Title: Talk 2 - Large N Matrix Quantum Mechanics as a Quantum Memory

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Collection: It from Qubit 2023

Date: August 04, 2023 - 9:30 AM

URL: https://pirsa.org/23080026

Abstract: In this paper, we explore the possibility of building a quantum memory that is robust to thermal noise using large N matrix quantum mechanics models. First, we investigate the gauged SU(N) matrix harmonic oscillator and different ways to encode quantum information in it. By calculating the mutual information between the system and a reference which purifies the encoded information, we identify a transition temperature, Tc, below which the encoded quantum information is protected from thermal noise for a memory time scaling as N^2. Conversely, for temperatures higher than T_c, the information is quickly destroyed by thermal noise. Second, we relax the requirement of gauge invariance and study a matrix harmonic oscillator model with only global symmetry. Finally, we further relax even the symmetry requirement and propose a model that consists of a large number N^2 of qubits, with interactions derived from an approximate SU(N) symmetry. In both ungauged models, we find that the effects of gauging can be mimicked using an energy penalty to give a similar result for the memory time. The final qubit model also has the potential to be realized in the laboratory.

It from qubit

“Large N Matrix Model as Quantum Memory”

Gong Cheng
Collaborate with C.J. Cao and B.Swingle

Based on arxiv:2211.08448

Aug 04, 2023

Motivation

What is Quantum Memory, why do we need it?

- Quantum information is typically fragile. Physical qubits decohere quickly.
- Many tasks require storing information for an arbitrary long time.
- Key idea is to use redundancy of physical system to encode small number of logical qubits.
- Quantum error correction codes (QECC) are quantum memory.

Motivation

Active vs. Passive Memory

- Active memory: requires actively detecting errors and correcting them.
- Passive memory (self-correcting memory): leave the system coupled with environment and information still preserved.
- Memory time $t_{mem} \to \infty$, as $N \to \infty$.

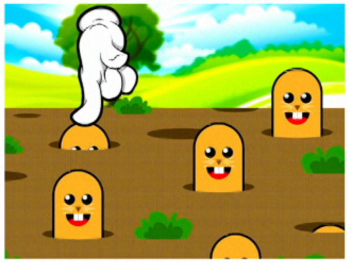


Figure: Acive quantum memory

Figure: Classical hard drive

Background

Why Gauge theory?

- Many important codes can be viewed as gauge theories, e.g. toric code is a lattice gauge theory. A 4d version of it forms a passive memory at finite temperature.
- Deconfined lattice discrete gauge theories are associated with high degree of entanglement and topological order.
- Linked to semi-classical gravity through holographic duality. A canonical example: 4d $SU(N)$ gauge theory.

Question:

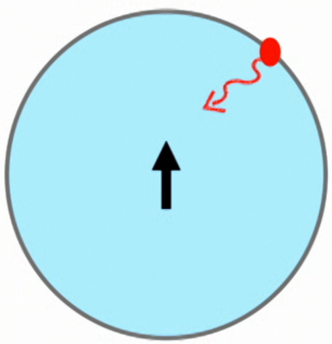
- Can we build a Passive Quantum Memory by gauge theory model?

Kitaev 2003, Kitaev/Preskill 2006, Alicki/Horodecki 2008

Bulk Calculation

How is information protected in bulk?

- Hide encoded qubit deep into the bulk.
- Bulk interaction is proportional to the gravitational coupling constant $G_N \propto \frac{1}{N^2}$.
- Model the boundary noise as thermal radiation in the bulk.



$$V_{int} = \frac{g}{N^2}\vec{S}\cdot\bar{\psi}\vec{\gamma}\psi$$

Figure: Protect qubit in the bulk

Cao/Cheng/Swingle coming soon

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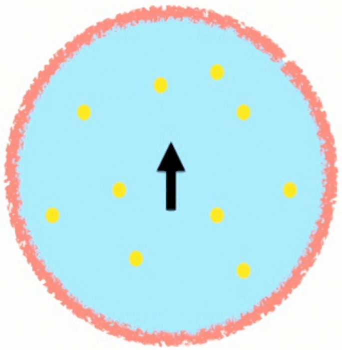

Figure: Protect qubit in the bulk

$$T < T_c,$$

$$|\rho(t) - \rho(0)| < \frac{t^2}{N^4} \sum_k n(k) e^{-\beta k}$$

$$t_{mem} \sim N^2$$

Cao/Cheng/Swingle coming soon

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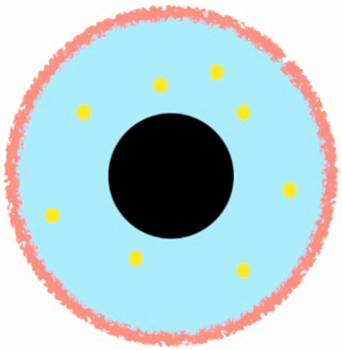

$$T > T_c,$$

$$\rho \to e^{-\beta H}$$

Figure: Protect qubit in the bulk

Cao/Cheng/Swingle coming soon

Conclusion

Bulk is a passive quantum memory with $t_{mem} \sim N^2$.

End of our "It from qubit" story

Thank you!

What from gravity?

Key ingredients we learn:

- Small gravitational coupling.
- Finite partition function at large N limit.

Can we have more quantum memory?

Background

Detour to quantum error correction:

- Encoding quantum information in code subspace $\mathcal{C}$.
- Error channel and recovery

$$\mathcal{R}(\mathcal{E}(\rho)) = \rho, \qquad \forall \rho \in \mathcal{C}.$$

- Knill-Laflamme condition (KL).

$$\langle i|E_a^\dagger E_b|j\rangle = c_{ab}\delta_{ij}, \qquad E_a \in \mathcal{E},\ |i\rangle \in \mathcal{C}$$

- No error can create transition between logical states. KL implies existence of recovery channel.

Boundary Viewpoint

Large N factorization:

- The n-point function of gauge invariant operators factorizes into product of two point functions, with corrections suppressed by powers of $\frac{1}{N}$:

$$\langle 0|O_i O_k O_l^\dagger O_j^\dagger|0\rangle \sim \langle 0|O_i O_j^\dagger|0\rangle\langle 0|O_k O_l^\dagger|0\rangle + \langle 0|O_i O_l^\dagger|0\rangle\langle 0|O_k O_j^\dagger|0\rangle + O\left(\frac{1}{N^2}\right)$$

$$\langle 0|O_i O_k O_j^\dagger|0\rangle \sim O\left(\frac{1}{N}\right).$$

- Find a set of errors and set of logical operators, with almost vanishing overlaps.

$$\mathcal{E} = \{E_a\}, \quad \mathcal{L} = \{O_i\}, \quad \langle 0|E_a O_i^\dagger|0\rangle = \frac{1}{N}, \quad |i\rangle = O_i^\dagger|0\rangle,$$

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$$\langle 0|O_i E_k O_j^\dagger|0\rangle \sim O\left(\frac{1}{N}\right).$$

- Approximate Knill-Laflamme condition (aKL):

$$\langle i|E_b^\dagger E_a|j\rangle = f_{ab}\delta_{ij} + \frac{g_{ab}^{ij}}{N^2}$$

$$\langle i|E_a|j\rangle = \frac{e_a^{ij}}{N}$$

Main Result

Theorem

The mutual information $I(S, R)$ is given by

$$I(S:R)(t) = 2\ln d - K\left(\frac{t}{N^2}\right),$$

where $K(x)$ is a polynomial function of x, for temperature $T < \frac{1}{\mu}$.
If the following conditions are satisfied:

1. *(Sparse spectrum) The number $n(\epsilon)$ of error operators with energy ϵ is bounded by* $\exp(\mu\epsilon)$ *for some $\mu > 0$,*
2. *(Uniform coupling) The coupling to thermal bath is $O(1)$,*
3. *(Approximate error correction) $\forall E_a, E_b \in \mathcal{E}$, the approximate Knill-Laflamme condition is satisfied.*

Application:Model I

Gauged Matrix Oscillator:

- Harmonic oscillator action with matrix d.o.f,

$$L = \frac{1}{2}\,\mathrm{Tr}[(\partial_t X)^2 - \omega^2 X^2]. \qquad X = \begin{pmatrix} x_1^1 & x_1^2 & \cdots & x_1^N \\ x_2^1 & x_2^2 & \cdots & x_2^N \\ \vdots & \vdots & \ddots & \vdots \\ x_N^1 & x_N^2 & \cdots & x_N^N \end{pmatrix}$$

- The action has a symmetry under unitary transformations $X(t) \to UX(t)U^\dagger$, with $U \in SU(N)$. Promote to a local symmetry $U \to U(t)$ by coupling to gauge field A,

$$D_t X = \partial_t X - [A, X].$$

Itzhaki/McGreevy 2005, Berenstein 2004

Application:Model I

Physical states

- A is a Lagrangian multipler. Integrating it out:

$$G = [X, \dot{X}] = 0.$$

$$G|\psi\rangle_{phy} = 0, \qquad [G, O_{phy}] = 0$$

- Physical Hilbert space basis,

$$a_i^{\dagger j} = \frac{1}{\sqrt{2\omega}}(X_i^j - iP_i^j)$$

$$|\psi\rangle = \mathrm{Tr}(a^{\dagger n_1})\,\mathrm{Tr}(a^{\dagger n_2})\cdots\mathrm{Tr}(a^{\dagger n_k})|0\rangle$$

Model I: Encoding

- Two Matrix oscillators a_1 and a_2, separated by a large distance.
- Logical operators are non-local and involve both a_1 and a_2 in a single trace:

$$|\tilde{1}\rangle = \frac{\mathrm{Tr}(a_1^{\dagger 2} a_2^{\dagger 2})}{N^2}|0\rangle_{12},$$

$$|\tilde{2}\rangle = \frac{\mathrm{Tr}(a_1^{\dagger} a_2^{\dagger})^2}{\sqrt{2}N^2}|0\rangle_{12}$$

$$O_{mn} = \frac{\mathrm{Tr}(a_1^{\dagger m} a_2^{\dagger n})}{N^{\frac{m+n}{2}}}$$

- Error operators only act locally.

$$\langle 0|E_{l,r}O_{m,n}|0\rangle \sim \frac{1}{N}$$

$$E_{l,r} =: \prod_{(l,r)} \frac{\mathrm{Tr}(a_1^{\dagger l})\,\mathrm{Tr}(a_2^{\dagger r})}{N^{\frac{l+r}{2}}}:$$

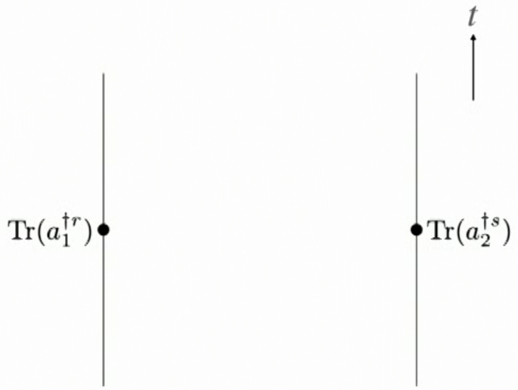


Figure: Two oscillators separated

Application:Model I

Check the conditions

- Approximate Knill-Laflamme condition:

$$\langle 0|O_{n'm'}E_{l',r'}E_{l,r}O_{n,m}|0\rangle = f_{lr,l'r'}\delta_{nm,n'm'} + O(\frac{1}{N^2})$$

- Couple with thermal bath

$$H = \omega\,\mathrm{Tr}(a_1^\dagger a_1) + \omega\,\mathrm{Tr}(a_2^\dagger a_2) + \lambda_{1,0}Tr(a_1^\dagger)b_{1,0} + \lambda_{1,1}\,\mathrm{Tr}(a_1^\dagger)\,\mathrm{Tr}(a_2^\dagger)b_{1,1} + \cdots$$

- Counting operators:

$$\mathrm{Tr}(P\{a_1^{\dagger n_1}a_2^{\dagger n_2}\cdots a_k^{\dagger n_k}\}), \qquad n := \sum_i n_i, \qquad \epsilon = n\omega$$

Number of single trace operator is bounded by $e^{\frac{\epsilon}{\omega}logk}$

What from gravity?

This is not an end of the question.

The final destination is "qubit".

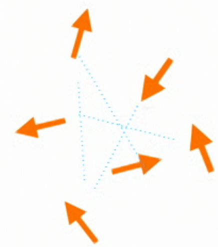
Main obstacles to "qubits":

- No large N gauge symmetry in nature.
- Finte Hibert space.

Application: Model II

Qubit model with approximate symmetry:

- The model consists of large N number of qubits,



$$H_0 = (\sum_{ij} S_j^{iz} - S_{\text{tot}})^2, \qquad [S_i^{j+}, S_j^{i-}] = 2S_i^{jz}$$

- Equipped with approximate $SU(N)$ symmetry:

$$\tilde{G}_i^j = \sum_k S_k^{j+} S_i^{k-} - S_i^{k+} S_k^{i-}$$

$$[\tilde{G}_i^j, S_k^{l\pm}] = (\delta_i^l S_k^{j\pm} - \delta_k^j S_i^{l\pm}) S_k^{lz}.$$

- Requires non-local strong couplings between spins

$$\Delta H = \frac{\mathcal{J}}{N} \sum_{ijkl} (S_j^{i+} S_k^{j-} S_l^{k+} S_i^{l-} - S_j^{i+} S_k^{j-} S_i^{l+} S_l^{k-} + \cdots)$$

$$\mathcal{J} \sim \log N$$

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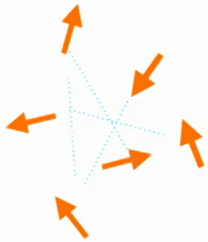

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$$\mathcal{J} \sim \log N$$

Limitations and future directions

Limitations:

- Require Gauge symmetry or strong all-to-all interactions
- Hard to implement logical operation
- Information slowly decay at zero temperature.

Future directions:

- Recovery channel of approximate QECC
- BFSS model as quantum memory
- Gauge theory with large N Abelian symmetry

Related works:

Related works:

- Quantum error correction and large N. Milekhin 2020
- Deconfinement and Error Thresholds in Holography. Bao/Cao/Zhu 2022
- Finite rank gauge theory models as QECC. Bao/Cao/Chatwin-Davies/Cheng/Zhu 2023

Thank You!