Title: Talk 22 - Microstates of a 2d Black Hole in string theory

Speakers: Panagiotis Betzios

Collection: It from Qubit 2023

Date: August 03, 2023 - 9:30 AM

URL: https://pirsa.org/23080018

Abstract: We analyse models of Matrix Quantum Mechanics in the double scaling limit that contain non-singlet states. The finite temperature partition function of such systems contains non-trivial winding modes (vortices) and is expressed in terms of a group theoretic sum over representations. We then focus on the model of Kazakov-Kostov-Kutasov when the first winding mode is dominant. In the limit of large representations (continuous Young diagrams), and depending on the values of the parameters of the model such as the compactification radius and the string coupling, the dual geometric background corresponds either to that of a long string (winding mode) condensate or a 2d (non-supersymmetric) semi-classical Black Hole competing with the thermal linear dilaton background. In the matrix model we are free to tune these parameters and explore various regimes of this phase diagram. Our construction allows us to identify the origin of the microstates of the long string condensate/2d Black Hole arising from the non trivial representations.

Microstates of a $2d$ Black Hole in String Theory

Panos Betzios

University of British Columbia

Work in collaboration with O. Papadoulaki
arXiv: 1711.04369 and arXiv: 2210.11484

See also related work by
A. Ahmadain, A. Frenkel, K. Ray, R. M. Soni arXiv: 2210.11493

It From Qubit 2023 - Perimeter Institute for Theoretical Physics

Microscopic Models for Black Holes

- Black Holes constitute some of the most fascinating but confusing objects in (Quantum) Gravity
- Pertinent questions: The Information Paradox, Microstructure, Interior, Nature of their Singularities
- It is possible that their complete understanding will involve "a complete theory of Quantum Gravity" (whatever this might entail)
- It is reasonable to attack and resolve these questions in lower dimensions: Even in 2d there do exist manifolds with similar characteristics - horizons and singularities
- Many efforts to construct microscopic models for Black Holes ("It from Qubit"!) that are "solvable" (at large-N) `[various authors ...]`
- We would like to have such a model in some version of string theory (our best bet for a theory of QGR) - the Black Hole background should be the target space of the string!

Today we will describe such a model in the context of the duality between $c = 1$ (Liouville) string theory and Matrix Quantum Mechanics (MQM)
- our "Qubit model" -

Matrix Quantum Mechanics (MQM)

Reviews by: [Kazakov, Ginsparg-Moore, Klebanov, Martinec,...]

- MQM (gauged) is a $0+1$ dimensional quantum mechanical theory of $N \times N$ Hermitian matrices $M(t)$ and a non dynamical gauge field $A(t)$

$$S = \int_{t_{in}}^{t_f} dt \, \mathrm{Tr} \left(\frac{1}{2} (D_t M)^2 - V(M) \right)$$

- Diagonalise $M(t) = U(t)\Lambda(t)U^\dagger(t)$, with $\Lambda(t)$ diagonal and $U(t)$ unitary
$\Rightarrow$ Presence of a Vandermonde determinant $\Delta(\Lambda) = \prod_{i<j}(\lambda_i - \lambda_j)$
i.e. The Hamiltonian is

$$H = -\frac{1}{2\Delta^2(\lambda)} \frac{d}{d\lambda_i} \Delta^2(\lambda) \frac{d}{d\lambda_i} + \sum_{i<j} \frac{J_{ij} J_{ji}}{(\lambda_i - \lambda_j)^2} + V(\lambda_i) \ ,$$

- J_{ij} are "momenta" conjugate to $SU(N)$ rotations

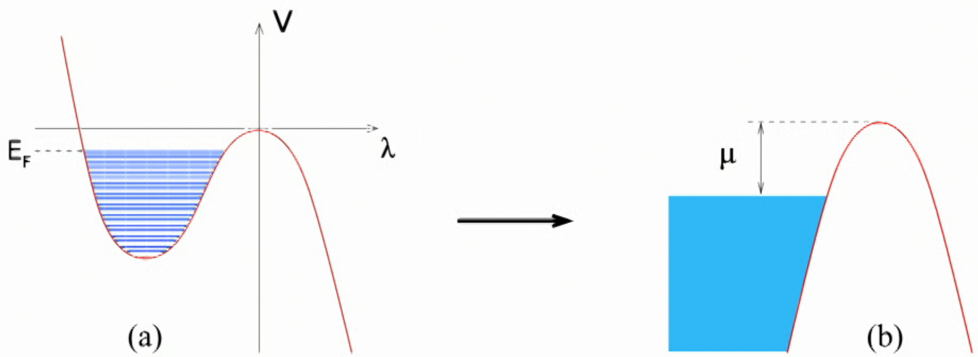
- Gauged model: Impose the Gauss-law constraint
$\delta S / \delta A = i[M, \dot{M}] \sim J = 0$ (*singlet sector projection*)

- Redefining the wavefunction as $\tilde{\Psi}(\lambda) \equiv \Delta(\lambda)\Psi(\lambda)$, the Schrödinger equation describes N non interacting fermions in a potential $V(\lambda)$

Double scaling limit in MQM

[Kazakov, Migdal...]

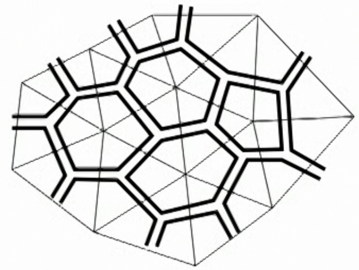
- Consider an initial state where the energy levels are populated up to some Fermi energy E_F below the top of the barrier, and send $\hbar \to 0$, $N \to \infty$ ("WKB").
- We focus near the quadratic maximum of the potential. We hold $\mu = -E_F/\hbar$ fixed in the limit
- The system is just quantum mechanics of free fermions in an inverted harmonic oscillator potential, with states filled up to $-\mu < 0$



Connection with the quantum gravity path integral

The connection with 2d random surfaces (string theory) is via this double scaling limit, where we tune our system near criticality.

- The double scaling limit produces smooth surfaces out of the Matrix fat-graphs while at the same time keeping all higher genera. The genus expansion is (roughly) in terms of $g_{st} \sim 1/\mu$



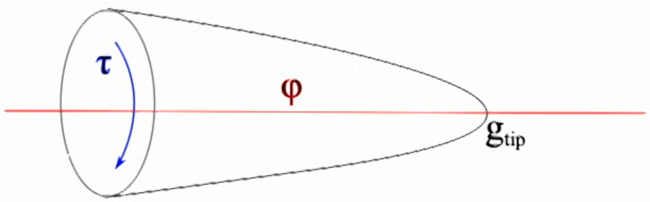
- The matrix eigenvalues λ introduce an emergent coordinate ϕ
- An archetypal form of Holography! "Geometry from the Matrix Qubits"
- The QG theory is called $c = 1$ *Liouville theory*. The target space is $2d$: a time direction t and an emergent space direction ϕ. The background is asymptotically flat and contains a linear dilaton and exponential tachyon
- What about other string backgrounds? Black holes?

(Euclidean) 2d black hole

Elitzur-Forge-Rabinovici, Mandal-Sengupta-Wadia

- (ST) low energy effective action: $S = \int d^2x\sqrt{G}e^{-2\Phi}(R - 4\nabla\Phi^2 - \frac{8}{\alpha'})$
- $2d$ "Cigar" solution ($-\infty < \phi < 0$ and $\phi = 0$ is the tip of the cigar)

$$ds^2 = (1 - e^{2Q\phi})d\tau^2 + \frac{d\phi^2}{1 - e^{2Q\phi}}\,, \quad \Phi = \Phi_0 + Q\phi\,,\ Q^2 = 4/\alpha'$$



- $g_{tip} = e^{\Phi_0}$ is the integration constant of the solution
- The weak string coupling region is at the "boundary of the cigar"
- The string coupling becomes strongest near the tip
- It has a fixed temperature (smoothness at the tip)

- The entropy and mass of the black hole scale as [Gibbons, Nappi, Kazakov-Tseytlin]

$$S \sim M \sim \frac{1}{g_{tip}^2} \sim e^{-2\Phi_0}$$

Exact (CFT) $2d$ black hole background

Witten, Dijkgraaf - Verlinde^2, ...

- The WZW model action is ($g \in G$ is a Lie group element)

$$S_k(g) = -\frac{k}{8\pi}\int d^2\sigma \, \mathrm{tr}\left[g^{-1}(\partial_\mu g)g^{-1}(\partial^\mu g)\right] + 2\pi k S_{WZ}(g)$$

- The worldsheet (exact) CFT describing the 2d black hole: a WZW coset model, the target space coset being $SL(2,R)_k/U(1)$
- The compactification radius of the Euclidean black hole is $R^2 = k\alpha'$, with k being also the level of the associated $SL(2,R)_k/U(1)$ WZW model
- The $SL(2,R)_k$ algebra dictates the central charge $c_{\text{cigar}} = \frac{3k}{k-2} - 1$, and the spectrum of primaries for the coset CFT
- Conformal invariance of the worldsheet theory requires $k = 9/4$
- In order to change the radius we need to append to this model additional degrees of freedom of an "internal" CFT

FZZ duality and the black-hole string transition

- FZZ duality: coset CFT is dual to *Sine-Liouville theory*, Fateev-Zamolodchikov[2]
- Sine-Liouville:
  $L_{SL} = \frac{1}{4\pi}\Big((\partial x)^2 + (\partial\phi)^2 + Q\hat{R}\phi + \xi e^{b\phi}\cos R\,(x_L - x_R)\Big)$
- Matches with the coset when the radius of x is $R = \sqrt{k}$ and

$$c_{\text{cigar}} = c_{SL} = 2 + 6Q^2 \;\Rightarrow\; Q^2 = \frac{1}{k-2}\,, \quad b = \sqrt{k-2} = \frac{1}{Q}$$

- The asymptotic weakly coupled region is $\phi \to -\infty$
- The strongly coupled region is for $\phi \to \infty$ near the potential wall
- The duality is a strong-weak duality
- For small radii, the black hole is better described in terms of a condensate of winding-strings, while for large radii the black hole description is the simplest ($Q \to \infty\,, k \to 2$ vs. $k \to \infty\,, Q \to 0$)
- A transition between such two behaviours is usually termed the black hole/string transition, Susskind, Horowitz-Polchinski, Sen, Kutasov ...

Sine-Gordon coupled to $2d$ QG

[Kazakov-Kostov-Kutasov , Moore]

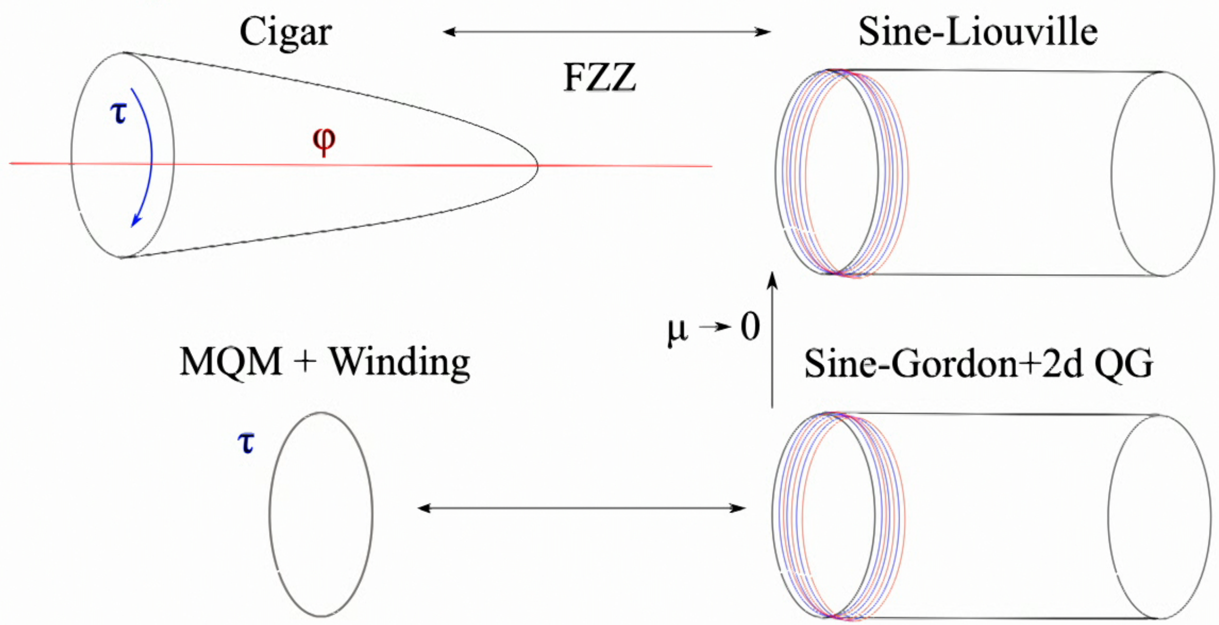
- Deform the linear dilaton background ($c = 1$ Liouville/MQM) including in the Lagrangian the first winding operators - $\xi(\mathcal{T}_{+R} + \mathcal{T}_{-R})$

$$L_d = \frac{1}{4\pi}\Big((\partial x)^2 + (\partial\phi)^2 + 2R\phi + \mu e^{2\phi} + \xi e^{(2-R)\phi}\cos R\,(x_L - x_R)\Big)$$

  This is a Sine-Gordon model coupled to $2d$ quantum gravity (μ-term)

- Approach the $\xi \to \infty$, $\mu \to 0$ region of parameters (SL-point)
- The target space winding modes correspond to vortices on the worldsheet [Kogan, Sathiapalan]
- What are these winding/vortex deformations in Matrix Quantum Mechanics?

The duality web

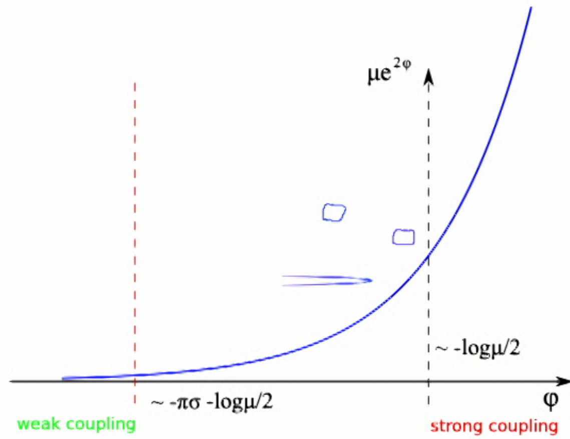


- Caveat I: From the gravity side it seems that we cannot change the radius of the cigar, while in the MQM/SG/SL description we can tune it
- Caveat II: These dualities are inherently Euclidean. Lorentzian description for the deformation? (long-strings...)

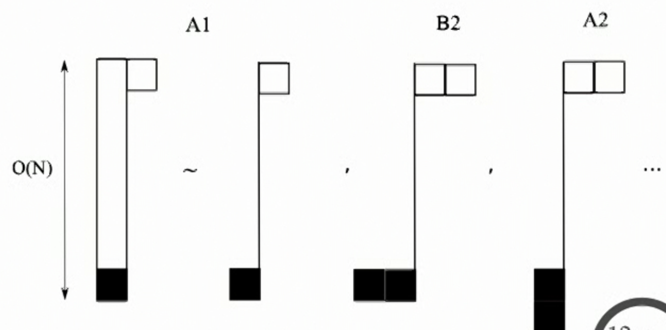
Non-singlet sectors in MQM - Long strings in Liouville

- The singlet sector of gauged Matrix Quantum Mechanics (MQM) cannot describe black holes [Martinec, Karczmarek-Maldacena-Strominger ...]

- Proposals that black holes could exist in the non-singlet sector of MQM [Kazakov-Kostov-Kutasov, Klebanov, ...]
- Adjoint representation: related to a long folded string that extends along Liouville [Maldacena, ...]
- Long strings are also related to the presence of FZZT branes (retracted at the weak coupling region)



- States containing n folded strings $\Rightarrow$ Irreps with a Young-Tableaux of n-boxes and n-anti-boxes [Maldacena]
- Can we keep $SU(N)$ gauged and still have long strings?



A dynamical "N-ZZ N_f-FZZT" matrix model

[P.B. Papadoulaki (2017), see also Gaiotto (2005)]

- Start with the (gauged) MQM action (N - ZZ/$D0$ branes)
- Introduce open strings between N-ZZ and N_f-FZZT branes, by adding $N_f \times N$ 'bi-fundamental fields $\chi_{\alpha i}$, $\psi_{\alpha i}$ ("quarks")

$$S_f = \int dt \sum_{\alpha}^{N_f} \mathrm{tr}\left(i\psi^\dagger_\alpha D_t \psi_\alpha - m_\alpha \psi^\dagger_\alpha \psi_\alpha + i\chi^\dagger_\alpha D^*_t \chi_\alpha - m_\alpha \chi^\dagger_\alpha \chi_\alpha \right) ,$$

- The $SU(N)$ Gauss' law constraint becomes

$$: i[M\,,\dot{M}]_{ij} :=: J_{ij} := \sum_{\alpha}^{N_f} \left[\psi^\dagger_{\alpha j} \psi_{\alpha i} - \chi^\dagger_{\alpha i} \chi_{\alpha j} \right]$$

- The Hamiltonian is now

$$\hat{H} = \sum_i^N -\frac{1}{2}\frac{\partial^2}{\partial \lambda_i^2} + V(\lambda_i) + \frac{1}{2}\sum_{i\neq j} \frac{J_{ij}J_{ji}}{(\lambda_i - \lambda_j)^2} + \sum_{i,\alpha}^{N,N_f} m_\alpha \psi^\dagger_{\alpha i}\psi_{\alpha i} + m_\alpha \chi^\dagger_{\alpha i}\chi_{\alpha i}$$

- The bi-fundamentals thus "feed" non-trivial representations (J_{ij})

Limit of [Kazakov, Kostov, Kutasov] Matrix Model

- The canonical partition function is expressed in terms of holonomy/Wilson loop zero modes ($U = \mathrm{tr}\, P e^{i \oint A}$)

$$Z_N^{(N_f)} \sim \int_{U(N)} \mathcal{D}U \, \frac{\exp\left[N_f \sum_{l=1} \frac{(-1)^{l+1}}{l} e^{-\beta m} \left(\mathrm{tr}\, U^l + \mathrm{tr}\, U^{-l}\right)\right]}{\exp\left(\sum_l \frac{q^l}{l} \mathrm{tr}\, U^l \, \mathrm{tr}(U^{-1})^l\right)}$$

- Take a double scaling limit ("heavy/quenched quarks")

$$N_f \to \infty, \quad m \to \infty, \quad \text{with} \quad N_f e^{-\beta m} = \tilde{t}, \quad \text{finite}$$

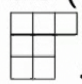
- The only surviving winding modes in this case: $\exp\left(\tilde{t}\, \mathrm{tr}\, U + \tilde{t}\, \mathrm{tr}\, U^\dagger\right)$, are identical to those studied in the matrix model of [Kazakov, Kostov, Kutasov]
- We can therefore obtain the later by taking a limit of a model that does admit a Lorentzian description and has a natural Liouville (FZZT brane - long string) interpretation

Partition function - Microstates from representations

[PB, Papadoulaki (2022)]

- The grand canonical partition function can be expanded as a statistical sum over representations/partitions (labelled by λ)

$$\mathcal{Z}_{MQM}(\mu, R; \tilde{t}) = \sum_{\lambda} s_\lambda(t_+) s_\lambda(t_-) \langle \lambda | \mathbf{G}(\mu, R) | \lambda \rangle$$

- $s_\lambda(t_\pm)$ are Schur polynomials (combinatorial part - Schur measure [Okounkov ...]) and $|\lambda\rangle \equiv$  ... are states/Young diagrams
- The operator $\mathbf{G}(\mu, R)$ is the "T-dual of the S-matrix" and incorporates all the MQM dynamics
- In this formalism the microstates of the winding condensate/black hole are manifest ($GL(\infty)$ representations/partitions)
- Our idea is to define a coarse graining/thermodynamic limit, considering large Young diagrams, that acquire a "continuous limiting shape"

  Many inequivalent representations have the same "limiting shape"

Plancherel measure

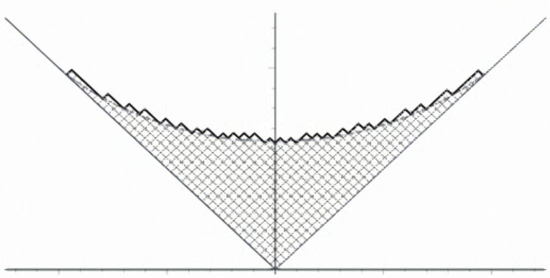
- Turn on only the first winding mode $t^+ = t^- = \xi$, $\Rightarrow$ the Schur measure specialises to "Plancherel measure" on partitions of n

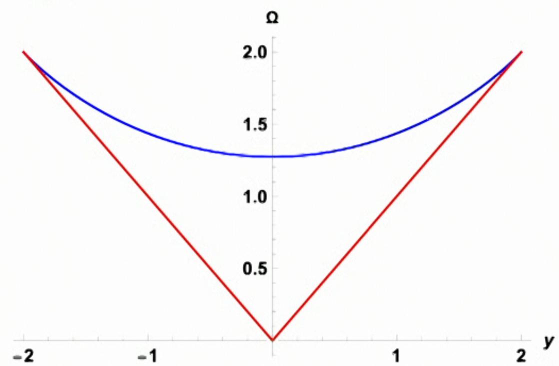
$$\mathfrak{M}_\lambda(\xi) = \sum_{n=0}^{\infty} \frac{\xi^{2n}}{n!} M_n(\lambda)\delta(|\lambda| - n)\,, \quad M_n(\lambda) = \frac{(\dim\lambda)^2}{n!}$$

- As the size of the partitions goes to infinity $n \to \infty$, the Plancherel measure exhibits a Cardy-like growth

$$\lim_{n\to\infty} M_n(\lambda) \sim \exp\left(2\sqrt{n}\right)$$

and concentrates to a universal limiting Young diagram shape the *Vershik-Kerov-Logan-Shepp limiting shape* $\mathbf{\Omega}$





The limiting shape for the complete partition function

- Our limiting shape should be found self consistently by including the contribution of the amplitude $\langle\lambda|\mathbf{G}|\lambda\rangle$ together with the Schur measure
- Solve a minimization problem in the space of highest weights [Douglas Kazakov, ...] $\Rightarrow$ determine the resolvent and leading shape
- The leading shapes (for $R < 2$) are found to be

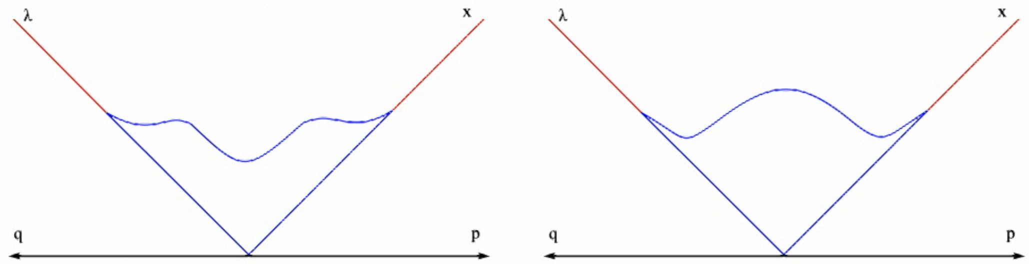


Figure: Left figure: large ξ_{eff} (small g_s) Right: small ξ_{eff} (large g_s). There is a third order phase transition between the two behaviours

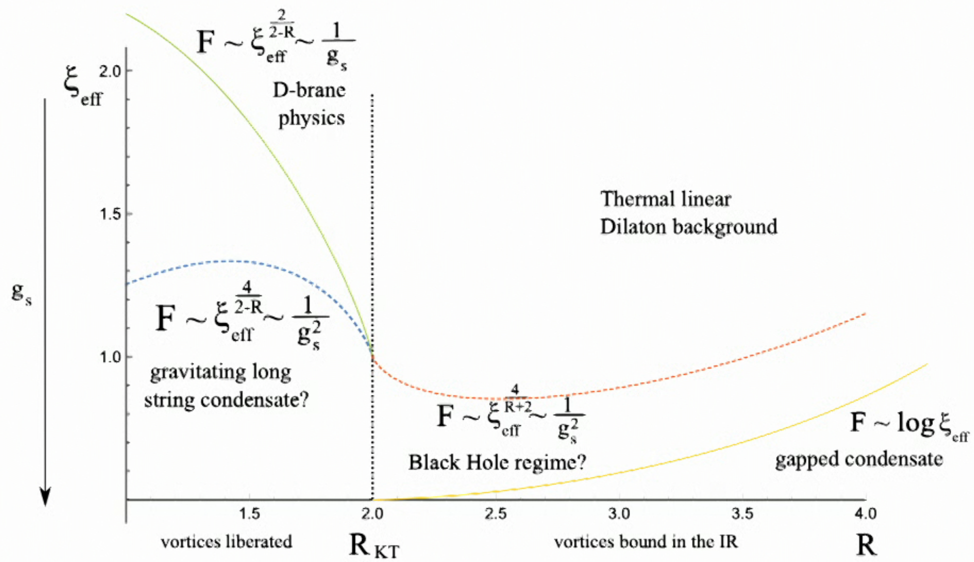
- The genus zero free energy near the transition (right fig.) behaves as

$$\mathcal{F}_0 \simeq -\frac{(2-R)^2}{8}\xi_{eff}^{\frac{4}{2-R}} \sim -\frac{(2-R)^2}{8}\frac{1}{g_s^2}$$

and coincides with the result of [Kazakov-Kostov-Kutasov, Tseytlin]

The phase diagram

Generalising the findings of [Moore, Kazakov-Kostov-Kutasov]



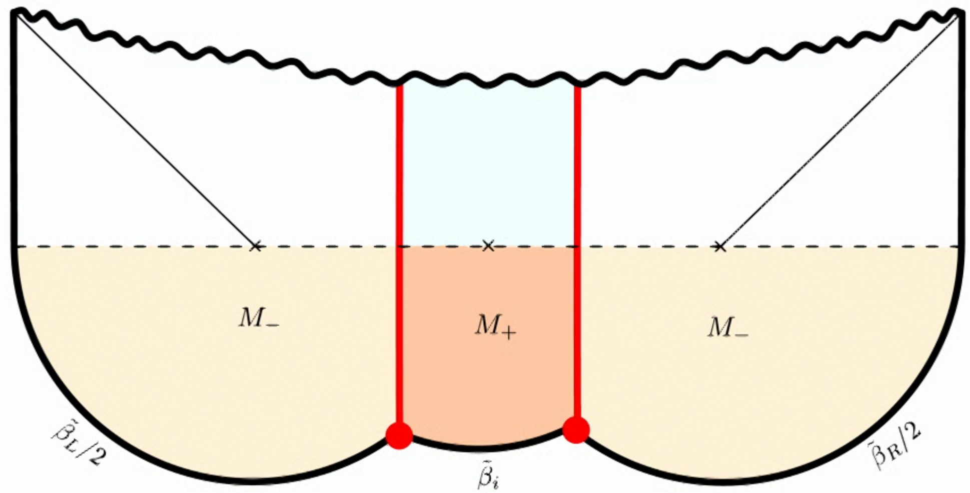
- The dashed lines signal phase transitions
- $T_{KT} \sim 1/R_{KT}$ is the Kosterlitz-Thouless temperature, above which worldsheet vortices get liberated and proliferate
- The continuous lines are regimes of different behaviour (cross-over)

Future Directions

- Analysis of observables that can distinguish a black hole from a gravitating long string condensate (absorption, QNMs, chaos?)
- The $2d$ black hole appears in higher dimensional (non-supersymmetric) asymptotically flat black holes taking a large D limit, [Emparan-Grumiller-Tanabe ...]
- Can we formulate a matrix model for these black holes? Any connection with the present work?
- Understand the physics of the Black Hole/String transition from a microscopic model
- Study a $2d$ version of charged/near extremal black holes and wormholes ($0A$ MQM model) - In progress with [N. Gaddam - O. Papadoulaki]
- Coarse graining in the space of Young diagrams - Other applications? (Wormholes [P.B.-Kiritsis-Papadoulaki (21)] - Quantum Information?)

Thank You!

On the nature of black hole microstates and the origin of black hole entropy



Javier M. Magán
Instituto Balseiro

In collaboration with V. Balasubramanian, A. Lawrence and M. Sasieta

Questions

- What is a microstate?
- Is it simple to construct microstates in general? And black hole microstates?
- What is their nature? (geometric properties and universal features)
- How do we count them?

Definition and examples of microstates

Strong notion of microstate: Indistinguishability from the thermal ensemble

$$\langle\Psi|\hat{Q}_i|\Psi\rangle = Q_i \to \mathrm{Tr}(\rho_\beta \hat{Q}_i)$$

$$\langle\Psi|\mathcal{O}(t)|\Psi\rangle \to \mathrm{Tr}(\rho_\beta \mathcal{O}(t))$$

$$\langle\Psi|\mathcal{O}(t)\,\mathcal{O}(0)|\Psi\rangle \to \mathrm{Tr}(\rho_\beta \mathcal{O}(t)\,\mathcal{O}(0))$$

Obvious candidates are the energy eigenstates $|E_i\rangle \to$ Eigenstate Thermalization Hypothesis

[Deutsch, 1991] [Srednicki, 1994]

Examples in SYK $\to$ $|B(l)\rangle = e^{-lH}|B\rangle$

[Kourkoulou, Maldacena, 2017]

Examples in JT gravity $\to$ $|PETS\rangle = \sum_{mn} e^{-\frac{1}{2}\tilde{\beta}_L E_m - \frac{1}{2}\tilde{\beta}_R E_n}\,\mathcal{O}_{nm}|m,n\rangle$

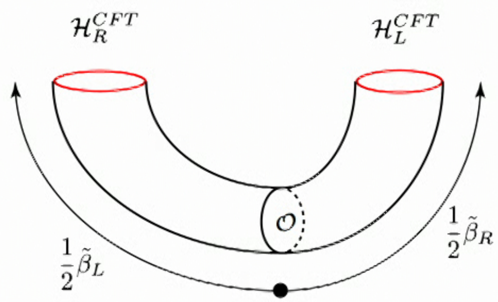
[Goel, Lam, Turiaci, Verlinde, 2018]

A CFT method to construct microstates?

An insightful set of quantum states in CFT's arise by considering dust shell operators

$$\mathcal{O}_{dust} = \prod_{i}^{N} \mathcal{O}(\theta_i)$$

Using these operators we can create states in the double Hilbert space as



$$= \sum_{mn} e^{-\tilde{\beta}_L E_n/2 - \tilde{\beta}_R E_m/2}\, \mathcal{O}_{nm} |n, m\rangle$$

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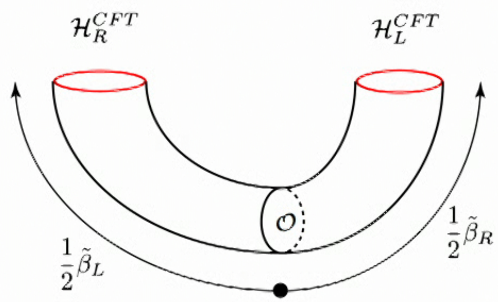

$$= \sum_{mn} e^{-\tilde{\beta}_L E_n/2 - \tilde{\beta}_R E_m/2}\, \mathcal{O}_{nm} |n, m\rangle$$

Still, how do we show the weak or strong notion?

Trick: Tune the states to lock the information of the perturbation behind the horizon

Claim: For specific preparation temperatures $\tilde{\beta}_L, \tilde{\beta}_R$, and for holographic CFT's, dust shell states for any N are microstates in the strong sense