Title: Talk 33 - Microscopic origin of the entropy of black holes

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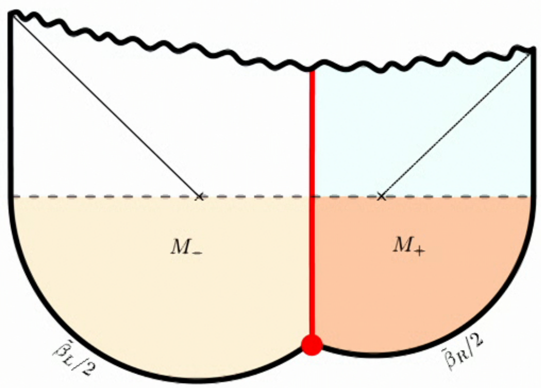
Abstract: We present a construction in which the origin of black hole entropy gets clarified. We start by building an infinite family of geometric microstates for black holes in general relativity. This construction naively overcounts the Bekenstein-Hawking entropy. We then describe how wormholes in the Euclidean path integral for gravity cause these states to have exponentially small, but universal, overlaps. These overlaps recontextualize the Gibbons-Hawking thermal partition function. We finally show how these results imply that the microstates span a Hilbert space of log dimension equal to the Bekenstein-Hawking entropy, and how they clarify the nature of the volumes of Eisntein-Rosen bridges.

A CFT method to construct black hole microstates

This follows because these CFT microstates have effective geometric descriptions as a domain wall

$$T_{\mu\nu}\Big|_{\mathscr{W}} = \sigma\, u_\mu u_\nu$$



The consistency relations between the preparation temperatures and the physical temperatures are given by the simple saddle point equations

$$\tilde{\beta}_L + \Delta\tau_- = \beta_L \qquad \tilde{\beta}_R + \Delta\tau_+ = \beta_R$$

The proper mass of the shell, related to number of operator insertions, is unconstrained from above

The volume of the Einstein-Rosen bridge roughly proportional to the mass of the shell $V_{ren} = ml/(d-1)$

A CFT method to construct black hole microstates

- We only needed a (not so) heavy scalar operator. These types of microstates are there for GR plus a scalar field. They are also there in String Theory as well (N=4 SYM)
- They have a precise semiclassical description that is geometric. It has singularities and horizons
- The Einstein-Rosen bridge grows indefinitely with the scaling dimension of the operator
- These infinite families naively overcount the Bekenstein-Hawking entropy
- Wheeler's bags of gold **[Wheeler, Frolov and Novikov, Fu and Marolf]**
- In this AdS/CFT construction it is difficult to argue these states do not contribute to the Black Hole Hilbert space
- Do they really overcount? How do we count them?

Wormholes, overlaps and universality

We then seek to compute the following quantities

$$\overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}} \rangle}\,, \qquad \overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} | \Psi_{\mathbf{m}} \rangle}\,, \qquad \overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} \Psi_{\mathbf{m}''} \rangle \langle \Psi_{\mathbf{m}''} | \Psi_{\mathbf{m}} \rangle}\,, \quad \cdots$$

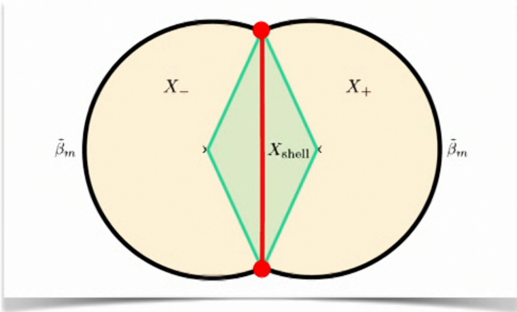
The overline notation means that we compute these quantities using the gravitational action

$$I[X] = -\frac{1}{16\pi G}\int_X (R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial X} K + \int_{\mathscr{W}} \sigma + I_{ct}$$

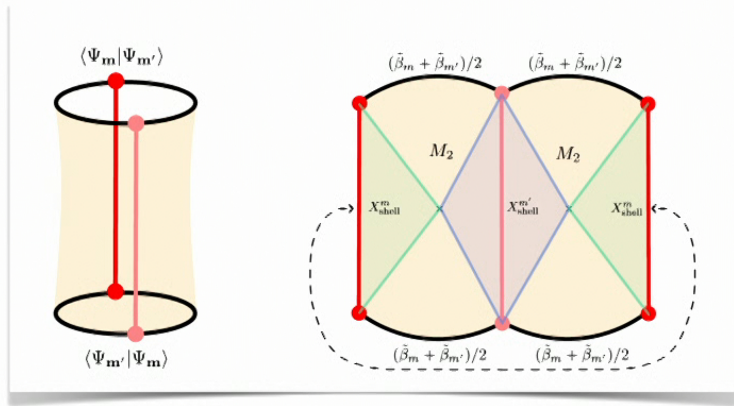
$$\overline{\langle \Psi_{\mathbf{m}} | \Psi_{\mathbf{m}'} \rangle \langle \Psi_{\mathbf{m}'} | \Psi_{\mathbf{m}''} \rangle \ldots \langle \Psi_{\mathbf{m}'_{\ldots}{}'} | \Psi_{\mathbf{m}} \rangle}\,\Big|_c = \frac{Z_n}{Z_1 Z_1' \cdots Z_1^{\prime\cdots\prime}}$$

[Saad, Shenker, Stanford, 2019]

Wormholes, overlaps and universality



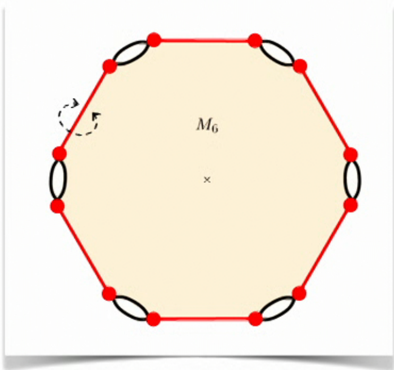
$\to Z_1$ This computes the norm of the microstates



Action of two boundary wormhole $Z_2 \leftarrow$

[Chandra, Hartman, 2022] [Sasieta, 2022]

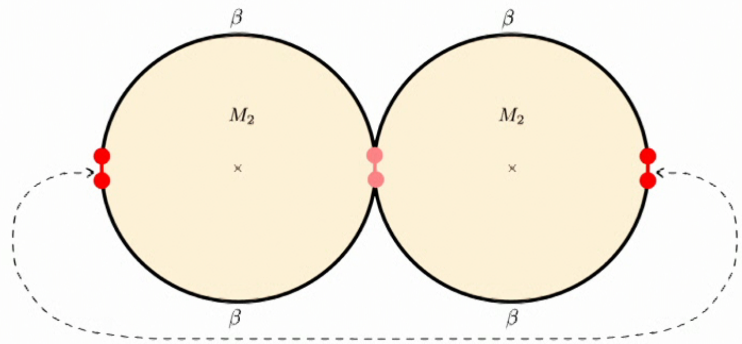
[Stanford, 2020]



$\to Z_6$ Action of six boundary wormhole

Wormholes, overlaps and universality

The wormhole action can be computed for any mass. A simplification occurs in the limit of large mass.



The shell trajectory pinches the geometry and the inverse temperature of each wormhole black hole is twice the original black hole inverse temperature. The inner product squared simplifies to

$$\overline{|\langle\Psi_m|\Psi_{m'}\rangle|^2}|_c = \frac{Z_2}{Z_1 Z_1'} \approx \frac{Z(2\beta)^2}{Z(\beta)^4}$$

This wormhole provides the 'plateau' of the spectral form factor of the black hole.

[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka, 2017]

Similar simplification occurs for the n-boundary wormhole

$$\overline{\langle\Psi_{m_1}|\Psi_{m_2}\rangle\langle\Psi_{m_2}|\Psi_{m_3}\rangle\ldots\langle\Psi_{m_n}|\Psi_{m_1}\rangle}|_c = \frac{Z_n}{Z_1^{(m_1)}\ldots Z_1^{(m_n)}} \approx \frac{Z(n\beta)^2}{Z(\beta)^{2n}}$$

The Hilbert space dimension

For a set of states $|\Psi_p\rangle$, the Hilbert space dimension is rank of the Gram matrix $G_{pq} = \langle\Psi_p|\Psi_q\rangle$.

Choose a set of Ω shell states with separated masses: $m_p = pm$, with $p = 1,\cdots,\Omega$ and sufficiently large m

The goal is to compute the rank of the Gram matrix for such set of states

From the gravity computation we know that

$$\overline{G^n_{pq}} = \frac{Z(n\beta)^2}{Z(\beta)^{2n}}\,\delta_{pq} \equiv \frac{Z_n}{Z_1^n}\,\delta_{pq} \qquad\qquad \frac{Z_n}{Z_1^n} \approx \frac{Z(n\beta)^2}{Z(\beta)^{2n}}$$

The rank is the number of non-zero eigenvalues. It follows from the density of states

[Pennington, Shenker, Stanford, Yang, 2019]

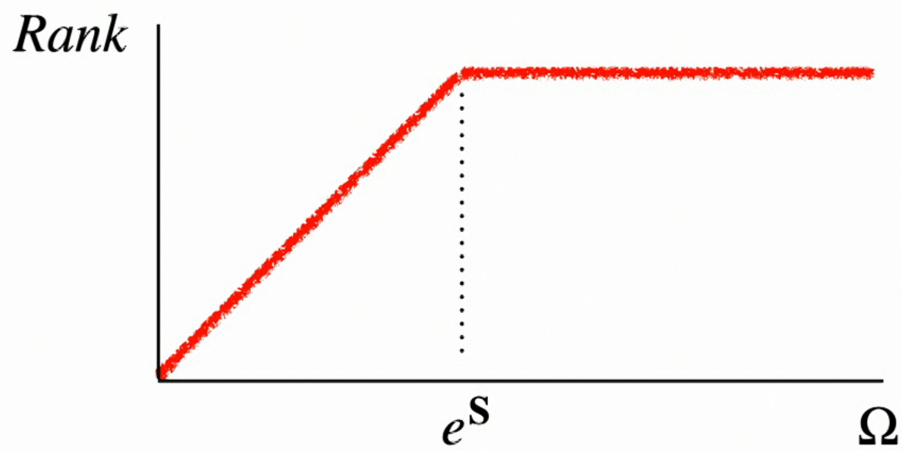
The Hilbert space dimension

The final form of the density of states is

$$D(\lambda) = \frac{e^{\mathbf{S}}}{2\pi\lambda}\sqrt{\left[\lambda - \left(1 - \Omega^{1/2}e^{-\mathbf{S}/2}\right)^2\right]\left[\left(1 + \Omega^{1/2}e^{-\mathbf{S}/2}\right)^2 - \lambda\right]} + \delta(\lambda)\left(\Omega - e^{\mathbf{S}}\right)\theta(\Omega - e^{\mathbf{S}})$$

where $\mathbf{S} \equiv 2\frac{A}{4G}$ is twice the Bekenstein-Hawking entropy

The rank of the Gram matrix as a function of the number of states Ω is then



Remark 1: Extensions and quantum corrections

[Climent, Emparan, J.M.M, Sasieta, Vilar Lopez, To appear soon]

- Charged (Reissner-Nordström) black holes
- Rotating (Myers-Perry) black holes
- Quantum corrections: Near extremal non-BPS
- Quantum corrections: Near extremal BPS
  [Boruch, Iliesiu, Yan, 2023]
- Quantum corrections: general quantum fields

Remark II : Unifying versions of the information paradox

Long times

[Maldacena, 2001]

[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, Tezuka, 2017]

[Saad, Shenker, Stanford, 2019]

mass as time

$|TFD(t_L)\rangle$

Page curve

[Page, 1993]

[Pennington, Shenker, Stanford, Yang, 2019]

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2019]

G as ρ

Origin of BH entropy

$|BH_{Super-exp}\rangle = \sum |BH_{exp}\rangle$

[Balasubramanian, Caputa, J.M, Wu]

[Balasubramanian, J.M, Wu]

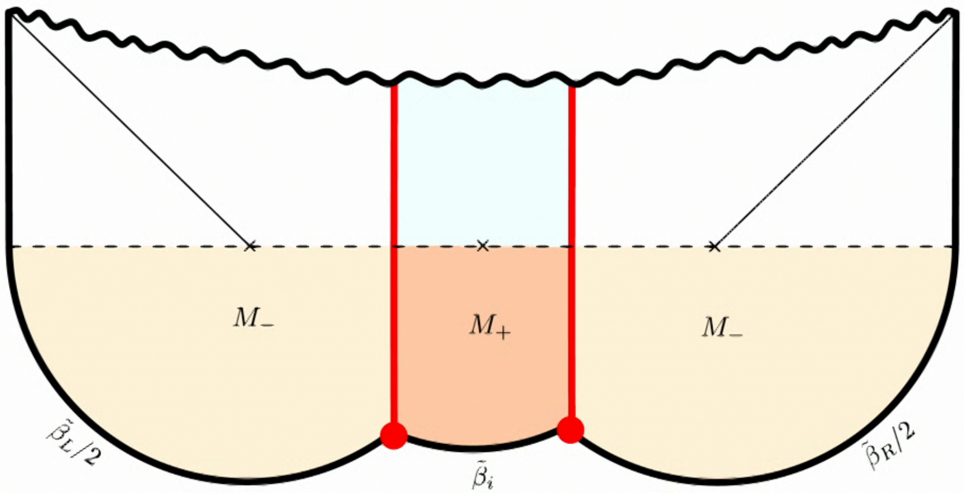
Saturation of ER bridges

[Stanford, Susskind, 2014]

[Brown, Roberts, Susskind, Swingle, Zhao, 2016]

[Belin, Myers, Ruan, Sarosi, Speranza, 2021]

On the nature of black hole microstates and the origin of black hole entropy



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