Title: (VIRTUAL) Talk 63 - Measurement-induced phase transition in teleportation and wormholes

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Abstract: We demonstrate that some quantum teleportation protocols exhibit measurement induced phase transitions in Sachdev-Ye-Kitaev model. Namely, Kitaev-Yoshida and Gao-Jafferis-Wall protocols have a phase transition if we apply them at a large projection rate or at a large coupling rate respectively. It is well-known that at small rates they allow teleportation to happen only within a small time-window. We show that at large rates, the system goes into a new steady state, where the teleportation can be performed at any moment. In dual Jackiw-Teitelboim gravity these phase transitions correspond to the formation of an eternal traversable wormhole. In the Kitaev-Yoshida case this novel type of wormhole is supported by continuous projections. Based on https://arxiv.org/abs/2210.03083

Measurement-induced phase transitions in teleportation and wormholes

Alexey Milekhin
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arXiv: 2210.03083 (w/ Fedor Popov)

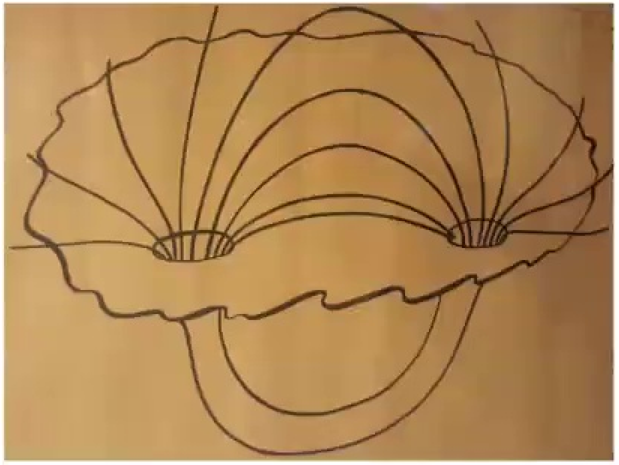


Figure: Drawing of a wormhole by John Wheeler

"It" motivation

- We know many static solutions of traversable wormholes.
- How can such objects form dynamically?
- Such process may involve topology change. Quantum information perspective should be very useful.
- The problem is easier if we already have connected space and we just want to stabilize the wormhole "handle" (e.g. two entangled black holes). Gao–Jafferis–Wall and Kitaev–Yoshida protocols. The second one involves measurements with postselection.

"Qubit" motivation

- Recently there has been a lot of discussions of measurement-induced phase transitions (MIPT) in condensed matter community.

  Li, Chen, Fisher'19; ...

- The upshot is that quantum systems are often resilient to measurements. One needs a finite measurement rate for something interesting to happen.

- Entanglement MIPT can be explained via emergent quantum error correction.

  Fan, Vijay, Vishwanath, You'20

- Can we learn anything about continuum (holographic) QFT? How do we describe measurements? Wormhole formation as MIPT?

Today

- I will explain that measurements are often equivalent to Euclidean evolution. Works well for QFT.
- There is a phase transition in Gao–Jafferis–Wall in Kitaev–Yoshida protocols.

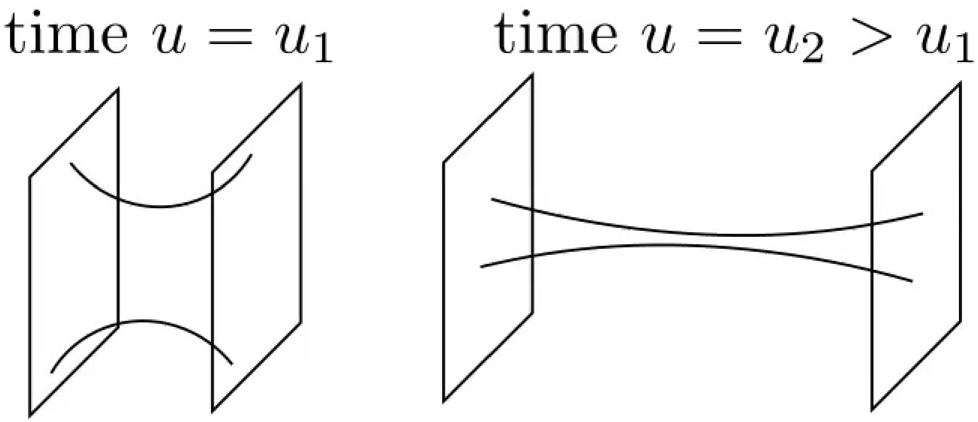
  Bulk: transition from two entangled black holes to an eternal traversable wormhole.

  Boundary perspective: MIPT in quantum teleportation (at least in SYK/JT gravity)

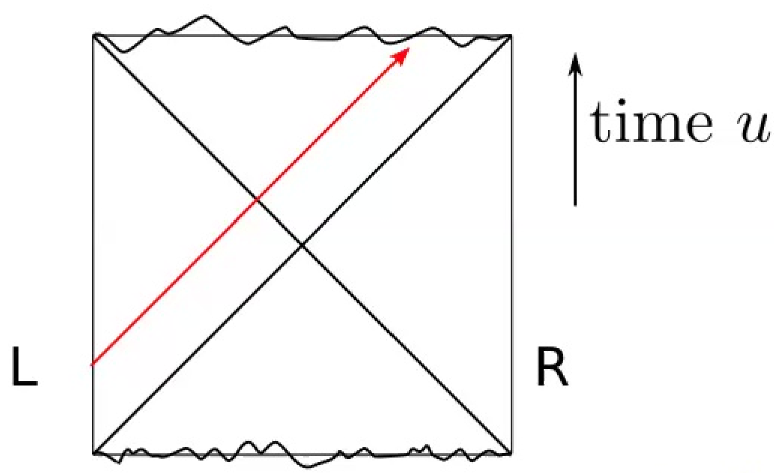
Plan

- A warmup with Gao–Jafferis–Wall: almost MIPT
- Kitaev–Yoshida I: path-integral approach to measurements
- Kitaev–Yoshida II: supporting an eternal wormhole with measurements

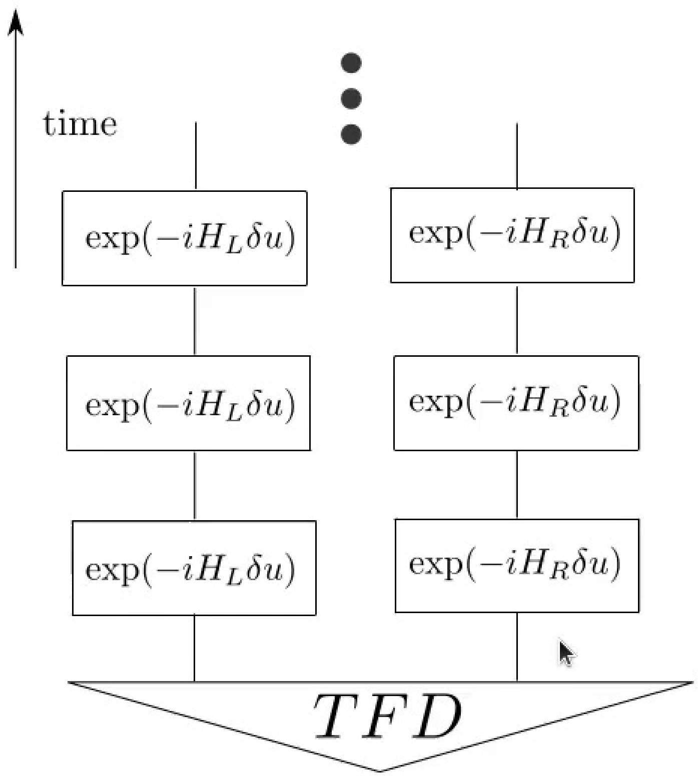
Holographic picture: non-traversability of Einstein–Rosen bridge



$$\langle \psi_L(u)\psi_R(u)\rangle \sim e^{-u/\beta}$$



Diagnostics of teleportation



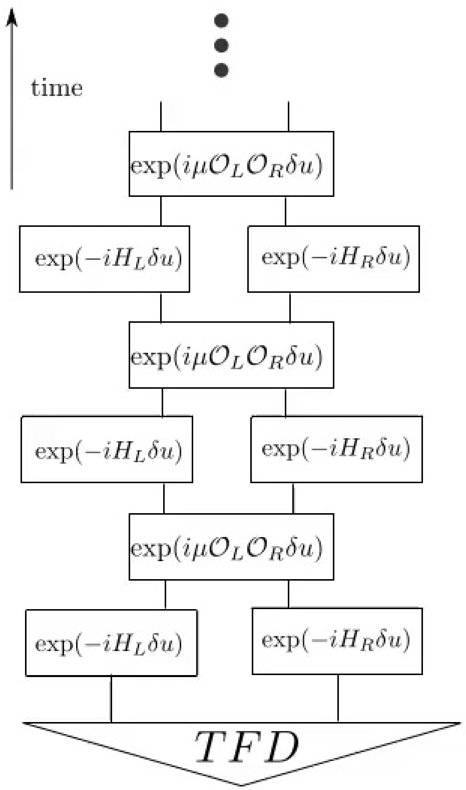
$$H = H_L + H_R,$$

$$|TFD\rangle = \sum_E e^{-\beta E/2}|E\rangle_L|E\rangle_R$$

No causal connection:
$\langle\{\psi_L(u_1), \psi_R(u_2)\}\rangle \equiv 0$

Diagnostics of teleportation/traversability
$\boxed{\langle\{\psi_L, \psi_R\}\rangle \sim \mathcal{O}(1)}$

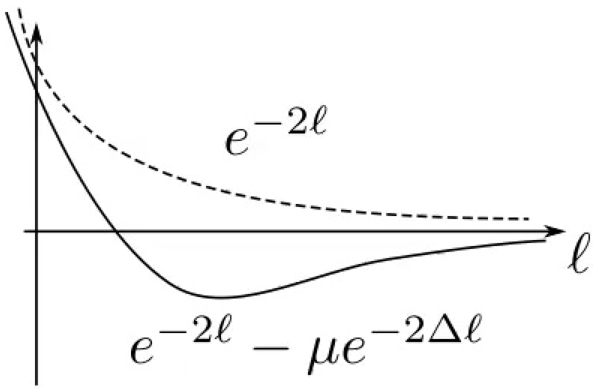
Continuous application of Gao–Jafferis–Wall.



Slows the expansion of ER bridge, making it traversable for a limited time.

Gao–Jafferis–Wall: the case of Sachdev–Ye–Kitaev model

- At low-energies has a dual description in terms of JT gravity. Only one bosonic classical degree of freedom is ℓ:

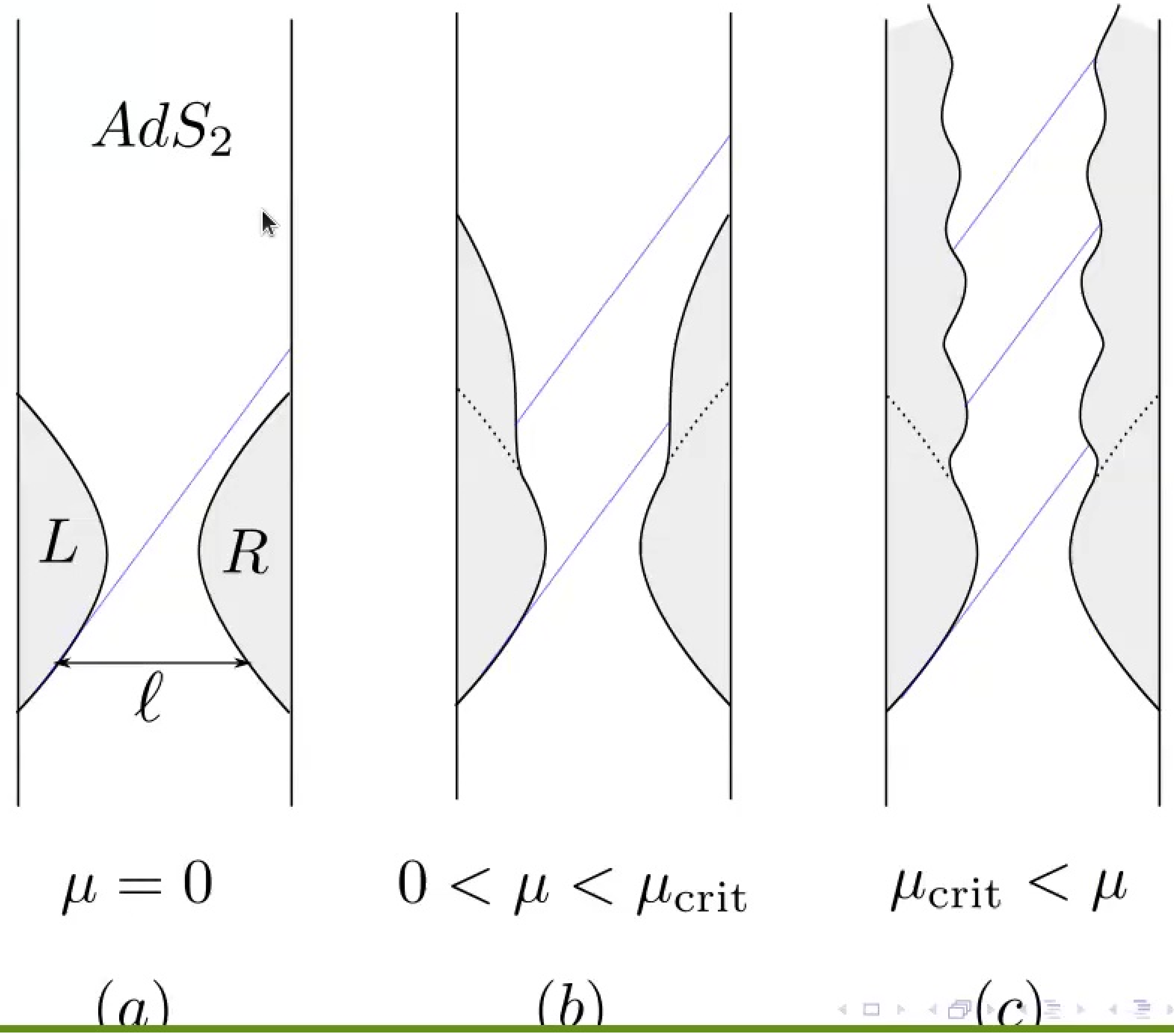


$$H_{eff} = \ell'^2 + e^{-2\ell} - \mu\theta(u)e^{-2\Delta\ell}$$

$$N \gg 1, J/N \ll 1/\beta \ll J$$

Maldacena, Qi'18

Phase transition: $\mu_{crit}(\beta, t, \Delta)$



Comments

- Such behavior was conjectured for (near-extremal) BTZ case based on perturbation theory.

  Fu, Grado-White, Marolf'18; Caceres, Misobuchi, Xiao'18

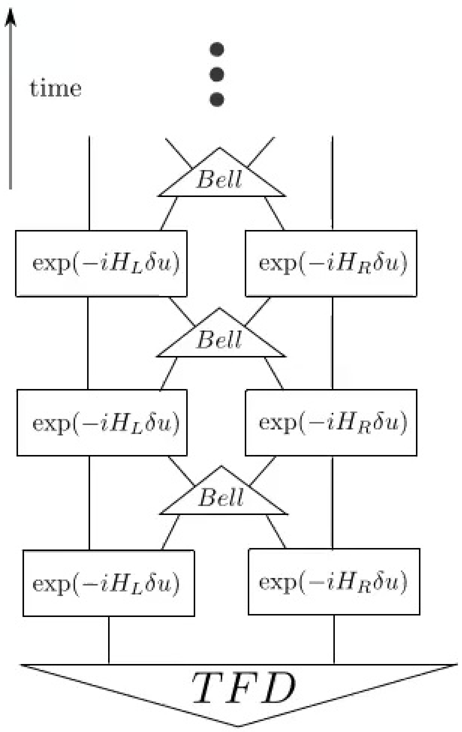
- The simplicity of JT allowed us to find a full solution

- Interpretation: GJW introduces negative energy inside the black hole. What if this negative energy is too big and makes the black hole superextremal? Do we get a naked singularity?

- No, we get an eternal traversable wormhole

- One can get an eternal traversable wormhole even if there was matter inside

Continuous application of Kitaev–Yoshida.



What if I do this frequently?

Kitaev–Yoshida: introduction

Every timestep du:

- with probability $1 - N\kappa du$ leave the system alone.
- otherwise pick up a random subsystem on each side and project them on Bell state with Π_j: that is, perform the measurement and throw away the sample if you did not like the outcome
- This is a physical operation, but the trace is not preserved

Introduce not-normalized $\widetilde{\rho}$:

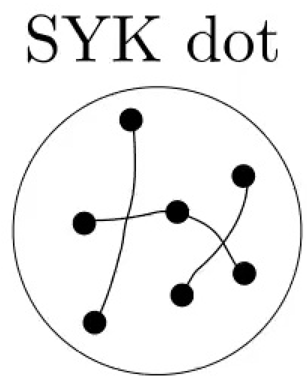
$$\widetilde{\rho}(u + du) = (1 - N\kappa du)\widetilde{\rho} + \kappa du \sum_j \Pi_j \widetilde{\rho} \Pi_j, \quad \Pi_j = 1 + i\psi_L^j \psi_R^j$$

+Hamiltonian part $i[H, \tilde{\rho}]$

Evaluate observables as

$$\frac{\mathrm{Tr}\,(\mathcal{O}\widetilde{\rho})}{\mathrm{Tr}\widetilde{\rho}}$$

Direct measurement introduces too much energy



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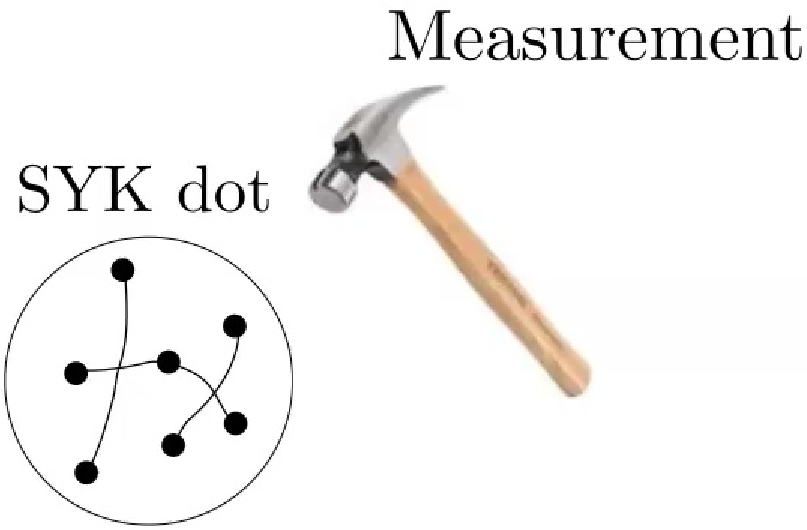


Use ancilla!

Weak measurement with post-selection as Euclidean evolution

- Couple system to an ancilla:

$$|0_A\rangle\langle 0_A| \otimes \rho,$$

- Evolution:

$$|0_A\rangle\langle 0_A| \otimes U_{00}^\dagger \rho U_{00} + |0_A\rangle\langle 1_A| \otimes U_{01}^\dagger \rho U_{01} +$$
$$+|1_A\rangle\langle 0_A| \otimes U_{10}^\dagger \rho U_{10} + |1_A\rangle\langle 1_A| \otimes U_{11}^\dagger \rho U_{11}$$

- Measure ancilla and keep only $|0_A\rangle$:

$$|0_A\rangle\langle 0_A| \otimes U_{00}^\dagger \rho U_{00}.$$

- U_{00} does not have to be unitary. It can be

$$U_{00} = e^{Sdu}$$

Kitaev–Yoshida: equations of motion

Final evolution equation:

$$\widetilde{\rho}(u + du) = \widetilde{\rho}(u) + \kappa du(S\widetilde{\rho}(u) + \widetilde{\rho}(u)S), \ S = i\sum_j \psi_L^j \psi_R^j$$

+Hamiltonian part $i[H, \tilde{\rho}]$

- Hermicity, complete positivity is preserved
- Similar coupling was analyzed for Brownian SYK

  Jian, Liu, Chen, Swingle, Zhang'21
- Compare with Gao–Jafferis–Wall coupling:

$$\rho(u + du) = \rho(u) + \mu du\,(iS\rho(u) - \rho(u)iS)$$

Schwinger–Keldysh in one slide

- "Normal" Hamiltonian evolution:

$$\underbrace{\exp(-iHdu)}_{\text{forward"+"part}}\ \rho\ \underbrace{\exp(iHdu)}_{\text{backward"}-\text{"part}}$$

- In our case:

$$\underbrace{\exp(-iHdu)\exp(\kappa Sdu)}_{\text{forward"+"part}}\ \widetilde{\rho}\ \underbrace{\exp(\kappa Sdu)\exp(iHdu)}_{\text{backward"}-\text{"part}}$$

- Effective hamiltonians:

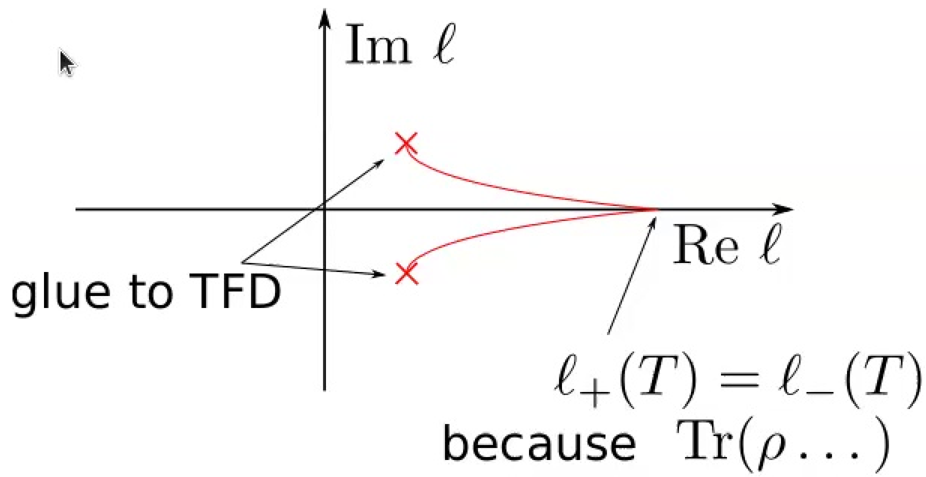
$$H_{+} = H_{SYK} + i\kappa S$$

$$H_{-} = H_{SYK} - i\kappa S$$

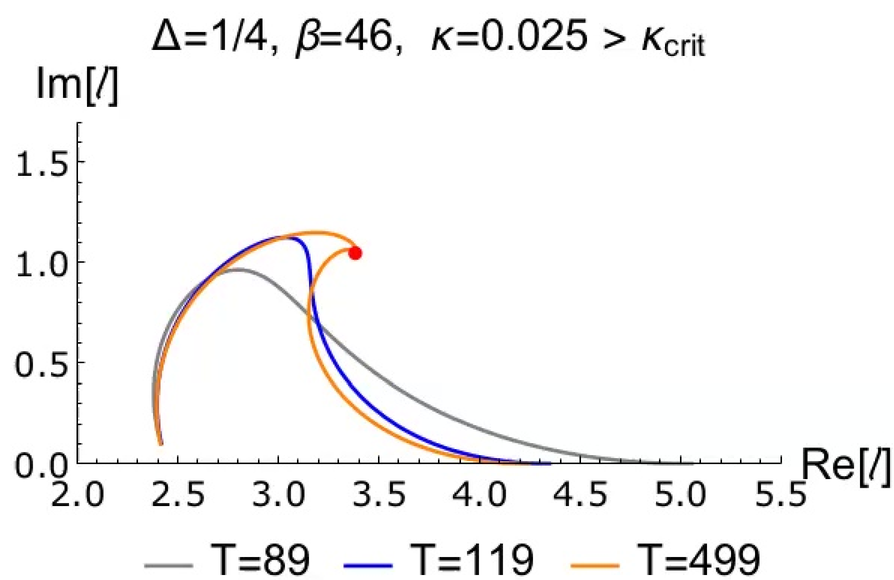
- Effective hamiltonian for complex length ℓ:

$$H_{eff}^{+} = \ell_{+}'^{2} + e^{-2\ell_{+}} - i\kappa e^{-2\Delta \ell_{+}}$$

$$H_{eff}^{-} = \ell_{-}'^{2} + e^{-2\ell_{-}} + i\kappa e^{-2\Delta \ell_{-}}$$

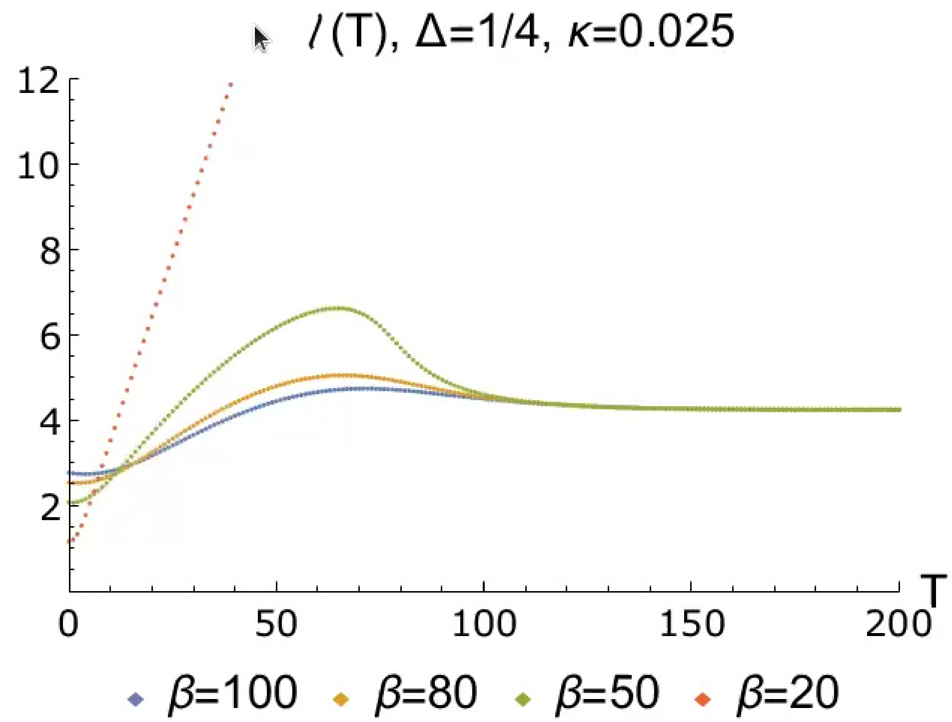


- For $\Delta \geq 1/2$ nothing interesting happens (in contrast with GJW)
- However, for $\Delta < 1/2$ and large κ, β the trajectory spends a lot of time near the critical point l_* (red dot)
- Numerics suggests that the phase is stable.



Wormhole length $\ell_+(T) = \ell_-(T)$ stops growing

$$\langle \psi_L(T)\psi_R(T)\rangle \sim \exp\left(-\Delta \ell(T)\right)$$



Takeaways

- Euclidean evolution can be recast as a measurement with post-selection. Accessible in a lab even now.

  Noel et al.'21; Koh et al.'22

- It is really easy to form an eternal traversable wormhole if you already have a TFD.

A comment on unentangled black holes

What if the black holes were initially unentangled but we couple them?

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A comment on unentangled black holes

What if the black holes were initially unentangled but we couple them?

In SYK the transition to eternal wormhole still happens, but this seem on rely on SYK microscopic details

Maldacena, AM'19

Open questions

- Explanation for the teleportation transition?
- Transition from TFD to eternal wormhole in higher dimensions
- Measurement without post-selection?
- Geometric description of measurements?

  Numasawa, Shiba, Takayanagi, Watanabe'16;
  Antonini, Bentsen, Cao, Harper, Jian, Swingle'22
- Understand complex reparametrizations better. They also govern large-q SYK dynamics

  Lensky, Qi'20

Thank you!

Figure: "Traversable wormhole supported by measurements" by Midjourney

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