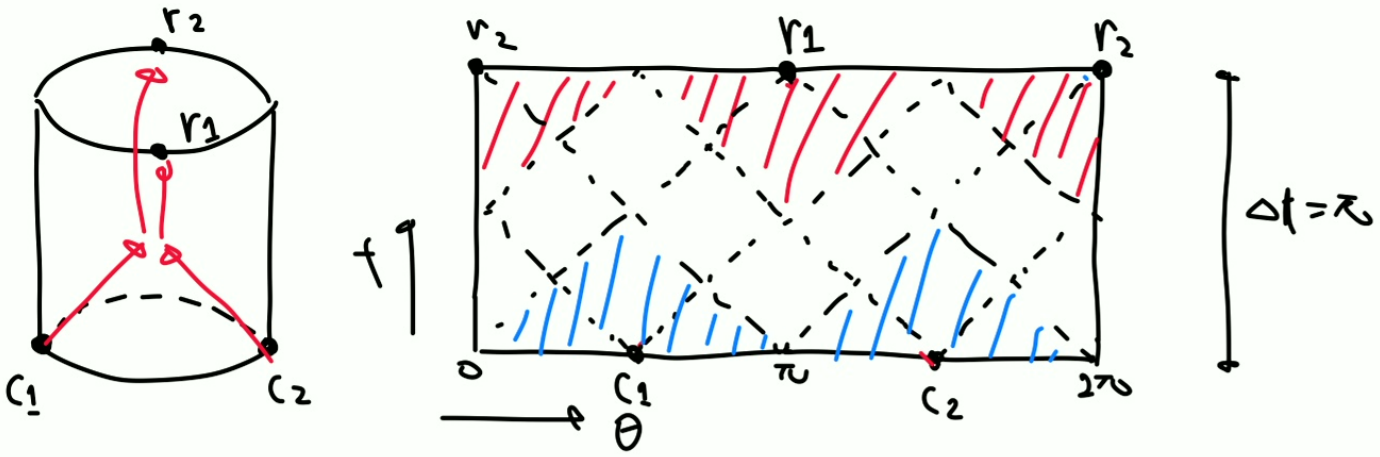
Boundary causality puzzle

✤ On the boundary, there is not enough time for particles to interact.

- Particles in the AdS/CFT correspondence must be non-interacting… !?



Future and Past light cones



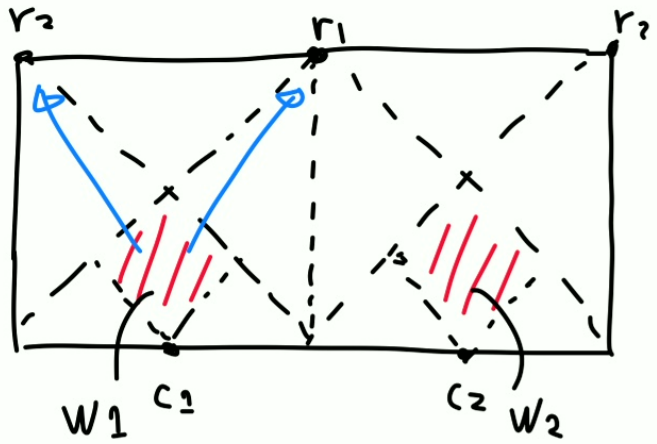
$J_+(C_1) \cap J_+(C_2)$ = Future of C_1, C_2.

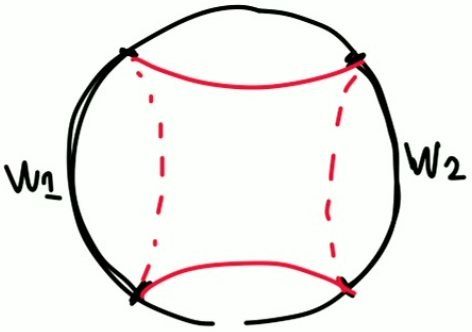
$J_-(r_1) \cap J_-(r_2)$ = Past of r_1, r_2.

No overlap !! (No direct boundary scattering)

Resolution : Entanglement as resource

[Alex May 2019]





$W_1 = J_+(c_1) \cap J_-(r_1) \cap J_-(r_2)$

$I(W_1, W_2) = O(\frac{1}{G_N})$

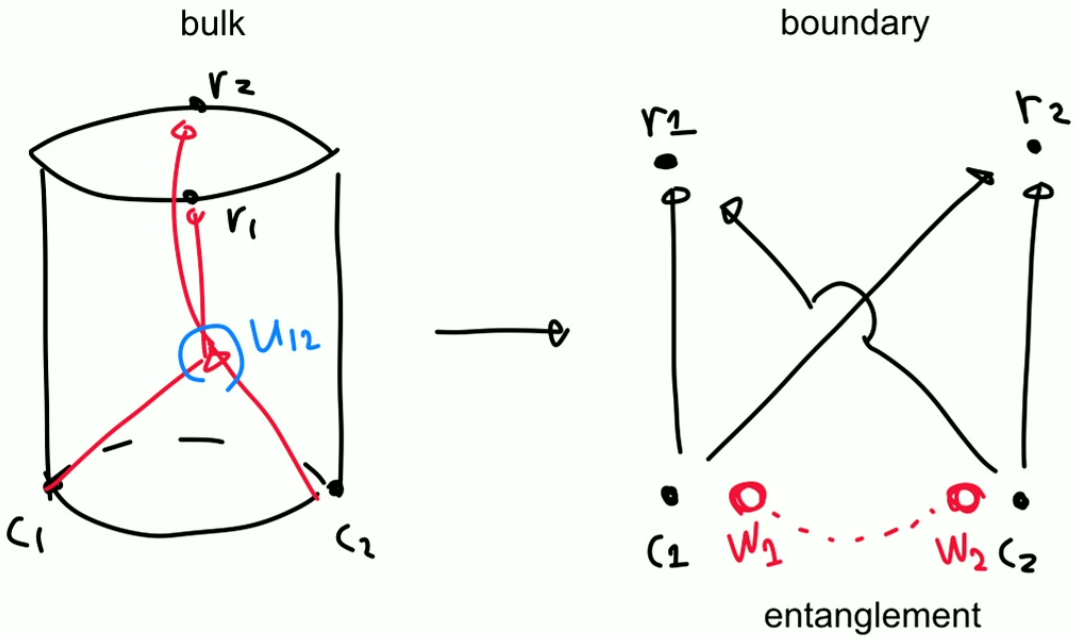
W_1 can signal to r_1, r_2

W_1 & W_2 entangled !

(via Ryu-Takayanagi formula)

Non-local quantum computation

✤ By using the pre-shared entanglement, (in principle) one can induce arbitrary unitary interactions.



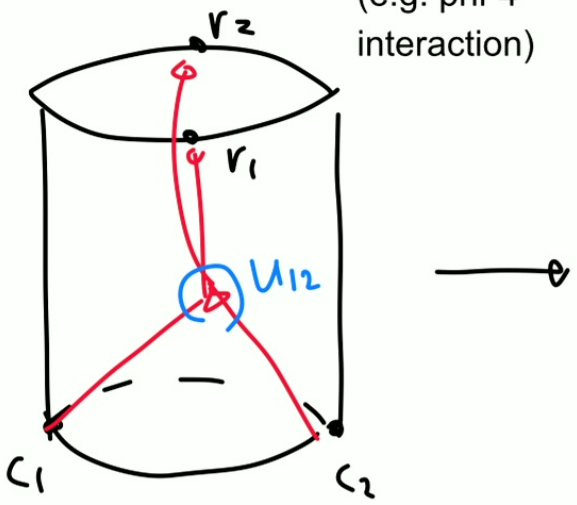
[May 2019]

What does AdS/CFT actually do?

✤ But probably, the AdS/CFT correspondence is not performing port-based teleportation…

bulk

(e.g. phi-4 interaction)



boundary

???

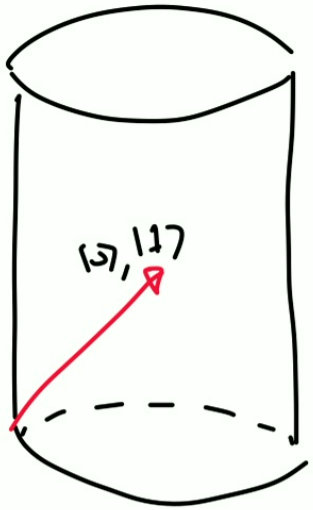
How does CFT utilize entanglement?

Entanglement + QECC =

Entanglement-assisted QECC

Quantum error-correction interpretation?

✤ Objects at the center of the AdS bulk are encoded into quantum error-correcting code.



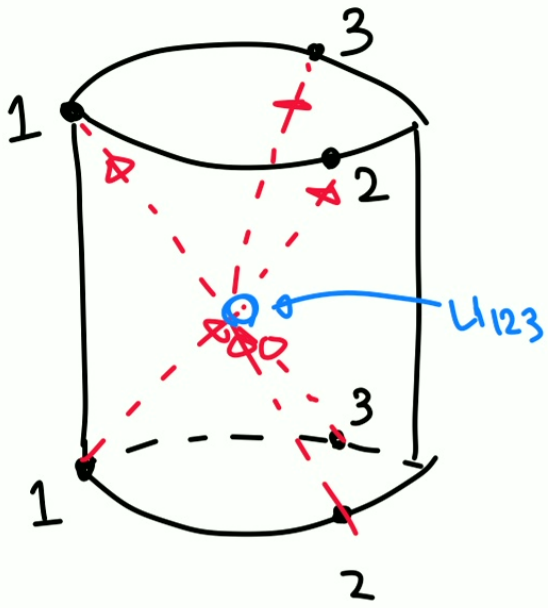
$|0\rangle \longrightarrow |\hat{1}\rangle$ in code subspace

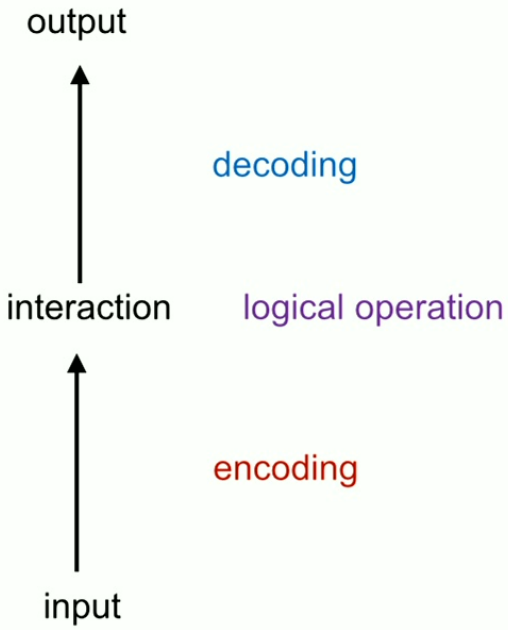
Change the states of particles

= changing the logical states of a quantum error-correcting code

Scattering and quantum error-correction

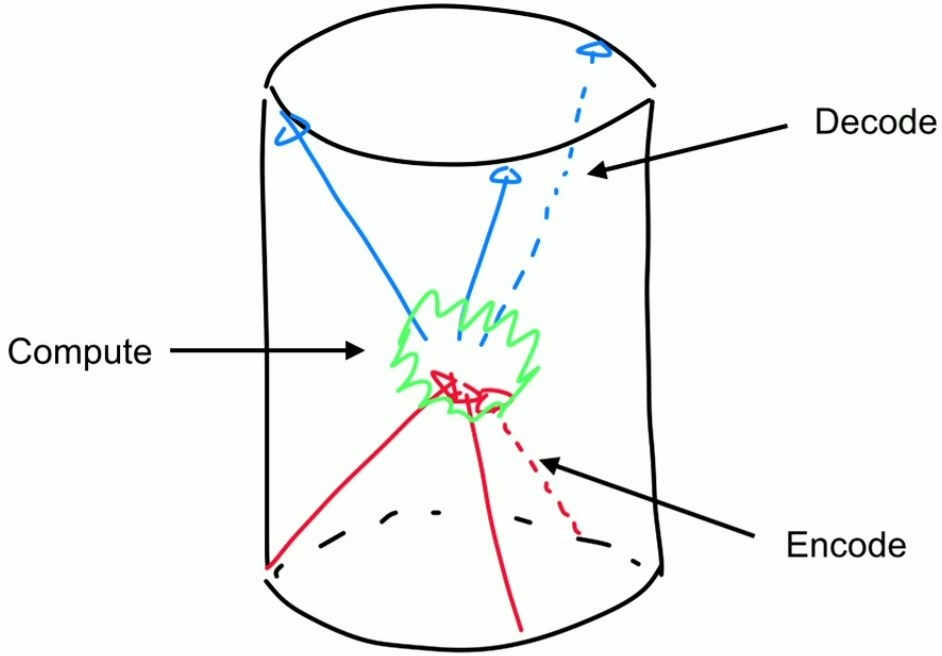
✤ Interactions must occur in a quantum error-correcting code.



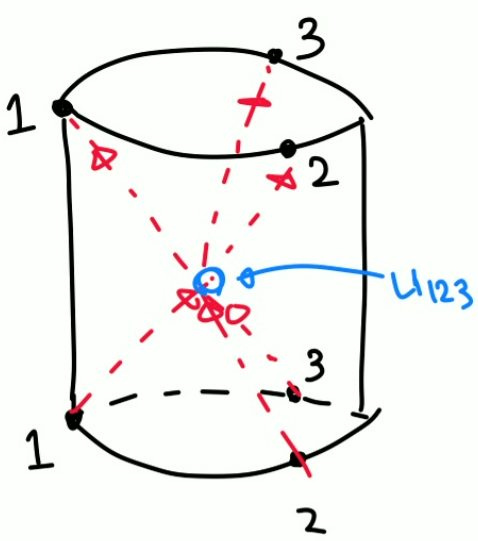


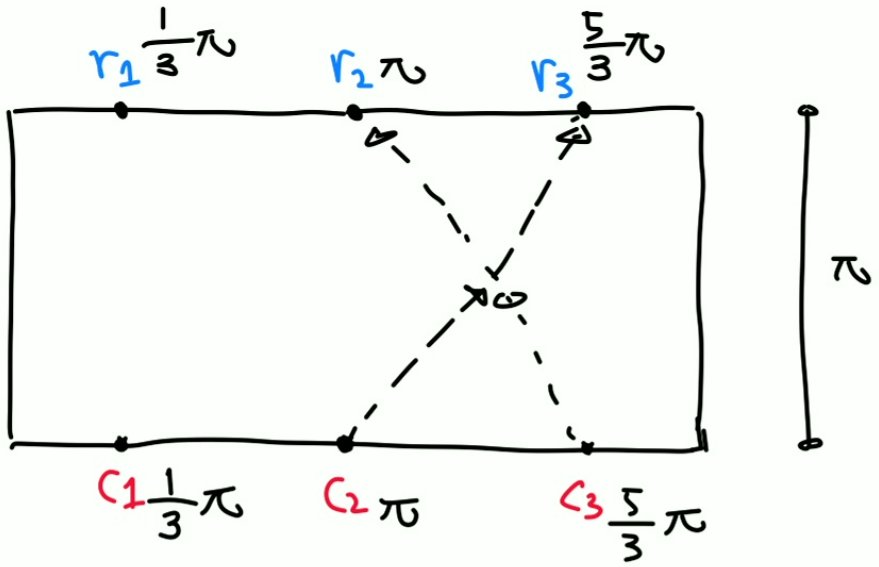
like a quantum computer…

✤ This is how a fault-tolerant quantum computer works.



3 particle Scattering (or $N \geq 3$)

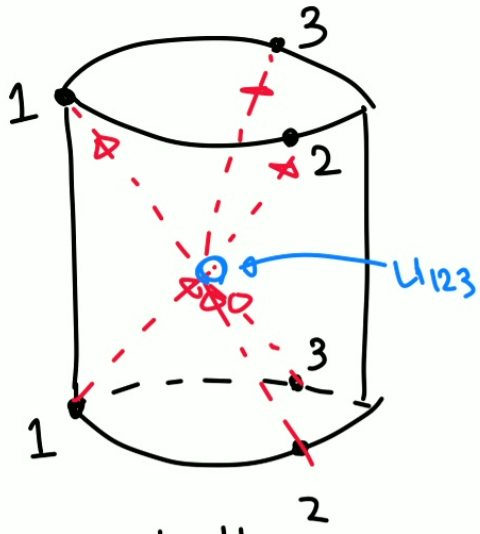


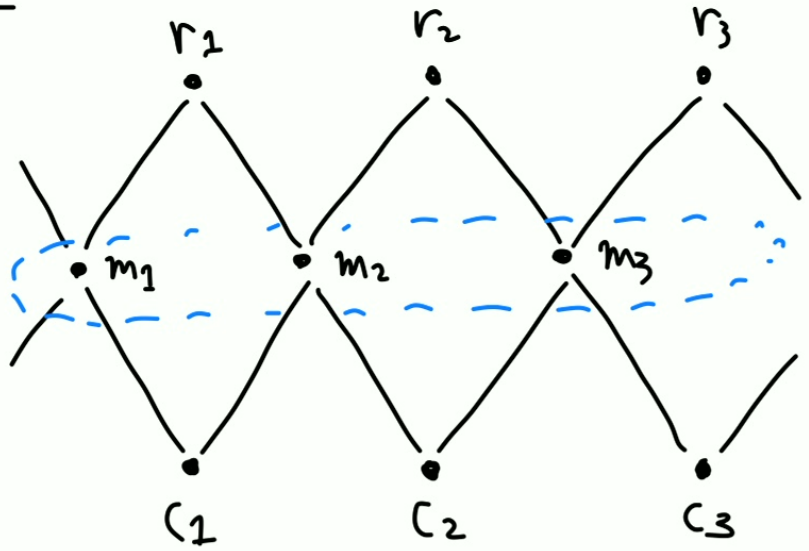


- 2-to-2 Scattering is possible

  3-to-3, not possible ....

Boundary Causality





bulk

boundary

QECCs ?

Input

c_j

Intermediate

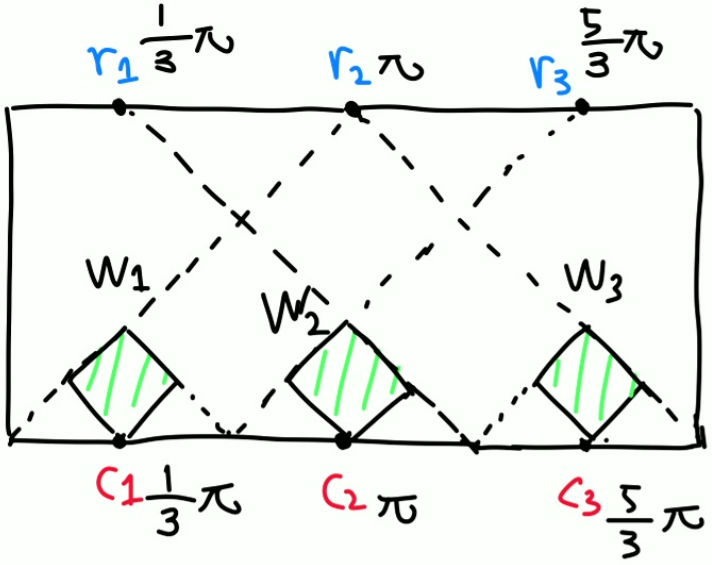
m_j

Output

r_j

Interaction happens here ?

Entanglement for 3 wedges ?



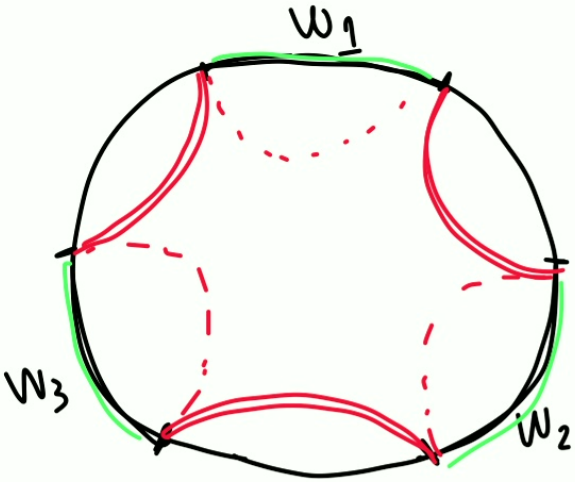
W_j's are entangled

How ??

$$W_2 \equiv J_-(r_1) \cap J_-(r_2) \cap J_-(r_3) \cap J_+(C_2)$$

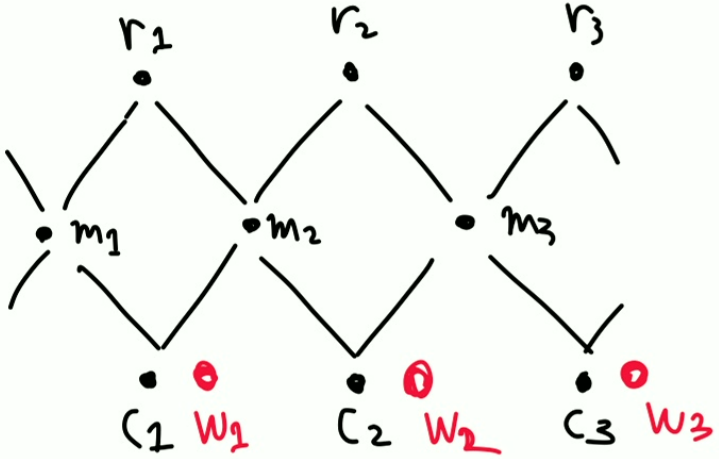
can signal to r_1, r_2, r_3

Connected wedge (Multi-party entanglement)



- $I(W_1, W_2W_3) = O(\frac{1}{G_N})$
- $I(W_1, W_2) \simeq 0$

No EPR pairs !!

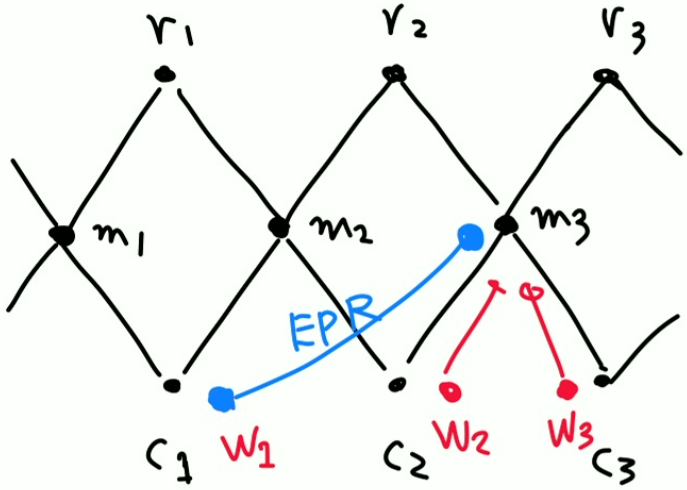


- "Teleportation" does not work ....

Entanglement in spacetime

EPR pair between different time slices

C_1 — m_1, m_2, m_3 encoding is possible!!

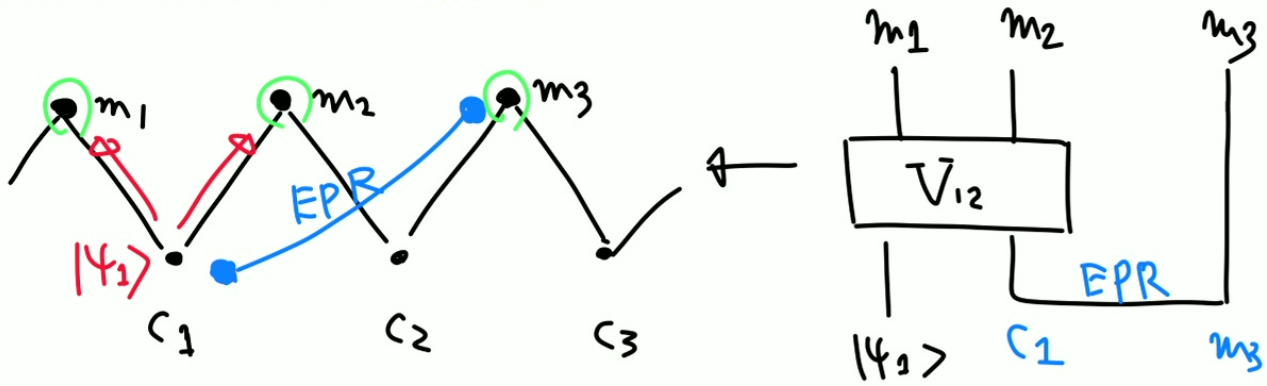


- C_1 and m_3 can share EPR pair

$$I(W_1, W_2 W_3) = O\left(\frac{1}{G_N}\right)$$

Entanglement-assisted QECCs

- Non-local encoding

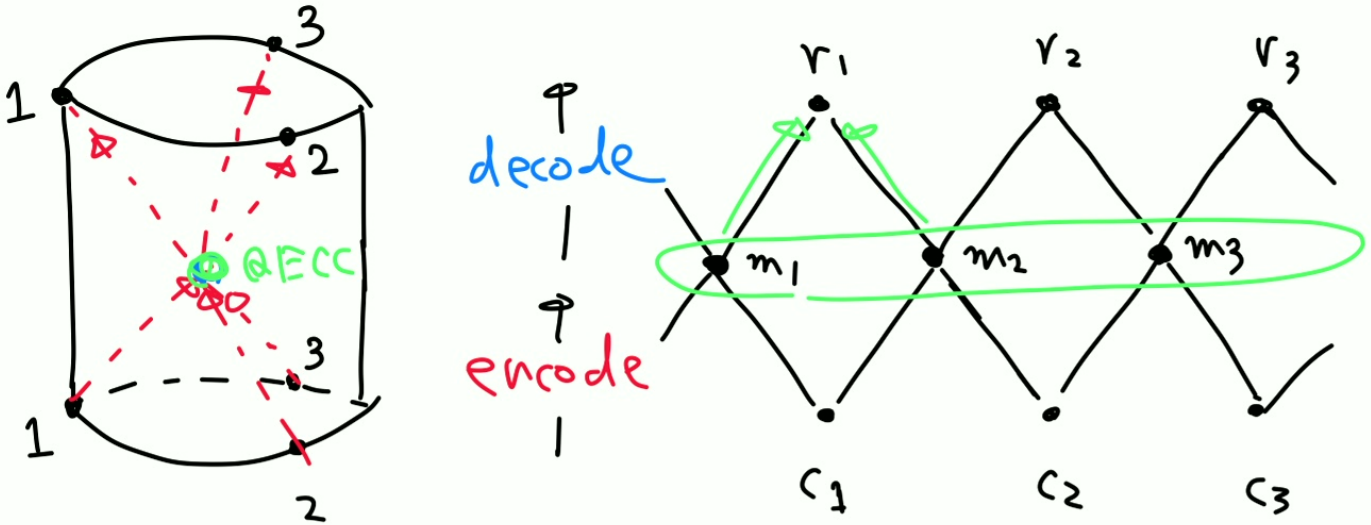


- V_{12} does not need to touch m_3

- Same mechanism as Hayden-Preskill protocol

  (BH – Early radiation entanglement)

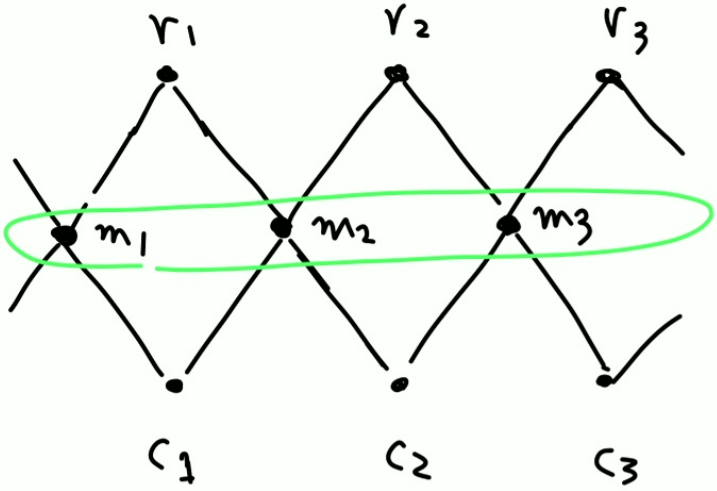
Encoding and Decoding



- r_2 has access to m_2 and m_3

  ⟶ $|\psi_1\rangle$ can be reconstructed on r_1, r_2, r_3.

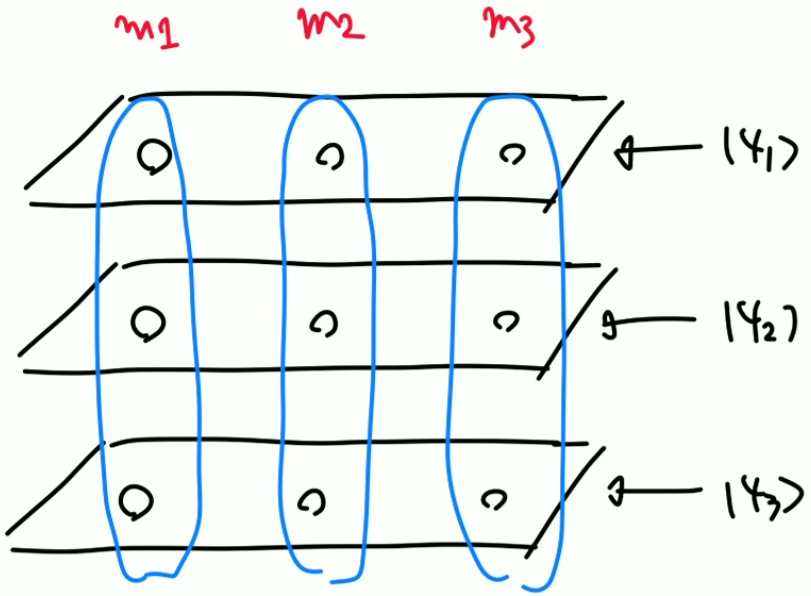
Interaction in QECCs



"Logical operators" act on m_1, m_2, m_3

Transversal logical operators

- Transversal (factorized) logical op only



$e^{i\theta(X_1X_2X_3 + Z_1Z_2Z_3)}$ is not possible

$V_1 \otimes V_2 \otimes V_3 = V$

Implementable Logical gates

- Arbitrary Clifford gate (maps Pauli to Pauli)

  n-qubit $O(n)$ EPR pairs

  Random Clifford encoding suffices.

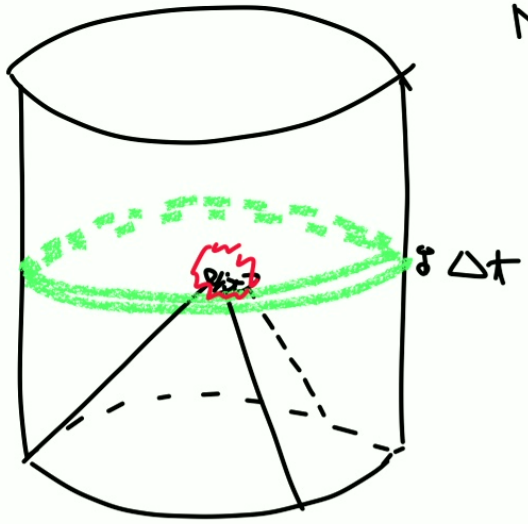
- Phase gate

  quadratic

  $$|x_1, x_2, x_3\rangle \longrightarrow e^{i\theta(x_1, x_2, x_3)} |x_1, x_2, x_3\rangle$$

  - $x_1 x_2 + x_2 x_3 + x_1 x_3$ can be generated
  - $x_1 x_2 x_3$?? (need 3rd level of Clifford hierarchy)

Interaction from transversal gate ?



Naively ...

- Interaction occurs in small Δt slice ?

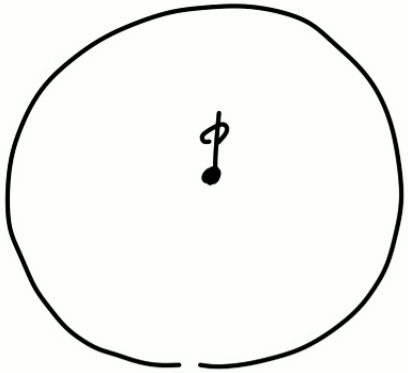
$$H = H_0 + \overbrace{V_{int}}^{\Delta t} \quad ?$$

- Logical operators are short-depth?

But ... - Not consistent with tensor network picture

- Various no-go result for transversal gate in QECCs.

No-go for transversal gate (causality constraint)



- Suppose $\phi \longrightarrow \hat{O}$ on boundary and $\hat{O}$ is "simple"

  e.g. $\hat{O} = O_1 \otimes O_2 \otimes \cdots \otimes O_n$.

- ϕ can be implemented in $O(1)$ time on boundary

  $\longrightarrow$ Instantaneous signalling to bulk center

- Hence, $\hat{O}$ should be "complex"

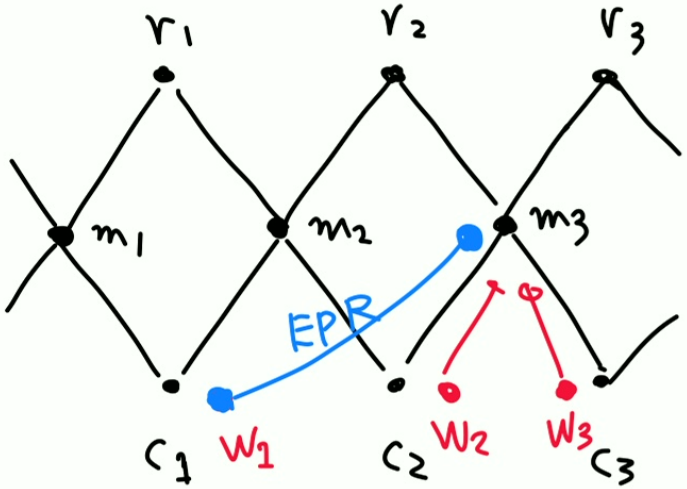
  (circuit depth should be high)

Is our protocol transversal ?

- Preparation at C_1, C_2, C_3 are needed.

  → Transversal operation on $C_1, C_2, C_3, m_1, m_2, m_3$

Transversality in spacetime ?



- Consistent with HKLL reconstruction.

Open Problems

1). Higher-order interactions

$\Theta(x_1, x_2, x_3, x_4, \cdots, x_M) = x_1 x_2 x_3 \cdots x_M$ (M-th order)

This requires us to go to M-th level of Clifford hierarchy. (less fault-tolerant, $\sim \frac{1}{M}$). ···.

2). 3-in-3-out EAQECC protocol, any application?

3). Transversal logical operator in spacetime?

4) Understand the mechanism in CFT.

Summary of protocol

- Logical op = Interaction in Hcode.