Title: Talk 118 - Overlapping qubits from non-isometric maps and de Sitter tensor networks

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Abstract: We construct approximately local observables, or "overlapping qubits", using non-isometric maps and show that processes in local effective theories can be spoofed with a quantum system with fewer degrees of freedom, similar to our expectation in holography. Furthermore, the spoofed system naturally deviates from an actual local theory in ways that can be identified with features in quantum gravity. For a concrete example, we construct two MERA toy models of de Sitter space-time and explain how the exponential expansion in global de Sitter can be spoofed with many fewer quantum degrees of freedom and that local physics may be approximately preserved for an exceedingly long time before breaking down. Conceptually, we comment on how approximate overlapping qubits connect Hilbert space dimension verification, degree-of-freedom counting in black holes and holography, approximate locality in quantum gravity, non-isometric codes, and circuit complexity.

Freie Universität Berlin

Caltech

Overlapping qubits from non-isometric maps and de Sitter tensor networks

[arXiv:2304.02673]

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Hilbert space dualities

When studying high-energy theory through the lens of quantum information, we often encounter maps like this:

Hilbert space $\mathcal{H}_m$	$\xrightarrow{\text{map } V}$	Hilbert space $\mathcal{H}_n$
Semiclassical AdS bulk	$V^\dagger V \approx \mathbb{1}$ *	Boundary CFT
dS timeslice at t	$V^\dagger V = \mathbb{1}$ †	dS timeslice at $t' > t$
Black hole interior	$V^\dagger V \neq \mathbb{1}$ ‡	Horizon theory
Semiclassical gravity	$V^\dagger V \stackrel{?}{=} \mathbb{1}$	Quantum gravity

* [Almheiri/Dong/Harlow '15]
† [Cotler/Strominger '22]
‡ [Akers/Engelhardt/Harlow/Penington/Vardhan '22]

Isometric vs. non-isometric

For a map $V : \mathcal{H}_m \to \mathcal{H}_n$ to be isometric, we require $\dim \mathcal{H}_m \leq \dim \mathcal{H}_n$.

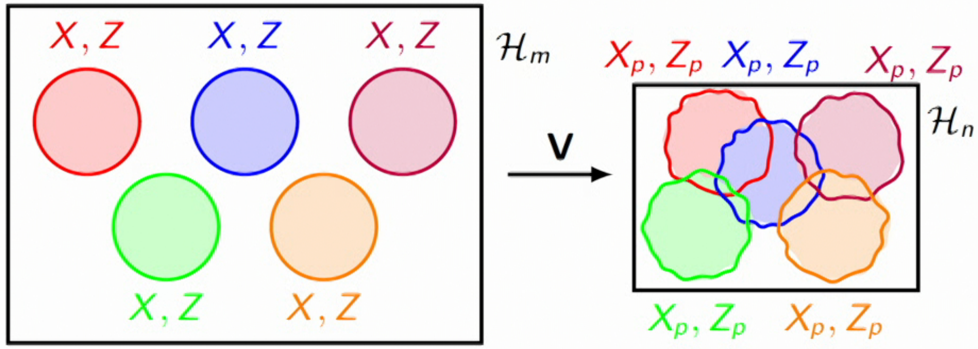
Isometric Hilbert space maps define **exact quantum codes**: Both the preparation of a logical state $|\bar{\psi}\rangle$ and its recovery as a physical state $|\psi\rangle$ can be performed by an exact circuit.

Non-isometric maps can only **approximately preserve information**:
$V^\dagger V \neq \mathbb{1}$ acts as a projector onto a preserved subspace.

Question: Hilbert space truncation in **quantum gravity**
$\leftrightarrow$ non-isometric projection?

Overlapping qubits

What happens to qubits under a non-isometric map V?



In the smaller Hilbert space, Paulis are mapped as

$$X \to X_p = VXV^\dagger \ , \quad Z \to Z_p = VZV^\dagger \ .$$

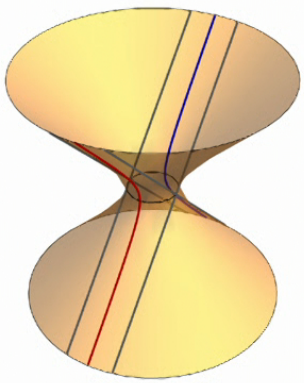
The n qubit algebra of the $X_p^{(j)}$ and $Z_p^{(k)}$ is *overlapping*, e.g.

$$|||[X_p^{(j)}, X_p^{(k)}]||| = O(\epsilon) \quad \text{for } j \neq k \ .$$

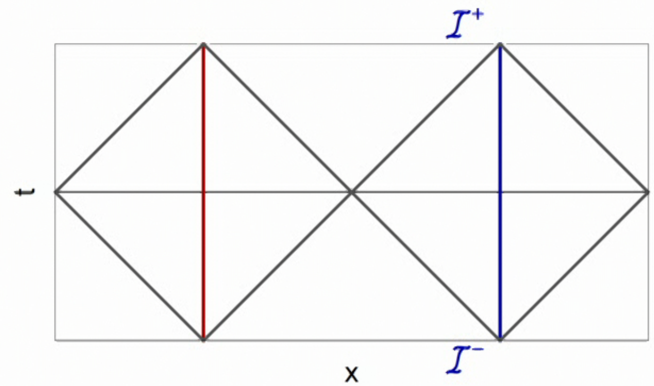
Surprisingly, small overlaps are possible for compression into **exponentially** fewer qubits: $\epsilon^2 \sim \frac{\log m}{n}$
[Chao/Reichardt/Sutherland/Vidick '17]

De Sitter spacetime

A physical setting with apparent Hilbert space growth:
dS spacetime, GR solution to pure $\Lambda > 0$ universe



(Higher-dim. embedding)



(Conformal coordinates)

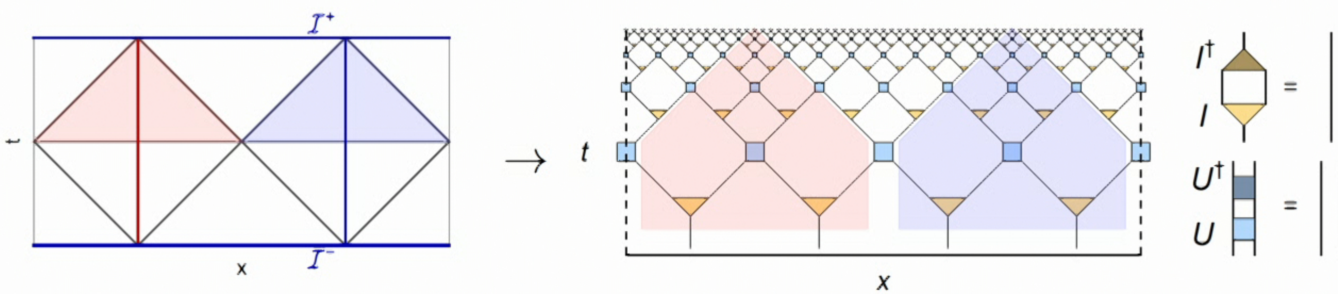
dS can be divided into *static patches* of stationary observers and rapidly expanding *exterior regions*.

De Sitter MERA

The *multi-scale entanglement renormalization ansatz* (MERA) [Vidal '06] can be used as a toy model of dS spacetime.

[Bény '11] [Czech/Lamprou/McCandlish/Sully '15]

[Bao/Cao/Carroll/Chatwin-Davis '17]

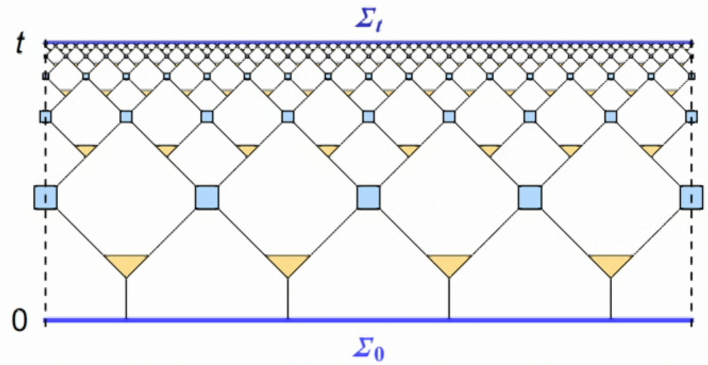


The causal structure of dS is reflected in the MERA tensors, built from isometries I and unitaries (disentanglers) U.

De Sitter MERA: non-isometric maps

The MERA construction allows us to define two types of non-isometric maps for qubits on dS time-slices:
A **global** and a **local map**.

The map $V_{\text{global}} : \mathcal{H}_{\Sigma_t} \to \mathcal{H}_{\Sigma_0}$ relates the global Hilbert space on a time-slice Σ_t to the "initial" one at $t = 0$.

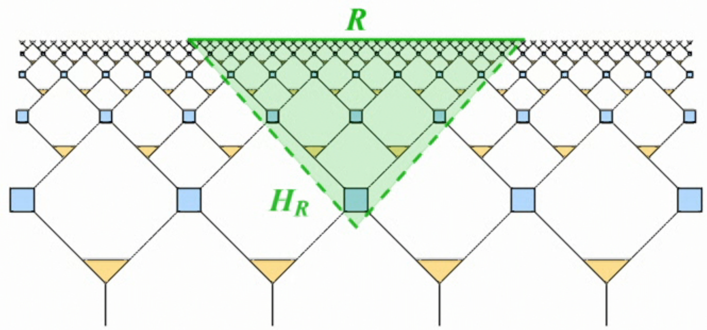


In a universe with such overlapping qubits in a fixed Hilbert space, non-local commutators become $O(1)$ at $\frac{t}{t_H} = O(S_{\text{dS}}) \approx 10^{120}$.

Curiously, this is consistent with models of slow-roll inflation.
[Dubovsky/Senatore/Villadoro '08]

De Sitter MERA: non-isometric maps

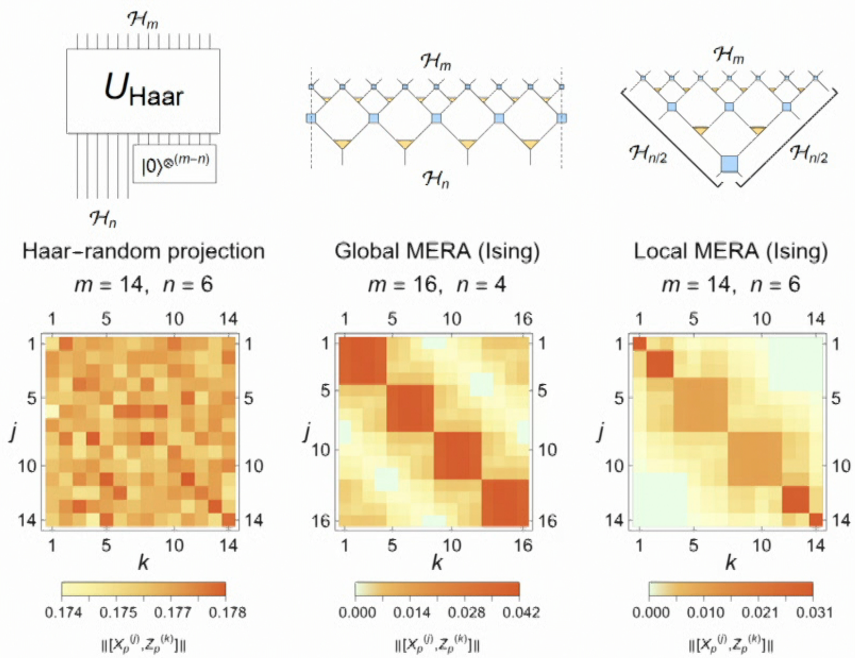
The alternative map $V_{\text{local}} : \mathcal{H}_R \to \mathcal{H}_{H_R}$ relates the Hilbert space on a time-slice *subregion* R to the horizon H_R of its past domain of dependence.



In this model, $\dim \mathcal{H}_R \sim e^{\dim \mathcal{H}_{H_R}}$. H_R is the "RT surface" of R, bounding its entanglement.

De Sitter MERA: Commutators

To preserve **approximate locality**, commutators $\|[X_p^{(j)}, Z_p^{(k)}]\|$ should decay with distance $|j - k|$.



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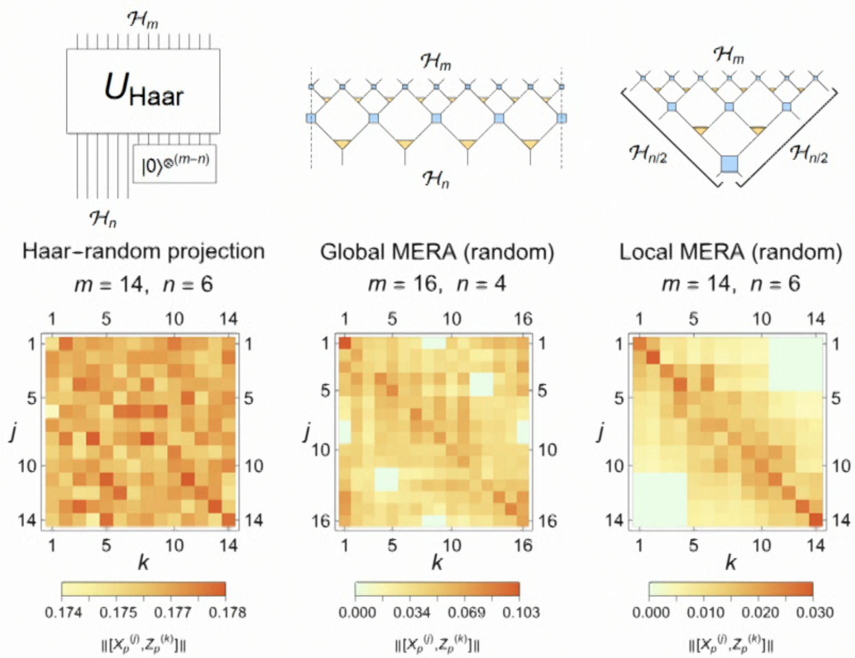

The causal structure of MERA/dS preserves approximate commutator locality!

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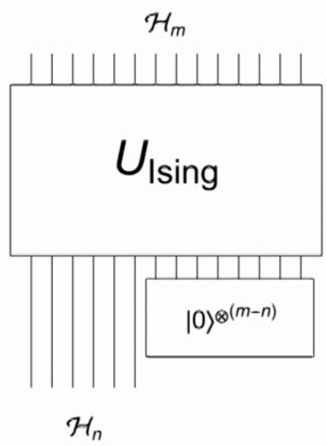
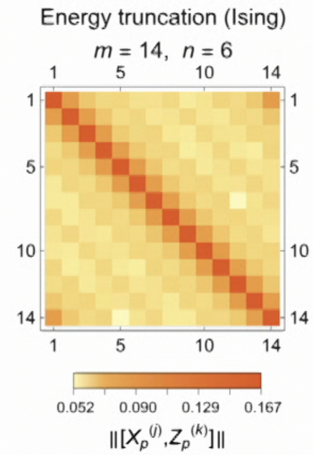
De Sitter MERA: Low-energy physics

Approximately local commutators can be achieved by truncation of high-energy states:

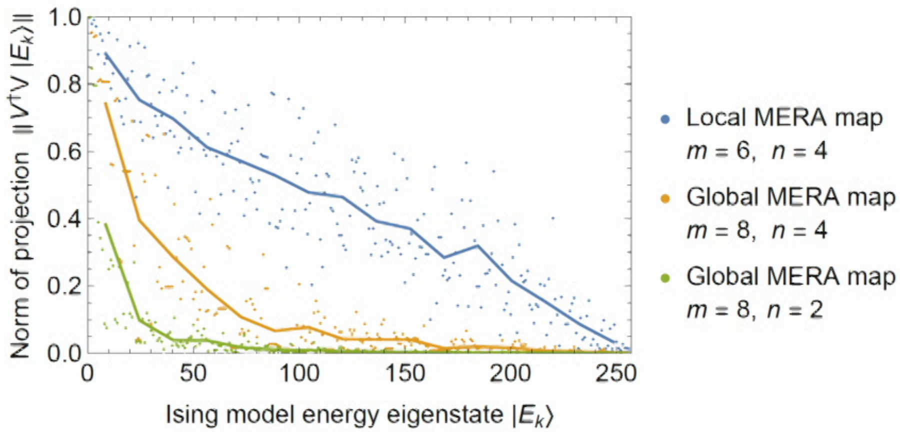




Small-distance corrections expected from gravitational dressing of operators. [Donnelly/Giddings '16]

De Sitter MERA: Low-energy physics

Global and local MERA models have the same effect:
Low-energy eigenstates are preserved,
high-energy states are represented with low fidelity.

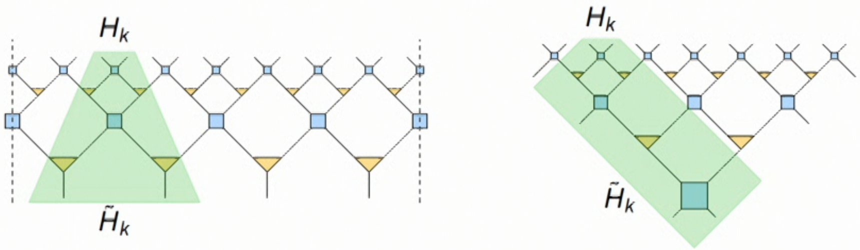


The global model sharply truncates after the first 2^n energy eigenstates, whereas the local model has a smooth behavior.

De Sitter MERA: Effective theories

Both the models describe an *effective theory* H of (subregions of) dS timeslices by a corresponding *fundamental theory* $\tilde{H}$.

- In the global MERA, $\tilde{H}$ is a truncation of H up to fixed scale/energy, **preserving locality**.
- In the local MERA, $\tilde{H}$ describes a very different theory: A local H leads to a **non-local** $\tilde{H}$.



Relationship to large-q SYK holographic proposal [Susskind '21]?

Discussion

Describing quantum gravity with overlapping qubits relates two old problems:

1. The problem of **degree of freedom counting** of Hilbert spaces when gravity is included
2. The problem of **Hilbert space dimension verification** for qubit systems

But this approach also raises lots of questions:

- Can we generally describe quantum gravity by quantum mechanics in a different (smaller) Hilbert space?
- What about flat spacetime?
- Relation to "static patch holography"?
- Low-energy CFT simulations with local MERA model?

Thank you for your attention!

“Expanding spacetime built from overlapping qubits”
(Midjourney v5)