Title: Supersymmetry in quantum complexity: clique homology is QMA_1-hard

Speakers: Tamara Kohler

Collection: It from Qubit 2023

Date: August 04, 2023 - 1:00 PM

URL: https://pirsa.org/23080030

Abstract: In this talk I will present recent results about the computational complexity of determining homology groups of simplicial complexes, a fundamental task in computational topology. In arXiv:2209.11793 we showed that this decision problem is QMA1-hard. Moreover, we showed that a version of the problem satisfying a suitable promise is contained in QMA. This suggests that the seemingly classical problem may in fact be quantum mechanical. In fact, we were able to significantly strengthen this by showing that the problem remains QMA1-hard in the case of clique complexes, a family of simplicial complexes specified by a graph which is relevant to the problem of topological data analysis. The proof combines a number of techniques from Hamiltonian complexity and homological algebra, and is inspired by a link with supersymmetric quantum mechanics. In this talk I will focus on how the link with supersymmetry inspired the result, and explain the intuition behind the proof.

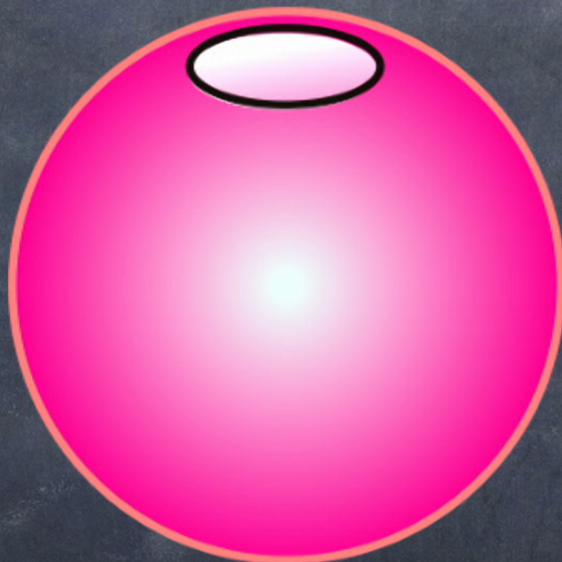
Supersymmetry in quantum complexity: clique Homology is QMA_1-hard

Joint work with Marcos Crichigno (Imperial College London & QC-ware)

arxiv:2209.11793

1

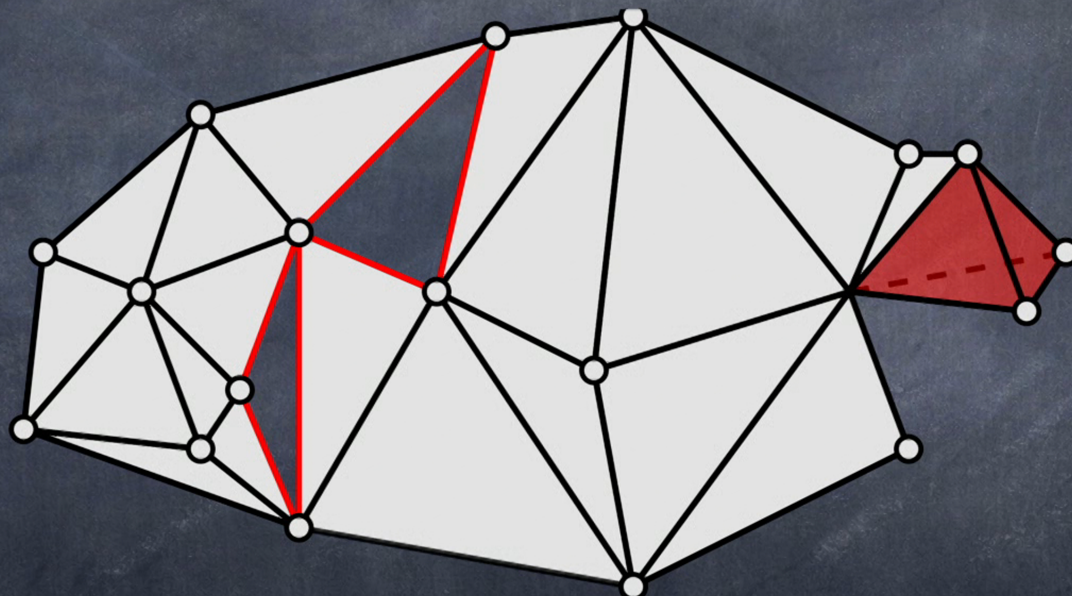
The homology problem



Does some manifold have an l-dimensional hole?

2

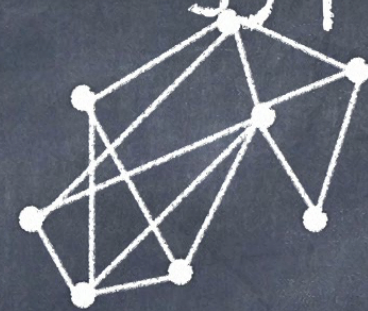
Simplicial homology problem



Does a simplicial complex have an l dimensional hole?

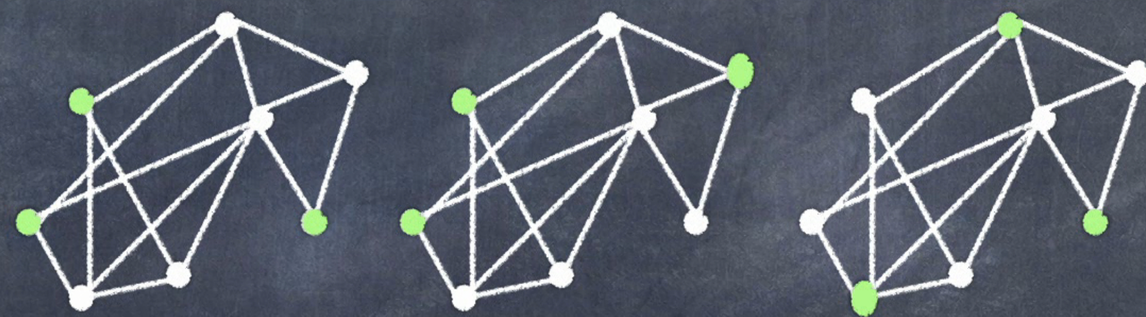
Independence homology problem

Given a (simple) graph G =



find all k-independent sets

3-independent:



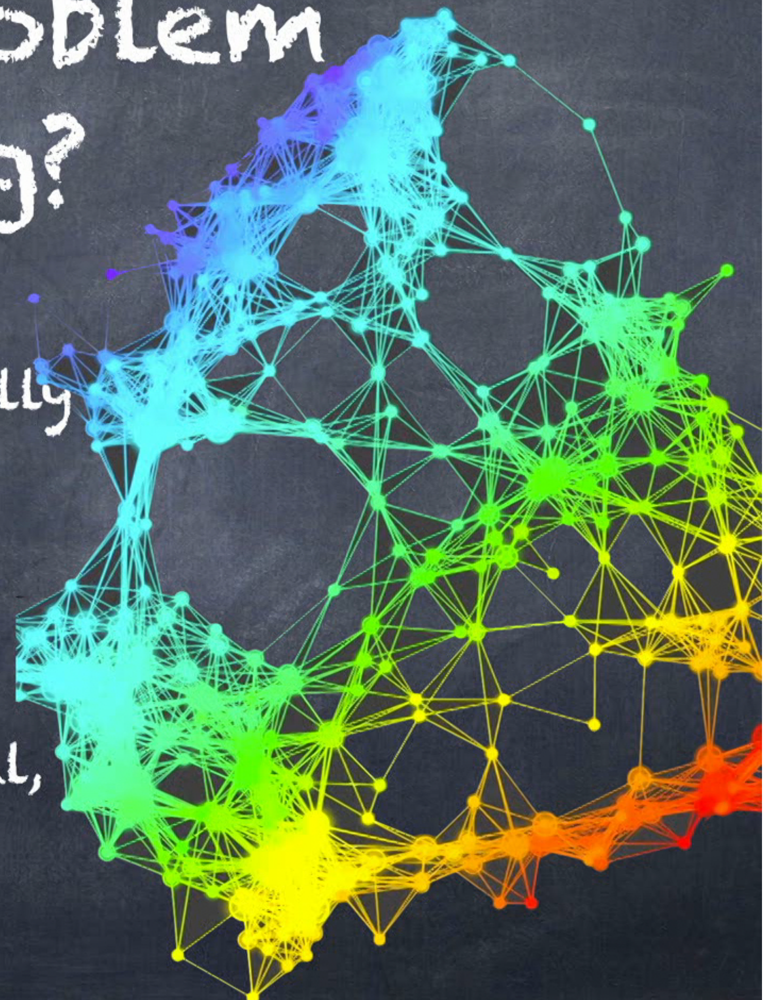
...

...

Does the independence complex of a graph G have an l-dimensional hole?

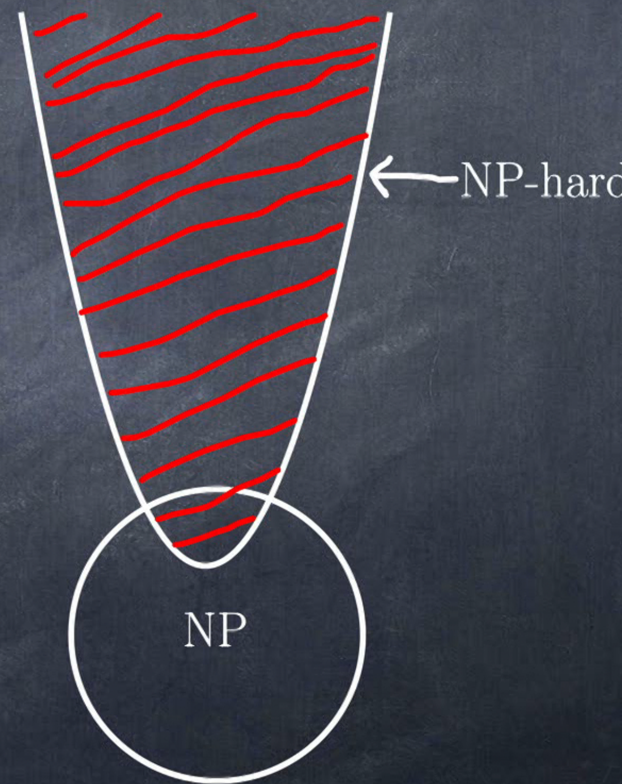
Why is this problem interesting?

- Clique homology has applications for topological data analysis - a practically useful problem! (Li et al, 2015)
- There is a quantum algorithm for a closely related problem - can that algorithm be dequantised? (Lloyd et al, 2014)



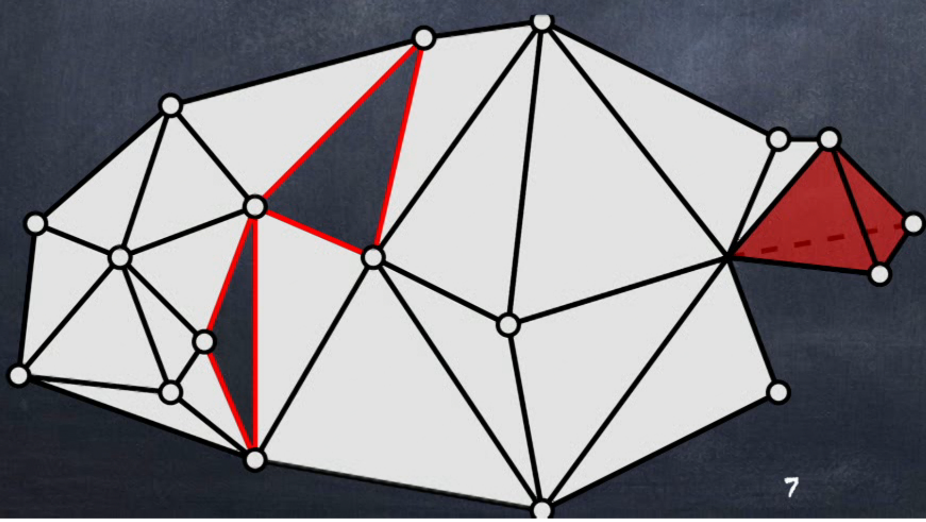
What was known about its complexity?

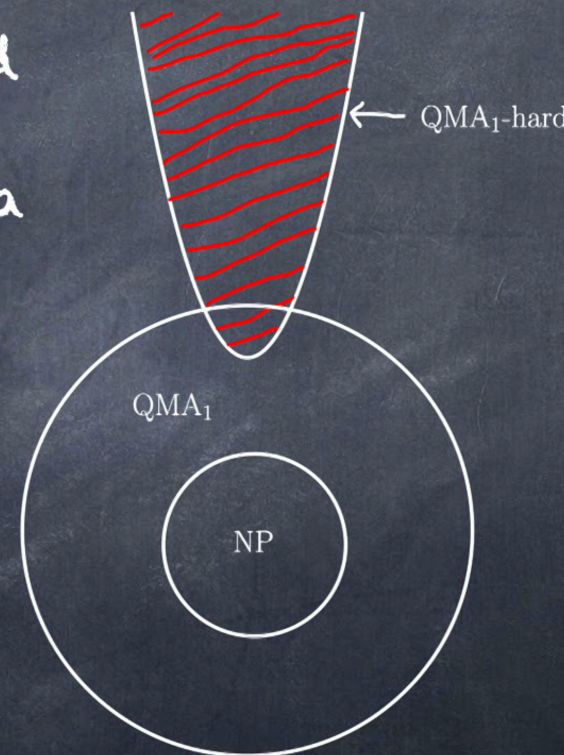
- The question of the complexity of simplicial homology was first defined formally in 2002 (Kaibel-Pfetsch, 2002).
- Clique homology was shown to be classically hard in 2016 (Adamaszek-Stacho) although it wasn't known whether this held in the regime where the quantum algorithm performed well (until Lloyd-Schmidhuber 2022).



Our main results

Clique homology is quantumly hard, and retains its hardness when restricted to cases where the quantum algorithm for a closely related problem performs well

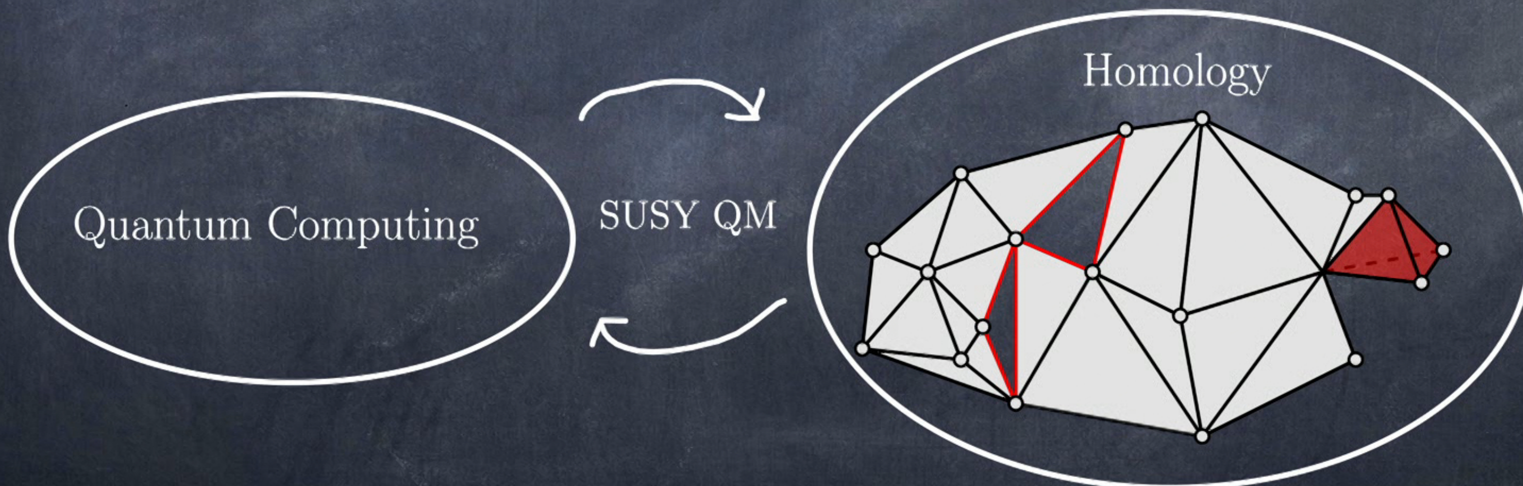




7

Our main results

Why should this (seemingly classical problem) be related to quantum complexity classes?



8

Outline of talk

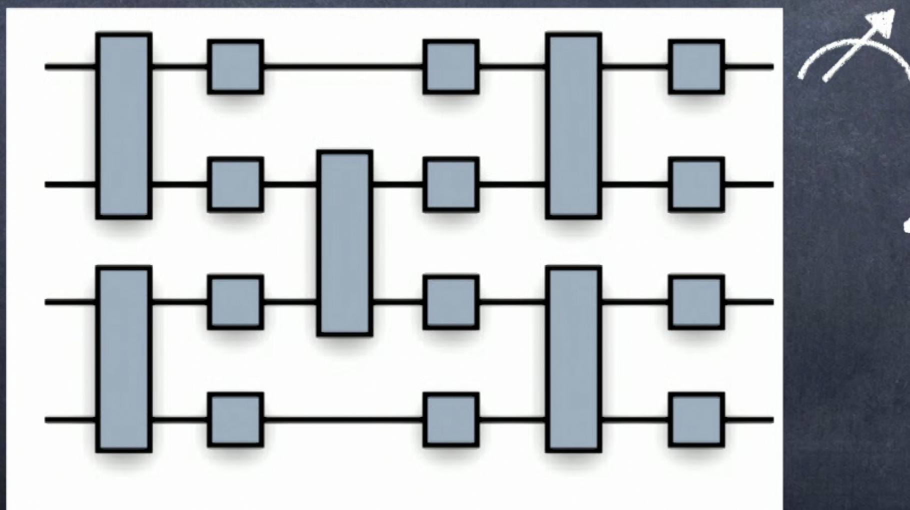
- Overview of quantum complexity theory
- The homology problem
- Supersymmetric quantum mechanics
- Overview of proof
- Future directions

Quantum complexity theory

Complexity class QMA

QMA is the class of problems that can be checked efficiently on a quantum computer

|W> →



Ux

P(YES) $\begin{cases} \geq 2/3 \text{ if yes} \\ \leq 1/3 \text{ if no} \end{cases}$

Complexity class QMA_1

A common choice of universal gate set is:

$$\mathcal{G} = \{\widehat{H}, T, \mathrm{CNOT}\},$$

$$\widehat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}, \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

However any set {CNOT, U} is universal if U is basis changing.
We choose {CNOT,U,Toffoli} with:

$$U = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$$ "Pythagorean gate"

(Rational coefficients - important for proof)

What sort of problems are characterised by QMA?

Local Hamiltonian problem

Given a k-local Hamiltonian acting on n spins $H = \sum_x h_x$

Promise: either $E_0 < a$ or $E_0 > b$ where $a - b \geq \frac{1}{\text{poly}(n)}$

Decide whether $E_0 < a$ or not

What sort of problems are characterised by QMA1?

Quantum satisfiability problem

Given sum of k-local projectors acting on n spins

$$H = \sum_x \Pi_x$$

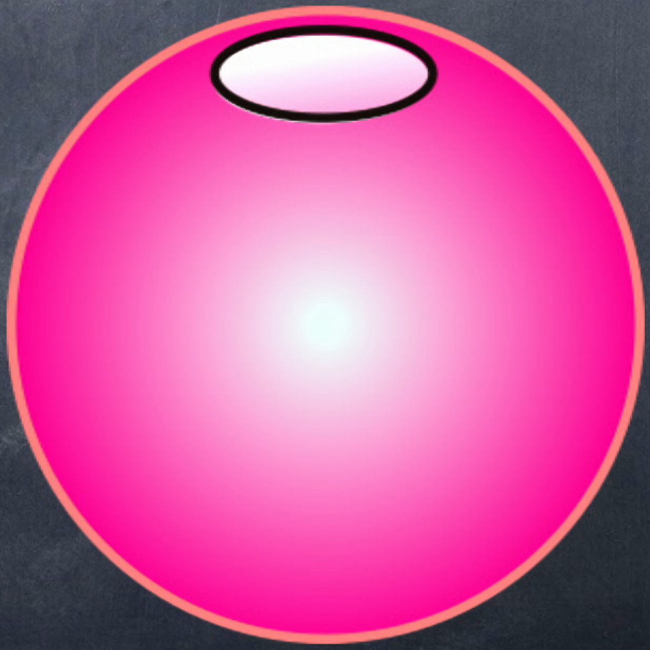
Promise: either $E_0 = 0$ or $E_0 > b$ where $b \geq \frac{1}{\mathrm{poly}(n)}$

Decide whether $E_0 = 0$ or not

Homology

How do we define holes rigorously?

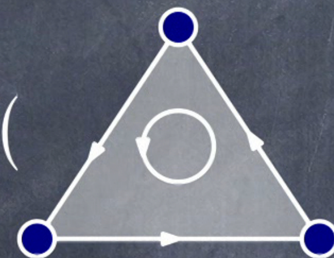
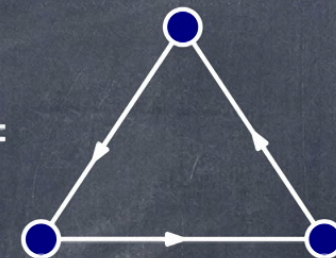
- holes are cycles
- holes arent the boundary of anything



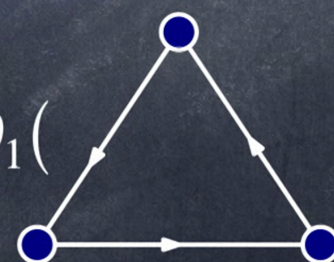
17

Introduce the boundary operator ∂_d

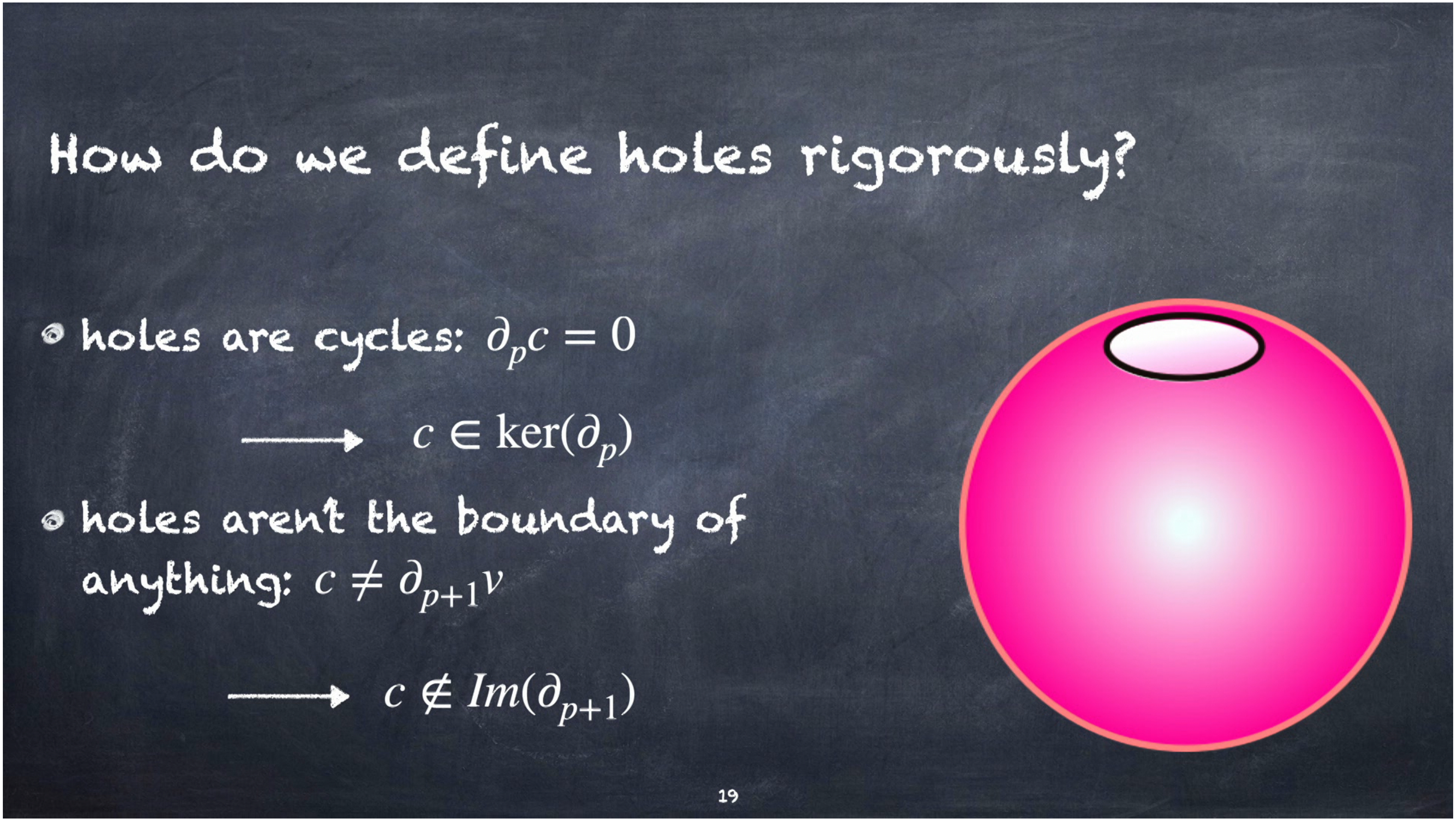
- cycles don't have boundaries $\partial(c) = 0$

$\partial_2($  $) =$ 

- the boundary of a boundary vanishes $\partial_{d-1}\partial_d = 0$

$\partial_1($  $) =$ 0

18


How do we define holes rigorously?
holes are cycles: $\partial_p c = 0$
$\longrightarrow c \in \ker(\partial_p)$
holes aren't the boundary of anything: $c \neq \partial_{p+1} v$
$\longrightarrow c \notin Im(\partial_{p+1})$
19

Supersymmetric Quantum Mechanics

N=2 SUSY quantum mechanics

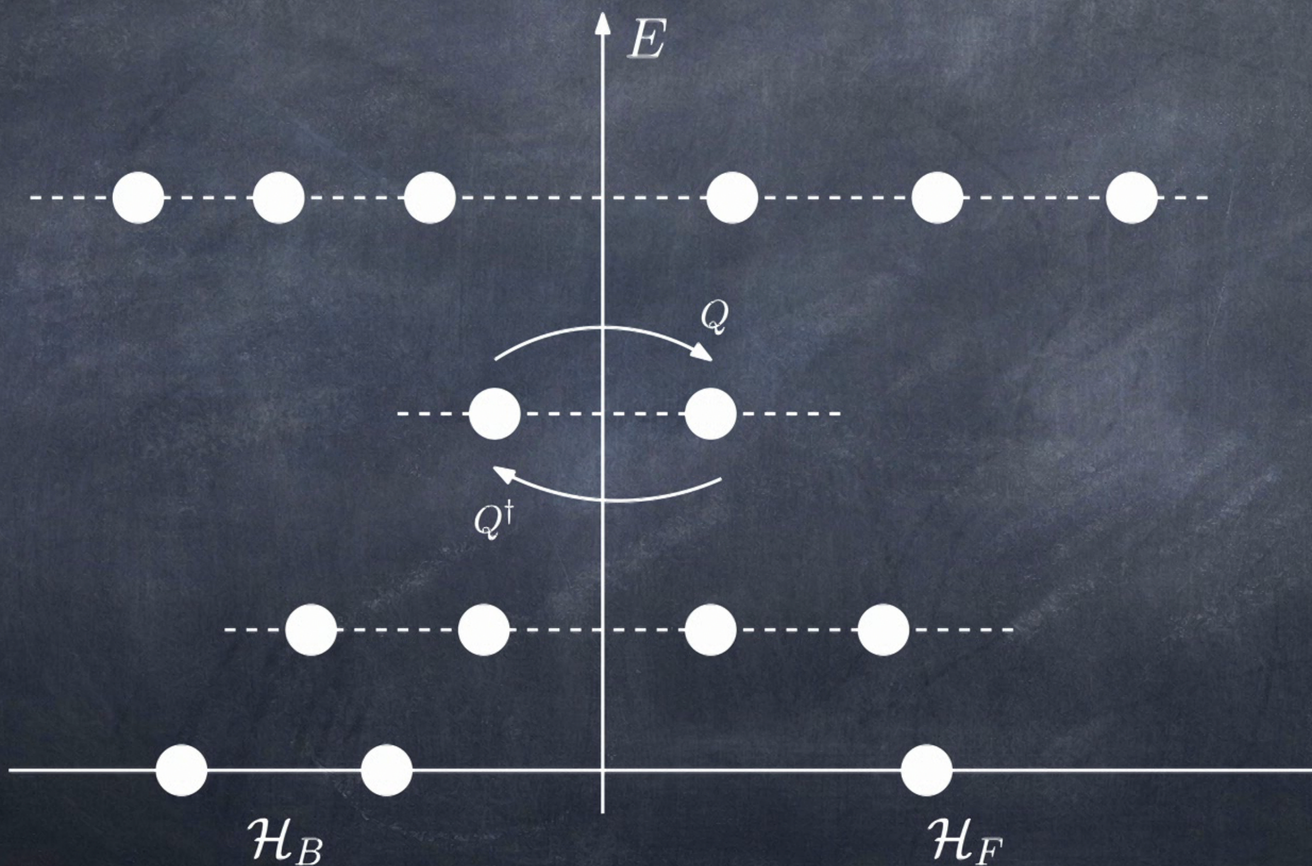
Hilbert space $\mathscr{H} = \mathscr{H}_B \oplus \mathscr{H}_F$

Supercharge $Q : (\mathscr{H}_B, \mathscr{H}_F) \to (\mathscr{H}_F, \mathscr{H}_B)$

where $Q^2 = 0$

Hamiltonian $H = \{Q, Q^\dagger\} = QQ^\dagger + Q^\dagger Q$

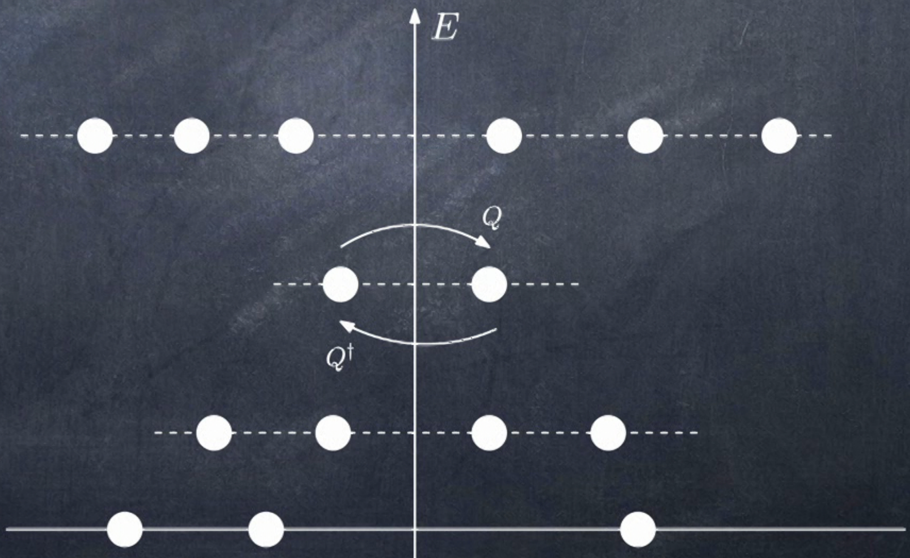
Hamiltonian $H = \{Q, Q^\dagger\} = QQ^\dagger + Q^\dagger Q$



23

Hamiltonian $H = \{Q, Q^\dagger\} = QQ^\dagger + Q^\dagger Q$

SUSY ground states satisfy $H|\psi_{SUSY}> = 0$



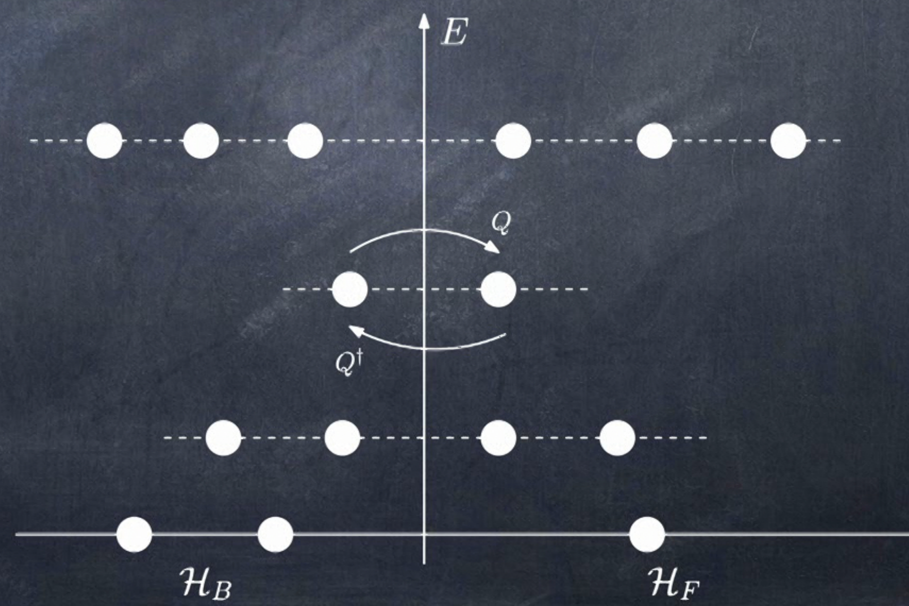
Hamiltonian $H = \{Q, Q^\dagger\} = QQ^\dagger + Q^\dagger Q$

SUSY ground states satisfy $H|\psi_{SUSY}> = 0$

$\longrightarrow \quad Q|\psi_{SUSY}> = Q^\dagger|\psi_{SUSY}> = 0$

SUSY ground states correspond to elements of homology groups

$$H := \frac{\ker(Q)}{Im(Q)}$$



Is the homology group

$$H_p(K) := \frac{\ker(\partial_p)}{Im(\partial_{p+1})}$$

non trivial?

$=$

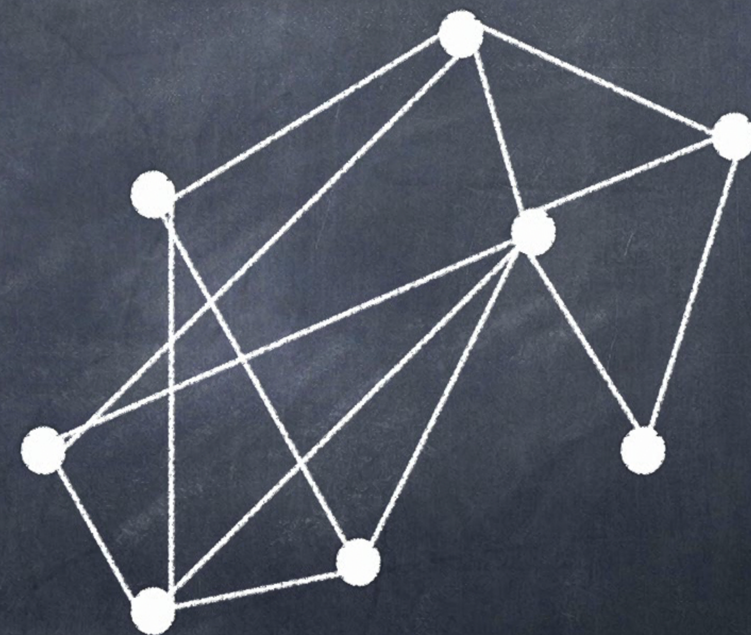
Is the SUSY Hamiltonian

$$H = \{\partial, \partial^\dagger\}$$

satisfiable?

Fermion hard core model

- Defined on a graph, G
- Hilbert space given by configurations of fermions on G such that no two fermions occupy adjacent sites
- Supercharge Q is given by the boundary operator of I(G)



25

Does the independence complex of a graph G have a hole?

$=$

Is the fermion hard core model defined on G satisfiable?

(Equivalently, does the clique complex of $\overline{G}$ have a hole?)

Overview of proof

Theorem: [Bravyi, 2006] [Gosset-Nagaj, 2013]

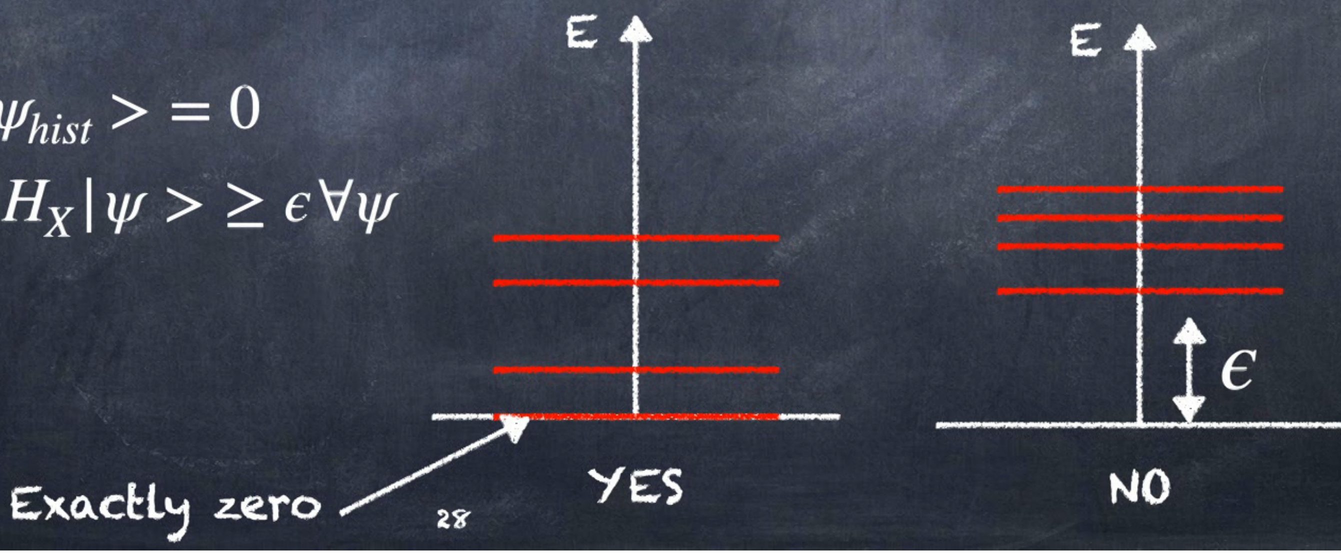
Quantum 4-SAT is QMA_1-complete

Given a QMA_1 verification circuit U_X construct a Hamiltonian:

$$H_X = \sum_a \Pi_a(U_X)$$

Such that:

- if $X \in L_{Yes}$, $H_X|\psi_{hist}> = 0$
- if $X \in L_{No}$, $<\psi|H_X|\psi> \geq \epsilon \forall\psi$



28

Theorem: [Bravyi, 2006] [Gosset-Nagaj, 2013]

Quantum 4-SAT is QMA_1-complete

The Hamiltonian $H_X = H_{in} + H_{clock} + H_{prop} + H_{out}$

$$\sum_{in} |011><011| \rightarrow H_{in}$$

$$H_{out} \leftarrow |011><011|$$

$$H^{(1)}_{\text{clock}} = |u\rangle\langle u|_1,$$
$$H^{(2)}_{\text{clock}} = |d\rangle\langle d|_L,$$
$$H^{(3)}_{\text{clock}} = \sum_{1\le j<k\le L} (|a1\rangle\langle a1| + |a2\rangle\langle a2|)_j \otimes (|a1\rangle\langle a1| + |a2\rangle\langle a2|)_k.$$
$$H^{(4)}_{\text{clock}} = \sum_{1\le j<k\le L} (|a1\rangle\langle a1| + |a2\rangle\langle a2| + |u\rangle\langle u|)_j \otimes |d\rangle\langle d|_k,$$
$$H^{(5)}_{\text{clock}} = \sum_{1\le j<k\le L} |u\rangle\langle u|_j \otimes (|a1\rangle\langle a1| + |a2\rangle\langle a2| + |d\rangle\langle d|)_k,$$
$$H^{(6)}_{\text{clock}} = \sum_{1\le j\le L-1} |d\rangle\langle d|_j \otimes |u\rangle\langle u|_{j+1}.$$

$$H_{\text{prop},t} = \frac{1}{2}\left[(|a1\rangle\langle a1| + |a2\rangle\langle a2|)_t \otimes I_{\text{comp}} - |a2\rangle\langle a1|_t \otimes \boxed{U_t} - |a1\rangle\langle a2|_t \otimes \boxed{U_t^\dagger}\right],\ .$$

$$H'_{\text{prop},t} = \frac{1}{2}(|a2,u\rangle\langle a2,u| + |d,a1\rangle\langle d,a1| - |d,a1\rangle\langle a2,u| - |a2,u\rangle\langle d,a1|)_{t,t+1} \otimes I_{\text{comp}}$$

29

Proof idea #1

Problem: Quantum satisfiability
Does a Hamiltonian have a zero energy ground state?

→ Use techniques from Hamiltonian complexity to show that the complexity holds even when restricting to the fermion hard core model.

Known to be QMA_1-complete for local Hamiltonians with a suitable promise.

Idea: use perturbation gadgets to simulate $H_X = H_{in} + H_{clock} + H_{prop} + H_{out}$ with fermion hard core Hamiltonians.

Proof idea #1

Problem: Quantu fiability
Does a Hamiltoni a
zero energy ground

Known to be QMA_1-complet
for local Hamiltonians
a suitable promise.

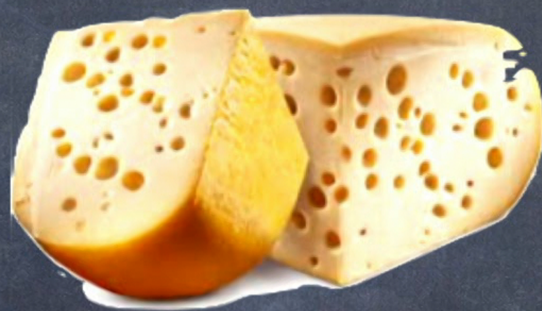
Use tec from
Ham complexity to show
complexity holds even
restricting to the fermion
hard core model.

Idea: use perturba ets to
simulate $H_X = H_{in} + H_{clock}$ H_{out}
with fermion hard core Ham

30

Proof idea #2

- Construct a graph where the independence complex has 2^n $(n-1)$-dimensional holes



← 2^n holes!

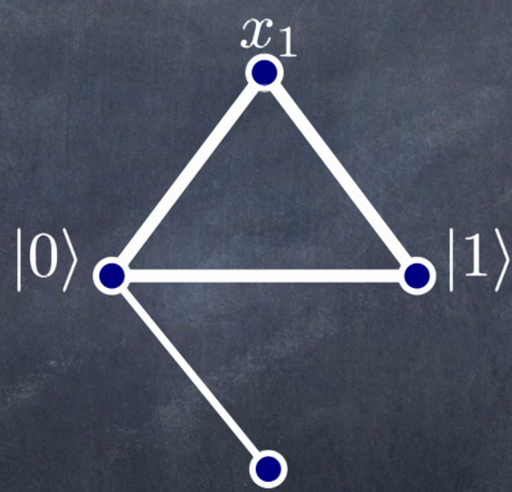
- Construct gadgets which 'fill in' holes corresponding to the projectors in quantum k-SAT
- Build up the graph corresponding to H_X - any remaining holes are satisfying solutions to quantum k-SAT

In order to reduce from quantum k-SAT to homology we need to encode the following projectors into an independence complex:

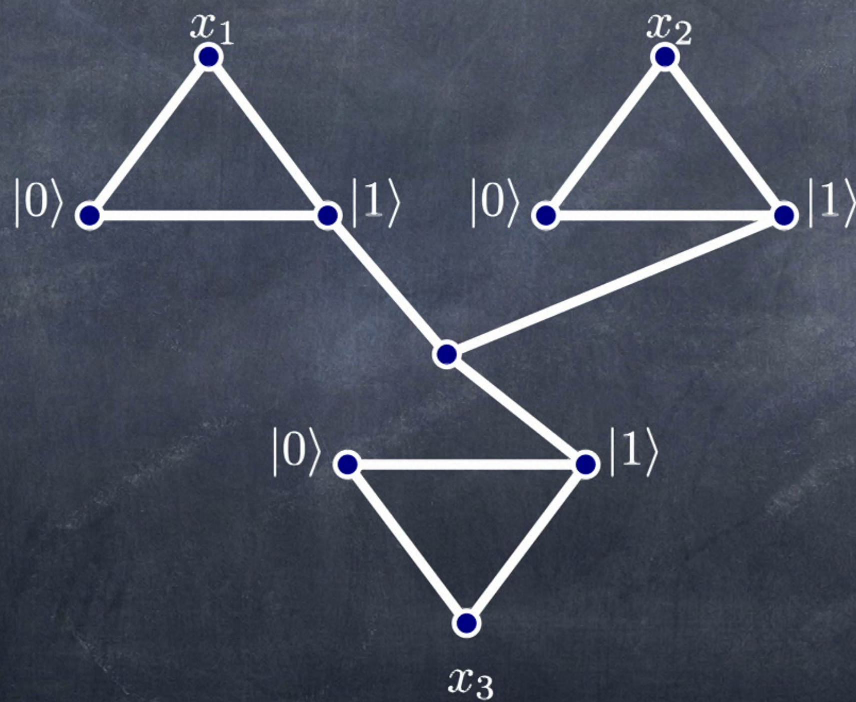
Term in H_{Bravyi}	*Penalizes state* $\|\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\|10\rangle(\|11\rangle - \|00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|01\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|00\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|000\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|101\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|010\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|011\rangle + 4\|100\rangle + 3\|101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|010\rangle + 3\|100\rangle - 4\|101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|011\rangle - \|100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|011\rangle - \|100\rangle)$
$H^{(1)}_{\text{clock}}$	$\|00\rangle$
$H^{(2)}_{\text{clock}}$	$\|11\rangle$
$H_{\text{in}}, H_{\text{out}}$	$\|011\rangle$
$H^{(6)}_{\text{clock}}, H^{(4)}_{\text{clock}}, H^{(5)}_{\text{clock}}, H^{(3)}_{\text{clock}}$	$\|1100\rangle$
$H^{(4)}_{\text{clock}}$	$\|0111\rangle$
$H^{(5)}_{\text{clock}}$	$\|0001\rangle$

Classical projectors

Classical projectors



Classical projectors



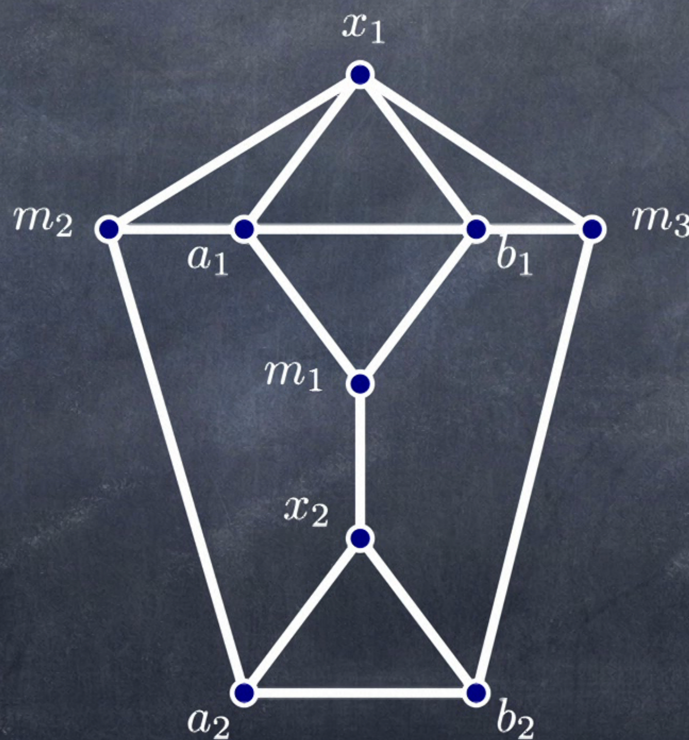
Taking stock...

Term in H_{Bravyi}	*Penalizes state* $\|\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\|10\rangle(\|11\rangle - \|00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|01\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|00\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|000\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|101\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|010\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|011\rangle + 4\|100\rangle + 3\|101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|010\rangle + 3\|100\rangle - 4\|101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|011\rangle - \|100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|011\rangle - \|100\rangle)$
$H^{(1)}_{\text{clock}}$	$\|00\rangle$
$H^{(2)}_{\text{clock}}$	$\|11\rangle$
$H_{\text{in}}, H_{\text{out}}$	$\|011\rangle$
$H^{(6)}_{\text{clock}}, H^{(4)}_{\text{clock}}, H^{(5)}_{\text{clock}}, H^{(3)}_{\text{clock}}$	$\|1100\rangle$
$H^{(4)}_{\text{clock}}$	$\|0111\rangle$
$H^{(5)}_{\text{clock}}$	$\|0001\rangle$

Taking stock...

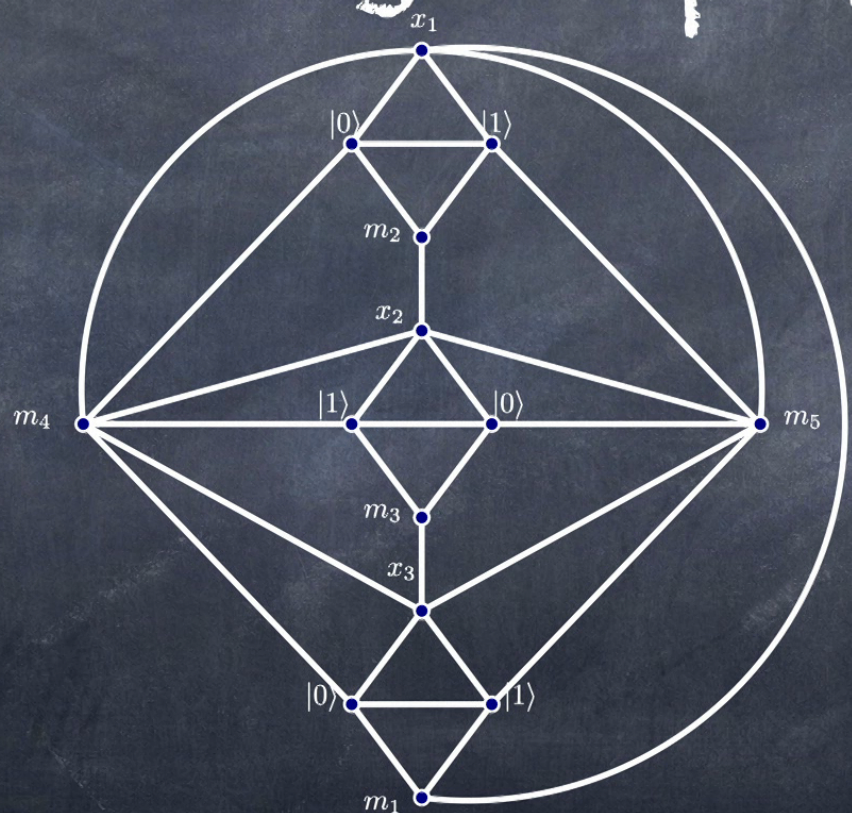
Term in H_{Bravyi}	*Penalizes state* $\lvert\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\lvert 10\rangle(\lvert 11\rangle - \lvert 00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 01\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 00\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 000\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 101\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 010\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 011\rangle + 4\lvert 100\rangle + 3\lvert 101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 010\rangle + 3\lvert 100\rangle - 4\lvert 101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H^{(1)}_{\text{clock}}$	$\lvert 00\rangle$
$H^{(2)}_{\text{clock}}$	$\lvert 11\rangle$
$H_{\text{in}},\ H_{\text{out}}$	$\lvert 011\rangle$
$H^{(6)}_{\text{clock}},\ H^{(4)}_{\text{clock}},\ H^{(5)}_{\text{clock}},\ H^{(3)}_{\text{clock}}$	$\lvert 1100\rangle$
$H^{(4)}_{\text{clock}}$	$\lvert 0111\rangle$
$H^{(5)}_{\text{clock}}$	$\lvert 0001\rangle$

Simple entangled projectors



$|00> - |11>$

Simple entangled projectors



$|101> - |010>$

Taking stock...

Term in H_{Bravyi}	*Penalizes state* $\lvert\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\lvert 10\rangle(\lvert 11\rangle - \lvert 00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 01\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 00\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 000\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 101\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 010\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 011\rangle + 4\lvert 100\rangle + 3\lvert 101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 010\rangle + 3\lvert 100\rangle - 4\lvert 101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H^{(1)}_{\text{clock}}$	$\lvert 00\rangle$
$H^{(2)}_{\text{clock}}$	$\lvert 11\rangle$
$H_{\text{in}},\ H_{\text{out}}$	$\lvert 011\rangle$
$H^{(6)}_{\text{clock}},\ H^{(4)}_{\text{clock}},\ H^{(5)}_{\text{clock}},\ H^{(3)}_{\text{clock}}$	$\lvert 1100\rangle$
$H^{(4)}_{\text{clock}}$	$\lvert 0111\rangle$
$H^{(5)}_{\text{clock}}$	$\lvert 0001\rangle$

36

Taking stock...

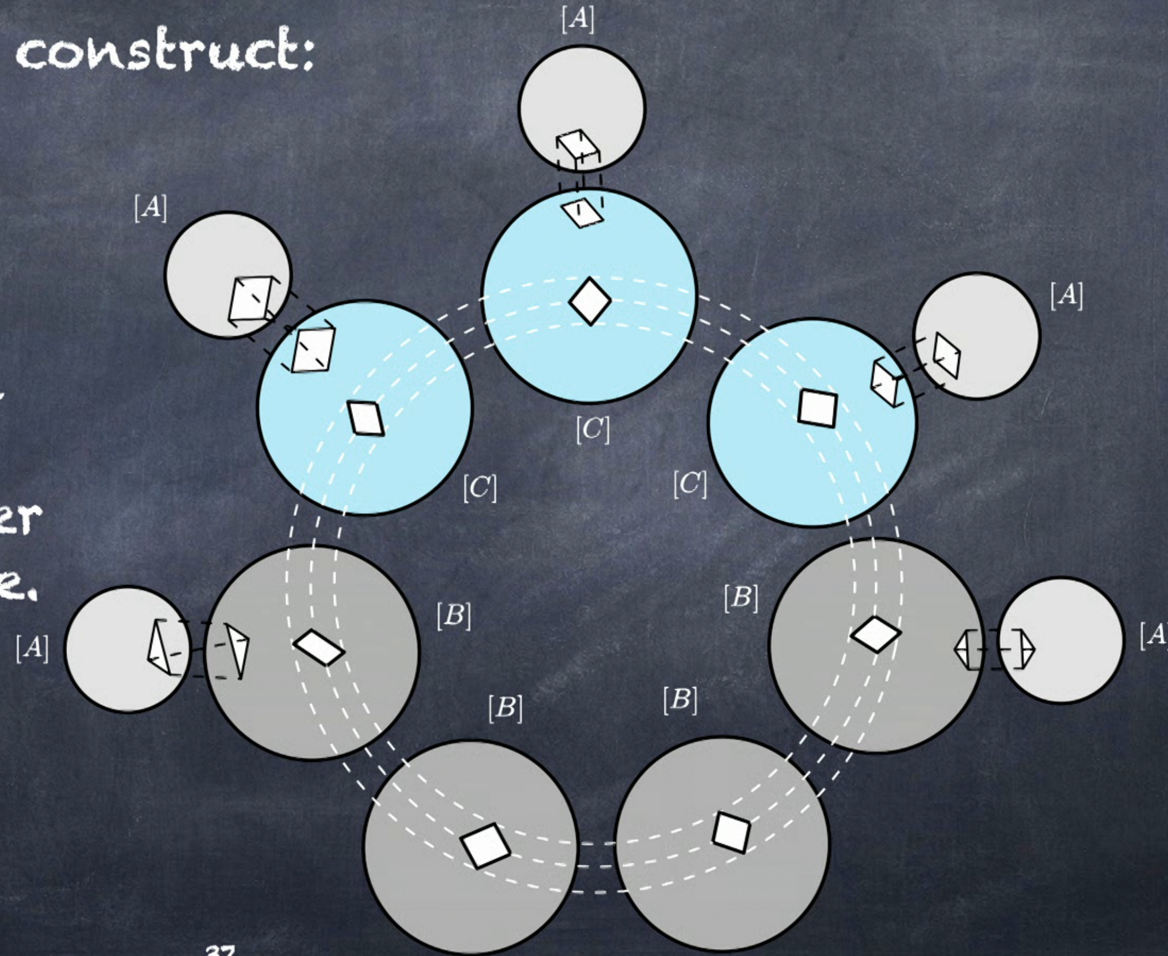
It looks like we've done a lot, but going this far only allows us to show classical complexity results...

Term in H_{Bravyi}	*Penalizes state* $\|\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\|10\rangle(\|11\rangle - \|00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|01\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|00\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|000\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|101\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|010\rangle(\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|011\rangle + 4\|100\rangle + 3\|101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\|010\rangle + 3\|100\rangle - 4\|101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\|1\rangle(\|011\rangle - \|100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\|11\rangle(\|011\rangle - \|100\rangle)$
$H^{(1)}_{\text{clock}}$	$\|00\rangle$
$H^{(2)}_{\text{clock}}$	$\|11\rangle$
$H_{\text{in}}, H_{\text{out}}$	$\|011\rangle$
$H^{(6)}_{\text{clock}}, H^{(4)}_{\text{clock}}, H^{(5)}_{\text{clock}}, H^{(3)}_{\text{clock}}$	$\|1100\rangle$
$H^{(4)}_{\text{clock}}$	$\|0111\rangle$
$H^{(5)}_{\text{clock}}$	$\|0001\rangle$

36

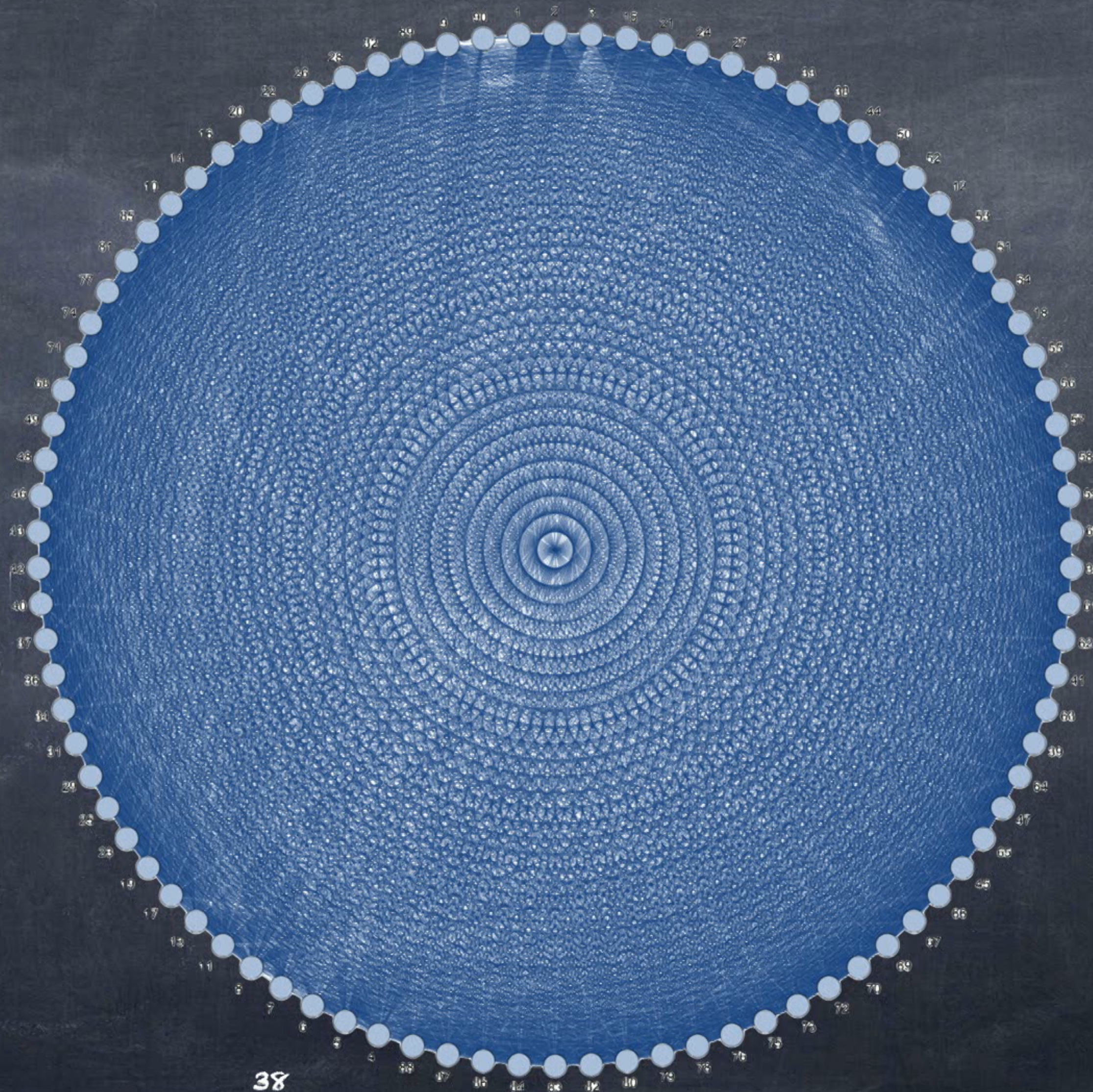
The Pythagorean projectors are the most technical challenging gadgets to construct:

We have to use 'simplicial surgery' to gut and glue simplices together in order to fill in the correct cycle.



37

This is implemented by this graph!

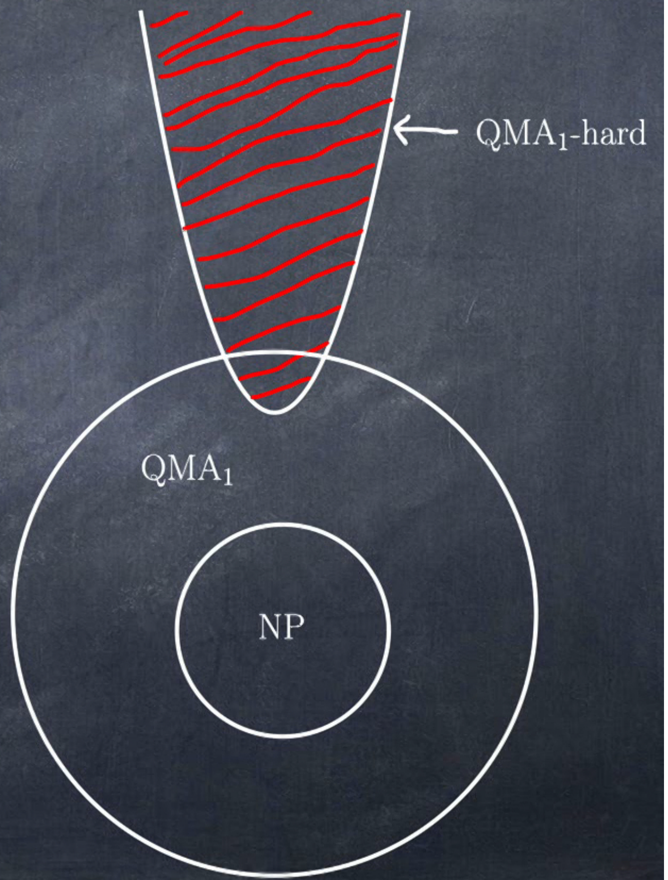


38

Putting everything together...

Clique / independence homology are `quantumly hard' with the graph, G, given as input.

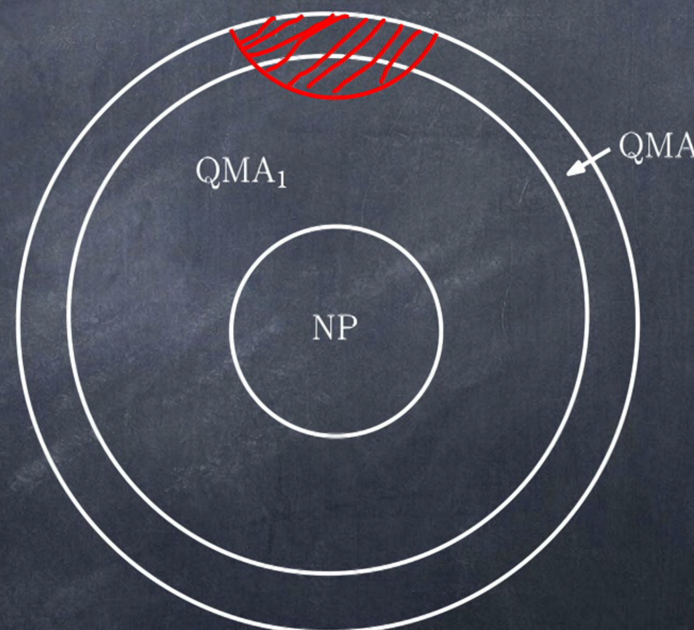
We also show that the problem remains quantumly hard when restricted to the regime where the Lloyd et al algorithm works well.



Future directions

Open questions

- Lower bound on complexity of the promise problem - current work, appearing shortly!
- Upper bound on complexity of the decision problem
- What can we say about the possibility of dequantising the Lloyd et al algorithm?



More speculatively...