Title: Euclidean Wormholes and Gravity as an Average

Speakers: Alexander Maloney

Collection: It from Qubit 2023

Date: August 02, 2023 - 1:00 PM

URL: https://pirsa.org/23080013

Euclidean Wormholes
&
Gravity as an Average

Alex Maloney, McGill University

It From Qubit 2023, Perimeter Institute

J. Chandra, S. Collier, T. Hartman, & A.M.
A.M., V. Meruliya & M. van Raansdonk (in progress)
J. Chakravarty, A.M., & K. Namjou (in progress)
A.M. (in progress)

Overview

Quantum Gravity and Conformal Field Theory

The most important tool in our study of gravity is holography:

- Gravity in Anti-de Sitter Space = Conformal Field Theory

This an equivalence between quantum theories:

$$\mathcal{H}_{GR} = \mathcal{H}_{CFT}, \qquad H_{GR} = H_{CFT}, \qquad Z_{GR} = Z_{CFT}, \qquad \ldots$$

which answers many deep questions (e.g. about black hole entropy, quantum space-time, . . .)

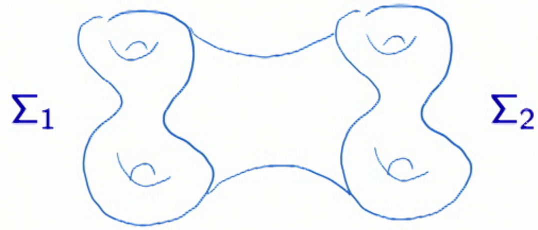
But the presence of Euclidean wormholes

- Euclidean solutions that connect disconnected boundaries

presents a puzzle for AdS/CFT.

Why should you care?

A Euclidean wormhole, like



would lead to non-factorization:

$$Z_{GR}(\Sigma_1 \cup \Sigma_2) \neq Z_{GR}(\Sigma_1) Z_{GR}(\Sigma_2)$$

But for every normal CFT, $Z_{\mathcal{C}}(\Sigma_1 \cup \Sigma_2) = Z_{\mathcal{C}}(\Sigma_1) Z_{\mathcal{C}}(\Sigma_2)$!

Maldacena & Maoz, ...

Ensemble Averaging

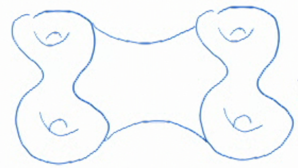
The interpretation is that

$$Z_{GR}(\Sigma) = \sum_{\text{CFTs } \mathcal{C}} Z_{\mathcal{C}}(\Sigma) p(\mathcal{C}) \equiv \overline{Z_{\mathcal{C}}(\Sigma)}$$

for some probability distribution $p(\mathcal{C})$ on the space of CFTs.

So, e.g. a two-boundary wormhole computes the variance:

$$\overline{Z_{\mathcal{C}}(\Sigma)^2} - \overline{Z_{\mathcal{C}}(\Sigma)}^2 =$$



$$+ \dots$$

What is $p(\mathcal{C})$?

Where does averaging come from?

Do we really need to care?

A word on Interpretation

An exact, UV complete theory is not an ensemble.

Ensembles seem to emerge when we approximate fancy UV complete theories by simple gravity path integrals.

Don't panic! In chaotic theories, this averaging is mostly harmless:

- Each member of the ensemble represents the whole,
- Up to $\mathcal{O}(e^{-S})$ microstructure which is removed by averaging.
- Even after averaging a lot of structure is visible (e.g. level repulsion)

How does the UV theory restore unitarity, i.e. recover a single CFT, if the low energy theory has a wormhole?

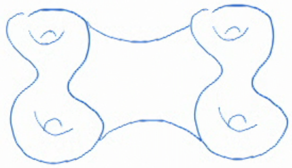
Outline

- Why you can't ignore Wormholes
- Why you are glad you didn't ignore Wormholes
- Wormholes and Baby Universes
- How Wormholes disappear in UV complete theories

You Can't Ignore Wormholes

Euclidean Wormholes are Ubiquitous

A Euclidean wormhole connects two different asymptotic regions:

Σ_1  Σ_2

$$ds^2 = dt^2 + a(t)^2 d\Sigma^2$$

They seem to be ubiquitous in theories of gravity. Why?

The FRW equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{\ell^2} + \rho \quad \xrightarrow[t \to it]{} \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{\ell^2} - \rho$$

will have a "bounce" with $\dot{a} = 0$ if there is enough energy

$$\rho_{bounce} = \frac{1}{\ell^2}$$

How to build a Wormhole

For example, for a matter field ϕ the energy density

$$\rho \sim \dot{\phi}^2 + (\nabla\phi)^2 + V(\phi) \underset{t \to it}{\longrightarrow} -\dot{\phi}^2 + (\nabla\phi)^2 + V(\phi)$$

leads to a bounce if ρ is sufficiently large and positive.

So we can build a wormhole from

- Potential Energy (e.g. mass),
- Gradient Energy, or
- Kinetic Energy (with an *axion* so that $\dot{\phi}^2 < 0$).

All three methods work!

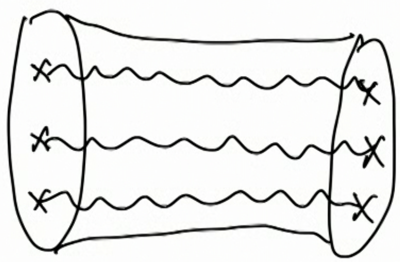
You can also add (negative) spatial curvature.

Marolf & Santos
A.M., V. Meruliya & M. van Raamsdonk
many others!

Wormholes with Particles

One simple wormhole involves point particles in 2+1 gravity.

Here $\rho = \sum_i m_i \delta(x - x_i)$ gives the wormhole:



$$ds^2 = dt^2 + cosh^2 t \; d\Sigma^2$$

where $d\Sigma^2$ solves $R_\Sigma = -2 + \rho$. We just need $\sum_i m_i > 1$.

Σ is locally hyperbolic with isolated conical defects at $x = x_i$.

This leads to a variance in correlation functions of heavy operators

$$\overline{\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle} - \overline{\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle}^2 \neq 0$$

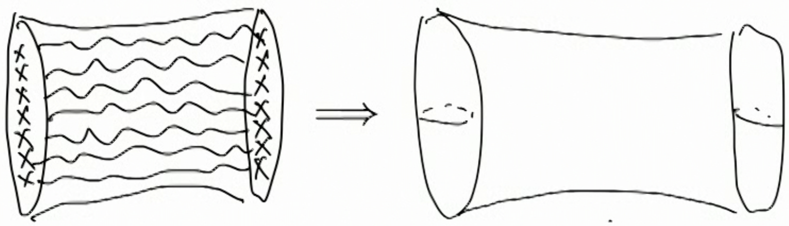
Smoother Wormholes

We can coarse grain

$$\rho = \sum_i m_i \delta(x - x_i) \approx \rho(x)$$

for some continuous density $\rho(x)$ with $\int \rho(x) dvol_\Sigma > 1$.

For example, if $\rho \approx$ constant then Σ looks hyperbolic at small scales, but like round S^2 at large scales:



You can approximate *any* two dimensional metric on Σ this way.

Averaging for Light Operators

In a 2d CFT dual to 2+1 gravity, this leads to a variance in correlation functions of light operators

$$\overline{\langle \underbrace{\mathcal{O}\dots\mathcal{O}}_{c} \rangle \langle \mathcal{O}\dots\mathcal{O}\rangle} - \overline{\langle \mathcal{O}\dots\mathcal{O}\rangle}^2 \neq 0$$

As the operators become lighter, we need many insertions.

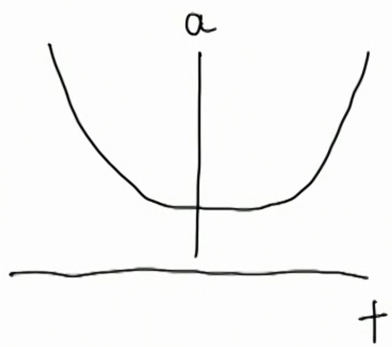
It's like the scattering of 137 electrons in QED:

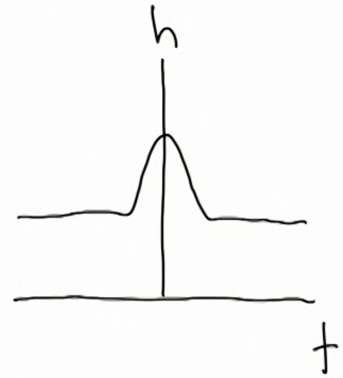
$$e^- \dots e^- \to e^- \dots e^-$$

Inhomogeneous Wormholes

It is also easy to find wormholes sourced by inhomogeneity, such as those with:

$$(\nabla\phi)^2 = k^2 h(t)\ dvol_\Sigma$$





These wormholes exist generically. No need to do anything special.

The upshot: wormholes are ubiquitous, and you can't ignore them!

Marolf & Santos
A.M., V. Meruliya & M. van Raamsdonk
...

You Should Be Glad

You Can't Ignore Wormholes

Random CFT

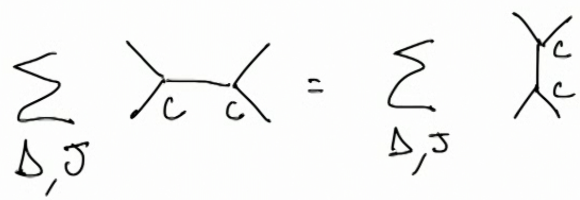
In a theory with wormholes, the dual is a distribution $p(\mathcal{C})$ over a space of CFTs. So we must develop a theory of random CFTs, analogous to the theory of random matrices. How?

The data that define a CFT are the:

- Operator dimensions $\Delta_i = h_i + \bar{h}_i$ and spins $J_i = h_i - \bar{h}_i$
- OPE coefficients C_{ijk}

of primary operators.

These are subject to bootstrap constraints, like modular & crossing symmetry:



The Space of CFTs

The space of CFTs is a weird and wild place.

It has both discrete and connected components, but a typical CFT is an isolated point.

Conjecture: the number of 2D CFTs grows like

$$N(c) \approx \sum_{\mathcal{C}} \frac{1}{|Aut(\mathcal{C})|} \approx e^{\# c^2 \log c} \qquad \text{as } c \to \infty$$

This is certainly a lower bound.

Here we count isolated CFTs, and integrate over spaces of CFTs with the natural Zamolodchikov metric.

What probability distribution can we define on this space?

What is the Probability Distribution?

The only sensible answer is a maximum entropy probability distribution.

Conjecture: General relativity in AdS is dual to the maximum entropy probability distribution over the space of solutions of the CFT bootstrap.

This would have striking implications for QFT, e.g. in CFT_2:

- A typical large c CFT_2 is sparse/extremal ($\Delta_{gap} = \mathcal{O}(c)$)
- Even though we know of no examples with $\Delta_{gap} > 2$!

More refined theories of gravity will be dual to maximum entropy distributions with more constraints.

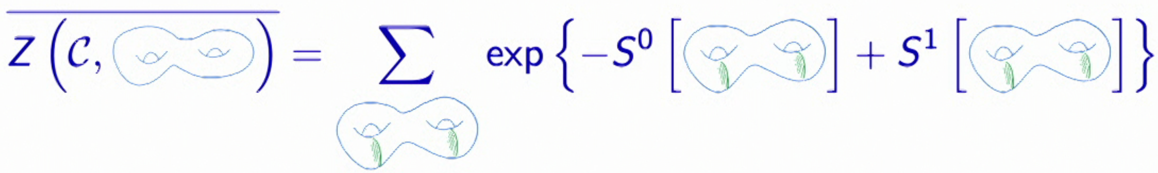
An Almost Integrable Example

One reasonably simple example is

- CFTs with $U(1)^N \times U(1)^N$ current algebra and $c = N$.

The space of CFTs $\mathcal{M}_N$ is a symmetric space.

With the homogenous (i.e. max entropy) probability measure:



Averages take the form of a simple theory of gravity where

- Gravity is one-loop exact,
- The saddle points are easy to understand

This is like an exact version of AdS/CFT.

There are also discrete versions of this average.

A.M. & E. Witten
N. Afkhami-Jeddi, H. Cohn, T. Hartman, A. Tajdini

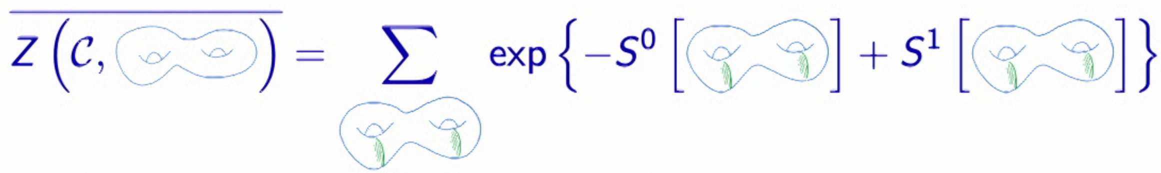
An Almost Integrable Example

One reasonably simple example is

- CFTs with $U(1)^N \times U(1)^N$ current algebra and $c = N$.

The space of CFTs $\mathcal{M}_N$ is a symmetric space.

With the homogenous (i.e. max entropy) probability measure:



Averages take the form of a simple theory of gravity where

- Gravity is one-loop exact,
- The saddle points are easy to understand

This is like an exact version of AdS/CFT.

There are also discrete versions of this average.

A.M. & E. Witten
N. Afkhami-Jeddi, H. Cohn, T. Hartman, A. Tajdini

An Ensemble that Solves Crossing

We now reinterpret these statistical constraints on individual CFTs as defining a *distribution* on the space of CFTs.

We want the *Maximum Entropy* distribution over the space of

$$\rho(h_i, \bar{h}_i), C_{ijk}$$

which solves S-channel crossing.

So we need a Cardy density of heavy states $(h, \bar{h} > \frac{c}{24})$, and a *Gaussian* distribution of OPE coefficients:

$$\overline{C_{123}} = 0$$

$$\overline{C_{123} C_{456}} = (\delta_{14}\delta_{25}\delta_{36} + \dots)\,|C_0(h_1, h_2, h_3)|^2$$

J. Chandra, S. Collier, T. Hartman, & A.M

Einstein Gravity in AdS_3 is an Average

Claim: This ensemble is dual to general relativity in AdS_3

$$S_{GR} = \frac{1}{16\pi G}\int\sqrt{g}\left(R+\frac{2}{\ell^2}\right)+\sum_i m_i\int d\ell + bdy$$

with particles of mass $m_i = \frac{1-\sqrt{1-16Gh_i}}{4G}$, with a caveat.

This is true in the sense that, as $c=\frac{3\ell}{2G}\to\infty$:

$$\overline{Z_{CFT}(\mathcal{O})} = e^{-c\,S_{GR}[g_o]+S^{(1)}[g_o]+\ldots}$$

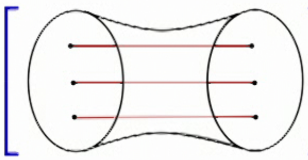
for *every* boundary observable $\mathcal{O}$ we can compute, up to a choice of channel.

Here g_o is a corresponding classical geometry.

The Maldacena-Maoz Wormhole

For example, the variance of C_{ijk} is the action of a wormhole

$$\overline{\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle^2} = \exp\left\{ -S \left[\quad \right] \right\}$$



with metric

$$ds^2 = dt^2 + \cosh^2 t \; d\Sigma^2$$

Here $d\Sigma^2$ is the hyperbolic metric on S^2 with three conical defects.

The wormhole is *necessary* to construct the dual of 2+1 gravity.

Aren't you glad you didn't ignore wormholes?

General Wormholes

This reproduces the action of single boundary solutions

$\overline{Z\left(\mathcal{C},\right.}$  $\left.\overline{}\right) = \overline{\sum C_{ijk}^2} = \exp\left\{-S\left[\right.\right.$  $\left.\left.\right]\right\} + \dots$

as well as wormholes with multiple boundaries:

$\overline{Z\left(\mathcal{C},\right.}$  $\left.\right)^2 = \overline{(\sum C_{ijk}^2)(\sum C_{ijk}^2)} = \overline{(\sum C_{ijk}^2)}^2 + \sum \overline{C_{ijk} C_{i'j'k'}}^2$

$= \exp\left\{-2S\left[\right.\right.$  $\left.\left.\right]\right\} + \exp\left\{-S\left[\right.\right.$  $\left.\left.\right]\right\} + \dots$

The sum over Wick contractions in the Gaussian ensemble reproduces (part of) the sum over geometries.

To reproduce more terms in the sum over geometries, we need to refine the ensemble to include more crossing constraints:

$$\rho_{Cardy} \to \rho_{MWK}, \qquad p(C_{ijk}) \to p(C_{ijk}) + \text{non-gaussianities}$$

An Aside on Interpretation

Where does ensemble averaging come from?

Consider a single, UV complete theory in higher dimensions.

The coarse-graining over (extremal) black hole microstates in this theory gives an emergent $AdS \times \mathcal{M}$ near horizon region.

- Low dimensional AdS gravity describes the near-horizon dynamics.

Conjecture: ensemble averaging in low-dimensional AdS is a remnant of this coarse graining over BH microstates in the parent theory.

We saw this explicitly in our 3D example, since we replaced an average over BH microstates by an average over an ensemble of couplings.

Wormholes and Baby Universes

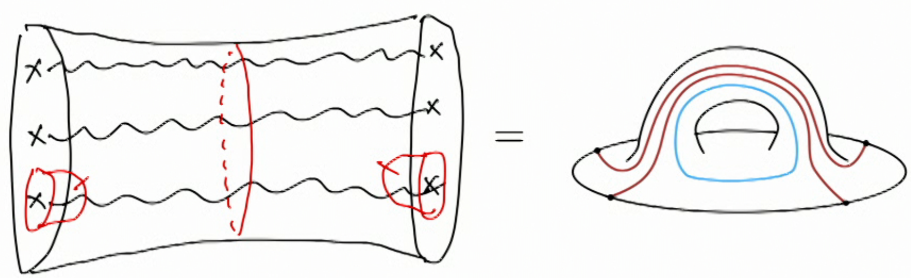
Single Boundary Wormholes & Bulk EFT

We have studied wormholes that connect multiple boundaries.

The wormholes we considered can also be interpreted as *single boundary* wormholes.

I expect that this is generically true: in any theory with multi-boundary wormholes, there are solutions with a single boundary and bulk handles.

For example, the quotient of the 3pt wormhole



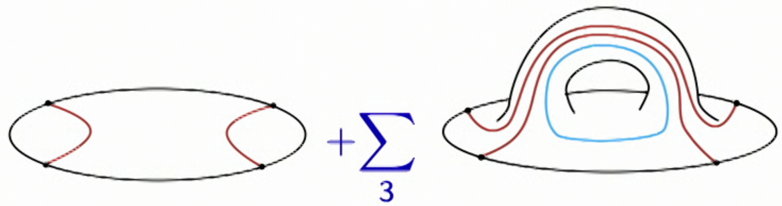
has a single boundary but a bulk handle.

Bulk Wormholes in 3D Gravity

Imagine starting with 3D gravity with *non-interacting* particles:

$$S_{GR} = \frac{1}{16\pi G}\int \sqrt{g}\left(R + \frac{2}{\ell^2}\right) + \sum_i m_i \int d\ell$$

Compute the four point correlator by summing over geometries

$\langle O_1 O_2 O_1 O_2 \rangle =$  $+\ldots$

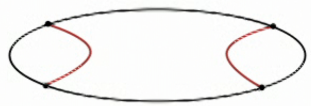
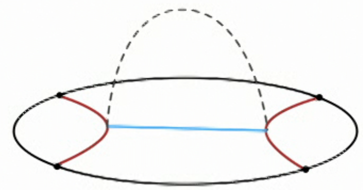
which are classical solutions to the equations of motion.

The particles never interact, they are only there to support a wormhole.

Gaussian Random Couplings

What does this look like from the low energy EFT?

It looks like the particles 1, 2, & 3 interact

$\langle O_1 O_2 O_1 O_2 \rangle =$  $+\sum_3$  $+\dots$

with bulk couplings drawn from a Gaussian distribution:

$$\overline{C_{123}} = 0, \qquad \overline{C_{123} C_{456}} = (\delta_{14}\delta_{25}\delta_{36} + \dots)\, |C_0(h_1, h_2, h_3)|^2$$

Conclusion: In a theory of gravity with wormholes, the coupling constants of the *bulk* EFT are random variables!

Bulk Effective Field Theory

This is a tractable (2+1) version of the classic story, where integrating out wormholes gives bilocal couplings to other fields:

$$Z = \int D\Phi \exp\left\{\int dx\mathcal{L} + \sum_i \int \mathcal{O}_i(x)\mathcal{O}_i(y)dxdy\right\}$$
$$= \int d\alpha^i P(\alpha_i) \int D\Phi \exp\left\{\int dx\,(\mathcal{L} - \alpha_i\mathcal{O}_i(x))\right\}$$

which are interpreted as Gaussian random couplings

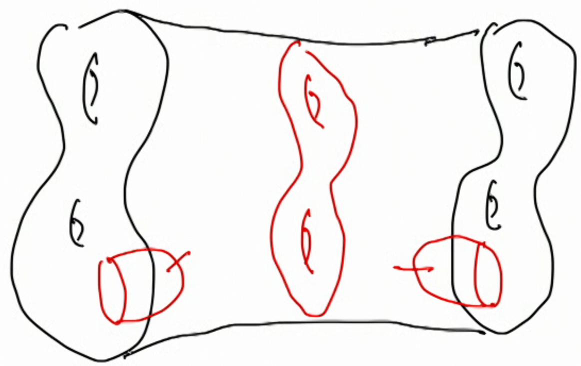
$$P(\alpha^i) \sim e^{-(\alpha^i)^2}$$

in the low energy EFT.

Coleman; Giddings & Strominger,...

Entangling AdS with a Baby Universe

What is the Lorentzian interpretation? Consider this wormhole with a single genus 4 boundary:



The surface of vanishing extrinsic curvature includes an AdS spatial slice and a baby universe with genus 2.

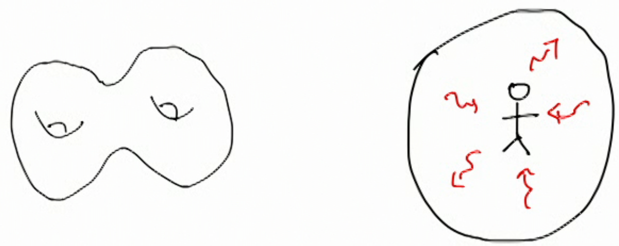
So the wormhole prepares an entangled state in the tensor product Hilbert space

$$|\psi\rangle = \sum_{i\alpha} \psi_{i\alpha} |i\rangle_{AdS} \otimes |\alpha\rangle$$

Chakravarty, Namjou & A.M.

Entangling with a Baby Universe

To an AdS observer this looks like a mixed state coming from entanglement with a disconnected baby universe.



It's not a thermofield double, but an observer sitting at the origin of AdS sees an effective temperature

$$T \lesssim \sqrt{g}$$

bounded by the topology of the baby universe. This is below the Hawking-Page transition, but can approach it as the baby universe gets more complicated ($g \to \infty$).

An observer can infer the structure of the baby universe from their "CMB" (i.e. the structure of their mixed state).

Chakravarty, Namjou & A.M.

How Wormholes Disappear in a UV Complete Theory

Restoring Unitarity

An actual UV complete theory of gravity should be dual to an individual CFT with fixed couplings.

How does this happen in, e.g. string theory, if the low energy theory includes wormholes?

We need a model which:

- Has wormholes in a supergravity approximation,
- that are removed by stringy effects.

Here is a speculative (but simple) proposal.

Axion Wormholes

In a typical SUGRA theory the effective action

$$S = -\frac{1}{2\kappa^2}\int \sqrt{g}\left(R + \frac{d(d-1)}{2\ell^2} - \frac{1}{2}h_{\alpha\beta}(\phi)\nabla_\mu\phi^\alpha\nabla^\mu\phi^\beta\right)$$

has massless scalars that parameterize a CFT moduli space $\mathcal{M}$ with metric $h_{\alpha\beta}$.

In Euclidean gravity, the metric on $\mathcal{M}$ becomes *Lorentzian*

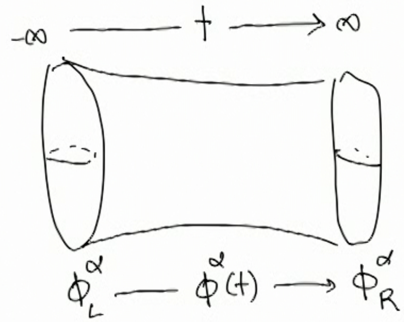
$$ds^2_{\mathcal{M}} = d\varphi^2 + e^{2\varphi}dC_o \underset{t\to it}{\longrightarrow} d\varphi^2 - e^{2\varphi}dC_o$$

for each axionic scalar.

These axions are dual to parity odd couplings in the boundary CFT, like $\theta F\wedge F$.

Axion Wormholes 2

You can find a $S^d \times \mathbb{R}_t$ wormhole that connects two boundaries with different values of couplings:



$$ds^2 = dt^2 + a(t)^2 \, d\Omega_d^2$$

The scalars $\phi^\alpha(t)$ follow a *Lorentzian* geodesic through $\mathcal{M}$, with

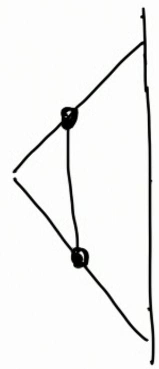
$$\frac{2(d-1)}{d}\pi^2 < -(\Delta s)^2_{\mathcal{M}} < \frac{2d}{d-1}\pi^2$$

Wormholes are *easier* to construct in lower dimensions. The lower bound gives a $\mathbb{T}^d \times \mathbb{R}_t$ wormhole.

Giddings & Strominger
Arkani-Hamed, Orgera & Polchinski
Bergshoeff, Collinucci, Ploegh, Vandoren &Van Riet
...

An Obstruction to Axion Wormholes

The classic axio-dilaton moduli space is the planar patch of AdS_2:



$$ds^2_{\mathcal{M}} = d\varphi^2 - e^{2\varphi} dC_o$$

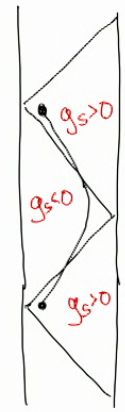
The longest timelike geodesic has length $(\Delta s_{\mathcal{M}})^2 = -\pi^2$, violating our bound.

This is why axion wormholes are hard to find in string theory.

Arkani-Hamed, Orgera & Polchinski

A Strawman Wormhole

In supergravity you can do a field redefinition to go to *global* coordinates on $\mathcal{M}$ and make the geodesic as long as you like.

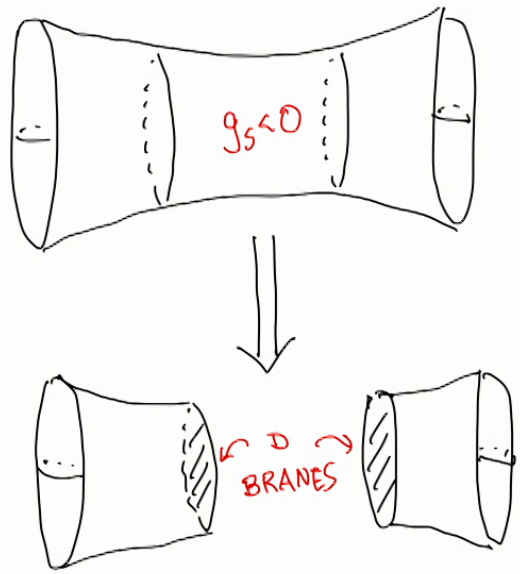


This leads to wormholes with *negative* g_s, but you could always keep $|g_s|$ small.

A naive supergravity observer might think this is fine since $G_N \sim g_s^2$ is small. They don't care about the sign of g_s.

A Stringy Resolution?

But string theory cares about the sign of g_s, since $T_{D-brane} \sim \frac{1}{g_s}$ becomes negative. So this strawman supergravity wormhole is killed by string theory.



Each side is like a half-wormhole, capped off by D-branes.

Perhaps this is a model for how string theory removes wormholes.

A.M. (in progress)

Conclusions

Wormholes, the good, the bad, and the ugly:

- Wormholes are ubiquitous, and you can't ignore them.
- They are crucial to explain the emergence of semi-classical gravity from a UV complete theory.
- Exactly how they are resolved in UV complete theories is still a bit of a mystery.

But there are simple models where we can understand this explicitly.