Title: Talk 17 - Channeling quantum criticality

Speakers: Yijian Zou

Collection: It from Qubit 2023

Date: August 02, 2023 - 11:00 AM

URL: https://pirsa.org/23080012

Abstract: We analyze the effect of decoherence, modelled by local quantum channels, on quantum critical states and we find universal properties of the resulting mixed state's entanglement, both between system and environment and within the system. Renyi entropies exhibit volume law scaling with a subleading constant governed by a "g-function" in conformal field theory (CFT), allowing us to define a notion of renormalization group (RG) flow (or "phase transitions") between quantum channels. We also find that the entropy of a subsystem in the decohered state has a subleading logarithmic scaling with subsystem size, and we relate it to correlation functions of boundary condition changing operators in the CFT. Finally, we find that the subsystem entanglement negativity, a measure of quantum correlations within mixed states, can exhibit log scaling or area law based on the RG flow. When the channel corresponds to a marginal perturbation, the coefficient of the log scaling can change continuously with decoherence strength. We illustrate all these possibilities for the critical ground state of the transverse-field Ising model, in which we identify four RG fixed points of dephasing channels and verify the RG flow numerically. Our results are relevant to quantum critical states realized on noisy quantum simulators, in which our predicted entanglement scaling can be probed via shadow tomography methods.

Channeling quantum criticality

Yijian Zou (Stanford), Shengqi Sang (Perimeter) and Tim Hsieh (Perimeter)

It From Qubit 2023, Perimeter Institute

Motivation I: Quantum simulation

Quantum simulators: promising tools to study quantum many-body physics

Pros:

- Going beyond classical capabilities
- Controlling individual qubits
- Dynamics with measurement, driving, or decoherence

Cons:

- Noisy. Large-scale quantum error correction is not yet available

Motivation II: Measurement-induced phases

POVM: $\left\{ M_a \middle| \sum_a M_a^\dagger M_a = I \right\}$ $\rho \to \{p_a, \rho_a\}$ $p_a = \mathrm{tr}(M_a^\dagger M_a \rho)$

$\rho_a = M_a \rho M_a^\dagger / p_a$

Random unitary gate + local projective measurement:

Volume law to area law phase transition by tuning measurement rate

Emergent conformal invariance at the critical point

Skinner, Ruhman, Nahum (2019)
Li, Chen, Fisher (2019)
Bao, Choi, Altman (2020)
Jian, You, Vasseur, Ludwig (2020)
Li, Chen, Ludwig, Fisher (2021)

Ground state of CFT + (weak) local measurement:

Logarithmic or area-law entanglement depending on measurement basis

Can enhance or destroy correlations

Garratt, Weinstein, Altman (2022)
Lin, Ye, YZ, Sang, Hsieh (2022)
Weinstein, Sajith, Altman, Garratt (2023)

Importantly: these phases exist only when we consider nonlinear observables of density matrix

Motivation III: Mixed-state phases of matter

Pure-state phases

Traditional view: phases are characterized by symmetry breaking order (Landau paradigm)

Modern view: phase are characterized by long-range entanglement (e.g., topological phase)

Mixed-state phases

Order-disorder transition at finite temperature (generalization of the Landau paradigm)

Another type of phase transition: transition in the information-theoretic properties

- Toric code under local decoherence — *Dennis, Kitaev, Landahl, Preskill (2002)*; *Bao, Fan, Altman, Vishwanath (2023)*
- **This work: CFT ground state under decoherence**

Setup

Given a spin-chain realization of the CFT

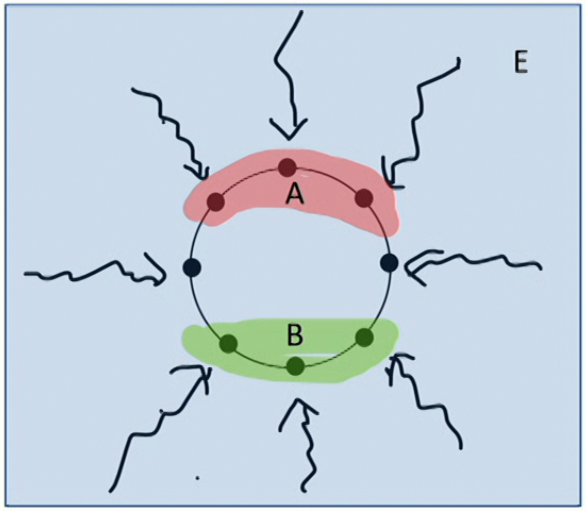
$$\rho = \mathcal{N}(|\psi\rangle\langle\psi|)$$

Dephasing noise: the environment measures the system at probability p

$$\mathcal{D}_{p,\vec{v}}(\rho) = \left(1 - \frac{p}{2}\right)\rho + \frac{p}{2}\sigma_{\vec{v}}\ \rho\ \sigma_{\vec{v}}.$$

$$\sigma_{\vec{v}} := v_x\sigma_x + v_y\sigma_y + v_z\sigma_z$$

What are the entanglement properties of the dephased state?



Independent noise model

$$\mathcal{N} = \otimes_{j=1}^{L}\mathcal{N}_j$$

Main result

- Entanglement quantities can **change abruptly** as we tune decoherence.

- Key: these entanglement quantities are **nonlinear in the density matrix**

  Entropy: $S(\rho_A) = -tr(\rho_A \log \rho_A)$ Logarithmic negativity: $N(A, \bar{A}) = \log tr\, |\rho^{T_A}|$

- New **entanglement phases of mixed states** connected by **RG flow**

Example: critical Ising model with dephasing noise

$$\begin{tikzcd}
\mathcal{D}_{p,y} \arrow[r] & \mathcal{I} \arrow[r, dash, dashed] \arrow[d] \arrow[dr] & \mathcal{D}_z \arrow[d] \\
(p \approx 0.5) & \mathcal{D}_x \arrow[r] & \mathcal{D}_{zx}
\end{tikzcd}$$

(Relevant) Weak X dephasing destroys “almost all” long-range entanglement
(Irrelevant) Weak Y dephasing preserves “almost all” long-range entanglement
(Marginal) Weak Z dephasing destroys “some” long-range entanglement

Renyi entanglement quantities

Renyi entropy: $S^{(n)}(\rho_A) = \frac{1}{1-n}\log tr\, \rho_A^n$

$$n \to 1:\ S(\rho_A) = -tr(\rho_A \log \rho_A)$$

Measures the correlation **between A and $\bar{A}E$**

Renyi Mutual information: $I^{(n)}(A,B) = S^{(n)}(\rho_A) + S^{(n)}(\rho_B) - S^{(n)}(\rho_{AB})$

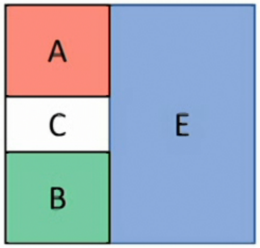
$$n \to 1:\ I(A,B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Measures the correlation **between A and B**

Renyi negativity: $N^{(n)}(A,\bar{A}) = \frac{1}{2-n}\log tr\,(\rho^{T_A})^n\ (n \in 2Z)$ or $\frac{1}{1-n}\log tr\,(\rho^{T_A})^n\ (n \in 2Z+1)$

$$n \in 2Z, n \to 1:\ N(A,\bar{A}) = \log tr\,|\rho^{T_A}|$$

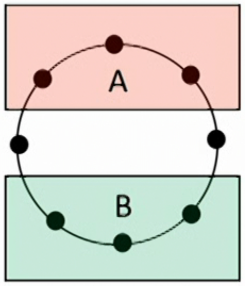
Measures the ***quantum*** correlation between A and $\bar{A}$



$\rho = \rho_{ABC}, \bar{A} = BC$

Entanglement in the ground state

For pure states, bipartite entanglement is completely characterized by Schmidt spectrum



$S^{(n)}(A) = \frac{c}{6}\left(1 + \frac{1}{n}\right)\log(\sin\frac{\pi L_A}{L}) + O(1)$, where c is central charge

$I^{(n)}(A, \bar{A}) = 2\, S^{(n)}(A)$

$N^{(n)}(A, \bar{A}) = S^{(n)}(A), n \in 2Z + 1$

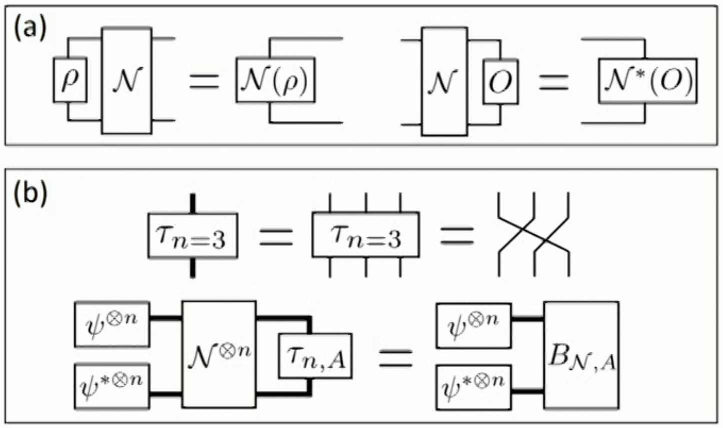
$I^{(n)}(A, B) \sim \eta^{2\Delta}(\eta \ll 1)$, where Δ is the scaling dimension of the lowest operator

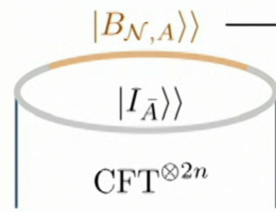
Cross ratio: $\eta = \frac{X_{12}X_{34}}{X_{13}X_{24}}, X_{ij} = \sin\frac{\pi x_{ij}}{L}$

Mapping to boundary criticality

$$S_A^{(n)}(\rho) := \frac{1}{1-n}\log \boxed{\mathrm{Tr}(\rho_A^n)}$$

$$Z_A^{(n)} = \mathrm{Tr}(\rho^{\otimes n}\tau_{n,A}) = \mathrm{Tr}(\mathcal{N}(|\psi\rangle\langle\psi|)^{\otimes n}\tau_{n,A})$$
$$= \mathrm{Tr}((|\psi\rangle\langle\psi|)^{\otimes n} B_{\mathcal{N},A})$$





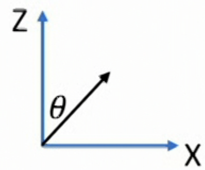
Boundary condition

$$S^{(n)}(\rho) = \alpha^{(n)}L - \frac{1}{n-1}\boxed{\log g_{\mathcal{N}}^{(n)}}$$

Boundary entropy

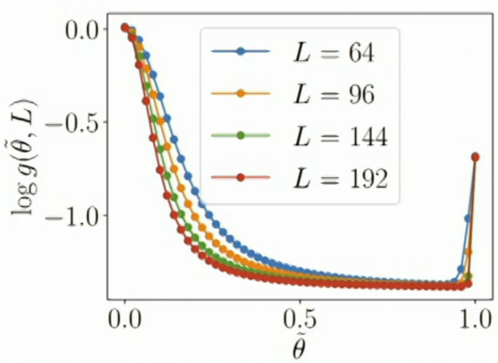
Numerical result: boundary entropy

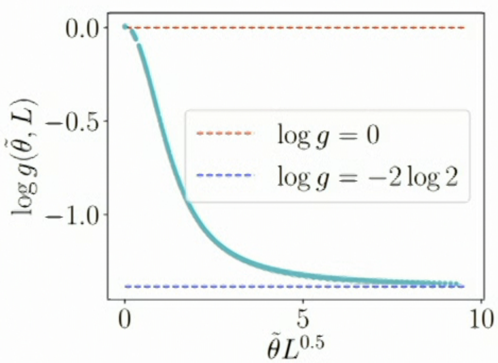
Dephasing in the ZX plane

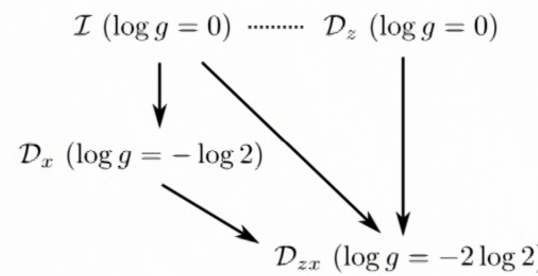


$$\mathcal{D}_{p,\vec{v}}(\rho) = \left(1 - \frac{p}{2}\right)\rho + \frac{p}{2}\sigma_{\vec{v}}\ \rho\ \sigma_{\vec{v}}.$$

$$\sigma_{\vec{v}} := v_x\sigma_x + v_y\sigma_y + v_z\sigma_z$$







Monotonicity under RG flow

Affleck, Ludwig (1989)

Friedan, Konechny (2004)

Renyi entanglement quantities

Mutual information measures both classical and quantum correlations

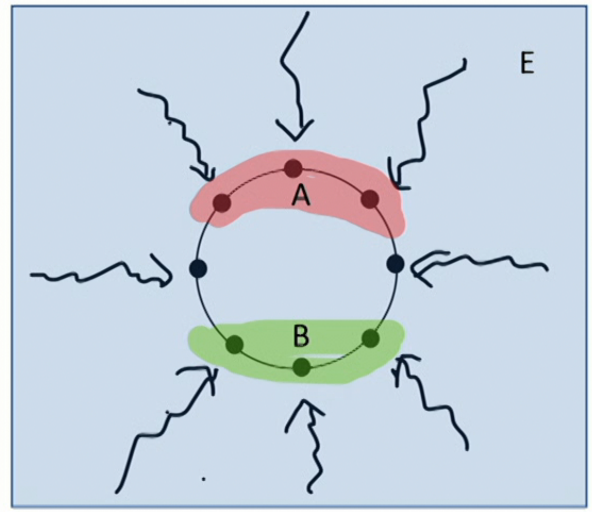
$$I^{(n)}(A,\bar{A}) = \frac{4\boxed{\Delta^{(n)}_{\mathcal{IN}}}}{n-1}\log\left(\frac{L}{\pi}\sin\left(\frac{\pi L_A}{L}\right)\right) + O(1)$$

$$I^{(n)}(A,B) = \text{const.} \times \eta^{\boxed{\Delta^{(n)}_{\mathcal{O}}}} \quad (\eta \ll 1)$$

Mutual negativity only measures quantum correlations

$$N^{(n)}(A,\bar{A}) = \frac{2\boxed{\Delta^{(n)}_{\mathcal{NT}}}}{n-1}\log\left(\frac{L}{\pi}\sin\frac{\pi L_A}{L}\right) + O(1)$$

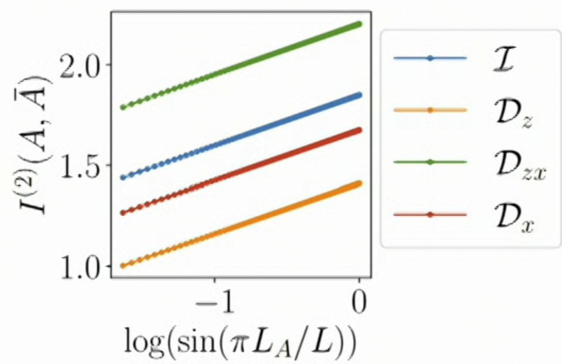
The scaling dimensions of boundary condition changing operators



Numerical result: Renyi mutual information

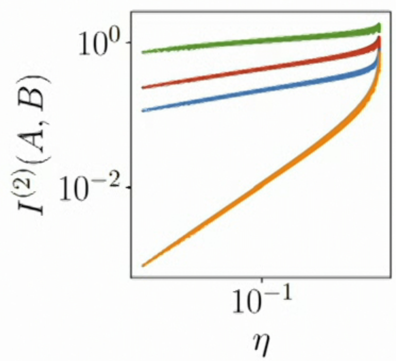
$$I^{(n)}(A,\bar{A}) = \frac{4\boxed{\Delta^{(n)}_{\mathcal{IN}}}}{n-1}\log\left(\frac{L}{\pi}\sin\left(\frac{\pi L_A}{L}\right)\right) + O(1)$$

Same exponent for all fixed points



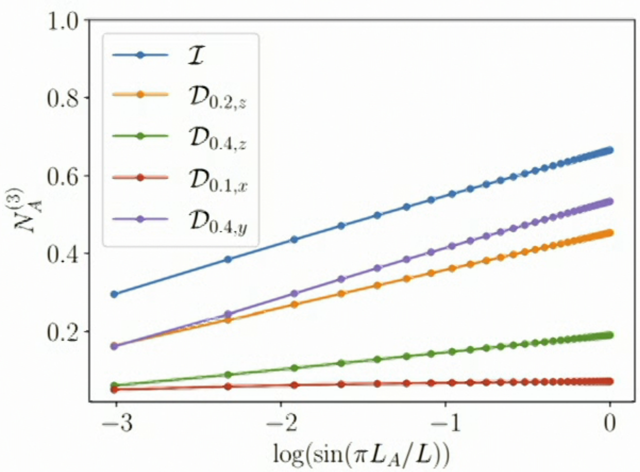
$$I^{(n)}(A,B) = \text{const.} \times \eta^{\boxed{\Delta^{(n)}_{\mathcal{O}}}} \quad (\eta \ll 1)$$

Different exponent among fixed points



Numerical result: Renyi negativity

$$N^{(n)}(A, \bar{A}) = \frac{2\boxed{\Delta_{NT}^{(n)}}}{n-1} \log\left(\frac{L}{\pi} \sin\frac{\pi L_A}{L}\right) + O(1)$$



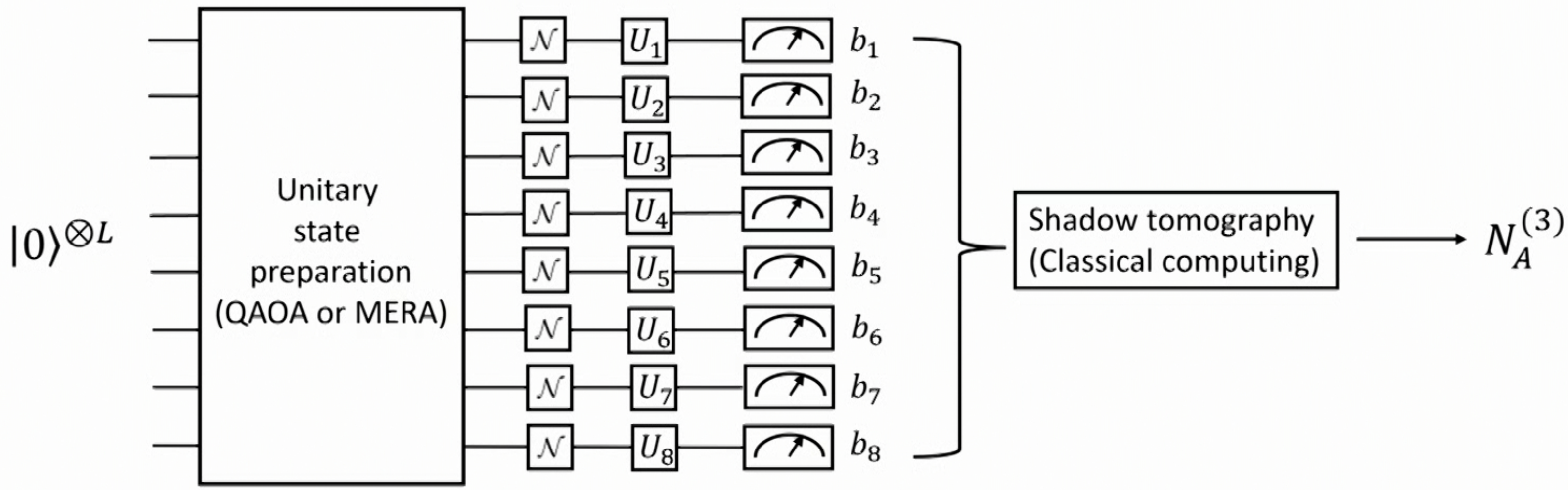
Weak X dephasing: Area law (relevant)

Weak Y dephasing: Logarithmic scaling with the same coefficient as the ground state (irrelevant)

Weak Z dephasing: Logarithmic scaling with continuously changing coefficient (marginal)

Experimental realization

Implementation on an ion-trap quantum simulator



Implication on code properties of CFT

Code subspace

$$H|\phi_\alpha\rangle = E_\alpha|\phi_\alpha\rangle$$

$$E_\alpha = \frac{2\pi}{L}\left(\Delta_\alpha - \frac{c}{12}\right)$$

$$\mathcal{H}_{\text{code}} = \text{span}\{|\phi_\alpha\rangle, \alpha = 1, 2, \cdots, d\}$$

$$|\psi\rangle_{QR} = \sum_\alpha |\phi_\alpha\rangle_Q |\alpha\rangle_R \qquad \rho_{QR} = \mathcal{N}_Q(|\psi\rangle\langle\psi|)$$

Coherent information $\quad I_c = S_Q - S_{QR} \qquad \text{Approximately decodable} \leftrightarrow I_c = \log d - \epsilon$

Relevant dephasing: not correctable error $\quad \lim_{L\to\infty} I_c = 0$

Irrelevant dephasing: correctable error $\quad \lim_{L\to\infty} I_c = \log d$

YZ, Sang (in preparation)

Caveat: have to be careful to replica limit

Summary

- Decoherence induces **new mixed-state phases** for a critical state
- These phases are distinguished by different **long-range quantum entanglement**
- These phases are connected by **RG flow (phase transitions)**
- These phases can be realized on a quantum computer
- Corollary: CFT is a **biased** quantum error correction code