Title: Talk 135 - Spectral properties of the sparse SYK model, with analysis of recent experimental simulation of holography

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Abstract: The Sachdev-Ye-Kitaev (SYK) model is a simple toy model of holography that has seen widespread study in the area of quantum gravity. It is particularly notable for its feasibility of simulation on near-term quantum devices. Recently, Swingle et al. introduced a sparsified version of the SYK model and analyzed its holographic properties, which are remarkably robust to deletion of Majorana interaction terms. Here we analyze its spectral and quantum chaotic properties as they pertain to holography as well as how they scale with sparsity and system size through large scale numerics. We identify at least two transition points at which features of chaos and holography are lost as the model is sparsified, and above which all important features are preserved, which may serve as guidelines for future experiments to simulate quantum gravity. Additionally, we apply these analyses to the SYK model that was recently experimentally simulated on the Google Sycamore quantum processor, which itself was a highly sparsified SYK model obtained through a machine learning algorithm incorporating mutual information signatures of a traversable wormhole.

Spectral properties of the Sparse SYK model, with analysis of recent experimental simulation of holography

Patrick Orman, Hrant Gharibyan, John Preskill

Sachdev–Ye–Kitaev model

$$H = \frac{1}{4!} \sum_{abcd} j_{abcd} \chi_a \chi_b \chi_c \chi_d$$

$$\overline{j_{abcd}} = 0, \quad \overline{j_{abcd}^2} = \frac{3! J^2}{N^3}$$

- *N* Majorana fermions interacting **all-to-all**, *q*=4
- Gaussian-random coupling, averaged over many samples
- Quantum chaotic & RMT; Holographic dual is JT gravity
- Feasible for simulation on near-term quantum devices

[1,2]

Sparse SYK

$$H = \frac{1}{4!} \sum_{abcd} x_{abcd} j_{abcd} \chi_a \chi_b \chi_c \chi_d$$

$$\overline{j_{abcd}} = 0, \quad \overline{j_{abcd}^2} = \frac{1}{p} \frac{3! J^2}{N^3}$$

$$\Pr(x_{abcd} = 1) \equiv p$$

- **Sparse** – each term is subject to probability of deletion [Xu, Susskind, Su, Swingle 2020]
- *p*=1 is full SYK
- Related to effective average degree *k* by $p = \frac{kN}{\binom{N}{4}}$
- Huge computational speedups (p=1 → p=0.1):
  - N=18: 2h→13m, N=20: 5h→27m, N=22: 12h→1h, N=26: 86h→11h

[3]

Nearest-neighbor gap ratio

$$r = \left\langle \min\left(\frac{\lambda_i - \lambda_{i-1}}{\lambda_{i+1} - \lambda_i}, \frac{\lambda_{i+1} - \lambda_i}{\lambda_i - \lambda_{i-1}} \right) \right\rangle_{\lambda, J}$$

$N =$	18	20	22	24	26	28	30
$N \mod 8$	2	4	6	0	2	4	6
Ensemble	GUE	GSE	GUE	GOE	GUE	GSE	GUE
Lit. r	0.5996(1)	0.6744(1)		0.5307(1)			
Lit. r, 2 blocks		0.4116(5)		0.4235(5)			

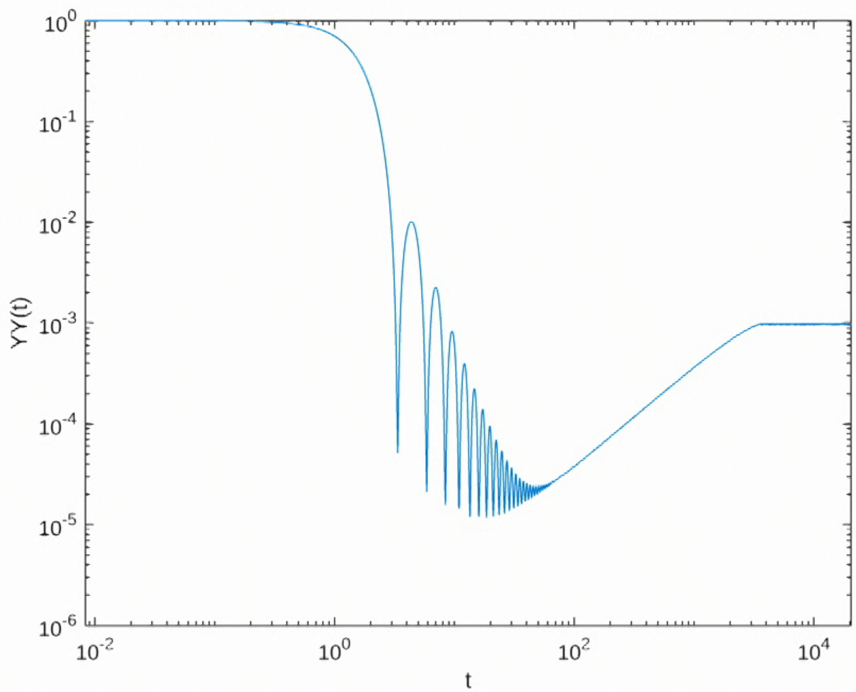
- Average of ratios of subsequent gaps in spectrum (each at most 1)
- Useful and common numerical diagnostic of RMT & quantum chaos; one established literature value per ensemble
- Robust for even a single sample, but we average over samples anyway
- Directly computed values differ due to “blocks” (parity sectors)

[4,5]

Spectral form factor (SFF)

$$g(t) = \frac{\langle |Z(t)|^2 \rangle}{L^2} = \frac{\langle |\operatorname{Tr} e^{-iHt}|^2 \rangle}{L^2} = \frac{\langle \sum_{ij} e^{i(E_i - E_j)t} \rangle}{L^2}$$

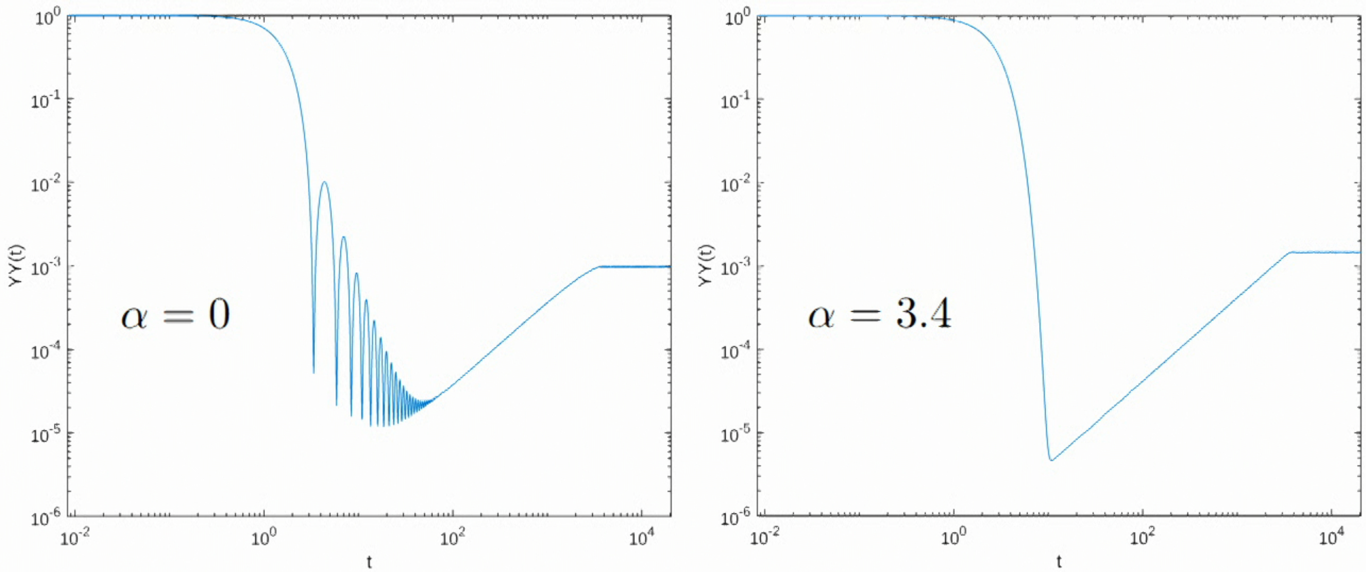
$$= \frac{1}{L^2} \int dE_1 dE_2 \, L^2 R(E_1, E_2) e^{i(E_1 - E_2)t}$$



- Fourier transform of the pair correlation function, normalized by L = dim H
- Another useful and common numerical diagnostic of RMT/quantum chaos, measurable in experiments [Joshi et al 2021]
- Dip, Ramp, Plateau
- Ramp time t_{ramp} corresponds to inverse of energy scale within which RMT is a well-fitting description, relevant in black hole duals

[6,7,12]

Gaussian-filtered SFF



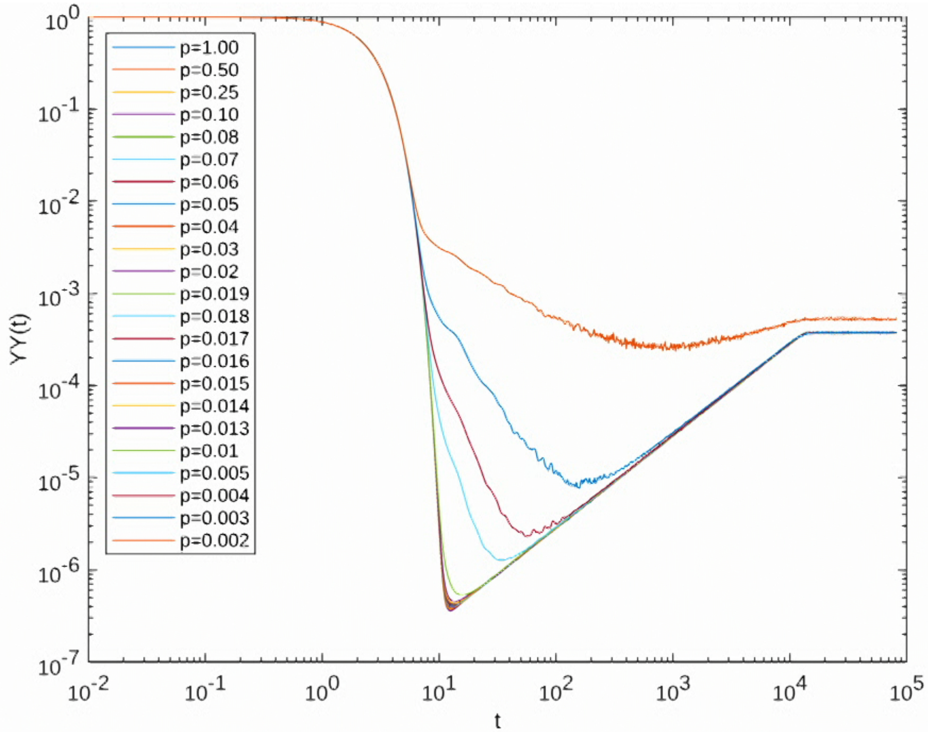
$$|Y(\alpha,t)|^2 = \left|\sum_i e^{-\alpha E_i^2} e^{-iE_i t}\right|^2, \quad h(\alpha,t) = \frac{|Y(\alpha,t)|^2}{|Y(\alpha,0)|^2}$$

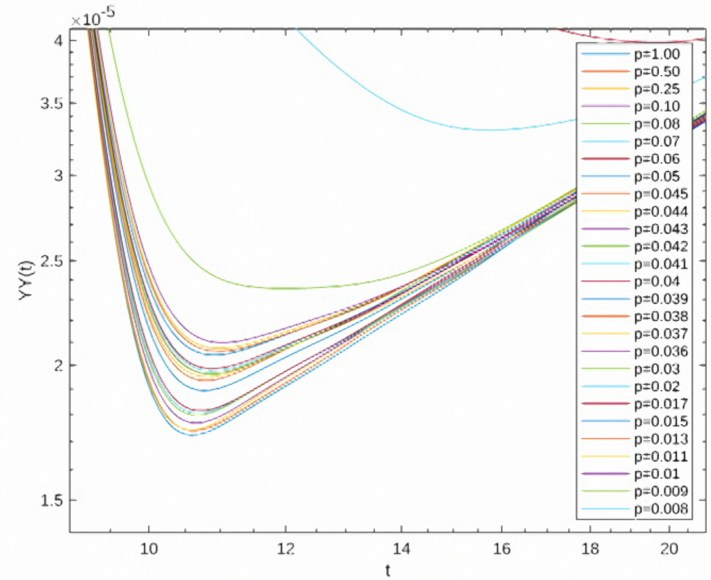
- Apply Gaussian window to spectrum, tune its width α
- Obfuscated early ramp revealed – increased sensitivity to detecting changes

[7]

SFF data

- N = 26
- α = 3.2
- 12,000 samples



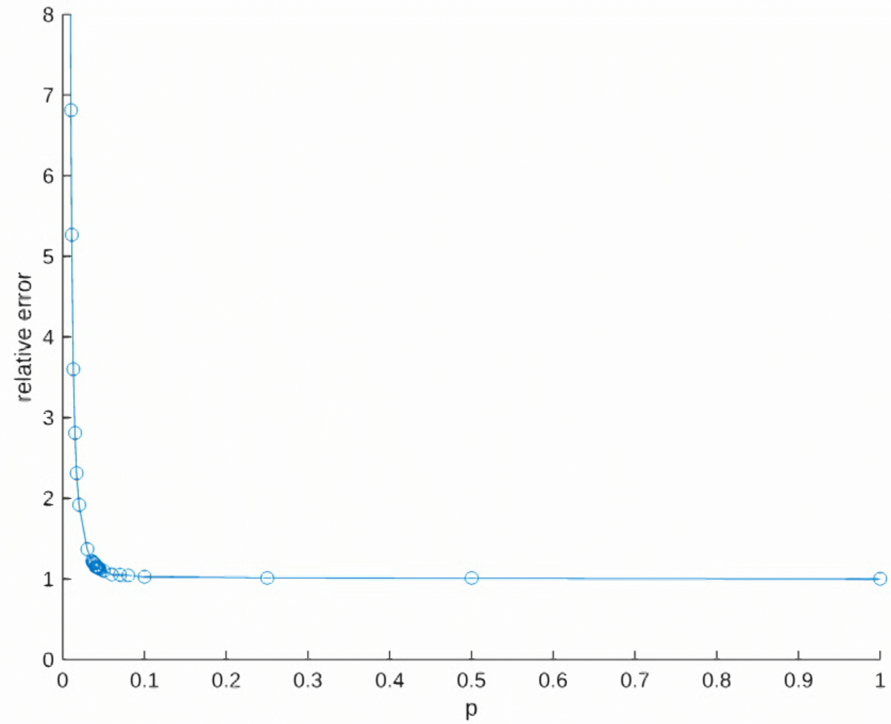


- Ramp deviates as model is sparsified, i.e. as *p* decreases
- Two transition points: (1) ramp first changes, (2) ramp gone
- To quantify deviation, take ratio of ramp minima (relative error)

SFF data, p_1

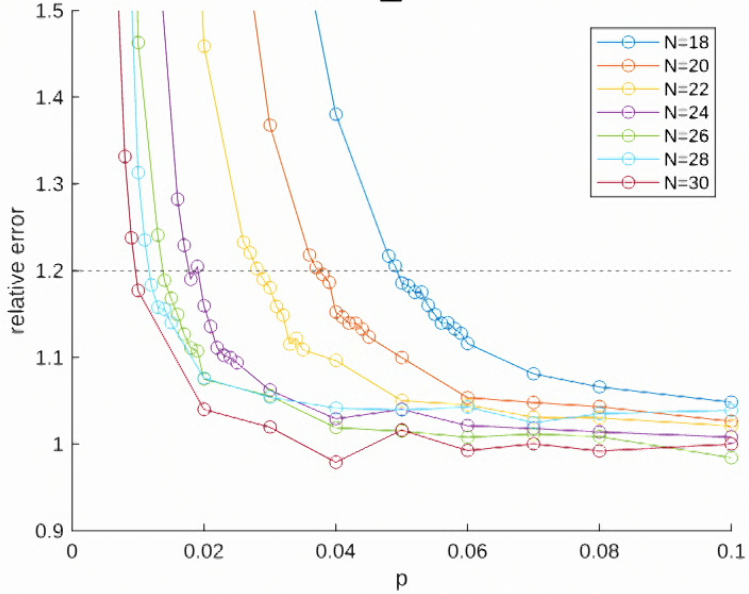
- N = 26
- α = 3.2
- 12,000 samples

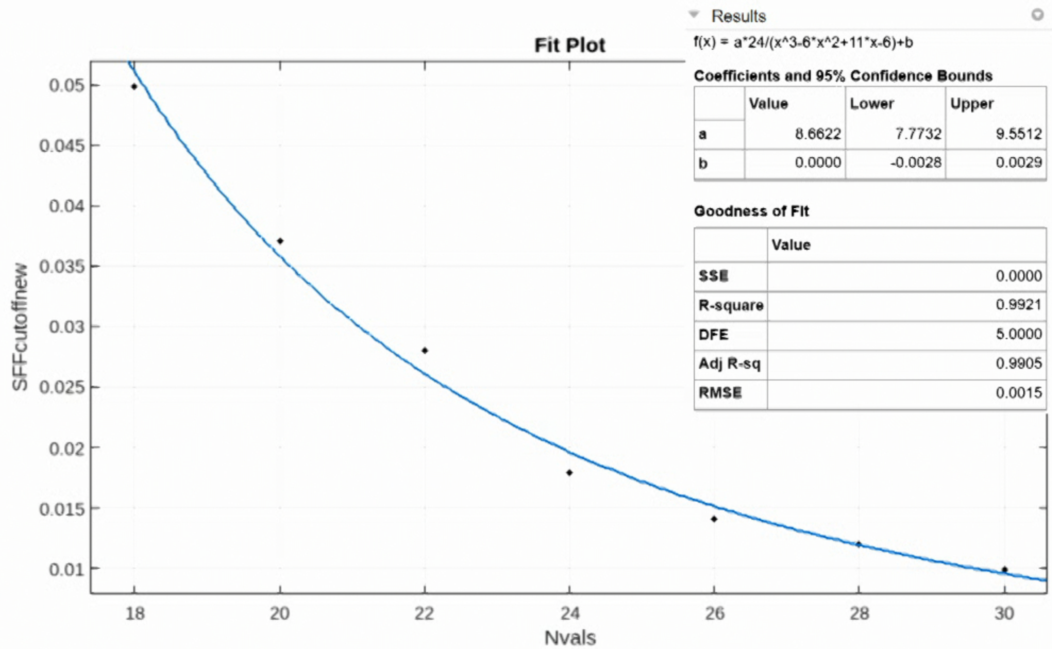
- $\min(SFF_i)/\min(SFF_{p=1})$



- Very resilient to sparsity, fairly sharp transition at which chaos first starts to change – What is really *needed* for emergent gravity?
- Significant implications for computational feasibility
- We call this point p_1

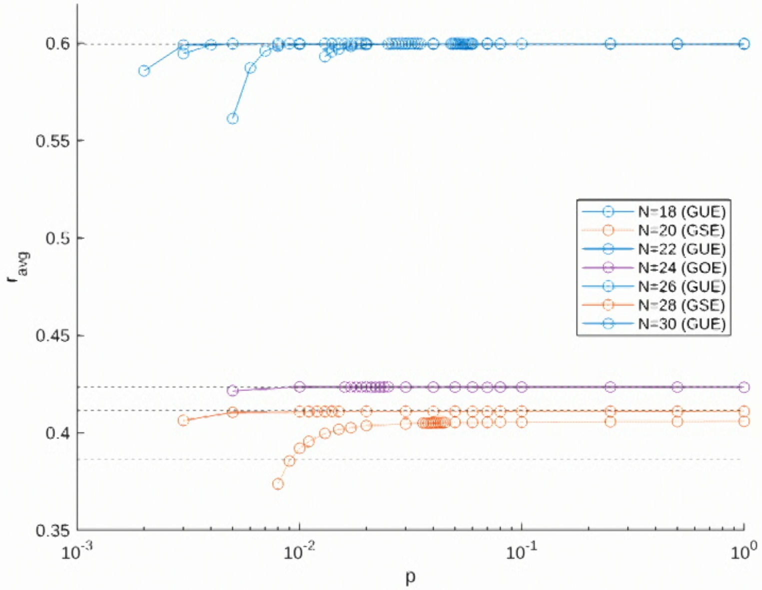
SFF data, p_1

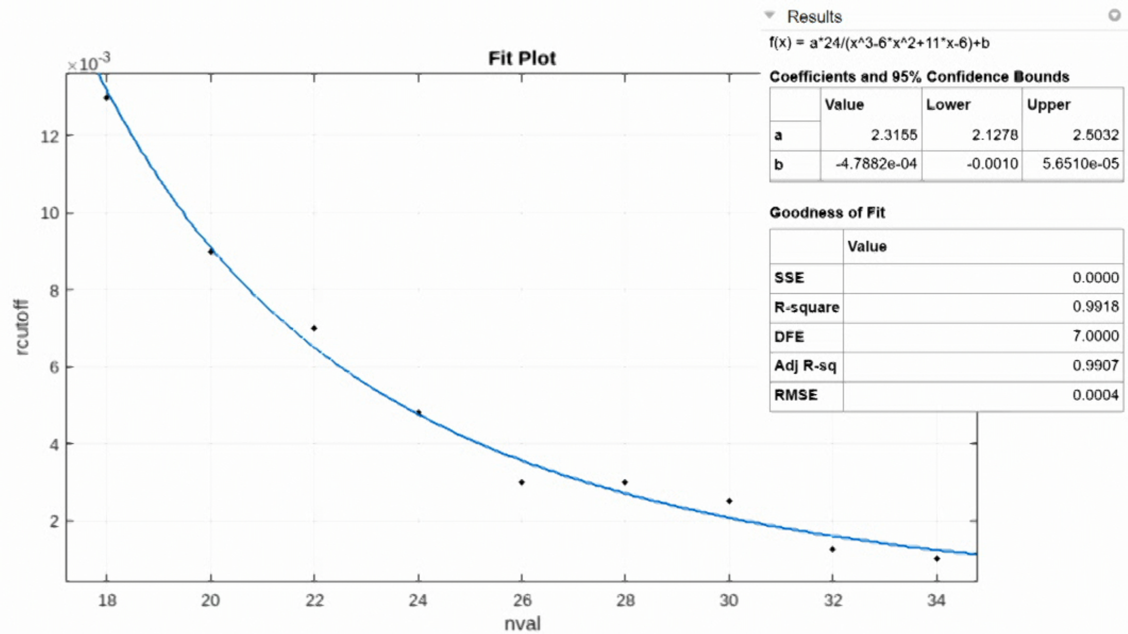




- Best curve fit is $p_1 \sim N/\binom{N}{4}$
- There appears to be a certain effective degree *k* at which chaos changes (~8.7)
  - Compare [3]'s conjectured min ~4 to exhibit maximally chaotic gravitational sector at low temp
- Can serve as a tangible guideline for faithful experimental simulation of holography on near-term quantum devices

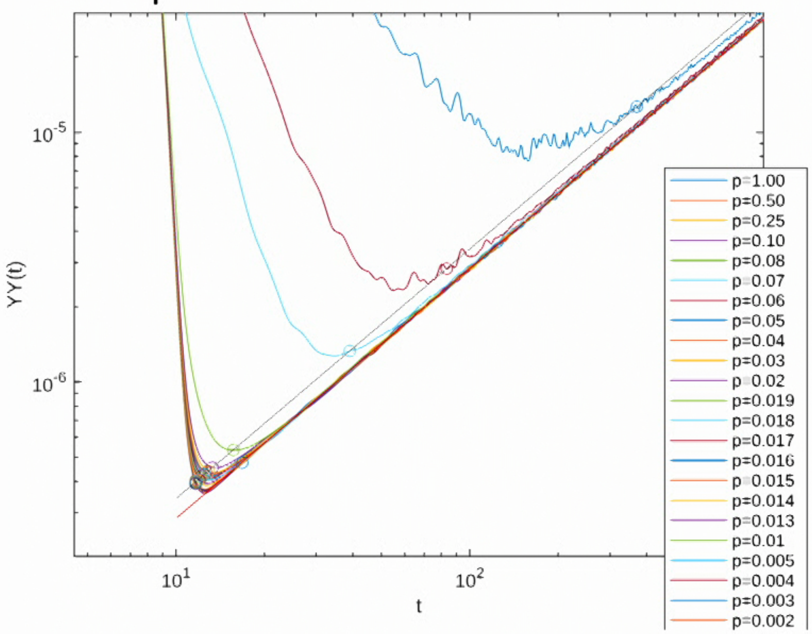
Gap ratio, p_2

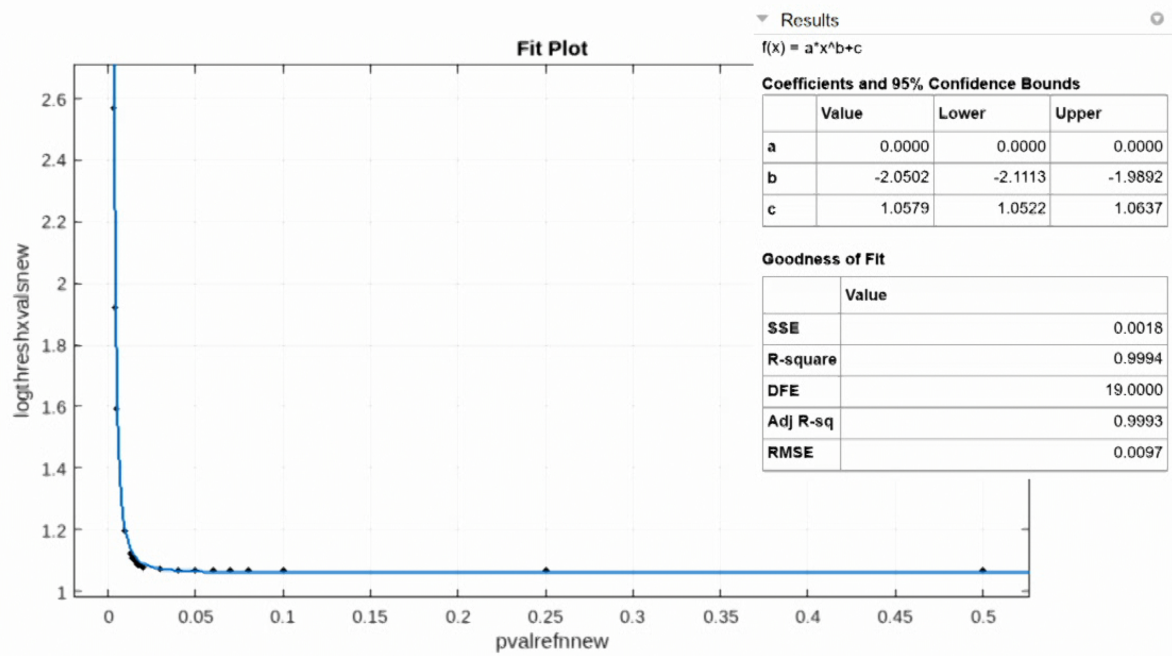




- Ramp's complete disappearance is complicated – cannot use SFF directly
- Instead, use gap ratio, which probes same phenomenon (spectral rigidity)
- We find an even sharper transition, so use a ~1% threshold relative error
- Best fit is $p_2 \sim N/\binom{N}{4}$ again, another average degree (~2.3)

t_{ramp} vs p





- Scaling of the ramp time with sparsity is also of interest
- Relative error threshold of 20% (but resulting fit is threshold-independent)
- Scaling is $t_{\mathrm{ramp}} \sim 1/p^2$ for all tested *N*

Emergence of degeneracies

N=22, p=1.00

Degeneracy	# samples	proportion	r	stdev
2-fold	20,000	1	0.5997	0.009

- When SYK gets very sparse (at p_2), each individual sample can have a different degeneracy, and accordingly a different r value, than expected
  - Gap ratios vary widely – standard deviation increases by factor of 10
  - Neither individual gap ratios nor average match expected literature values
  - Within each degeneracy class, avg r still decreases with sparsity
- What's going on here? Is it interesting?

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Degeneracy	# samples	proportion	r	stdev
2-fold	20,000	1	0.5997	0.009

N=22, p=0.01

Degeneracy	# samples	proportion	r	stdev
2-fold	19,999	0.99	0.5994	0.011
4-fold	1	0.00	0.6633	N/A

N=22, p=0.005

Degeneracy	# samples	proportion	r	stdev
2-fold	19395	1	0.5786	0.065
4-fold	597	0.03	0.5831	0.128
8-fold	7	0.00	0.2796	0.144
16-fold	1	0.00	0.5607	N/A

N=22, p=0.003

Degeneracy	# samples	proportion	r	stdev
2-fold	10537	1	0.4464	0.151
4-fold	5672	0.54	0.3563	0.170
8-fold	2205	0.21	0.2683	0.141
16-fold	1035	0.10	0.2492	0.140
$\geq$32-fold	551	0.05	0.1872	0.163

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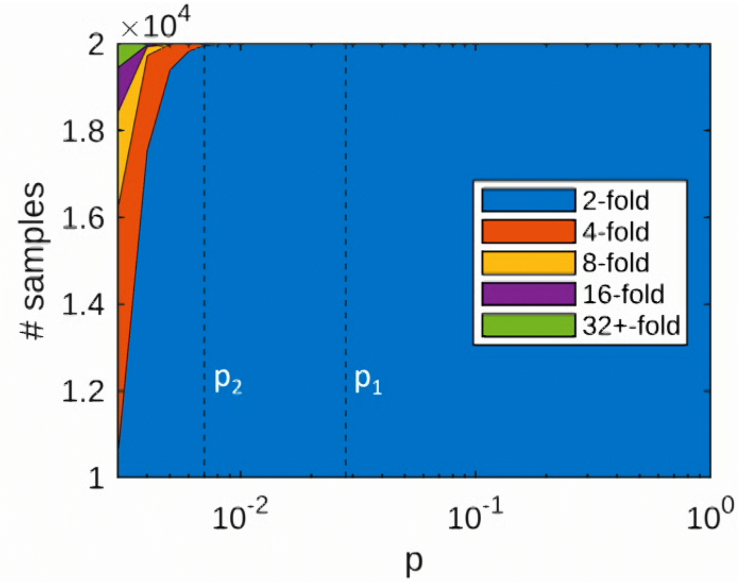
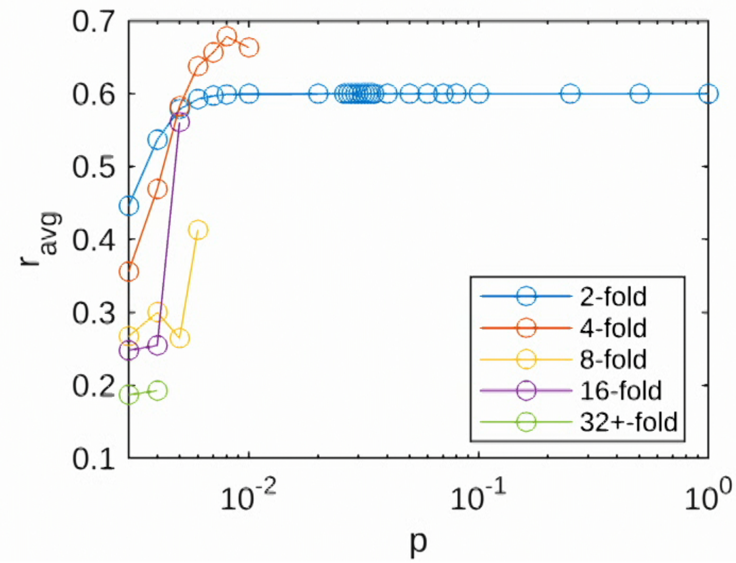




Analysis of recent experimental simulation of holography

"Traversable Wormhole Dynamics on a Quantum Processor"

Jafferis, Zlokapa, Lykken, Kolchmeyer, Davis, Lauk, Neven, Spiropulu
Nature 612 p.51–55, Dec 2022

(Based on [9,10], see also [11])

Three machine-learned models

Main:

$$H = -0.36\psi^1\psi^2\psi^4\psi^5 + 0.19\psi^1\psi^3\psi^4\psi^7 - 0.71\psi^1\psi^3\psi^5\psi^6 + 0.22\psi^2\psi^3\psi^4\psi^6 + 0.49\psi^2\psi^3\psi^5\psi^7$$

- 7/10 Majoranas
- 5/210 terms
- "a loss function based only on the error from the asymmetric **mutual information signature** of a traversable wormhole and on a regularization penalty to induce sparsity"

S16:

$$H = -0.35\psi^1\psi^2\psi^3\psi^6 + 0.11\psi^1\psi^2\psi^3\psi^8 - 0.17\psi^1\psi^2\psi^4\psi^7 - 0.67\psi^1\psi^3\psi^5\psi^7 + 0.38\psi^2\psi^3\psi^6\psi^7 - 0.05\psi^2\psi^5\psi^6\psi^7$$

- 8/10 Majoranas
- 6/210 terms
- same loss function as Main, but "unlike the Hamiltonian in the main text, **does not have all commuting terms**. It successfully preserves the mutual information dynamics demonstrating perfect size winding, and is consistent with other gravitational signatures"

S17:

$$H = +0.60\psi^1\psi^3\psi^4\psi^5 + 0.72\psi^1\psi^3\psi^5\psi^6 + 0.49\psi^1\psi^5\psi^6\psi^9 + 0.49\psi^1\psi^5\psi^7\psi^8 + 0.64\psi^2\psi^4\psi^8\psi^{10} - 0.75\psi^2\psi^5\psi^7\psi^8 + 0.58\psi^2\psi^5\psi^7\psi^{10} - 0.53\psi^2\psi^7\psi^8\psi^{10}$$

- 10/10 Majoranas
- 8/210 terms
- "a loss function designed to optimize for new physics. We learn a Hamiltonian with a teleportation signal larger than that of the SYK model by maximizing the difference in mutual information"

Initial observations

- Low N, only one sample – how to understand chaos & holography?
  - e.g. "Wormholes without averaging" – Saad, Shenker, Stanford, Yao 2021 (but only looks at typical members of ensemble)
- Very highly sparsified
  - $p_1 = 0.31$ for $N=10$
  - $p_2 = 0.11$ for $N=10$
  - Effective sparsity $5/210 = 0.024$
- Appears to be well past both transition points (although these are for ensembles)
  - Simply not enough terms for emergent gravity?

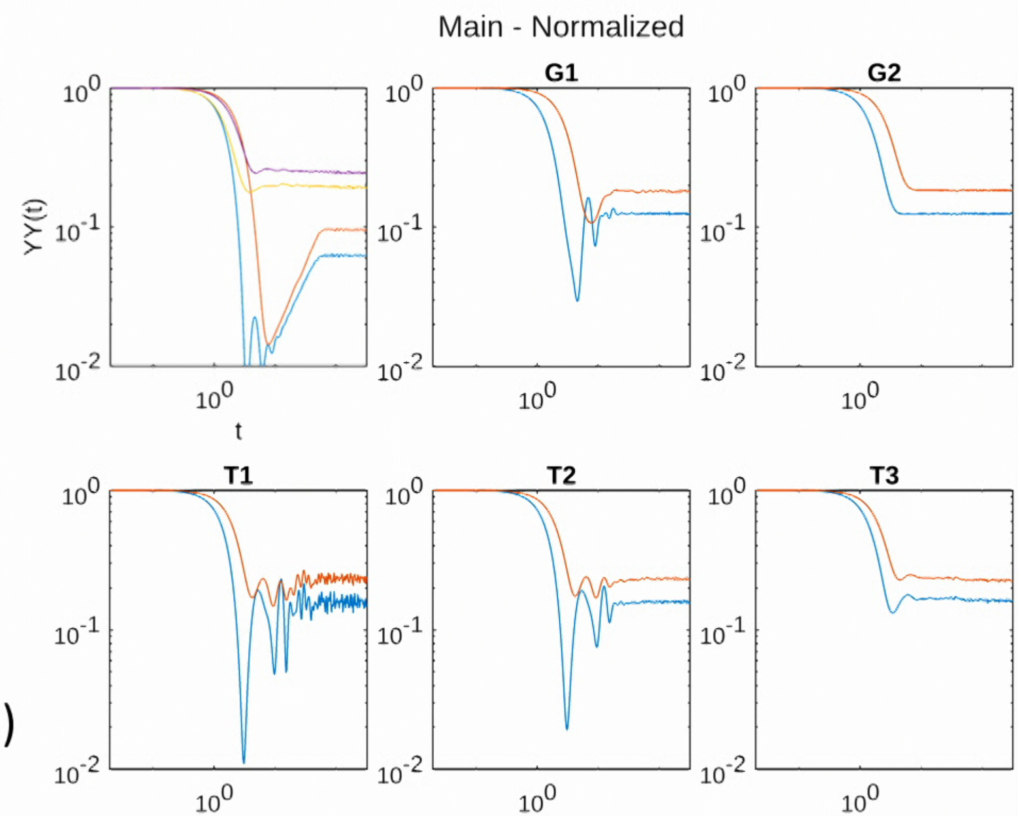
Gap ratio: full/sparse SYK vs. simulated

- Literature value for SYK N=10 (GUE): **2-fold** degeneracy, **r = 0.5996**
- **Recall: robust for even a single sample**

- **Main:** 4-fold, r = 0.5178
- **S16:** 2-fold, r = 0.5681
- **S17:** unusual symmetry

SFF

- Appears to be a **very atypical** member of the (sparse) SYK ensemble
- Sparse SYK at p=5/210=0.024: ensemble-averaged SFF has no ramp, only plateau
- Attempted to introduce ensemble averaging with many strategies
  - Ensemble-averaging in a neighborhood around the machine-learned sample's coefficients (shifting j's Gaussians' means) produces noisy SFF that only slightly resembles a tiny ramp-like dip
  - All others are just plateau



Open questions

- What is the *core necessity* for gravity to emerge from quantum mechanics, if all interactions are not?
- Can we consider a single un-averaged sample to be holographic or not (e.g., if it is not typical of the full ensemble)?
  - How should we think about averaging in ensembles whose members have differing properties from each other, i.e. near transition points?
  - What even is averaging?
- How do we understand holography of SYK for $p_2 < p < p_1$?
  - Clearly not the same as full SYK, but is it still holographic in a different way?
  - Machine-learned model in [8] may live in this regime
- How do we understand the emergence of degeneracies and fluctuation of gap ratio near p_2?

Future research

- SFF, gap ratio, and p_1 & p_2 may serve as diagnostic tools and tangible guidelines for how sparse one can go to enable faithful simulation of holography and quantum gravity
- Currently pushing numerics to larger N
  - Full SYK not even feasible – sparse SYK is feasible and has same properties
  - Finite size analysis – are $p_{1,2}$ critical?
- Explore $p_2 < p < p_1$ holography, and p_2 degeneracy emergence
- Carry out same ML model in [8] for larger system size
  - Aforementioned issues simply due to insufficiently large N?
- Modify ML model to account for presence of ramp and/or other quantum chaotic & RMT features
  - Could also introduce noise and average over many learned samples

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