Title: Talk 84 - Complementarity and the unitarity of the black hole S-matrix

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Abstract: Recently, Akers et al. proposed a non-isometric holographic map from the interior of a black hole to its exterior. Within this model, we study properties of the black hole S-matrix, which are in principle accessible to observers who stay outside the black hole. Specifically, we investigate a scenario in which an infalling agent interacts with radiation both outside and inside the black hole. Because the holographic map involves postselection, the unitarity of the S-matrix is not guaranteed in this scenario, but we find that unitarity is satisfied to very high precision if suitable conditions are met. If the internal black hole dynamics is described by a pseudorandom unitary transformation, and if the operations performed by the infaller have computational complexity scaling polynomially with the black hole entropy, then the S-matrix is unitary up to corrections that are superpolynomially small in the black hole entropy. Furthermore, while in principle quantum computation assisted by postselection can be very powerful, we find under similar assumptions that the S-matrix of an evaporating black hole has polynomial computational complexity.

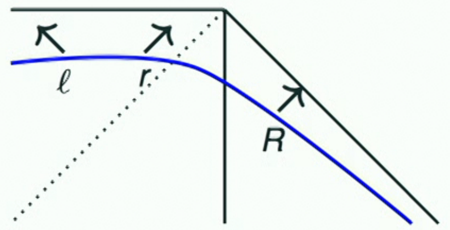
Quantum error correcting code and Holography

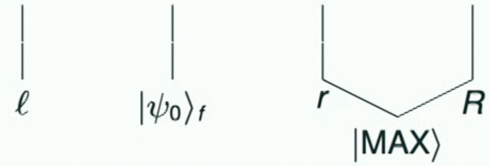
- AdS/CFT as a QECC: Almheiri, Dong, Harlow (2014)
- Holographic code: Pastawski, Yoshida, Harlow, Preskill (2015)
- Random tensor network: Hayden, Nezami, Qi, Thomas, Walter, Yang (2016)
- Alpha-bits: Hayden, Pennington (2018)
- Quantum minimal surface and QECC: Akers, Pennington (2021)

Recent advance: Non-isometric code, arXiv:2207.06536 [Akers, Engelhardt, Harlow, Penington, Vardhan (2022)]

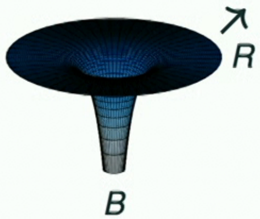
Non-isometric code

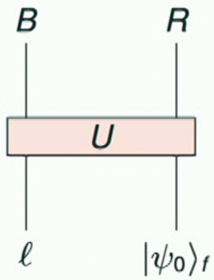
Effective Picture





Fundamental Picture





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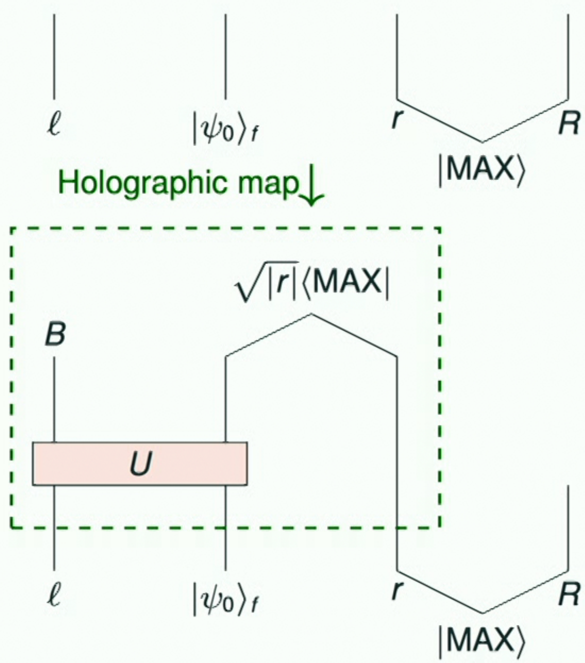
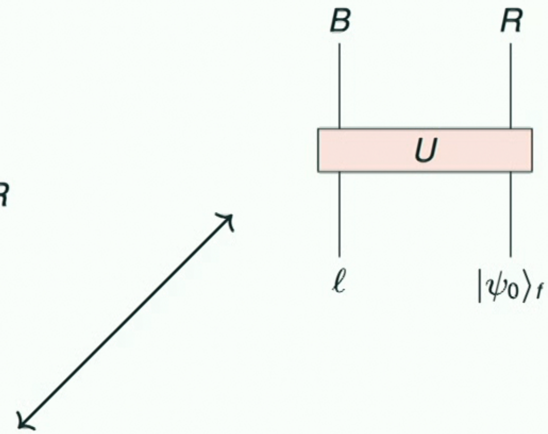
Non-isometric code [Akers et al. (2022)]





1. Finite BH entropy ✓
2. EFT for interior & exterior ✓
3. Unitarity ✓

Feature 1: Post-selection

One of the key features of the non-isometric code is post-selection. The rule is simple: For any state $|\psi\rangle_{AB}$, apply

$$|\psi\rangle_{AB} \rightarrow (I_A \otimes \langle\phi|_B)|\psi\rangle_{AB}$$

for some fixed vector $|\phi\rangle_B$.

[Horowitz, Maldacena (2003), Aaronson (2004), Preskill and Lloyd (2013)]

Post-selection

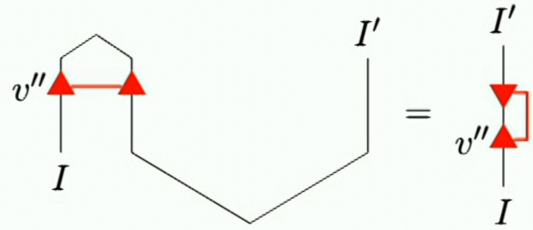
Solving NP-complete problems efficiently

$$U_f|x\rangle|0\rangle = \begin{cases} |x\rangle|1\rangle & \text{if } f(x) = 1, \\ |x\rangle|0\rangle & \text{otherwise.} \end{cases}$$

$$U_f\left(\frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle|0\rangle\right)$$

Post-select on observing $|1\rangle$.

Making unitary process non-unitary



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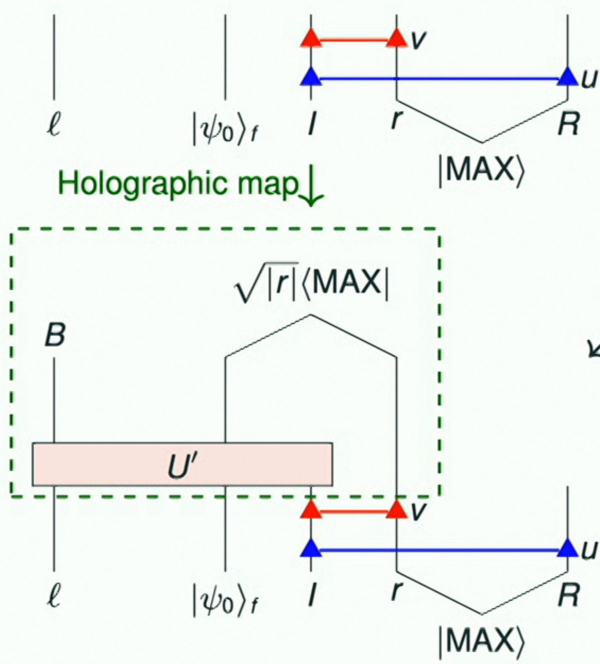
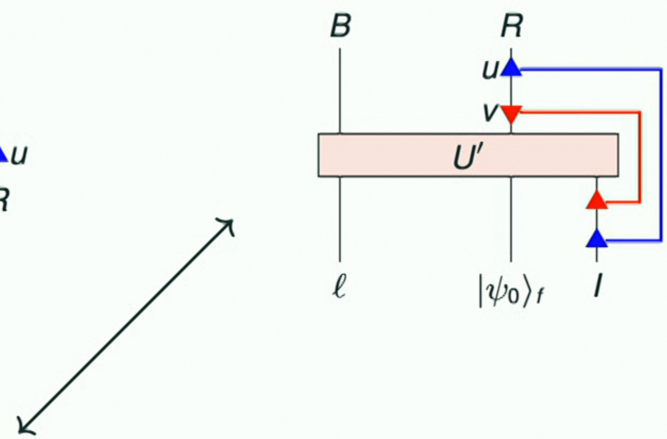
The model

Goal: Incorporate realistic features (e.g., Interaction, Polynomially complex U)





1. Unitarity preserved for all v and u?
2. Complexity still polynomial?

Pseudorandom Unitary

A family of efficiently implementable unitaries $\{U_k\}$ which is computationally indistinguishable from Haar-random unitaries.

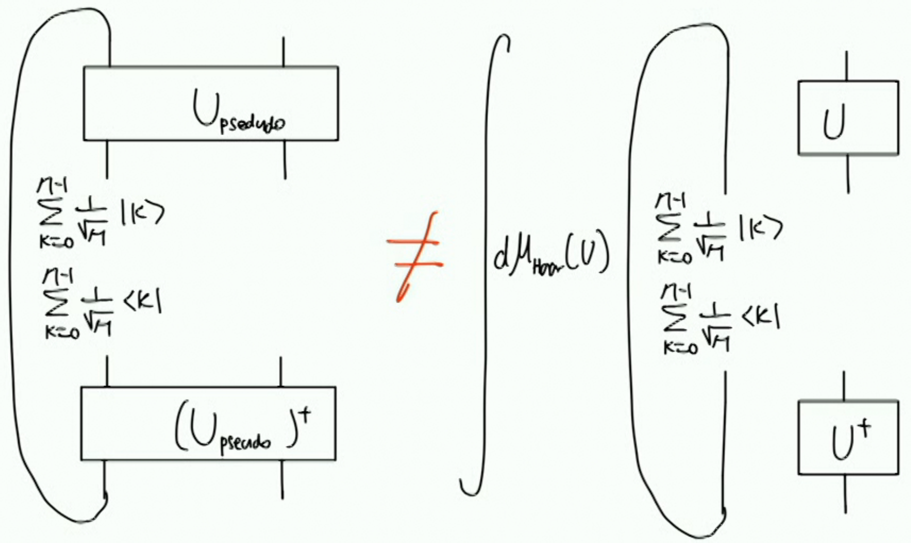
* Similar to pseudorandom states [Harlow, Hayden(2013), Aaronson(2015), Ji, Song, Liu(2017), Bouland, Fefferman, Vazirani (2021), **K**, Tang, Preskill(2021), Yang, Engelhardt (2023)], but stronger.

1. Pick random U_k.
2. Run a circuit consisting of poly-copies of U_k and polynomial # of gates.
3. Measure in a simple basis.
4. Average over k.

1. Pick a Haar-random U.
2. Replace U_k by U in the same circuit
3. Measure in a simple basis.
4. Average over the Haar measure.

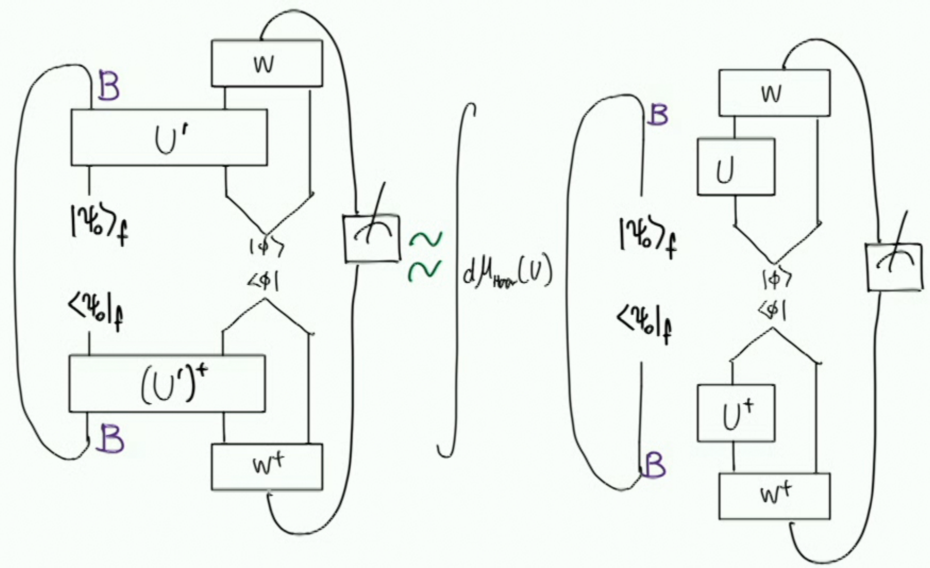
* If the measurement statistics are indistinguishable up to a superpolynomially small error and $\{U_k\}$ is efficiently implementable, $\{U_k\}$ *pseudorandom*.

Pseudorandom Unitary



$$U_{\text{pseudo}}(|k\rangle \otimes |\psi\rangle) = |k\rangle \otimes U_k|\psi\rangle$$

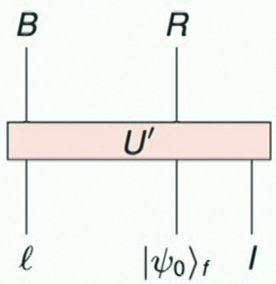
The central hypothesis: “Pseudorandomnesss” of U'



(for polynomially complex $|\phi\rangle$, W, and measurement.)

The central hypothesis: "Pseudorandomnesss" of U'

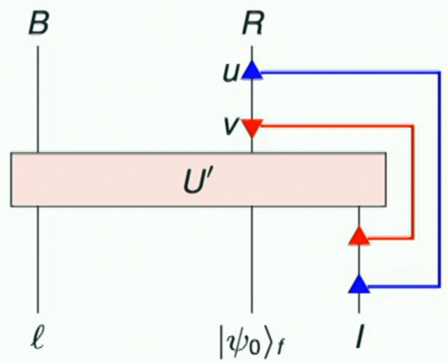
Hypothesis: Upon tracing out B, the resulting physical process is computationally indistinguishable (against polynomially complex experiments) from the process in which U' is replaced by a Haar-random unitary, up to an error exponentially small in $\log |B|$.



B: Remaining Black hole

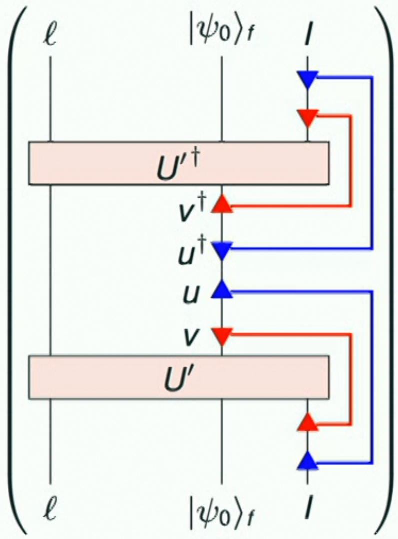
* This is certainly possible if U' is Haar-random. The nontrivial part of this hypothesis is that there is such U' of polynomial complexity.

Postselection vs. Unitarity



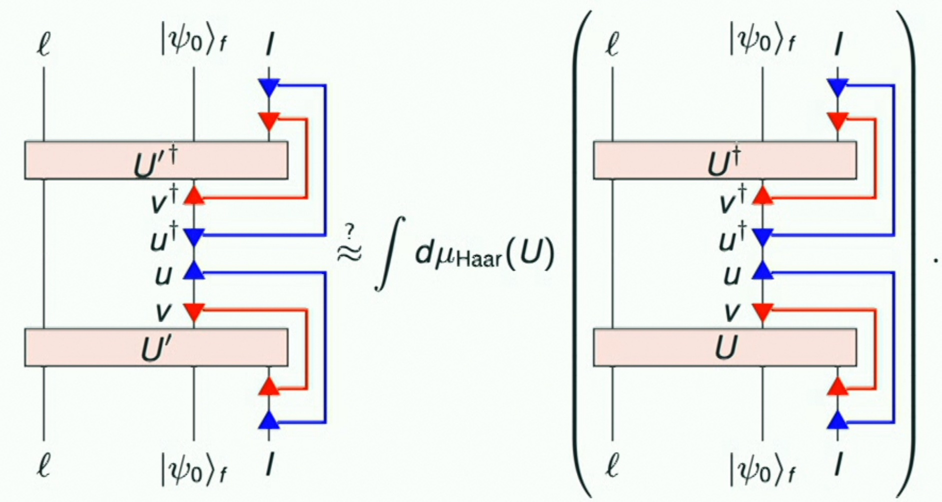
Is this process unitary (or more precisely, isometric) for all u and v?

A fact

$\int d\mu_{\text{Haar}}(U')$  $= I_\ell \otimes I_l$

[Akers et al. (2022)]

Invoking Pseudorandomness (almost)



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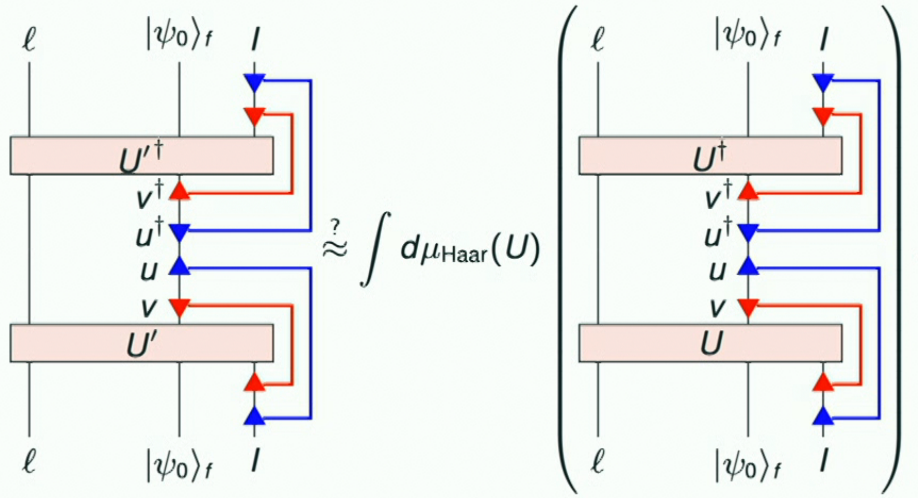
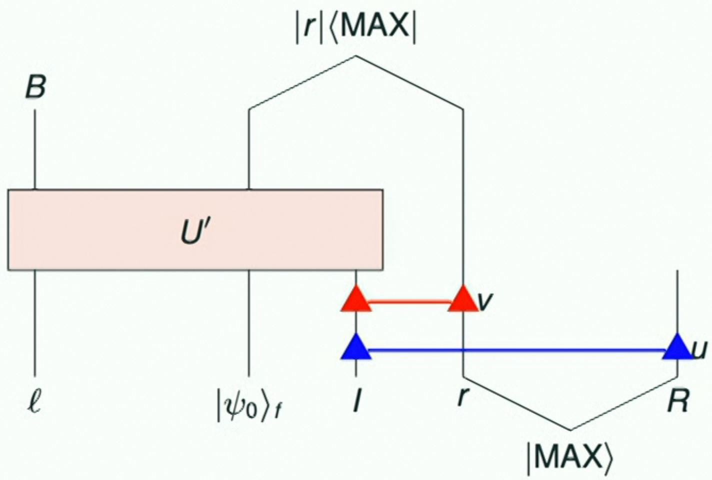
Invoking Pseudorandomness (almost)



Straightening the legs



- Pro: Everything is unitary (plus measurement)
- Con: Postselection is applied on a huge system.
  - This reduces the norm by a factor of $|r|$, yielding a $|r|$-fold blowup in the error bound.

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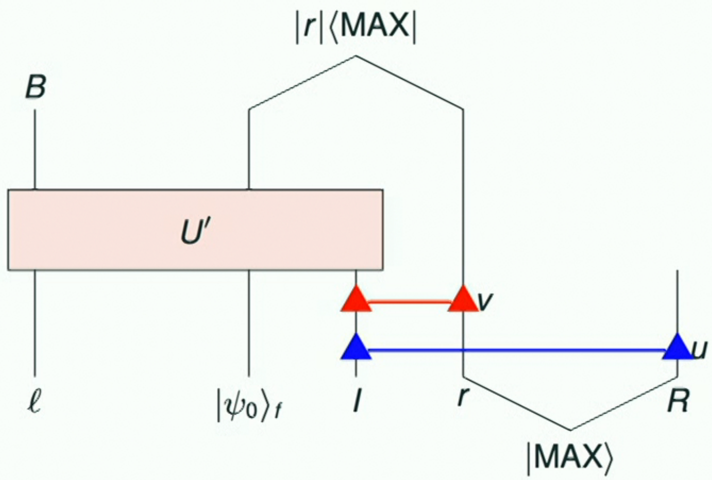
Straightening the legs



- Pro: Everything is unitary (plus measurement)
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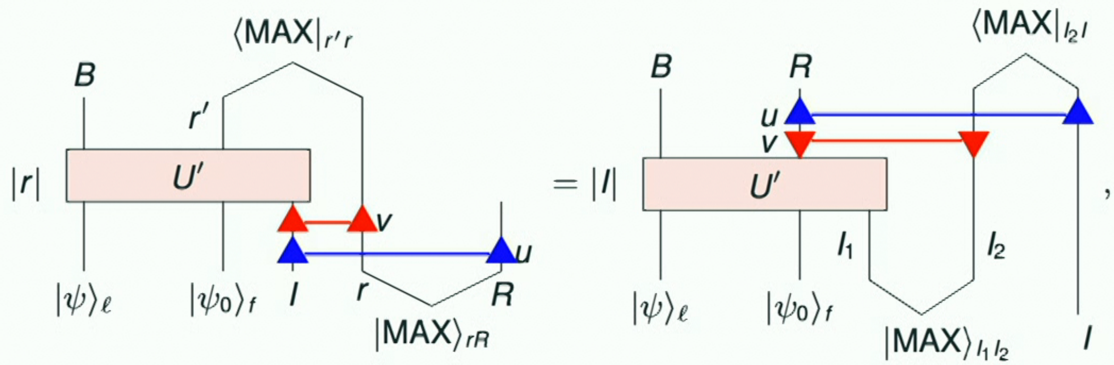
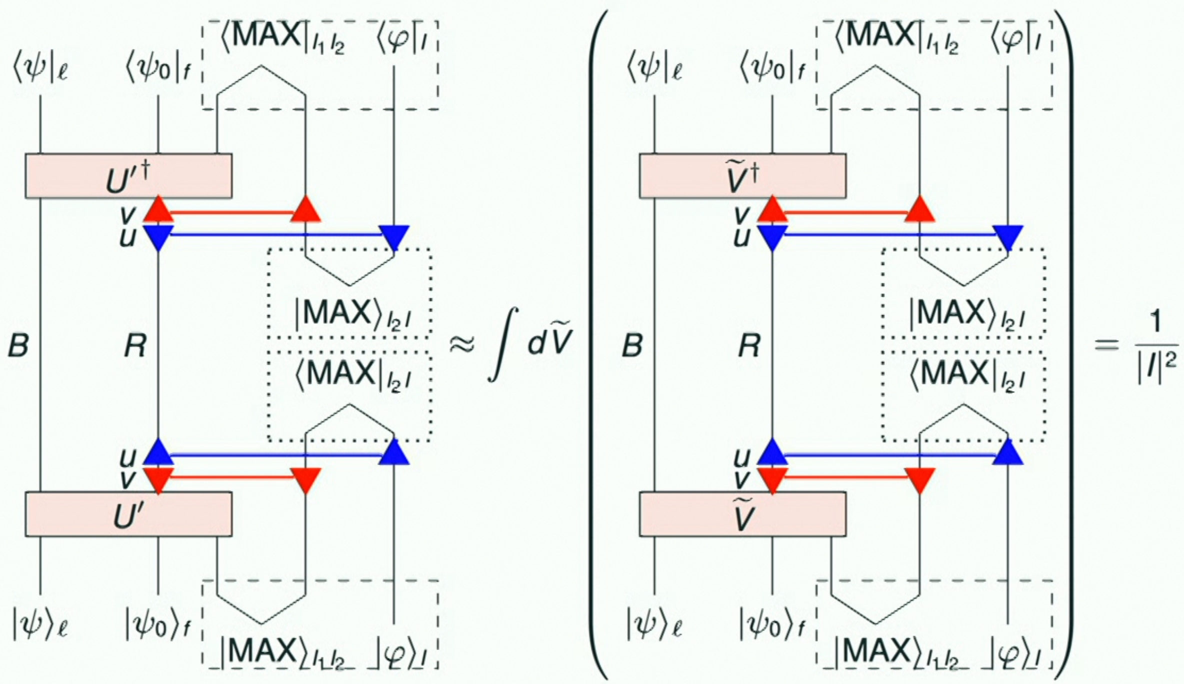
  - This reduces the norm by a factor of $|r|$, yielding a $|r|$-fold blowup in the error bound.

Straigtening the legs, correctly



Now everything is unitary, and the post-selection is applied to a small (infaller) subsystem.

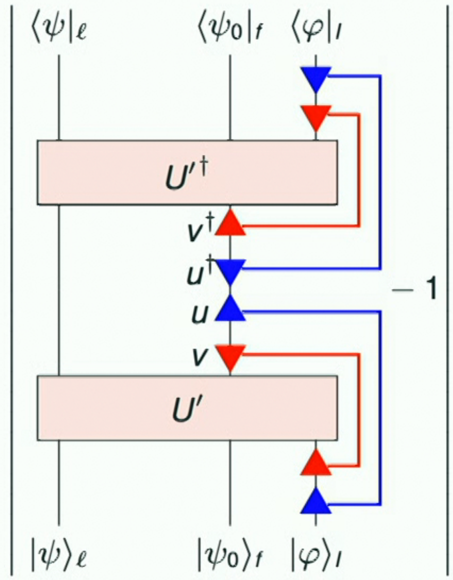
Invoking Pseudorandomness



Error: $\epsilon(\log|B|) \approx$ superpolynomially small in $\log|B|$.

(Approximate) Unitarity

For any low-complexity states $|\psi\rangle_\ell$ and $|\varphi\rangle_I$,



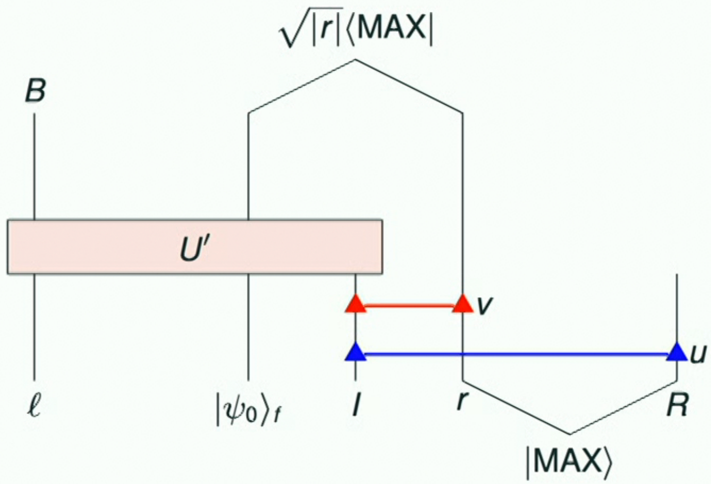
$$- 1 \Big| \leq |I|^2 \epsilon(\log |B|),$$

"Pseudorandomness" of U' $\rightarrow$ (Approximate) unitarity

Complexity

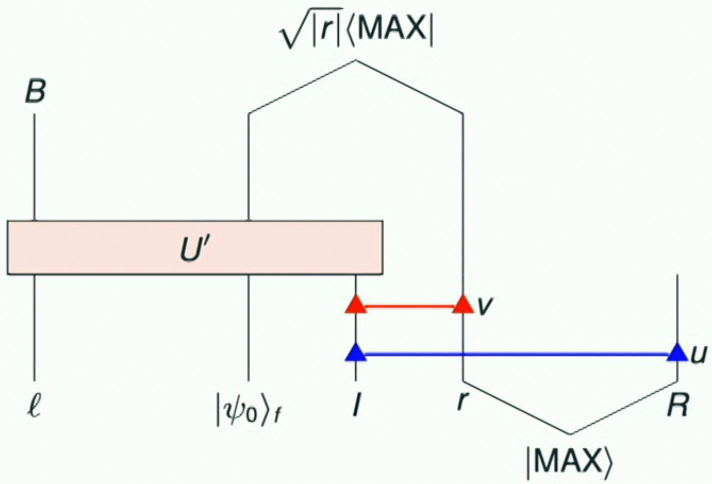
Q: If we throw a robot into the BH, would its evaporation process be suddenly exponentially complex?

Probably not, but then again, it is unclear why this should be polynomially complex.



Why? Generally postselection + BQP is PP. [Aaronson (2005)]

Complexity



Claim: The complexity of this map is at most

$$O(\dim(I)C),$$

where C is the complexity of U', v, u, and $|\psi_0\rangle_f$ altogether.

Idea: QSVT [Gilyen, Su, Low, Wiebe (2018)]

Basic Idea

In quantum computing, there is a very well-known procedure called amplitude amplification (AA). Suppose we are given a unitary U such that

$$U|0\dots0\rangle = \sqrt{p}|\psi\rangle + \sqrt{1-p}|\psi_\perp\rangle,$$

together with an ability to unambiguously distinguish $|\psi\rangle$ and $|\psi_\perp\rangle$. AA lets us prepare $|\psi\rangle$ almost deterministically, with the computational cost of $O(1/\sqrt{p})$.

[Brassard, Høyer, Mosca; Tapp (2001)]

* Modern version: Quantum Singular Value Transformation [Gilyen, Su, Low, Wiebe (2018)]

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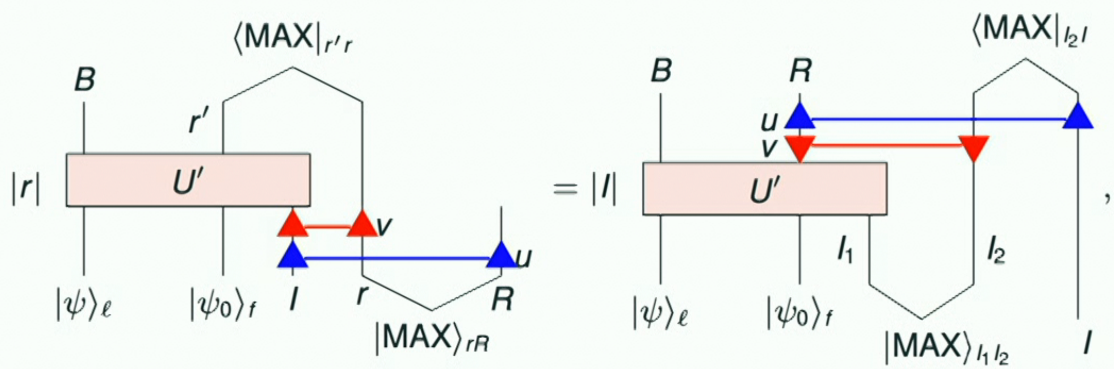
Probability



Postselection success probability $\approx 1/|I|^2 \rightarrow$ Overhead = $O(|I|)$.

Using Quantum Singular Value Transformation [Gilyen, Su, Low, Wiebe (2018)], the claim follows.

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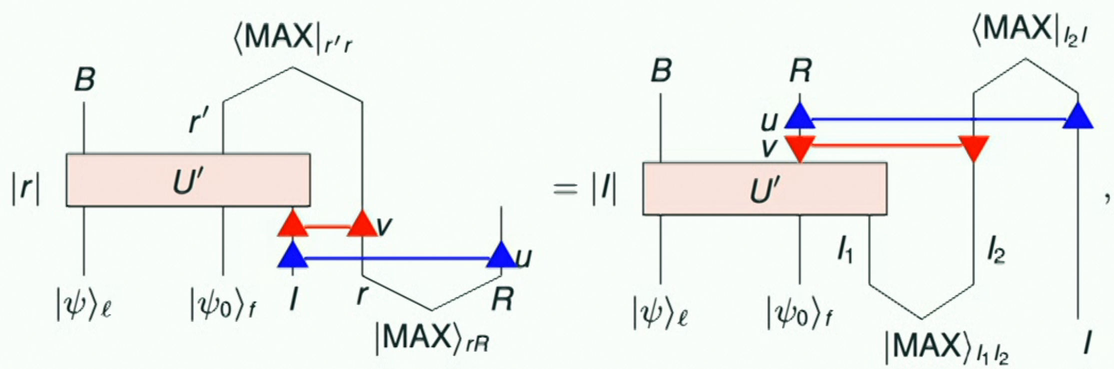
Probability



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Using Quantum Singular Value Transformation [Gilyen, Su, Low, Wiebe (2018)], the claim follows.

Problems

- The Rule: We applied the holographic map of Akers et al. to the setup with interaction. Is this okay?
  - The unitarity in the fundamental picture is violated, but by an exponentially small amount. Accounting for quantum gravity effects, the exact unitarity may be restored.
  - A different proposal: [DeWolfe, Higginbotham (2023)]
- If we throw in 1000 qubits interacting with R and r, our upper bound for complexity is $e^{O(1000)}\mathcal{C}$. Can we do better?
  - If we view U' as a black box, our bound is essentially optimal.
  - Polynomial (and additive) overhead is more physically reasonable, but it looks like doing so requires knowledge about the structures of U'.
- How to construct pseudorandom unitaries?
  - **Challenge**: Apply random phases in two complementary bases ℓ times. Quantify the closeness of this ensemble to Haar-random unitaries.
    - For instance, compute the frame potential.

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