Title: Non-local quantum computation meets quantum gravity

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Quantum gravity meets quantum cryptography

Alex May

Perimeter Institute

What is quantum gravity?

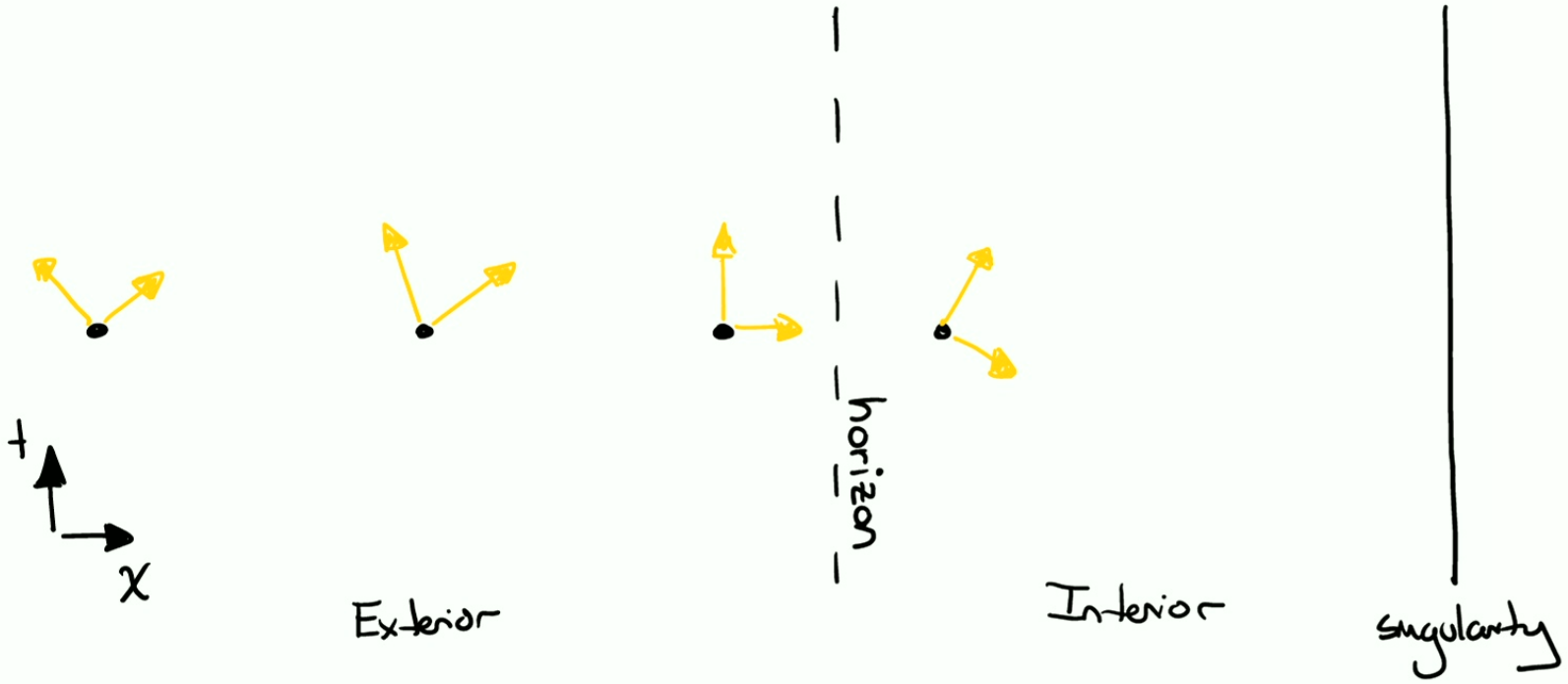
- Quantum gravity tries to fit gravitational physics into the framework of quantum mechanics

- This is a challenge not just to our understanding of gravity, but also our understanding of quantum systems.

↳ How do we construct quantum systems that exhibit gravitational physics?

Black holes

- A black hole is a region in spacetime from which it is impossible to escape to far away.

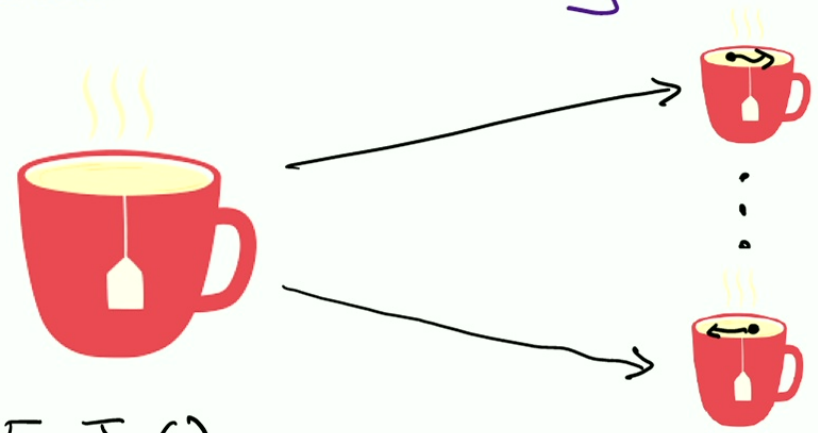


Properties of black holes

- Black holes are described very simply:

"No-hair" theorem: once M, J, Q are specified, there is a unique static solution to general relativity + E&M

- Analogous statement about ordinary matter is false:



Thermodynamic parameters E, J, Q

entropy

$$S = \log \Omega(E, J, Q)$$

of "microstates" consistent with our thermodynamic description

Do black holes have entropy?

- General relativity describes black holes only in terms of macroscopic / thermodynamic quantities M, J, Q

  ↳ Quantum gravity should provide the microstates

Analogy:

general relativity ⟷ quantum gravity

thermodynamics ⟷ statistical mechanics

- Can extract the entropy from thermodynamics, provides hints about stat. mech

  ↳ Can we also extract entropy from general relativity?

Black holes and entropy

Idea: Black holes have microstates, specifically

$$S_{bh} = \log \Omega_{bh} = \frac{A_{bh}}{4G_N}$$

Support for this:

- Black hole areas always increase in classical gravity
- Thermodynamic systems should have a temperature T, which we can relate to energy by $TdS = dE$ ($M = E$ here)

  ↳ Leads to $T = \frac{1}{8\pi G_N M}$ which Hawking verified explicitly!

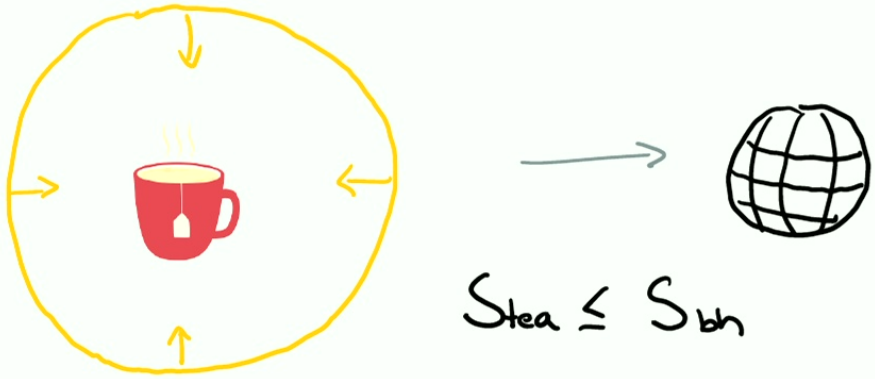
Entropy and area

- Learned that

$$S_{bh} = \frac{A_{bh}}{4G_N}$$

- Pretty weird! For ordinary matter, always find $S_{th} \propto$ Volume

- But, notice that we can turn ordinary matter into a black hole by <u>adding</u> entropy



$S_{tea} \leq S_{bh}$

Conclude that <u>all</u> matter has at most an areas worth of degrees of freedom.

The holographic principle

- Based on these ideas, t'Hooft and Susskind suggested the following:

> Quantum gravity in 3+1 dimensions should have an alternative, non-gravitational description in 2+1 dimensions

The holographic principle

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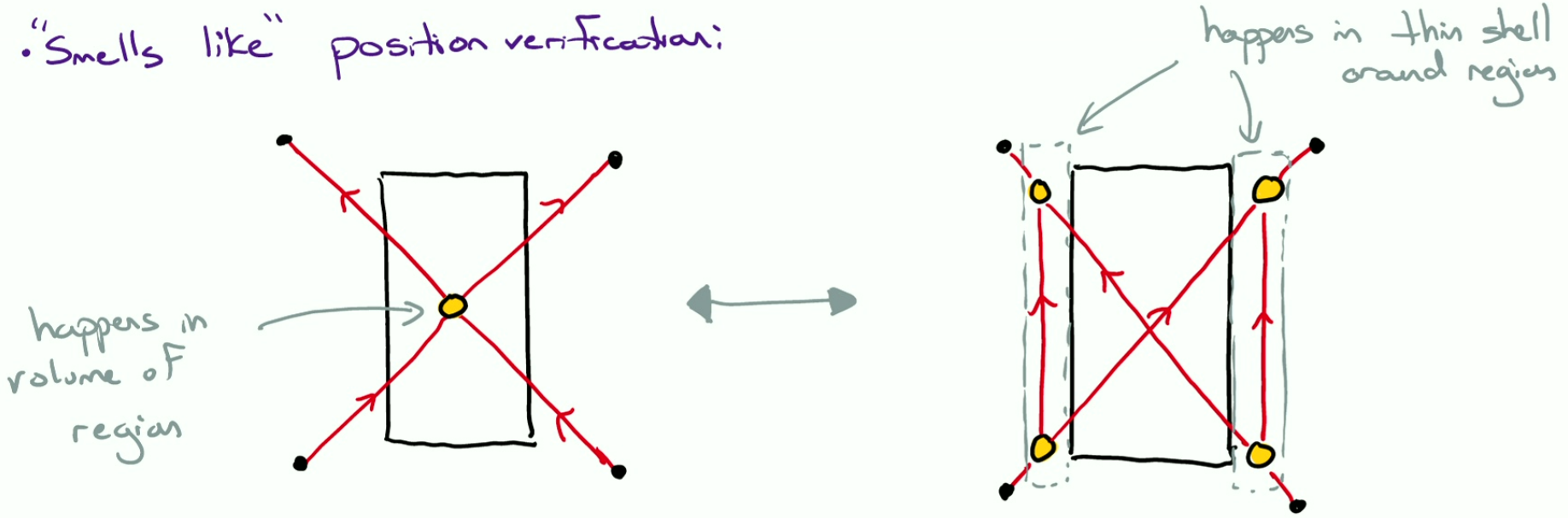
- "Smells like" position verification:



Towards holographic quantum gravity

AdS space and holography

- "Holography" is easiest to realize in asymptotically anti de Sitter space, which are solutions in classical gravity

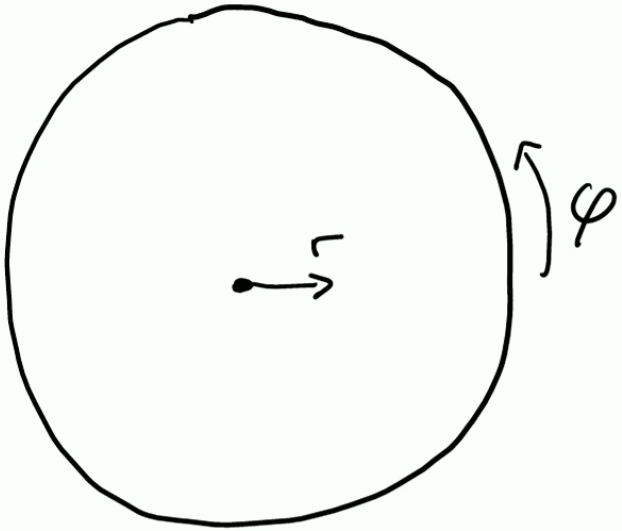
$$S_{grav} = \int d^d x \sqrt{g} (R + 2\Lambda) \qquad \text{with} \qquad \Lambda < 0$$

- For simplicity, we will study these spacetimes in $2+1$ dimensions, and look for a $1+1$ dimensional holographic theory.

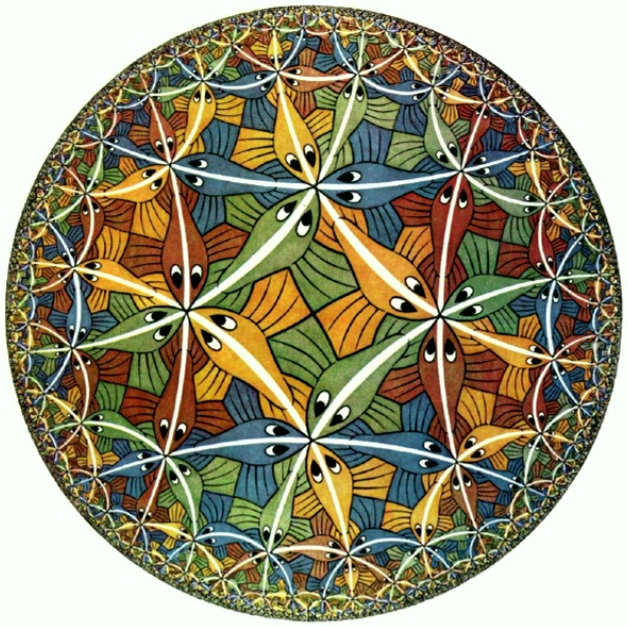
- AdS space is counterintuitive, but lets try to get a feel for it...

Hyperbolic disk

- First consider only the spatial directions:



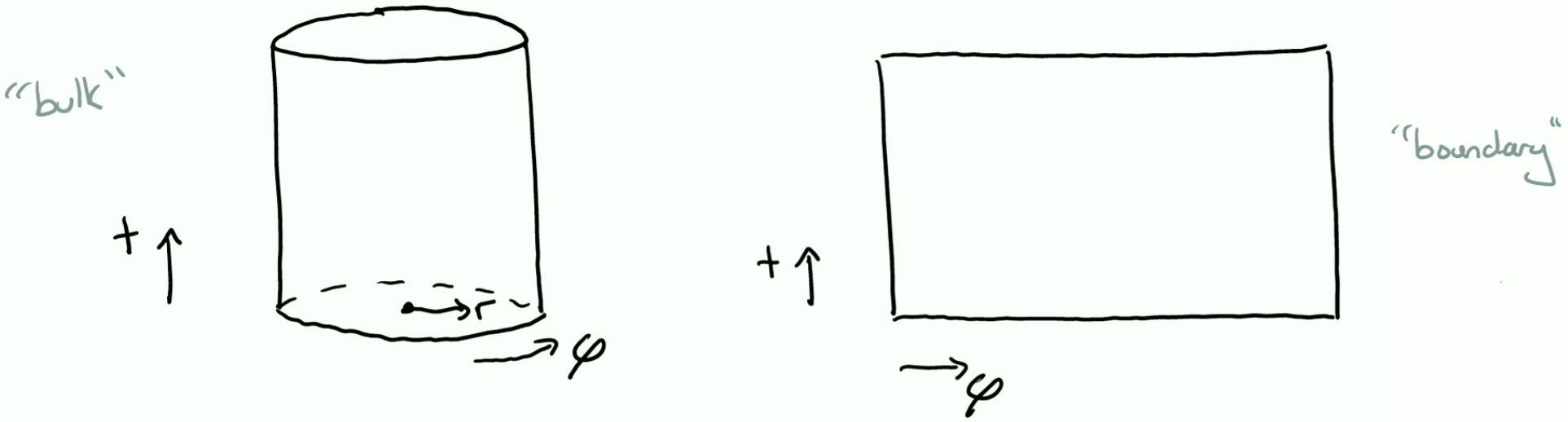
$t = 0$ "slice" of AdS



1 fish = 1 unit of physical distance

Global structure of AdS

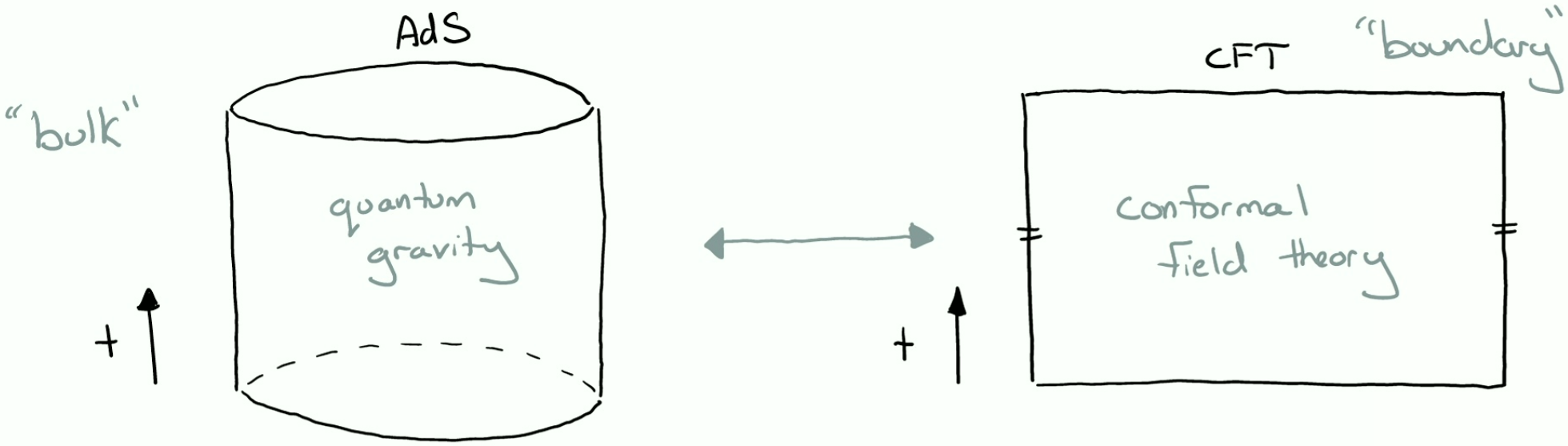
- If we put the time coordinate in, we get a picture like this?



- Note: "boundary" (at ∞) has a time direction.

  ↳ $\Lambda \geq 0$ we do <u>not</u> get this!

AdS/CFT Maldacena (1997)



- AdS/CFT states that quantum gravity in AdS_{d+1} is "equivalent" to a conformal field theory in the d dimensional boundary.

AdS/CFT correspondence

- AdS/CFT is (conjectured to be) a consistent theory of quantum gravity, valid at all energy scales.

- There are limitations to keep in mind, most importantly

  ↳ We don't live in an asymptotically AdS space!

- Still, theres a lot to learn from AdS/CFT

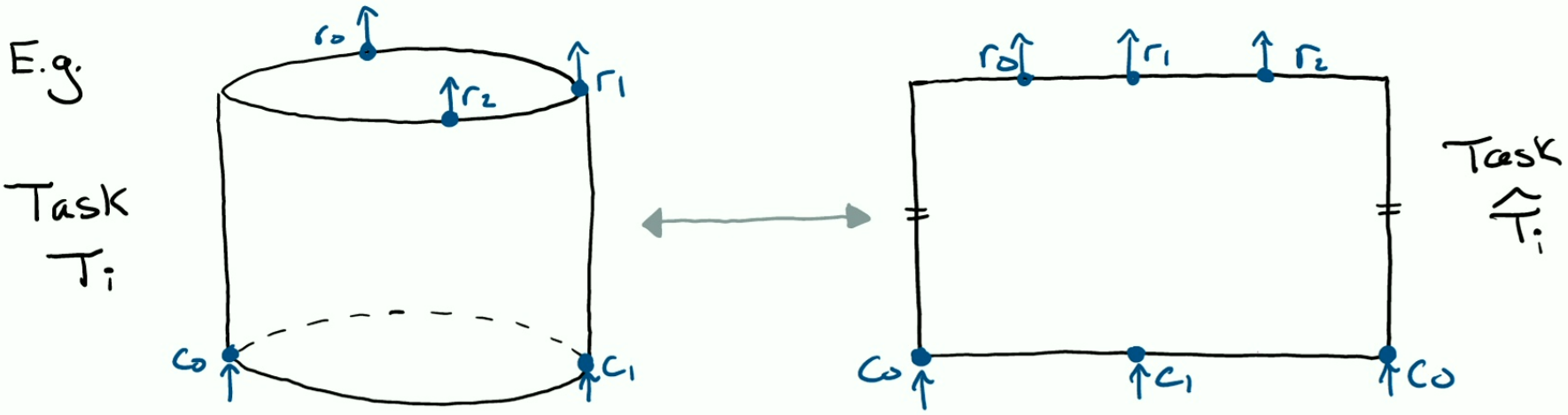
  (A) How can QM record a higher dimensional theory?

  (B) How can gravitational physics be recorded into QM?

Quantum tasks in holography AM (2019)

- To address (A) and (B), study quantum tasks in AdS/CFT

E.g.



$$\{(T_i, p_{success}(T_i))\} \longleftrightarrow \{(\hat{T}_i, p_{success}(\hat{T}_i))\}$$

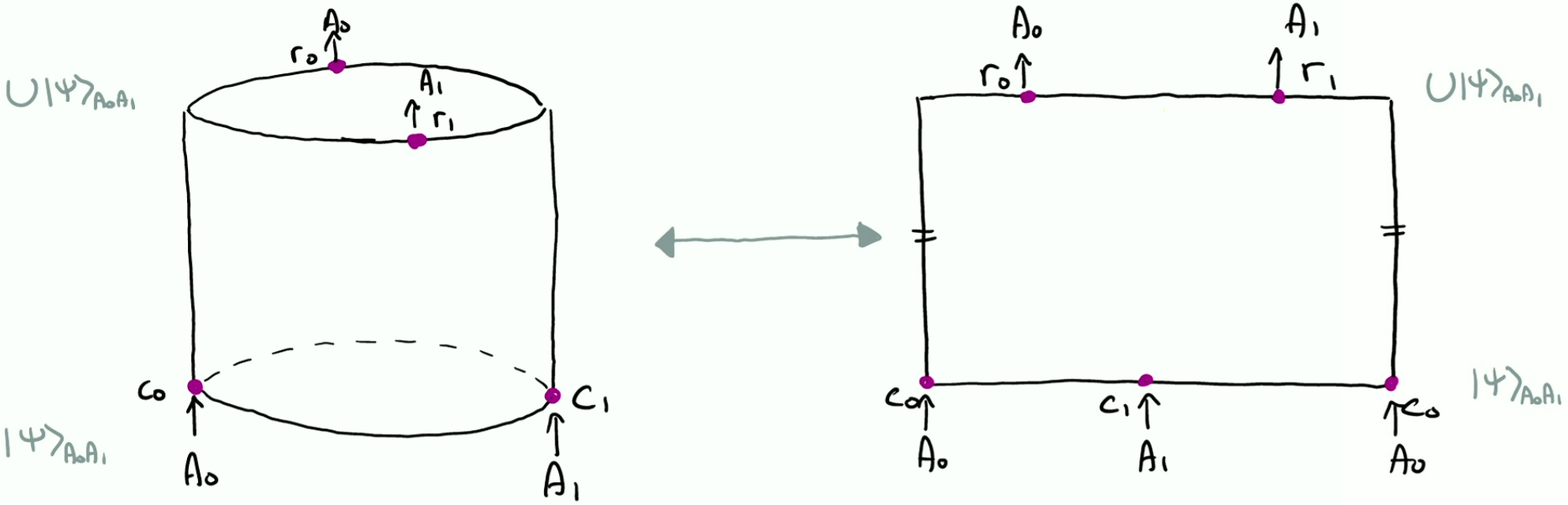
$$p_{suc}(T) = p_{suc}(\hat{T})$$

Operational perspective on AdS/CFT, inspired by Kent (2012)

Role of entanglement in holography

A puzzle

- To better understand how a theory can be "holographic", consider this task:



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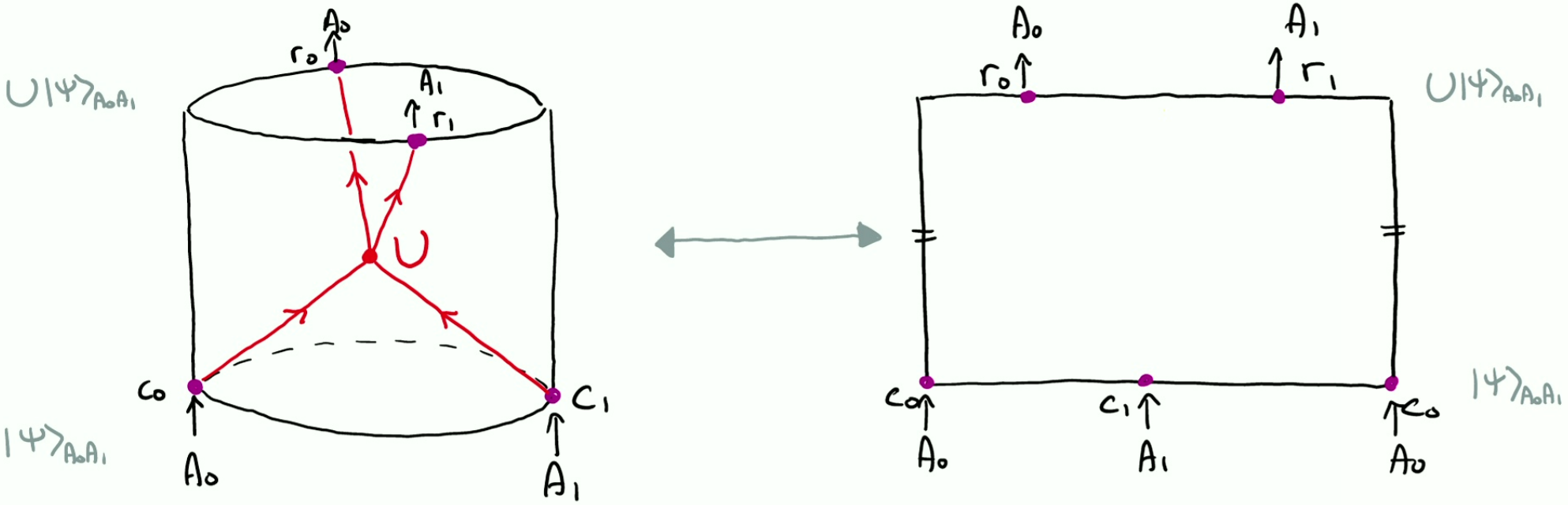

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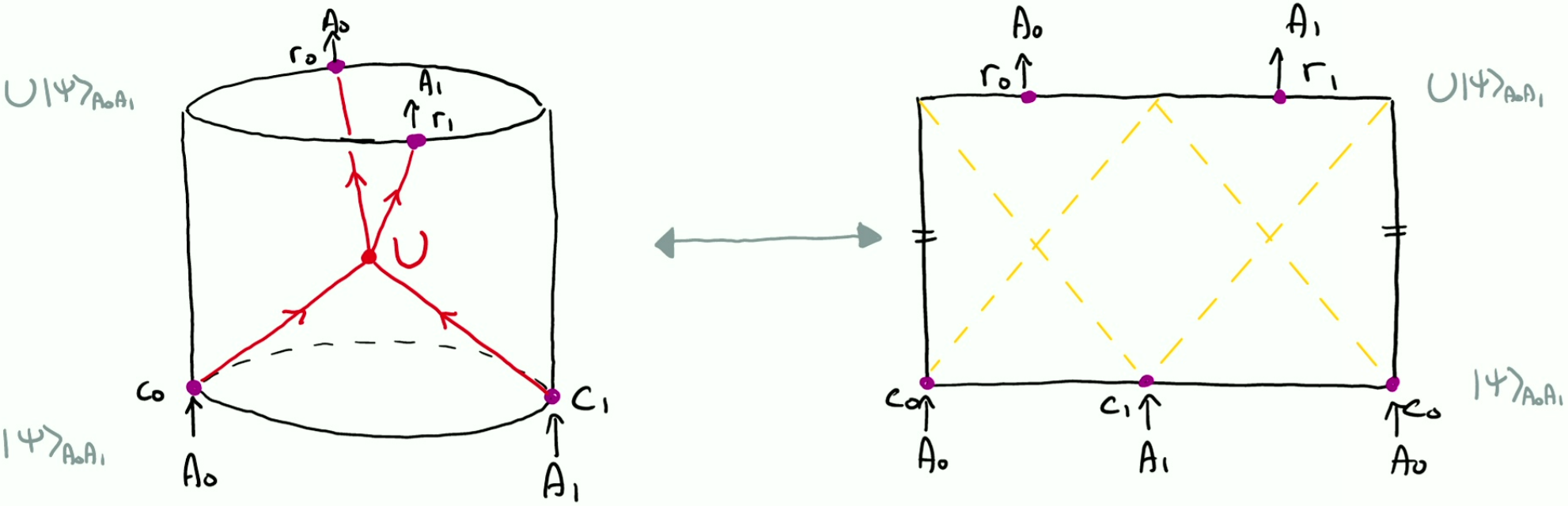

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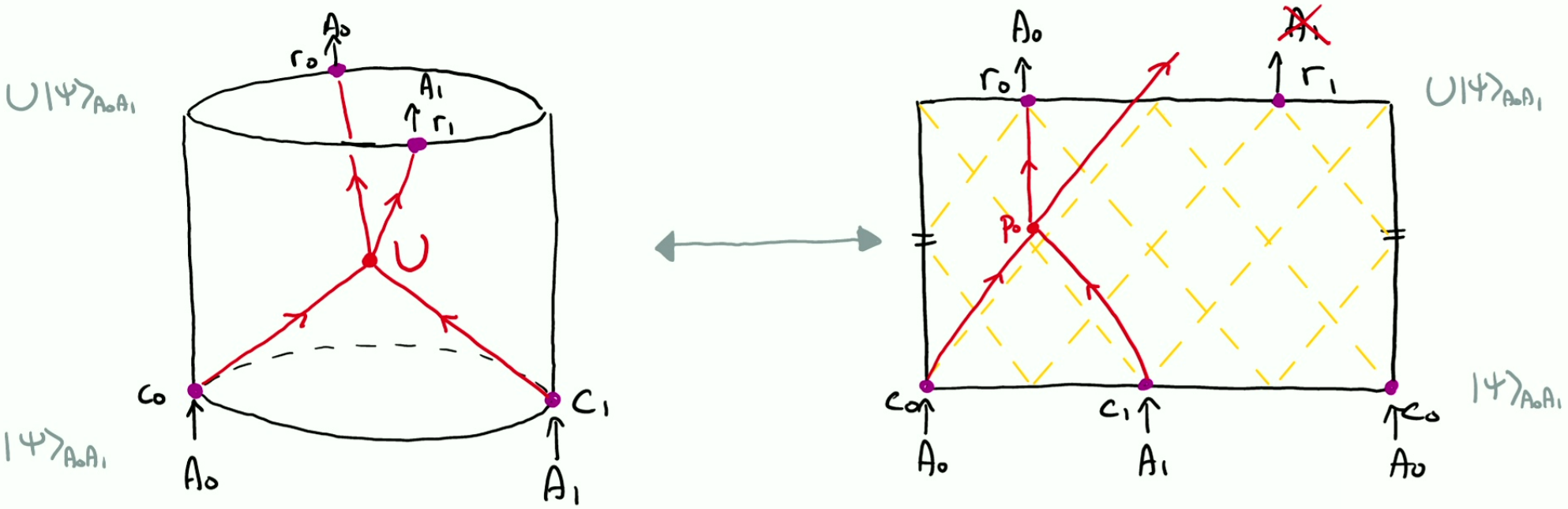

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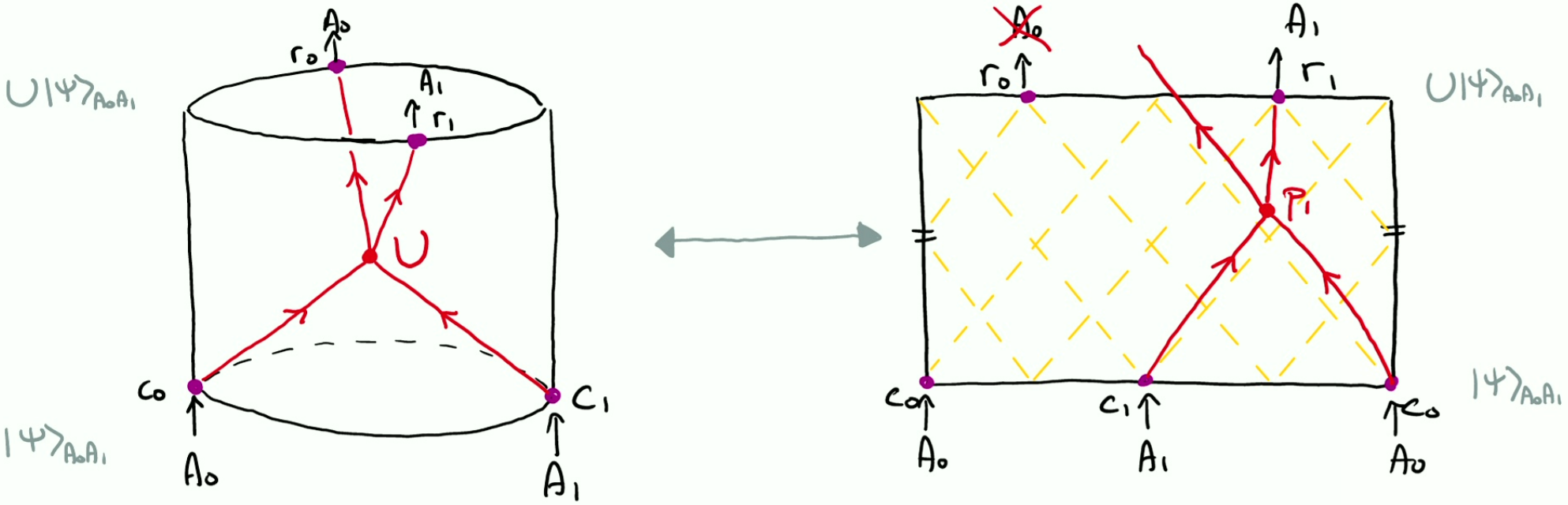
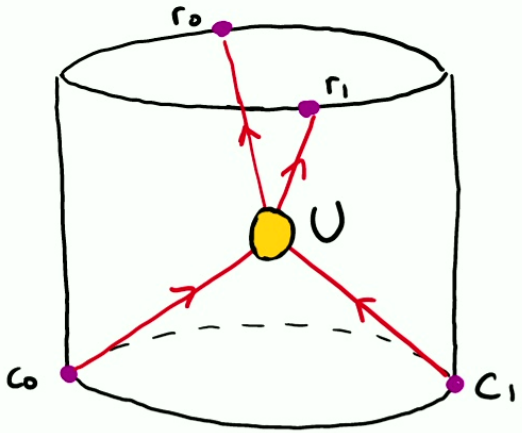
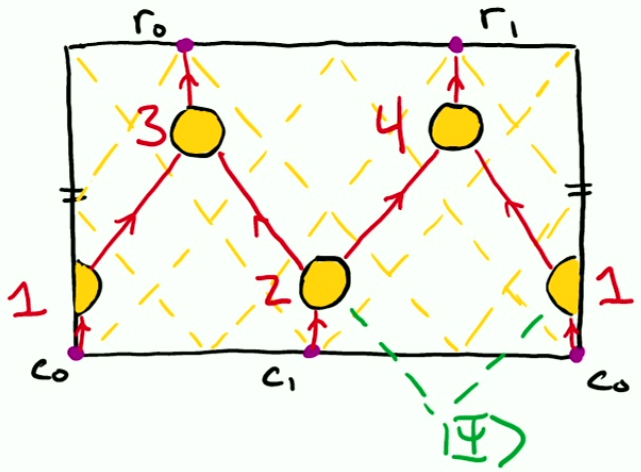


- What gives? Did we misunderstand AdS/CFT?

Position-verification in AdS/CFT (AM 2019)

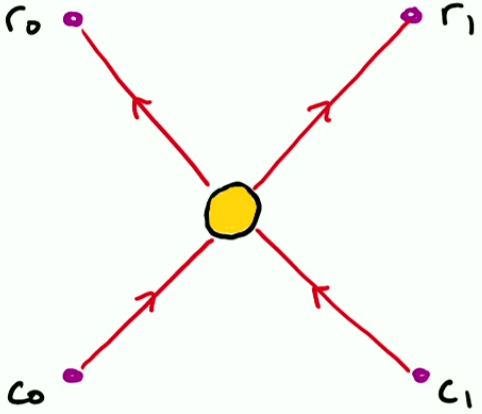
bulk

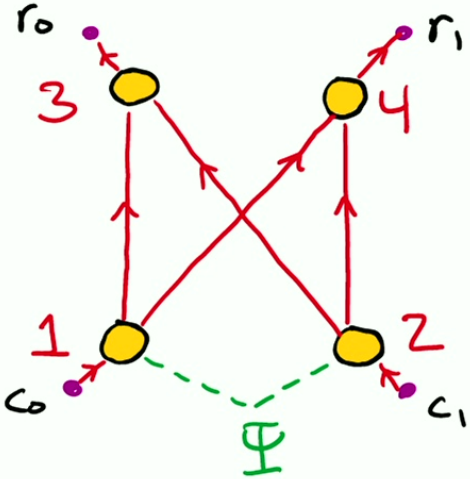




boundary

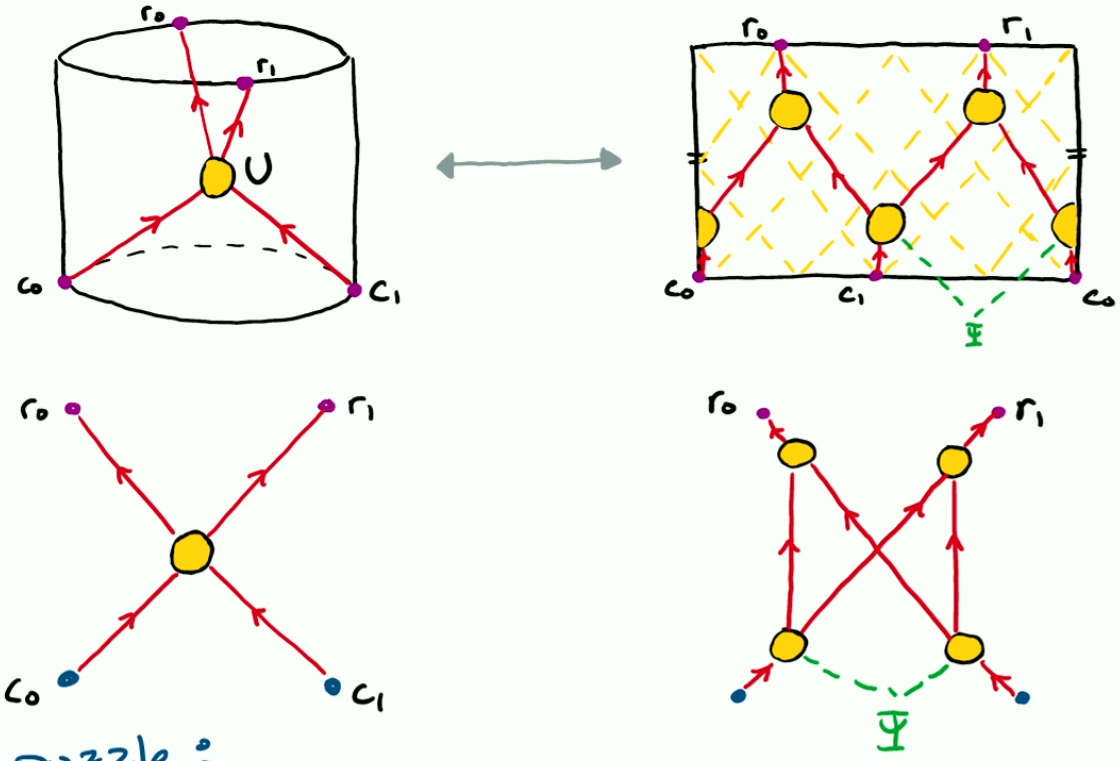
honest / local





cheating / non-local

Position-verification in AdS/CFT (AM 2019)



Resolves earlier puzzle:

Entanglement based, non-local procedure allows local bulk interactions to be reproduced in boundary

Entanglement in AdS/CFT

2→2 Connected wedge theorem

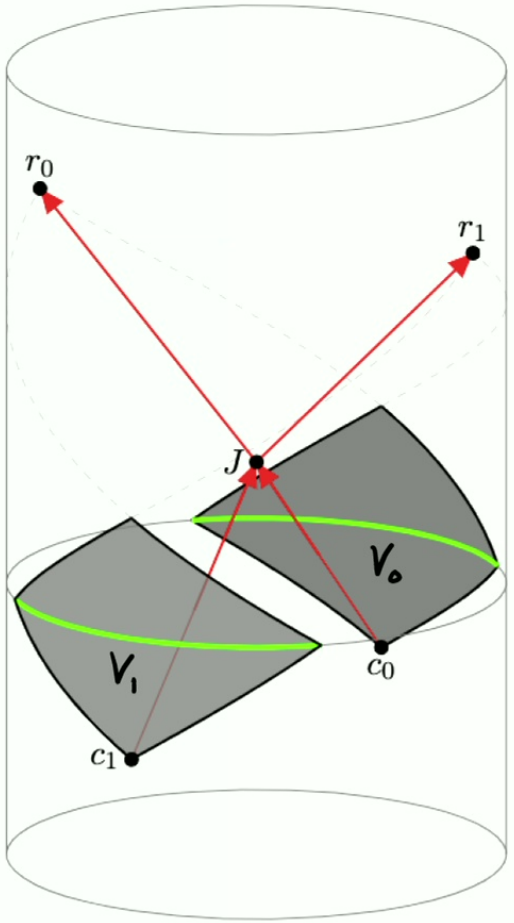
- Given this perspective on reproducing bulk physics using NLQC, expect the following:

If c_0, c_1, r_0, r_1 have a scattering region

(1)

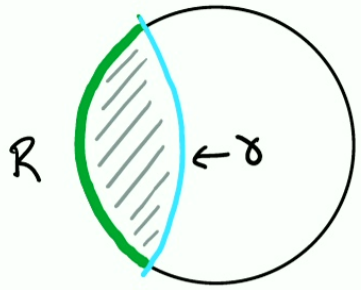
V_0 and V_1 are strongly correlated:

$I(V_0 : V_1) = O(1/G_N)$



Entanglement in AdS/CFT

- Mutual information : $I(V_0 : V_1) = S(V_0) + S(V_1) - S(V_0 V_1)$
- In AdS/CFT, the entropy plays another role:



$$S(R) = \min_{\gamma} \frac{\text{area}(\gamma)}{4G_N}$$

Ryu-Takayanagi formula (2006)

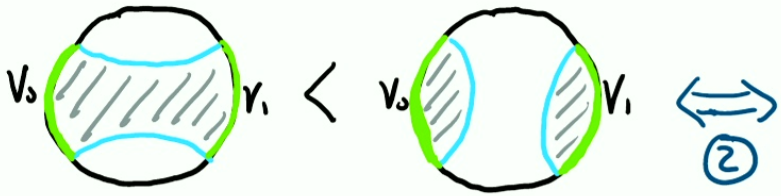
- Bulk gravitational data recorded into boundary correlation

↳ Bulk minimal surfaces ⟷ boundary correlation

2→2 Connected wedge theorem

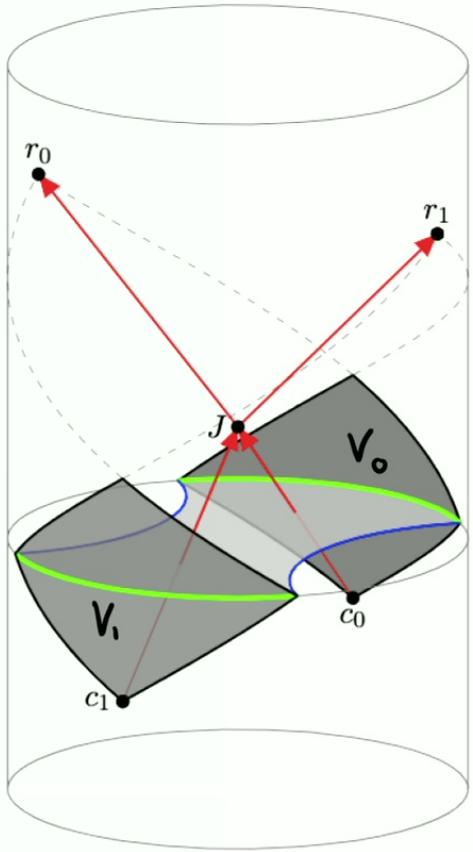
If c_0, c_1, r_0, r_1 have a scattering region

①



⟺ ②

V_0 and V_1 are strongly correlated, $I(V_0:V_1) = O(V/G_N)$

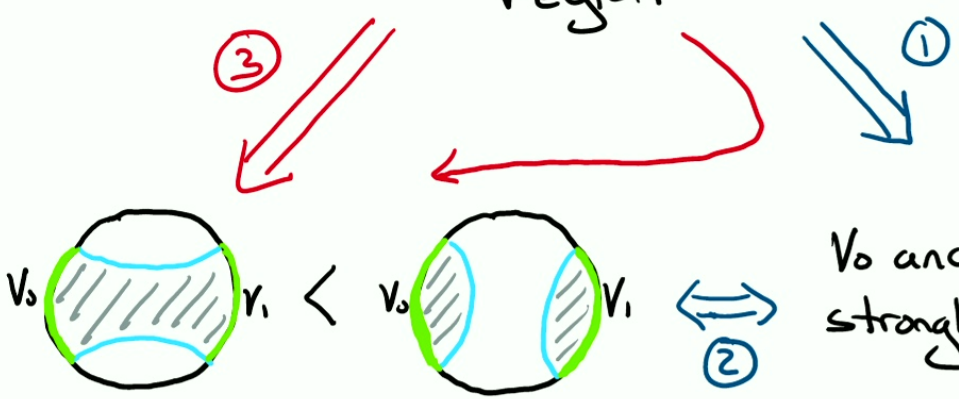


① Expected from necessity of entanglement

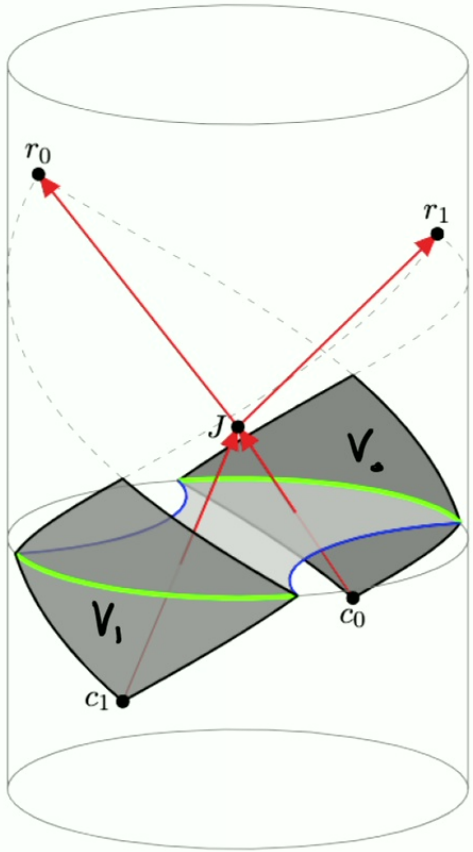
② Follows from Ryu-Takayanagi formula

2→2 Connected wedge theorem

If c_0, c_1, r_0, r_1 have a scattering region



V_0 and V_1 are strongly correlated, $I(V_0:V_1) = O(1/G_N)$



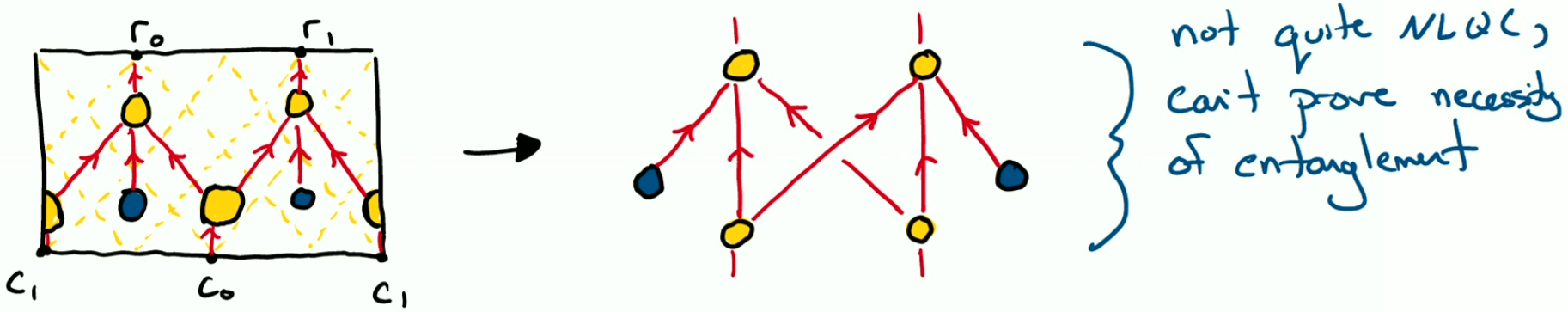
③ Proven directly in G.R.

↳ Assume NEC, use focusing theorem.

AM, Jon Sorce, Geoff Penington (2020) / AM, Jon Sorce, Beni Yoshida (2022)

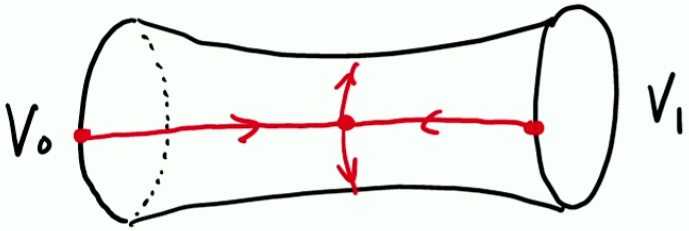
Higher dimensional interactions from NLQC

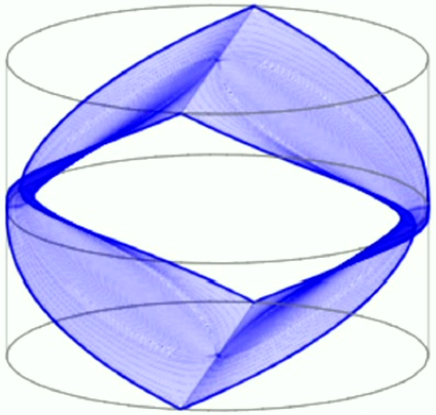
- CFT using NLQC to reproduce bulk interactions is one resolution to our puzzle, but is that really whats going on?
- Perspective is supported by CW theorem tummy out correct, but a loophole remains:



Interaction inside of black holes

- There are two approaches here, see also Cree + Dolev 2022
- To briefly mention one of them:
  go to a setting w/o "intermediate" regions, e.g. wormholes:





- Precise sense in which you can approximate the original geometry with a wormhole → AM, Van Raamsdonk 2021

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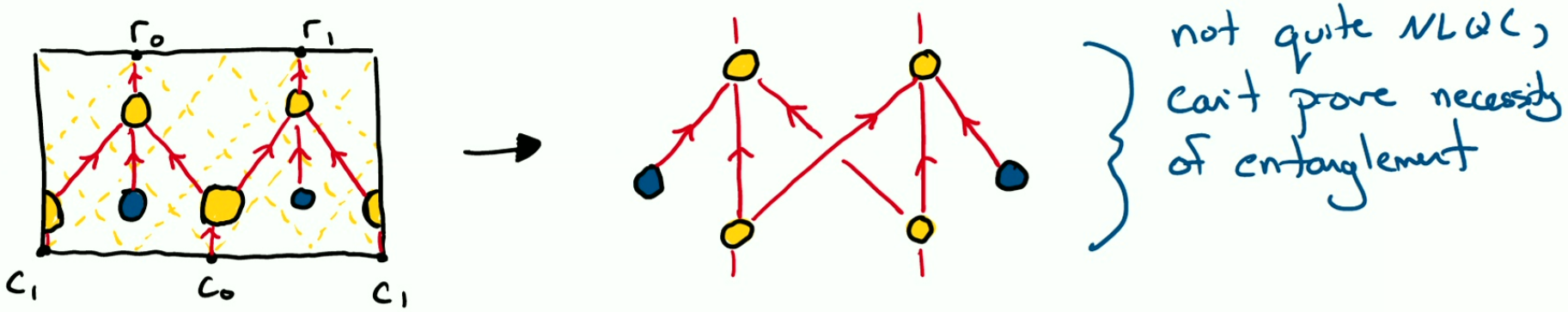


↳ Can we get back to the simpler NLQC setting? (or understand this new setting better?)

What would we like to know?

- One way to phrase what we have in AdS/CFT:

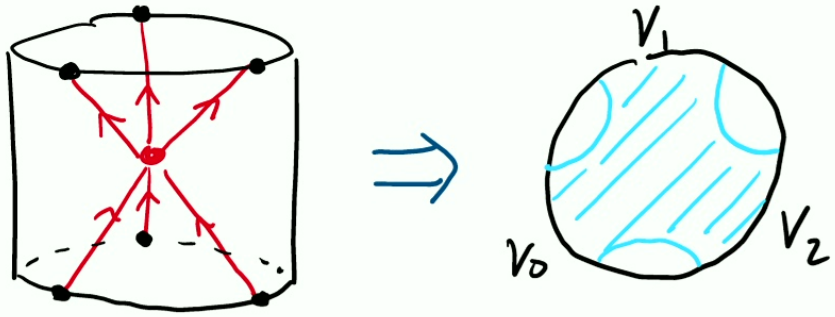
$$\left\{\begin{array}{c}\text{can compute in spacetime} \\ \text{region of area } A \\ \text{of holographic spacetime}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{can implement in NLQC} \\ \text{using } I \sim O(A/G_N)\end{array}\right\}$$

(1) Can we better understand what the set on the left is here? (K-frs talk)

(2) Can we get better constraints on RHS? (Discussed in AM 2021)
(and so constrain LHS)

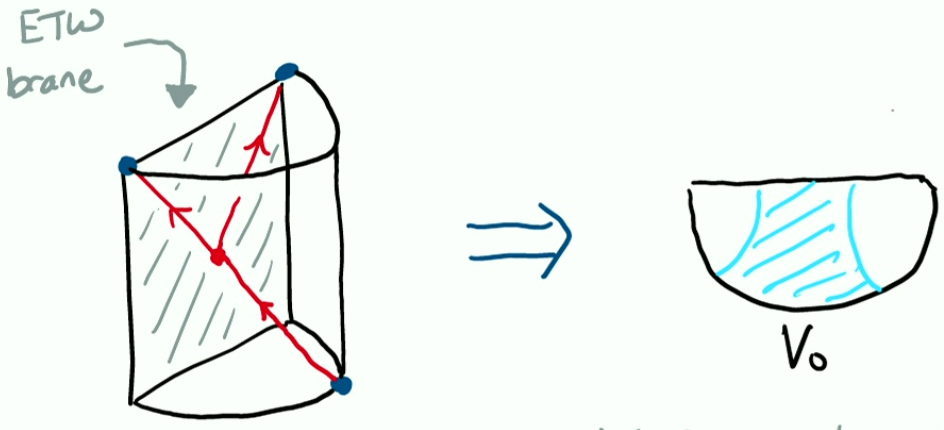
(3) Can we better understand how AdS/CFT does NLQC? (see Beni's talk)
(and get some insights towards better NLQC?)

n→n connected wedge theorem ①



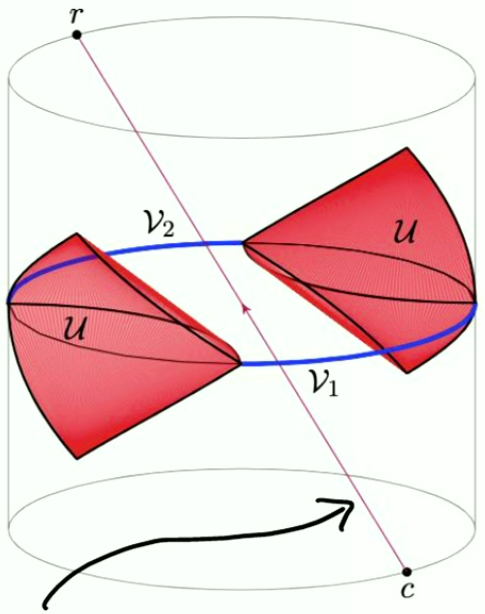
AM, J. Sorce, B. Yoshida (2022)

1→2 connected wedge theorem ②



AM, D. Wakeham (2021)

Privacy-duality theorem ③



Causal curve from $c \to r$ that avoids U

$$I(V_0 : V_1 | U) = O(1/G_N)$$

AM (2021)

Summary

- Holographic principle: QG in d dimensions has equivalent, QM description in $d-1$ dimensions

- Role of NLQC: At least in concrete realization of this principle, NLQC is how higher dimensional interactions are reproduced in QM.

One route to better understanding QG is via NLQC! (and vis versa?)

Thanks!

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