

Title: Non-local quantum computation meets quantum gravity

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Quantum gravity meets quantum cryptography
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What is quantum gravity?

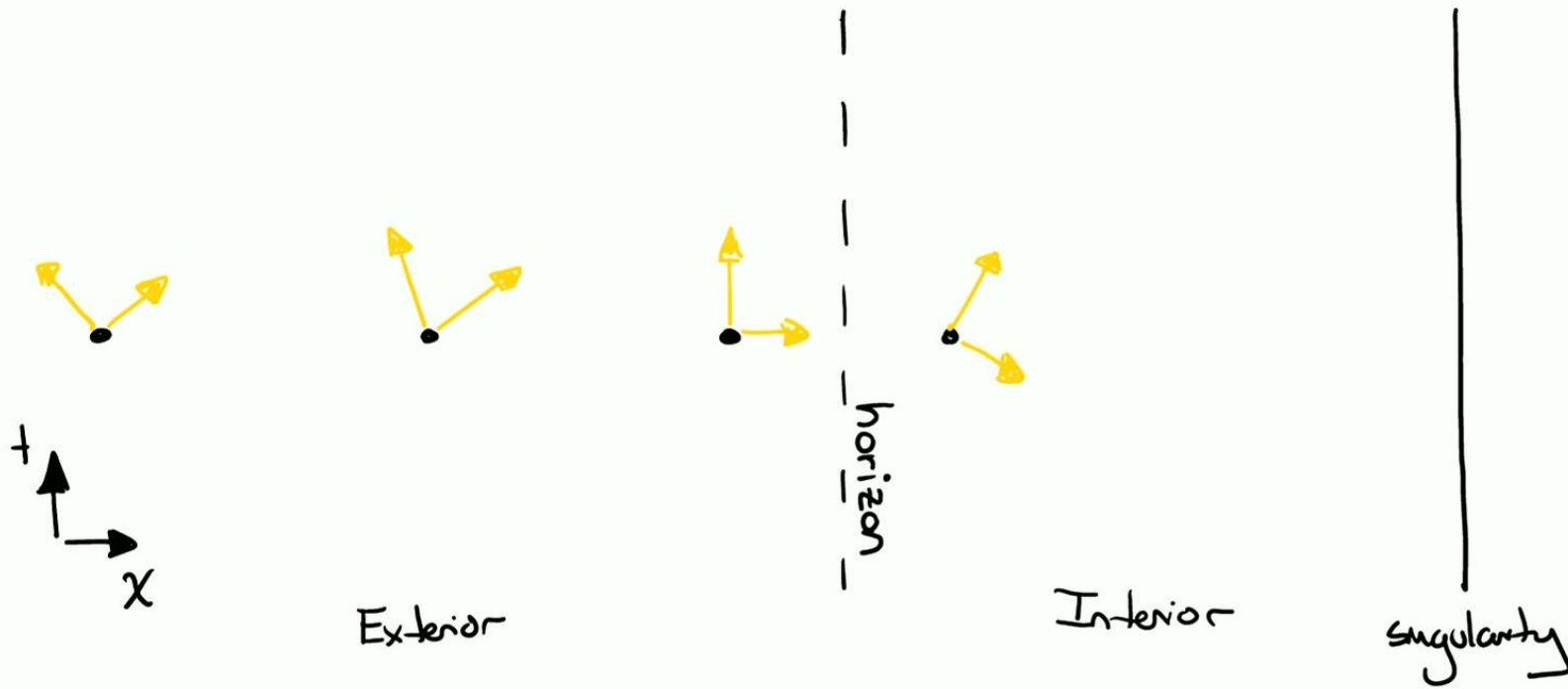
- Quantum gravity tries to fit gravitational physics into the framework of quantum mechanics
- This is a challenge not just to our understanding of gravity, but also our understanding of quantum systems.



How do we construct quantum systems that exhibit gravitational physics?

Black holes

- A black hole is a region in spacetime from which it is impossible to escape to far away.

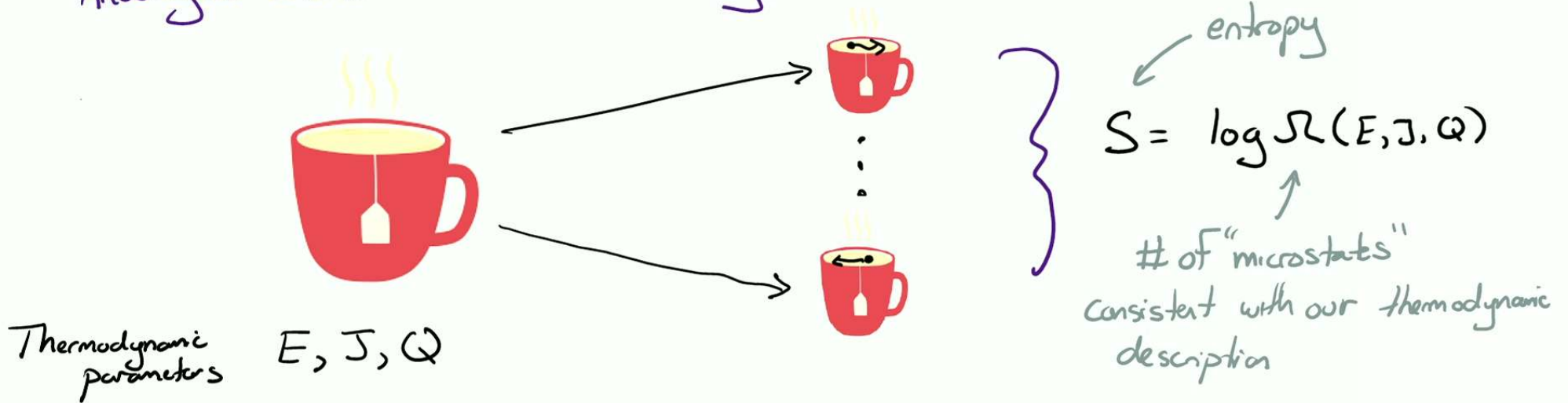


Properties of black holes

- Black holes are described very simply:

"No-hair" theorem: once M, J, Q are specified, there is a unique static solution to general relativity + E&M

- Analogous statement about ordinary matter is false:



Do black holes have entropy?

- General relativity describes black holes only in terms of macroscopic / thermodynamic quantities M, J, Q

↳ Quantum gravity should provide the microstates

Analogy:

general relativity	↔	quantum gravity
thermodynamics	↔	statistical mechanics

- Can extract the entropy from thermodynamics, provides hints about stat. mech

↳ Can we also extract entropy from general relativity?

Black holes and entropy

Idea:

Black holes have microstates, specifically

$$S_{bh} = \log \Omega_{bh} = \frac{A_{bh}}{4G\hbar}$$

Support for this:

- Black hole areas always increase in classical gravity
- Thermodynamic systems should have a temperature T , which we can relate to energy by $TdS = dE$ ($M=E$ here)

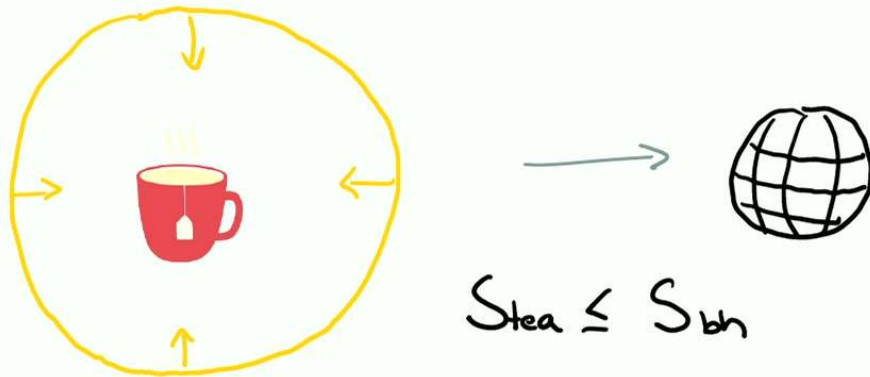
↳ Leads to $T = \frac{1}{8\pi G\hbar M}$ which Hawking verified explicitly!

Entropy and area

- Learned that

$$S_{bh} = \frac{A_{bh}}{4G_N}$$

- Pretty weird! For ordinary matter, always find $S_{th} \propto \text{Volume}$
- But, notice that we can turn ordinary matter into a black hole by adding entropy



Conclude that all matter has at most an area worth of degrees of freedom.

The holographic principle

- Based on these ideas, 't Hooft and Susskind suggested the following:

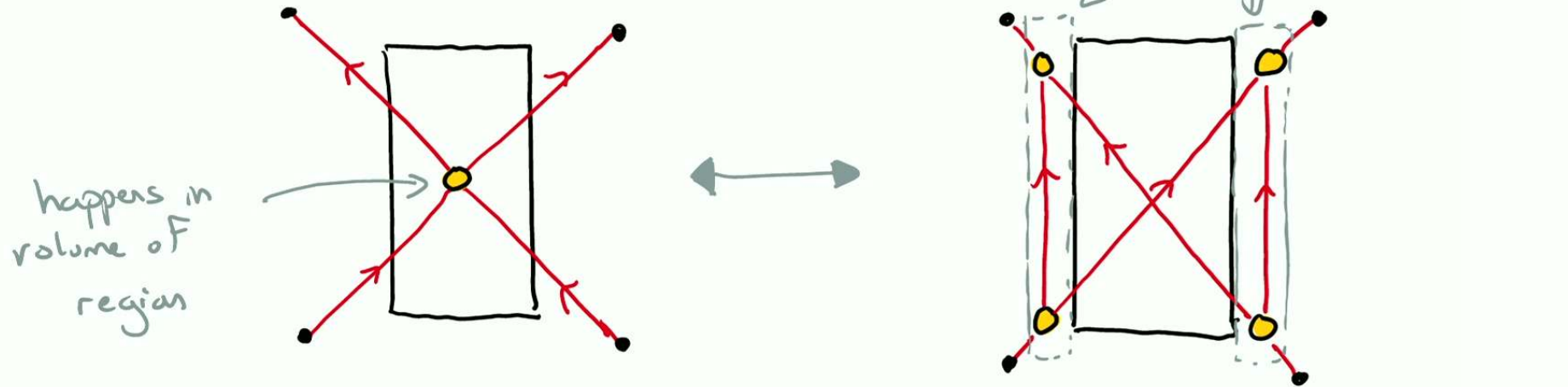
Quantum gravity in $3+1$ dimensions should have an alternative, non-gravitational description in $2+1$ dimensions

The holographic principle

- Based on these ideas, 't Hooft and Susskind suggested:

Quantum gravity in $3+1$ dimensions should have an alternative, non-gravitational description in $2+1$ dimensions

"Smells like" position verification:



Towards holographic quantum gravity

AdS space and holography

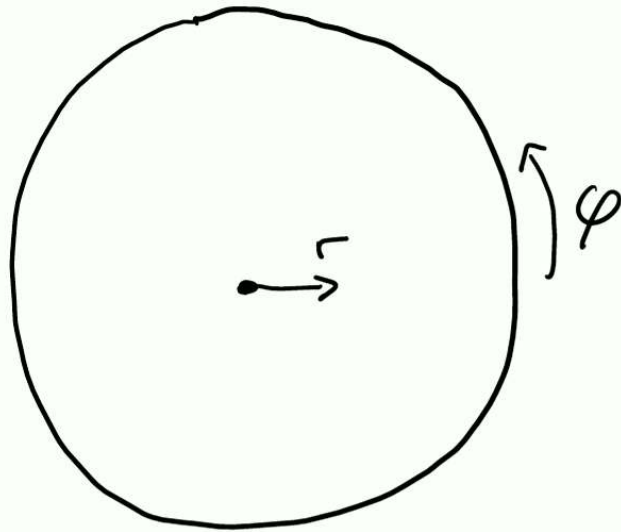
- "Holography" is easiest to realize in asymptotically anti de Sitter space, which are solutions in classical gravity

$$S_{\text{grav}} = \int d^d x \sqrt{g} (R + 2\Lambda) \quad \text{with} \quad \Lambda < 0$$

- For simplicity, we will study these spacetimes in $2+1$ dimensions, and look for a $1+1$ dimensional holographic theory.
- AdS space is counterintuitive, but lets try to get a feel for it...

Hyperbolic disk

- First consider only the spatial directions:



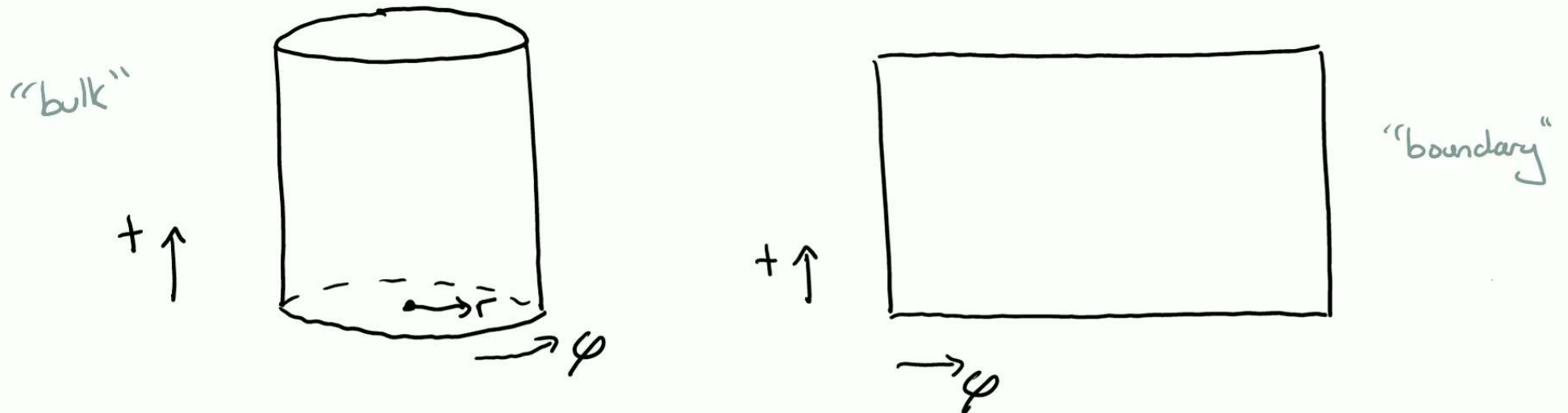
$t=0$ "slice" of AdS



1 fish = 1 unit of physical distance

Global structure of AdS

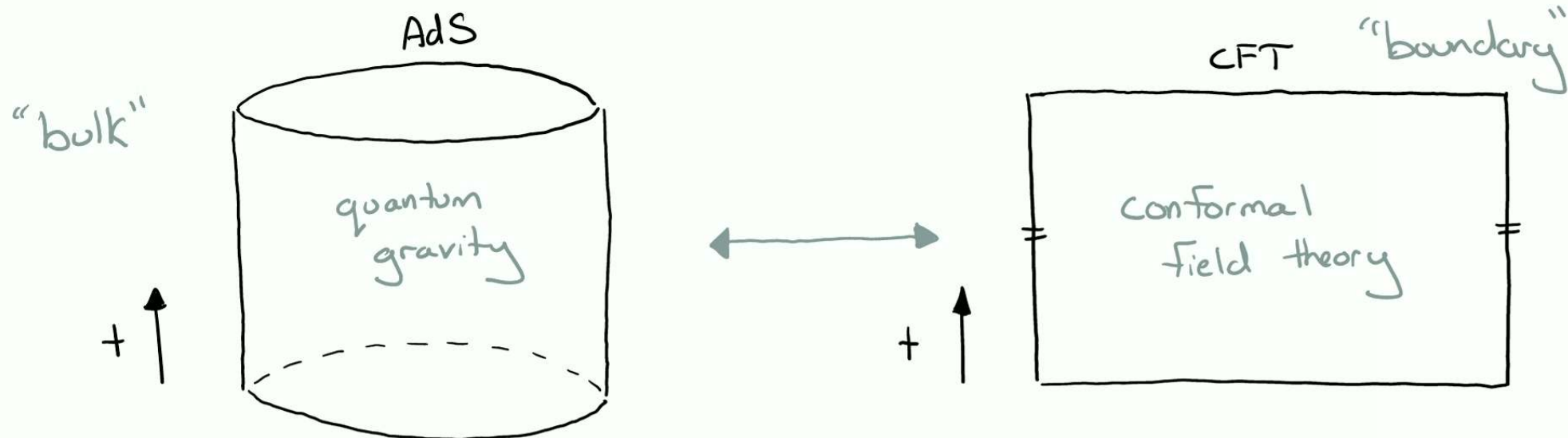
- If we put the time coordinate in, we get a picture like this?



- Note: "boundary" (at ∞) has a time direction.

↳ $\Lambda \geq 0$ we do not get this!

AdS/CFT Maldacena (1997)



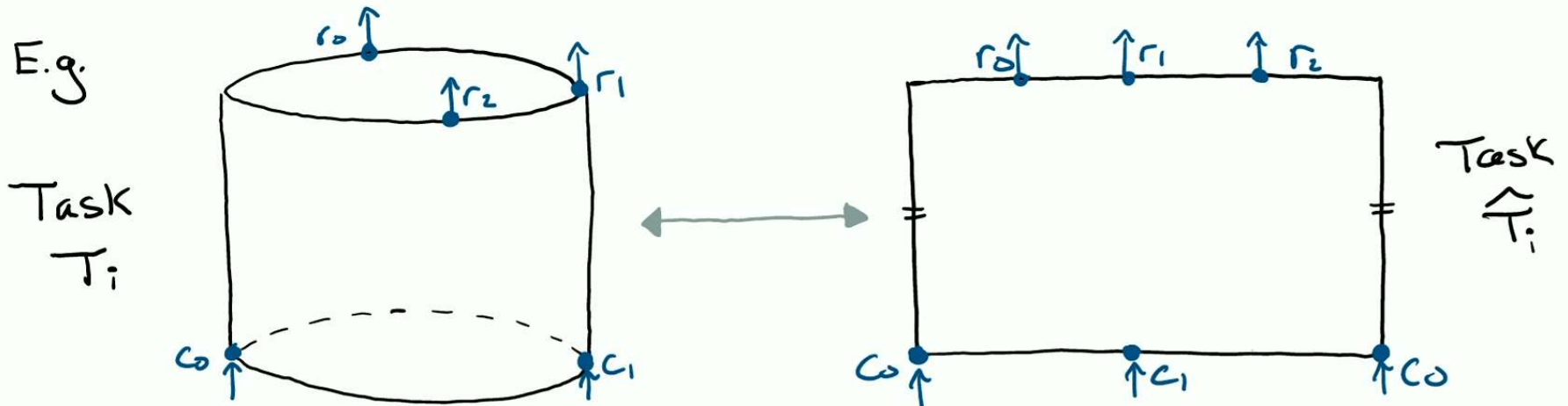
- AdS/CFT states that quantum gravity in AdS_{d+1} is "equivalent" to a conformal field theory in the d dimensional boundary.

AdS/CFT correspondence

- AdS/CFT is (conjectured to be) a consistent theory of quantum gravity, valid at all energy scales.
- There are limitations to keep in mind, most importantly
 - ↳ We don't live in an asymptotically AdS space!
- Still, there's a lot to learn from AdS/CFT
 - Ⓐ How can QM record a higher dimensional theory?
 - Ⓑ How can gravitational physics be recorded into QM?

Quantum tasks in holography AM (2019)

- To address (A) and (B), study quantum tasks in AdS/CFT



$$\{(T_i, p_{\text{success}}(T_i))\} \longleftrightarrow \{(\hat{T}_i, p_{\text{success}}(\hat{T}_i))\}$$

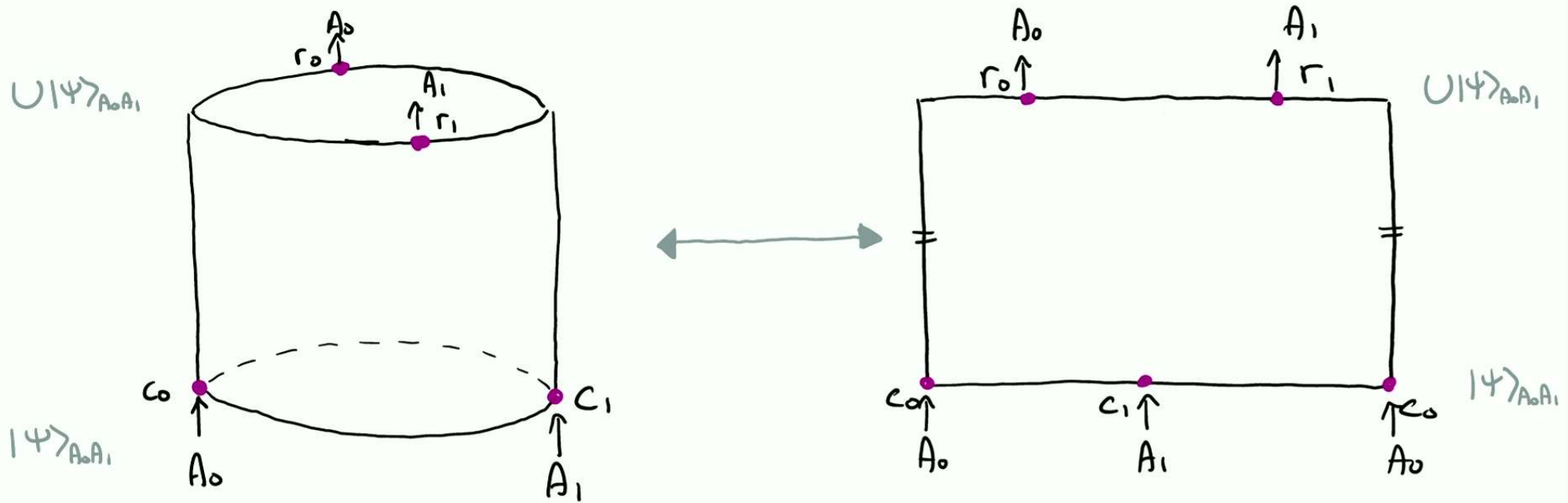
$$p_{\text{suc}}(T) = p_{\text{suc}}(\hat{T})$$

Operational perspective on AdS/CFT, inspired by Kent (2012)

Role of entanglement in holography

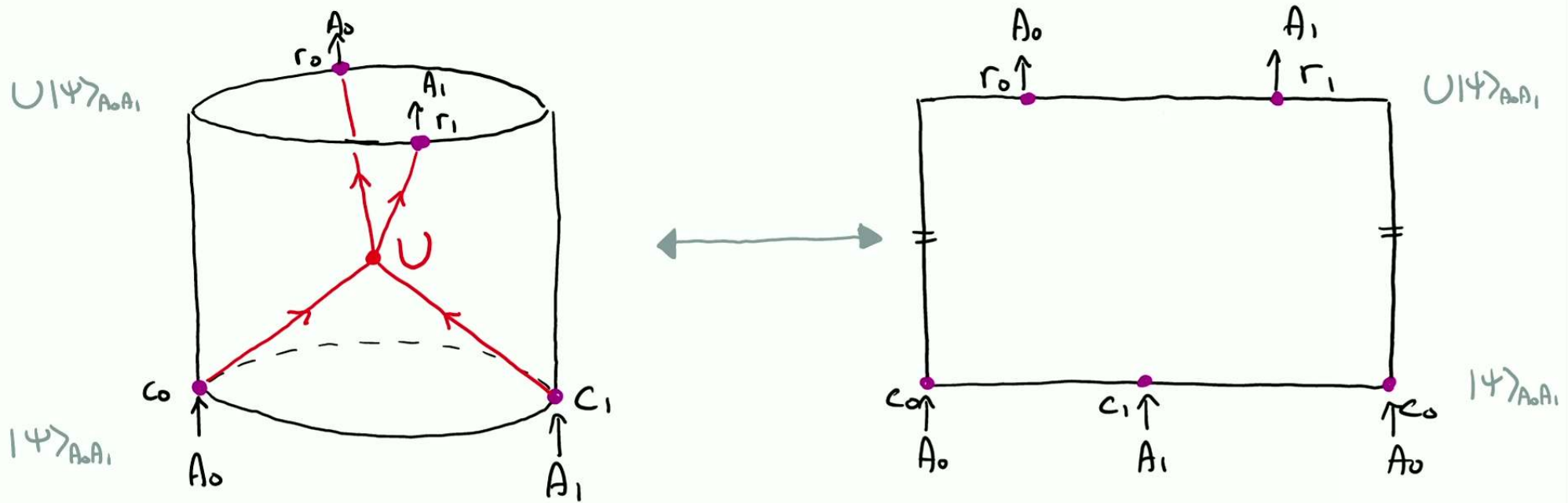
A puzzle

- To better understand how a theory can be "holographic", consider this task:



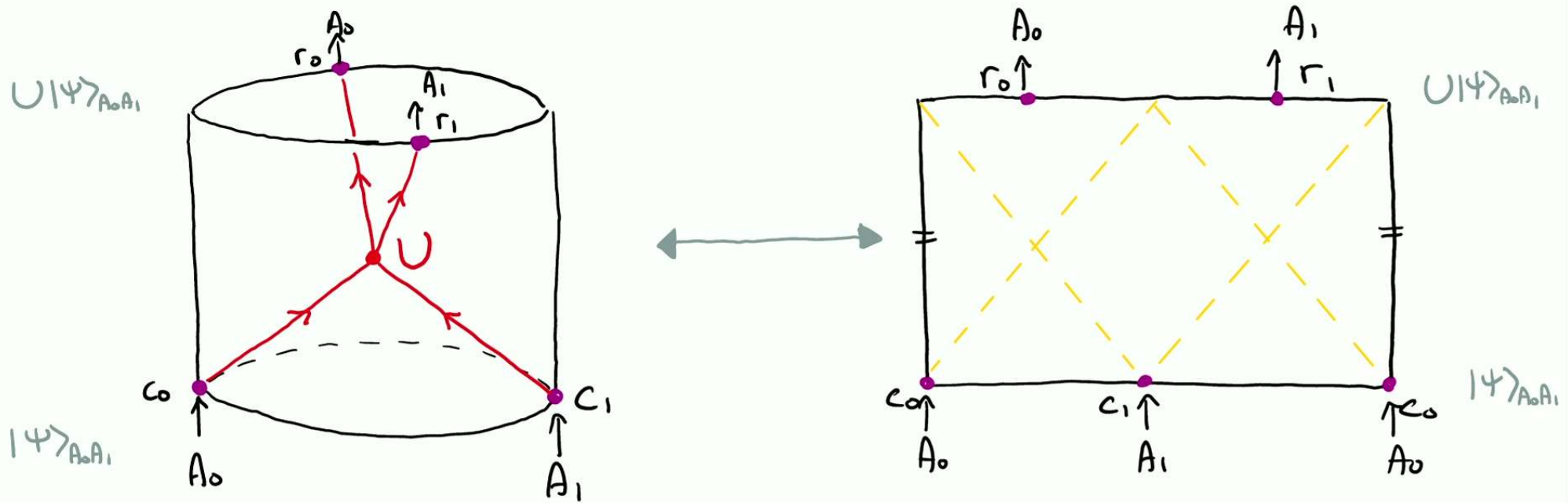
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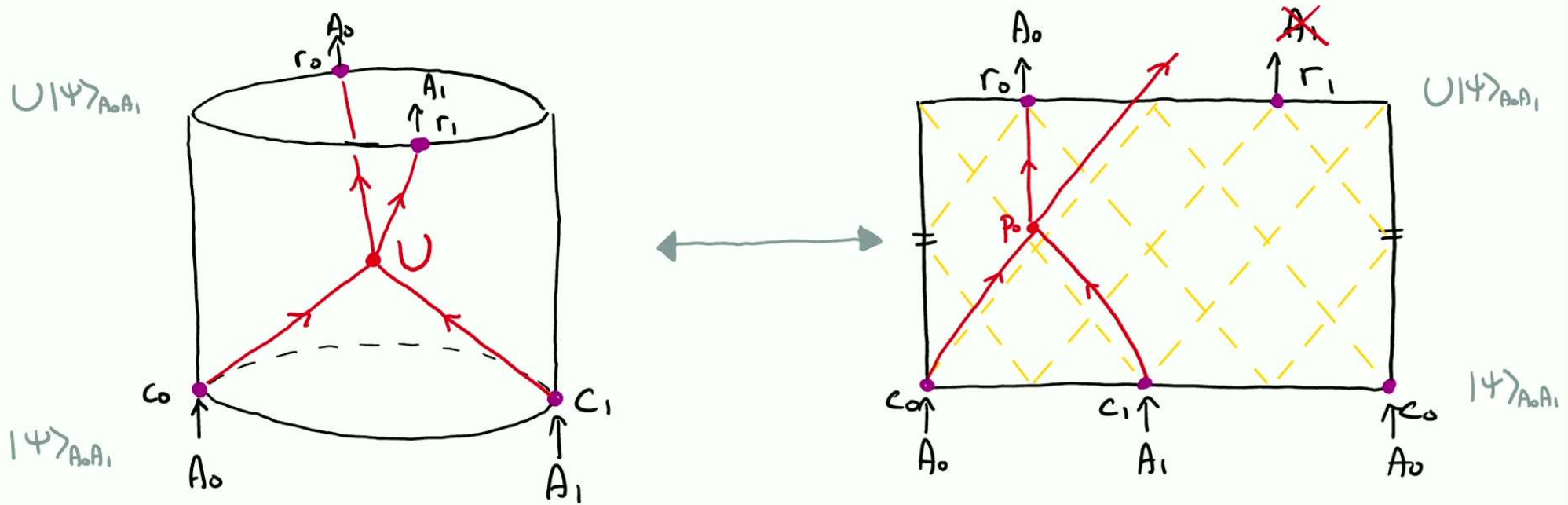
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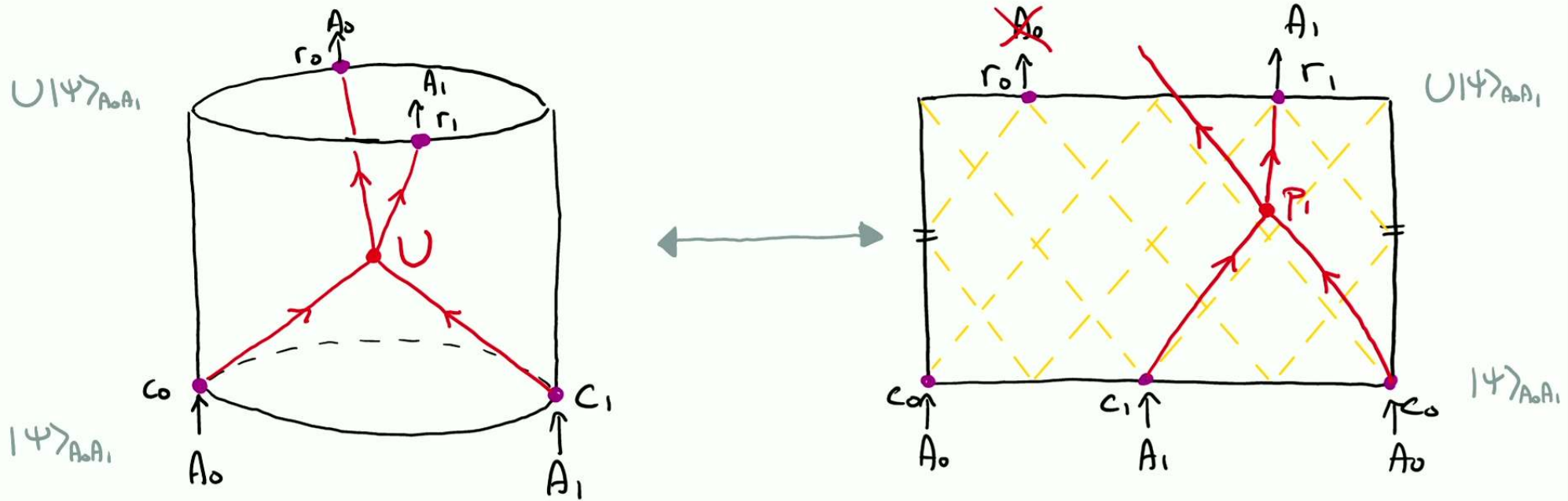
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A puzzle

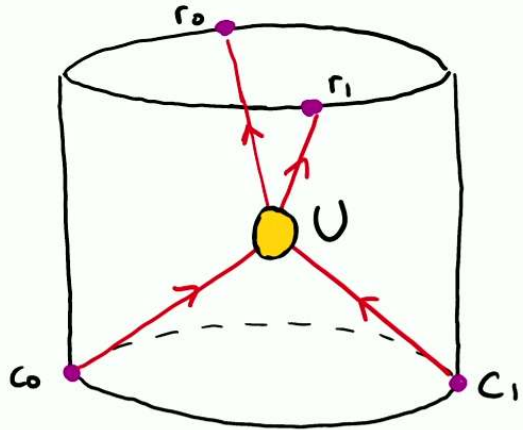
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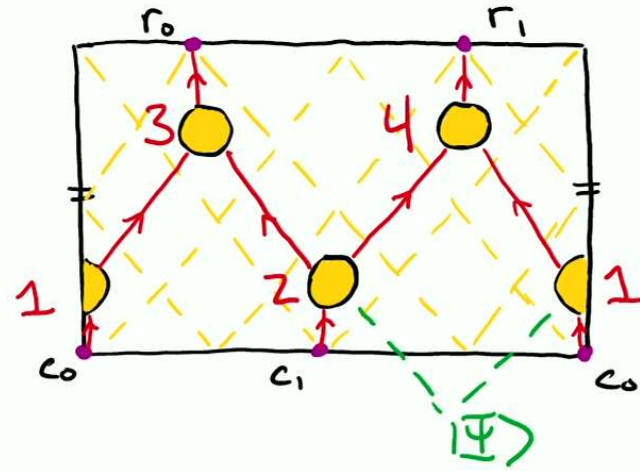
- What gives? Did we misunderstand AdS/CFT?

Position-verification in AdS/CFT (AM 2019)

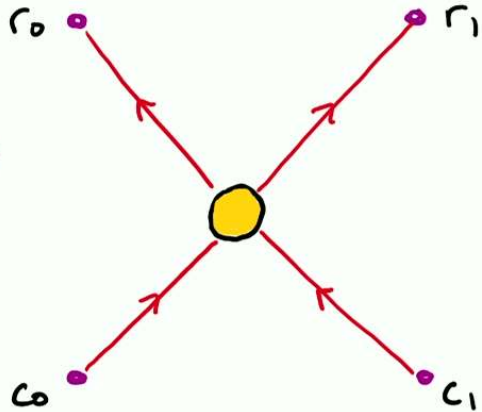
bulk



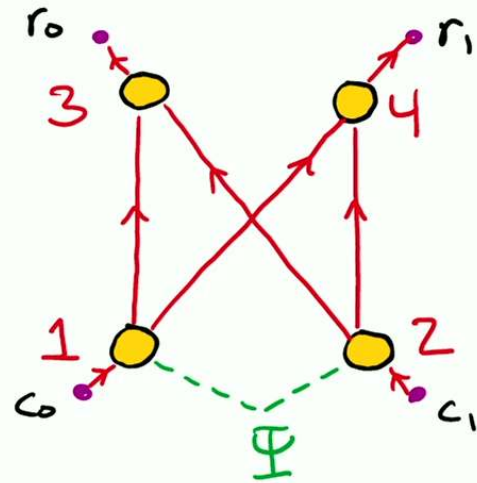
boundary



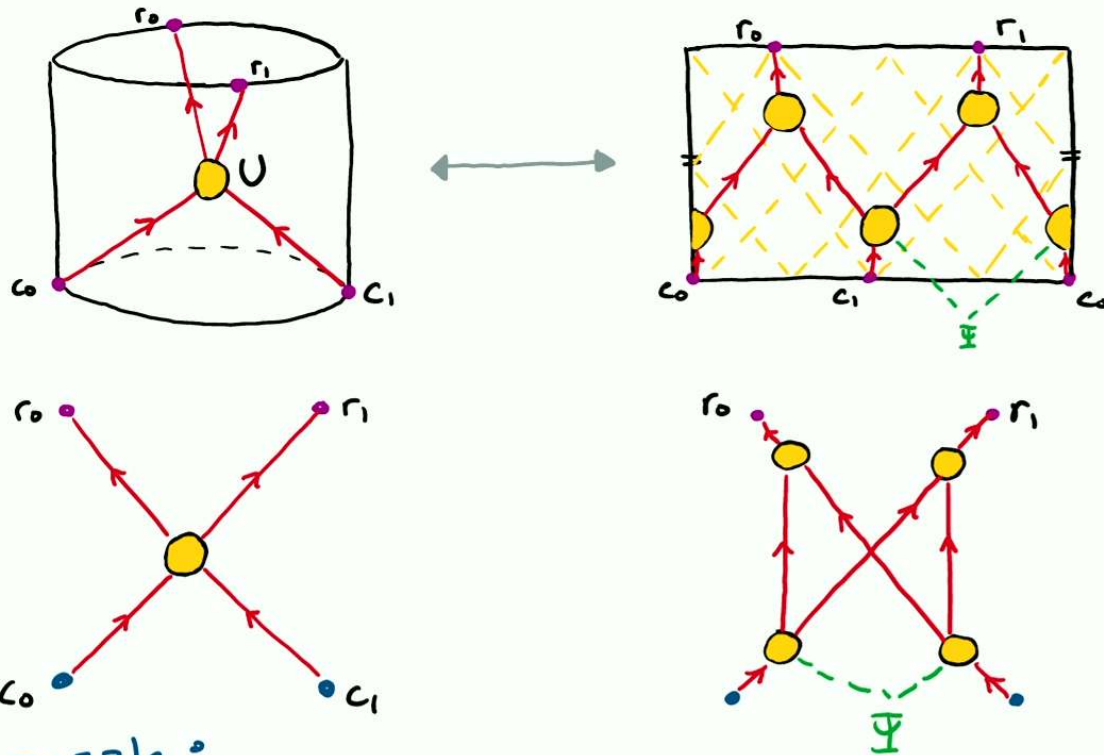
honest / local



cheating / non-local



Position-verification in AdS/CFT (AM 2019)



Resolves earlier puzzle:

Entanglement based, non-local procedure allows local bulk interactions to be reproduced in boundary

Entanglement in AdS/CFT

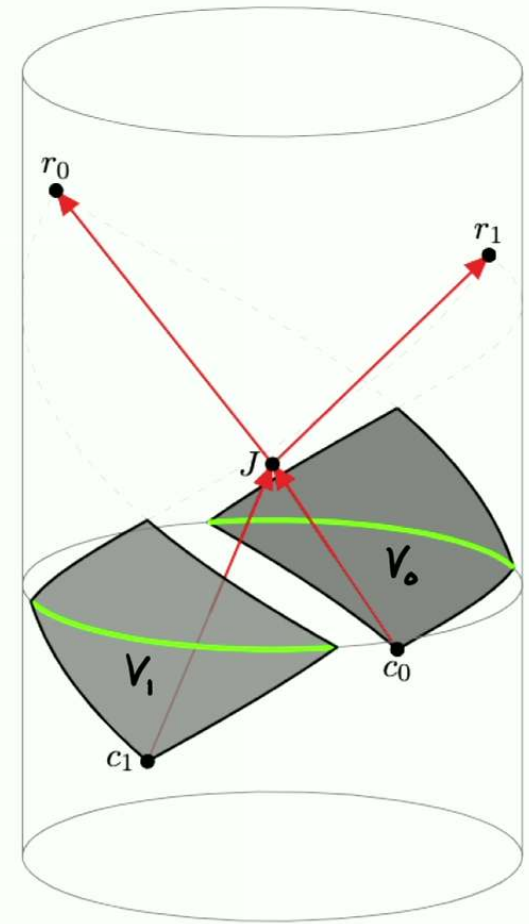
Z → Z Connected wedge theorem

- Given this perspective on reproducing bulk physics using NLQC, expect the following:

If c_0, c_1, r_0, r_1
have a scattering
region

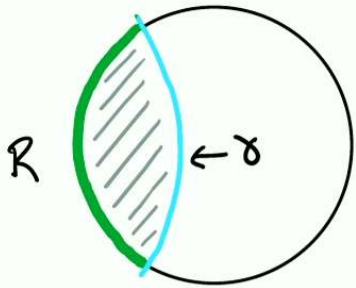


V_0 and V_1 are
strongly correlated:
 $I(V_0:V_1) = O(1/G_N)$



Entanglement in AdS/CFT

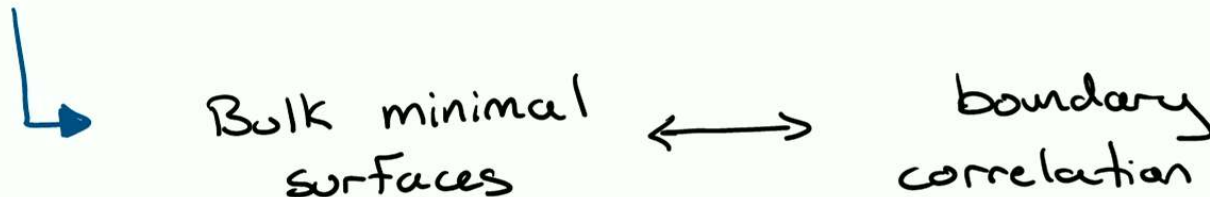
- Mutual information : $I(v_0:v_1) = S(v_0) + S(v_1) - S(v_0v_1)$
- In AdS/CFT, the entropy plays another role:



$$S(R) = \min_{\gamma} \frac{\text{area}(\gamma)}{4G_N}$$

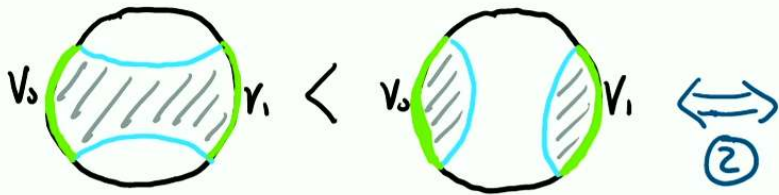
Ryu-Takayanagi
Formula
(2006)

- Bulk gravitational data recorded into boundary correlation



2→2 connected wedge theorem

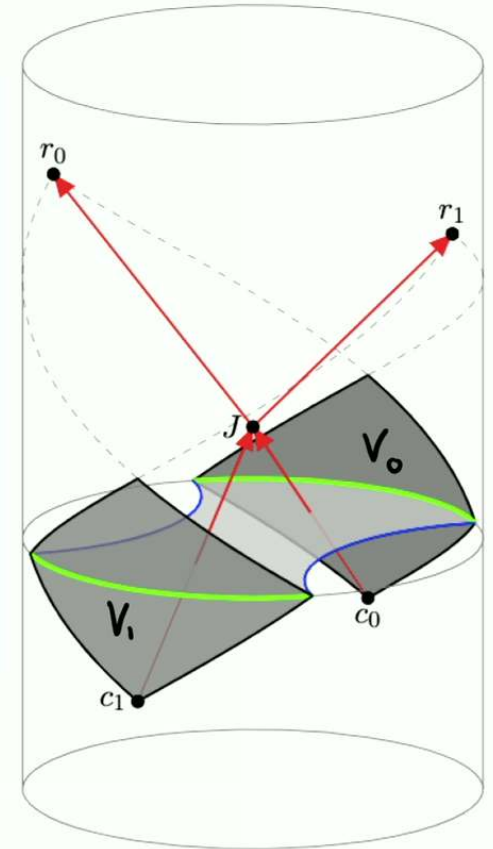
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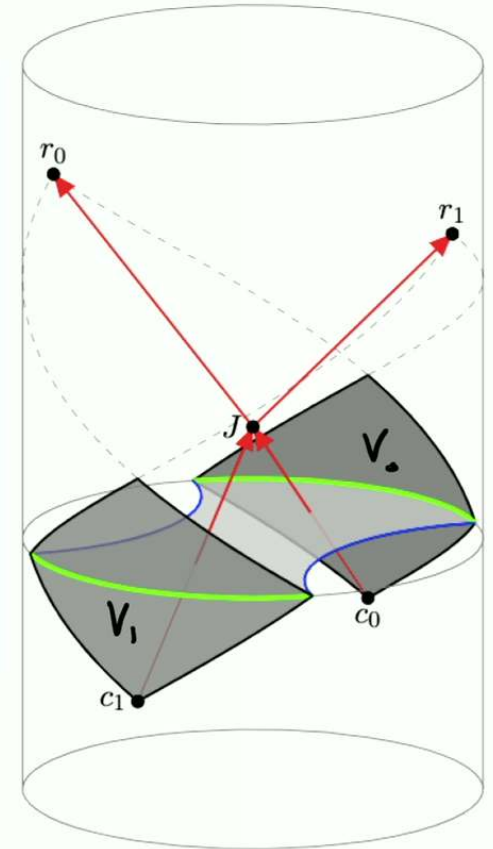
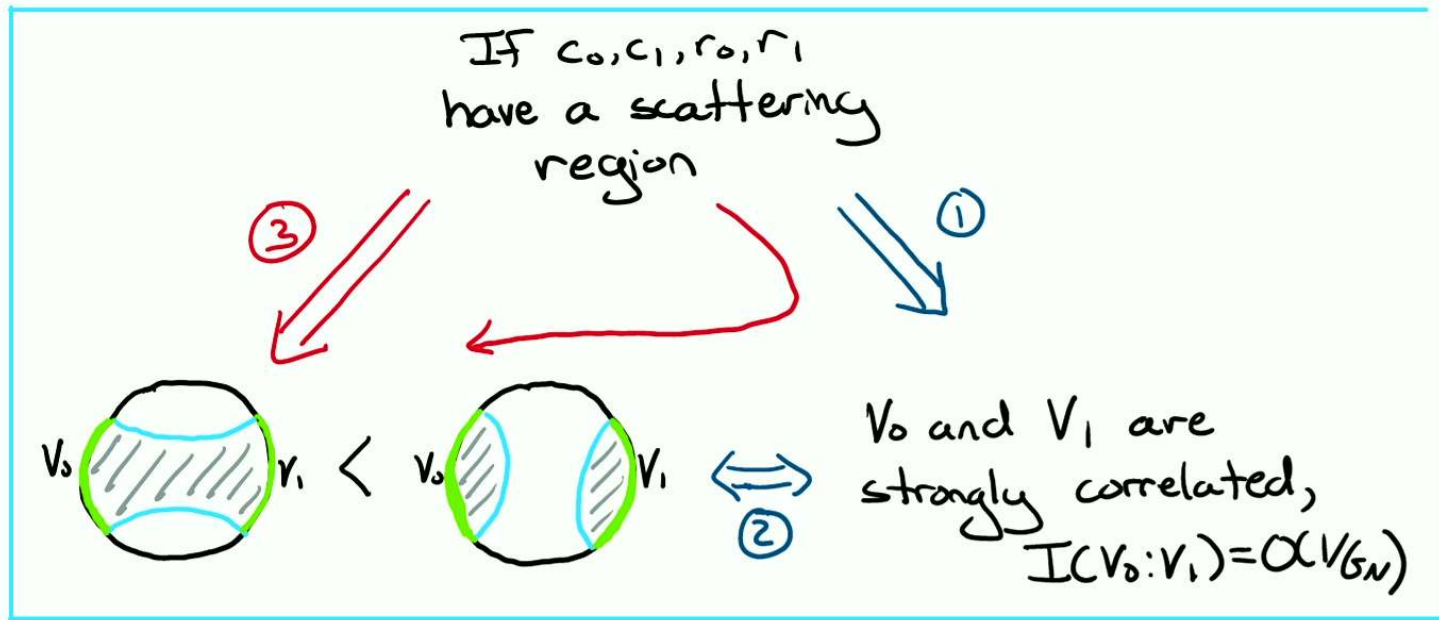
V_0 and V_1 are
strongly correlated,
 $I(V_0:V_1) = \mathcal{O}(V_{S^2})$

① Expected from necessity of entanglement

② Follows from Ryu-Takayanagi formula



2 → 2 connected wedge theorem



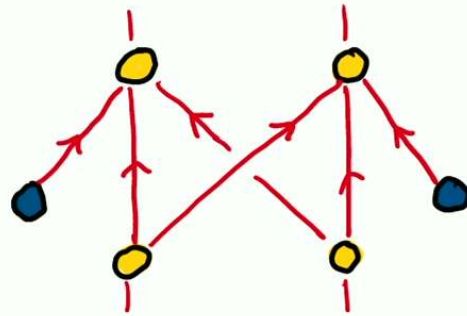
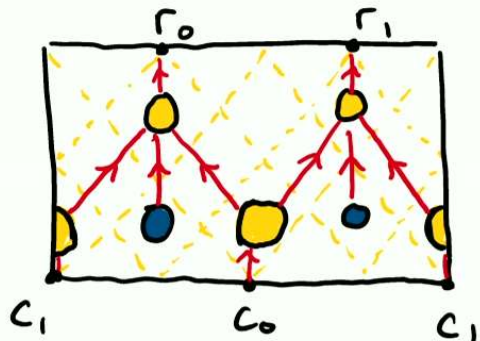
③ Proven directly in G.R.

↳ Assume NEC, use focusing theorem.

AM, Jon Sorce, Geoff Penington (2020) / AM, Jon Sorce, Beni Yoshida (2022)

Higher dimensional interactions from NLQC

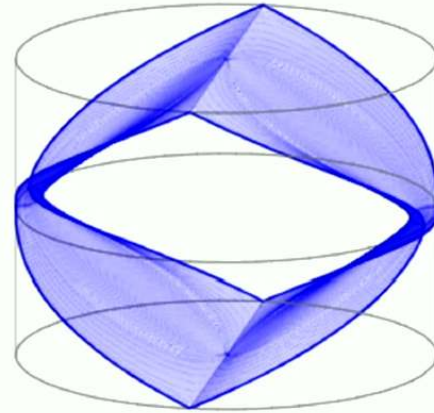
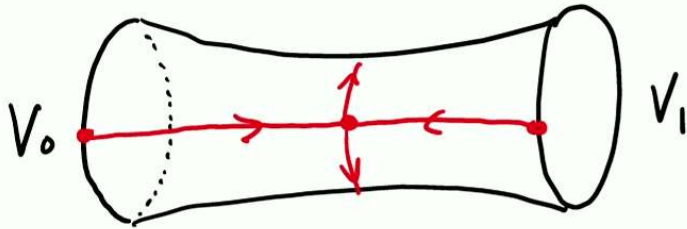
- CFT using NLQC to reproduce bulk interactions is one resolution to our puzzle, but is that really what's going on?
- Perspective is supported by CW theorem turning out correct, but a loophole remains:



} not quite NLQC,
can't prove necessity
of entanglement

Interaction inside of black holes

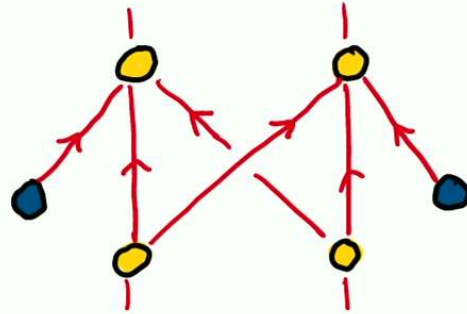
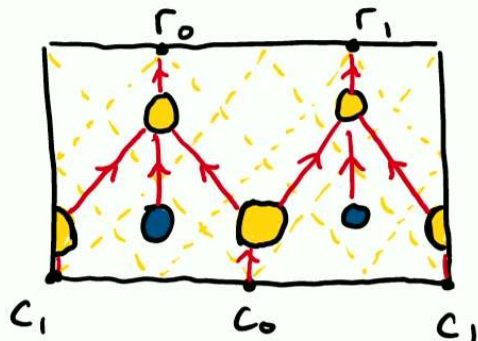
- There are two approaches here, see also Cree + Dolev 2022
- To briefly mention one of them:
go to a setting w/o "intermediate" regions, e.g. wormholes:



- Precise sense in which you can approximate the original geometry with a wormhole  AM, Van Raamsdonk 2021

Higher dimensional interactions from NLQC

- CFT using NLQC to reproduce bulk interactions is one resolution to our puzzle, but is that really what's going on?
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can't prove necessity
of entanglement



Can we get back to the simpler NLQC setting? (or understand this new setting better?)

What would we like to know?

- One way to phrase what we have in AdS/CFT:

$$\left\{ \begin{array}{l} \text{can compute in spacetime} \\ \text{region of area } A \\ \text{of holographic spacetime} \end{array} \right\} \subseteq \left\{ \begin{array}{l} \text{can implement in NLQC} \\ \text{using } I \sim O(A/G_N) \end{array} \right\}$$

- ① Can we better understand what the set on the left is here?

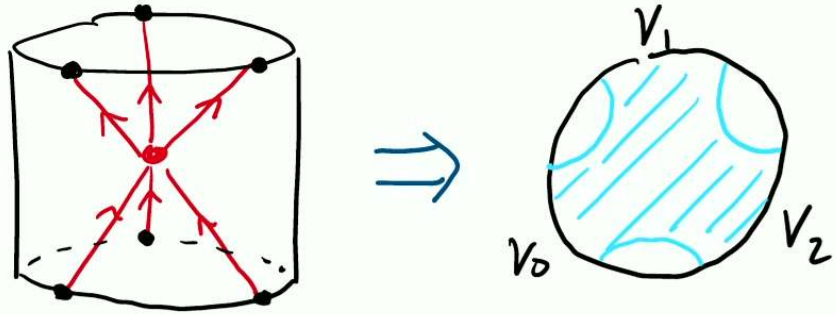
(Kfir's talk)

- ② Can we get better constraints on RHS? (Discussed in AM 2021)
(and so constrain LHS)

- ③ Can we better understand how AdS/CFT does (see Beni's talk)
NLQC? (and get some insights towards better NLQC?)

$n \rightarrow n$ connected wedge theorem

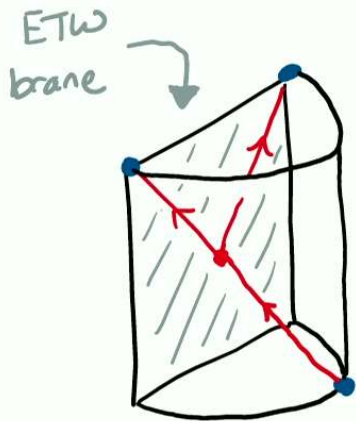
①



AM, J. Sorce, B. Yoshida (2022)

$1 \rightarrow 2$ connected wedge theorem

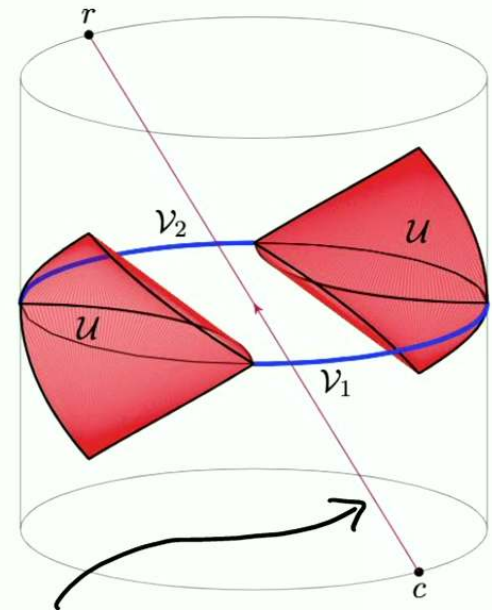
②



AM, D. Wakeham (2021)

Privacy-duality theorem

③



Causal curve from $c \rightarrow r$ that avoids U



$$I(V_0; V_1 | U) = O(1/G_N)$$

AM (2021)

Summary

- Holographic principle: QG in d dimensions has equivalent, QM description in $d-1$ dimensions
- Role of NLQC: At least in concrete realization of this principle, NLQC is how higher dimensional interactions are reproduced in QM.

One route to better understanding QG
is via NLQC! (and vis versa?)

Thanks!

What would we like to know?

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