Title: talk 30 - Measurement-based quantum simulation of Abelian lattice gauge theories

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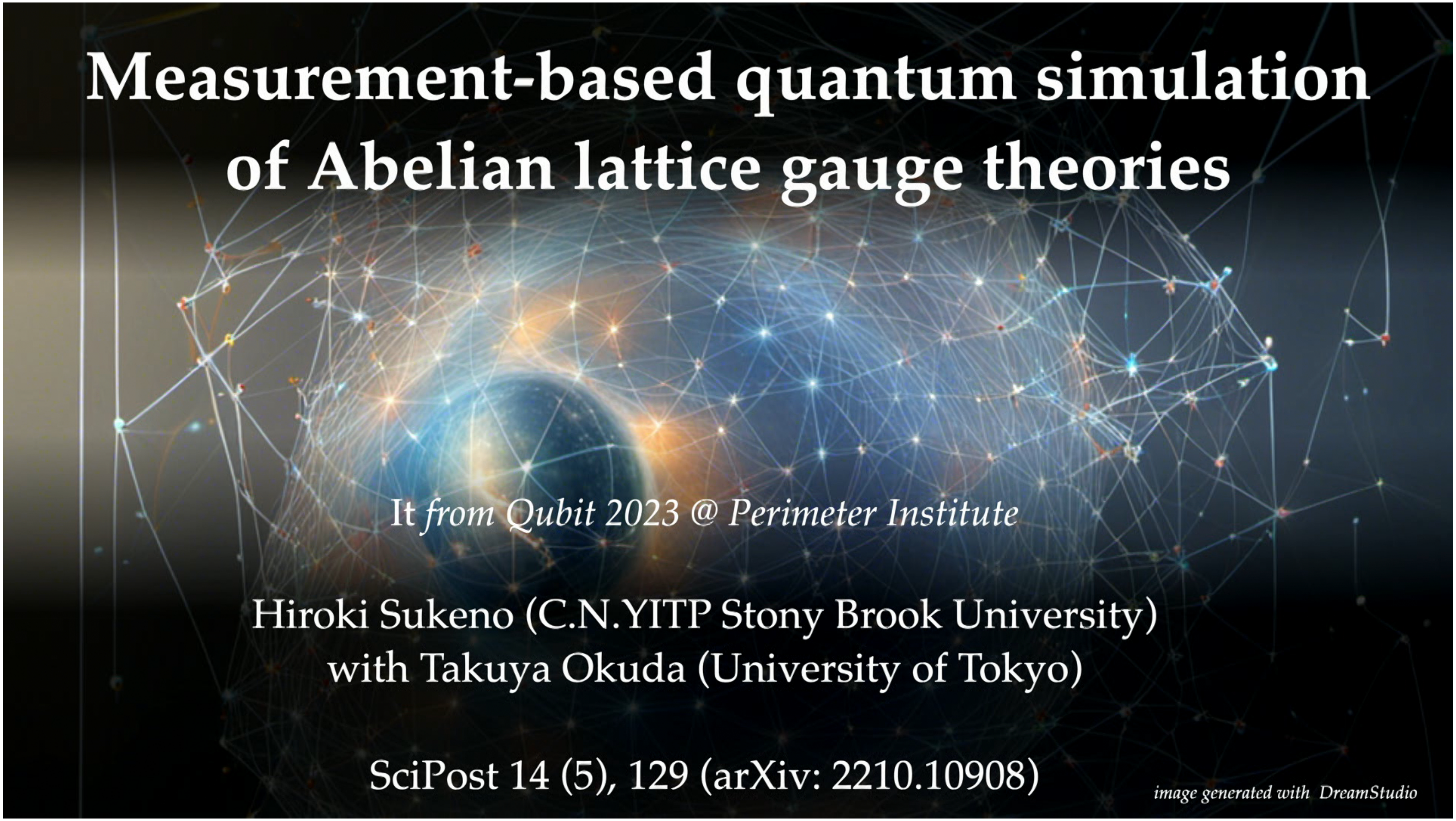
Abstract: Quantum simulation of lattice gauge theory is expected to become a major application of near-term quantum devices. In this presentation, I will talk about a quantum simulation scheme for lattice gauge theories motivated by Measurement-Based Quantum Computation [1], which we call Measurement-Based Quantum Simulation (MBQS). In MBQS, we consider preparing a resource state whose entanglement structure reflects the spacetime structure of the simulated gauge theory. We then consider sequentially measuring qubits in the resource state in a certain adaptive manner, which drives the time evolution in the Hamiltonian lattice gauge theory. It turns out that the resource states we use for MBQS of Wegner's models possess topological order protected by higher-form symmetries. These higher-form symmetries are also practically useful for error correction to suppress contributions that violate gauge symmetries. We also discuss the relation between the resource state and the partition function of Wegner's model. This presentation is based on my work with Takuya Okuda [2].

[1] R. Raussendorf and H. J. Briegel, A One-Way Quantum Computer, Phys. Rev. Lett. 86, 5188 (2001)
[2] H. Sukeno and T. Okuda, Measurement-based quantum simulation of Abelian lattice gauge theories, arXiv:2210.10908

Measurement-based quantum simulation of Abelian lattice gauge theories



It *from Qubit 2023 @ Perimeter Institute*

Hiroki Sukeno (C.N.YITP Stony Brook University)
with Takuya Okuda (University of Tokyo)

SciPost 14 (5), 129 (arXiv: 2210.10908)

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Introduction

- Quantum simulation of lattice gauge theories
- Most of works in the literature: so-called gate-based quantum computers.
- **Measurement-Based Quantum Computation (MBQC)** is also a model capable of quantum computation.
- In this vein, we have formulated an MBQC scheme for simulating a class of spin models that includes gauge theories.
- It turned out our construction exhibits some features which may be of interest to the It from Qubit community.
- In our construction, the spacetime structure of simulated theory is reflected in the entanglement structure of the state to be measured, i.e., the *resource state*.

Plan

- Essence of MBQC (MBQS) in (0+1)dimensions
- Wegner's generalized Ising models
- MBQS for Wegner's models
- Higher-form symmetries, an SPT order, and a holographic interplay.
- Some remarks

Essence of MBQC (MBQS) in (0+1)dimensions

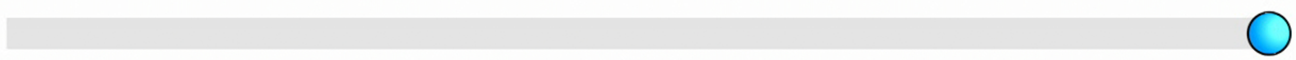
MBQC: (0+1)dimensions

$$X^{\#}Z^{\#} \cdot U_1 |\psi\rangle$$

MBQC: (0+1)dimensions

$$X^{\#}Z^{\#} \cdot U_1 \left| \psi \right\rangle$$

MBQC: (0+1)dimensions

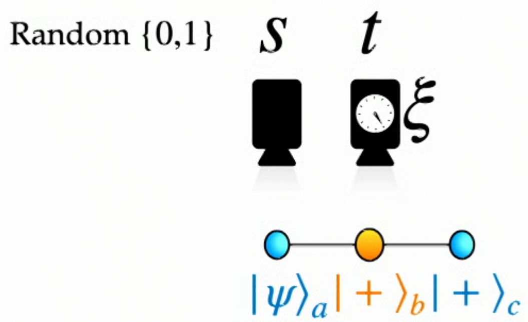


Post-measurement product state

$$U_N \cdots U_2 U_1 |\psi\rangle$$

Simulated state
(Cleaned up)

MBQC: (0+1)dimensions



- Consider a one-qubit "initial state" $|\psi\rangle$
- Prepare a "resource state" $CZ_{a,b}CZ_{b,c}|\psi\rangle_a|+\rangle_b|+\rangle_c$
- Measure: the a qubit with the basis $Z^s|+\rangle \quad (s = 0,1)$
- Measure: the b qubit with the basis $Z^t e^{i\xi Z}|+\rangle \quad (t = 0,1)$

$$\left[\langle +|_a Z_a^s \otimes \langle +|_b e^{-i\xi Z_b} Z_b^t\right] \times \left[CZ_{a,b}CZ_{b,c}|\psi\rangle_a|+\rangle_b\,|+\rangle_c\right] \propto X^t Z^s \cdot e^{-i(-1)^s \xi X}|\psi\rangle_c$$

Teleportation to the c qubit & rotation.

The rotation angle depends on the outcome of the first measurement s.

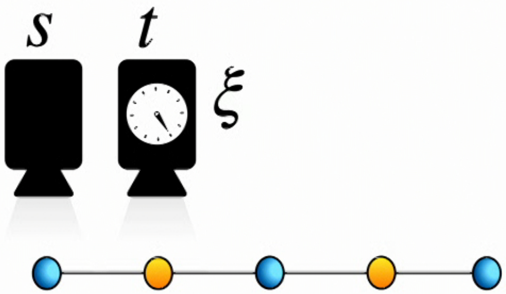
Notation:

$|+\rangle$

$CZ_{a,b} := |0\rangle\langle 0|_a \otimes I_b + |1\rangle\langle 1|_a \otimes Z_b = CZ_{b,a}$

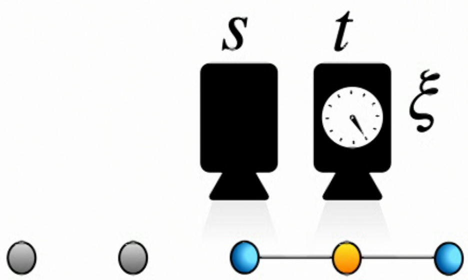
Single-qubit measurement

MBQC: (0+1)dimensions



$X^t Z^s \cdot e^{-i(-1)^s \xi X} | \psi \rangle \to X^t Z^s \cdot e^{-i\alpha X} | \psi \rangle$

by choosing $\xi = (-1)^s \alpha$. (α : desired angle)



One can choose angles $\{\xi_k\}$ adaptively to absorb effects from previous measurements.

After measuring all qubits except the last one, we will be left with

$$X^{\#} Z^{\#} e^{-i\alpha_k X} \cdots e^{-i\alpha_2 X} e^{-i\alpha_1 X} | \psi \rangle_{\text{right bdry}}$$

can be removed

MBQC

What we have just shown is a simple example of MBQC.

MBQC (measurement-based quantum computation)

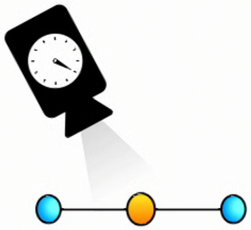
(Universal) quantum computation can be achieved by
(1) **preparing a resource state**
(2) **measuring the resource state in a certain adaptive pattern.**
(3) **post-processing (unwanted) byproduct operators**

[Raussendorf-Briegel (2001)]
Review article: e.g. [T.-C. Wei (2023)]

However, our goal below is not the universal quantum computation, but a quantum simulation of Wegner's Ising models.

MBQC: multi-body interaction term

For a multi-qubit quantum computation, we can use another building block of MBQC.



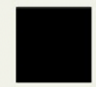
- Consider a general "initial state" $|\psi\rangle_{bc}$
- Prepare a "resource state" $CZ_{a,b}CZ_{a,c}|\psi\rangle_{bc}|+\rangle_a$
- Measure the middle qubit with $\{e^{i\xi X}|0\rangle, e^{i\xi X}|1\rangle\}$, i.e., $X^s e^{i\xi X}|0\rangle \quad (s = 0,1)$

$$\langle 0|_a e^{-i\xi X_a} X_a^s \cdot CZ_{a,b}CZ_{a,c}|\psi\rangle_{bc}|+\rangle_a = e^{-i\xi Z_b Z_c}(Z_b Z_c)^s|\psi\rangle_{bc}$$

→ **Multi-qubit rotation.**

Spin model to be simulated

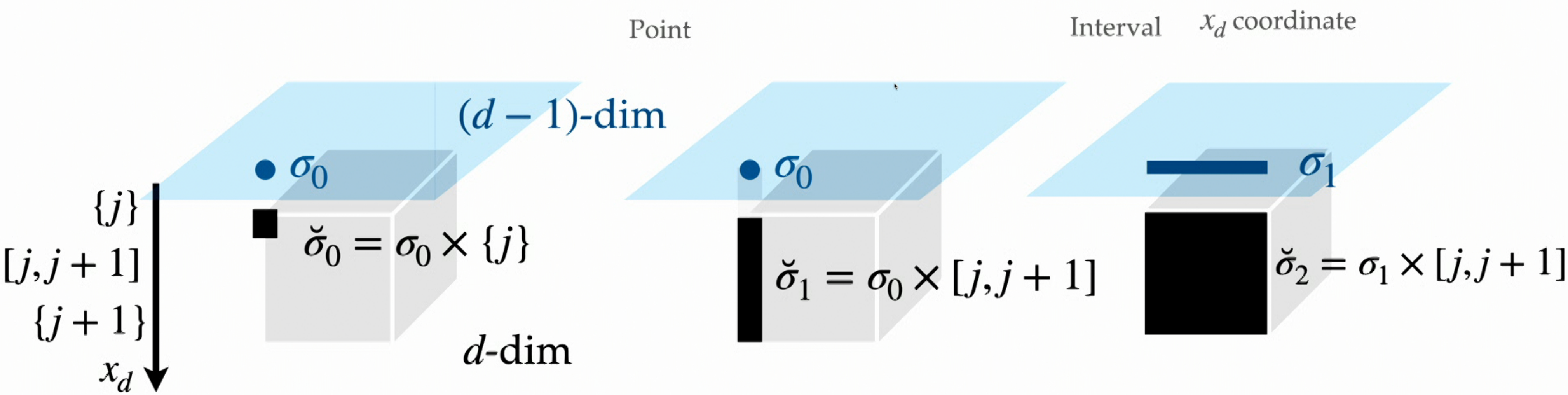
—Wegner's generalized Ising models—

Cell simplex σ_i	σ_0	σ_1	σ_2	σ_3
				

$\breve{\sigma}_i$: cell simplices in d dimensional hypercube lattice

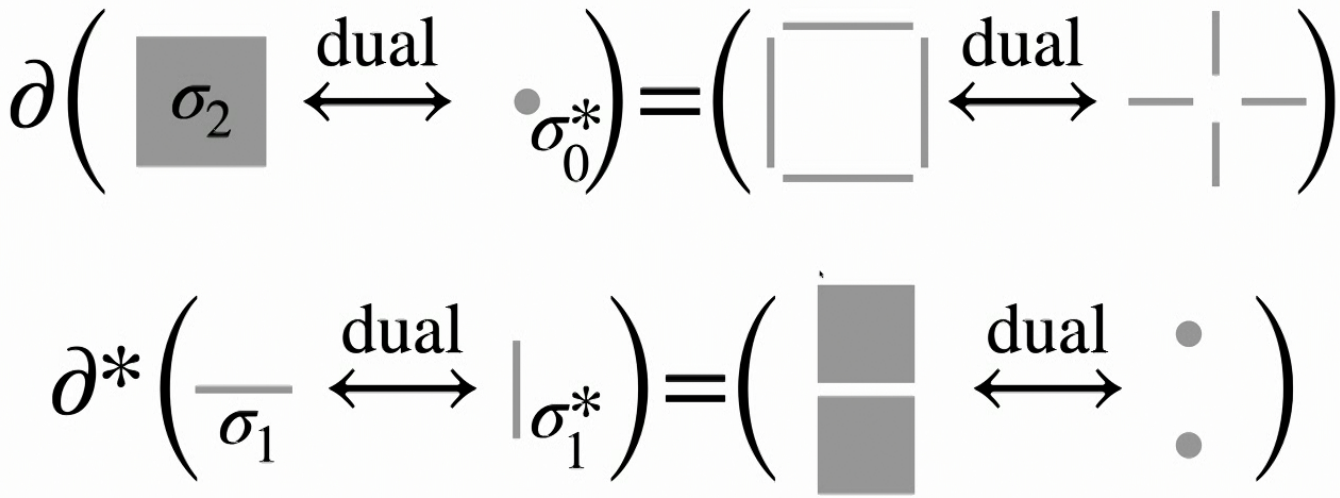
σ_i : cell simplices in $d-1$ dimensional hypercube lattice

$$\breve{\sigma}_i = \sigma_i \times \{j\} \text{ or } \breve{\sigma}_{i+1} = \sigma_i \times [j, j+1]$$



Similarly, we have cell simplices in the dual lattice with $\sigma_i \simeq \sigma^*_{d-i}$.

We have $\partial^2 = 0$ (and $(\partial^*)^2 = 0$) and a chain complex.



Wegner's generalized Ising model

Model $M_{(d,n)}$:
Classical spin variables $S_{\breve{\sigma}_{n-1}} \in \{+1, -1\}$ living on $(n-1)$-cells in the d-dimensional hybercubic lattice. [Wegner (1971)]

Euclidean action (classical Hamiltonian) I :

$$I = -J \sum_{\breve{\sigma}_n} \left(\prod_{\breve{\sigma}_{n-1} \subset \partial \breve{\sigma}_n} S_{\breve{\sigma}_{n-1}} \right) .$$

Via the transfer matrix formalism, we obtain a quantum Hamiltonian in $(d-1)$ dimensions with the continuous time. [Kogut (1979)]

$$H_{(d,n)} = -\sum_{\sigma_{n-1}} X(\sigma_{n-1}) - \lambda \sum_{\sigma_n} Z(\partial \sigma_n) .$$

Wegner's generalized Ising model

$M_{(d,1)}$

Classical Ising model

$$I = -J \sum_{\text{edge}} S(\partial \breve{\sigma}_1)$$

edge — site variable

Transverse field Ising model

$$H_{(d,1)} = -\sum_{\sigma_0} X(\sigma_0) - \lambda \sum_{\sigma_1} Z(\partial \sigma_1)$$

$M_{(d,2)}$

Gauge theory (Wilson's plaquette action for $G = \mathbb{Z}_2$)

$$I = -J \sum_{\text{plaquette}} S(\partial \breve{\sigma}_2)$$

plaquette — link variable

Quantum pure gauge theory

$$H_{(d,2)} = -\sum_{\sigma_1} X(\sigma_1) - \lambda \sum_{\sigma_2} Z(\partial \sigma_2)$$

Wegner's generalized Ising model

We wish to simulate a Trotterized (real) time evolution:

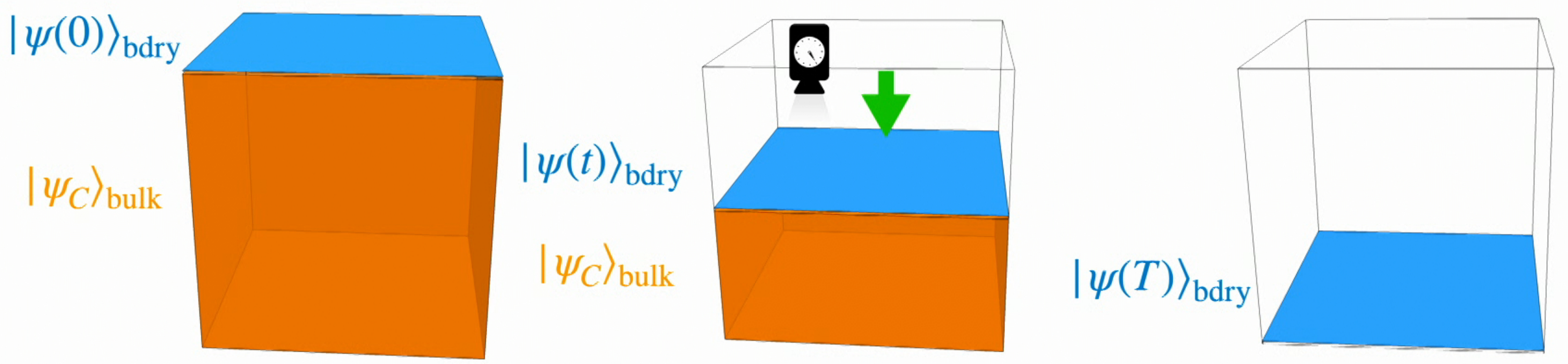
$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$$

with

$$T(t = j\Delta t) = \left(\prod_{\sigma_{n-1}} e^{i\Delta t X(\sigma_{n-1})} \prod_{\sigma_n} e^{i\Delta t \lambda Z(\partial\sigma_n)}\right)^j .$$

MBQS of lattice gauge theories

MBQS



$|\psi(t)\rangle_{bdry}$: **simulated state of $M_{(d,n)}$** with the Trotterized time evolution $T(t)$,

$$|\psi(t)\rangle_{bdry} = T(t)\,|\psi(0)\rangle \,.$$

$|\psi_C\rangle_{bulk}$: **resource state** to be measured — **generalized cluster state (gCS)**.

MBQS

Entanglement in our resource state, $\mathrm{gCS}_{(d,n)}$ (generalized cluster state), is **tailored** to reflect the space-time structure of the model $M_{(d,n)}$:

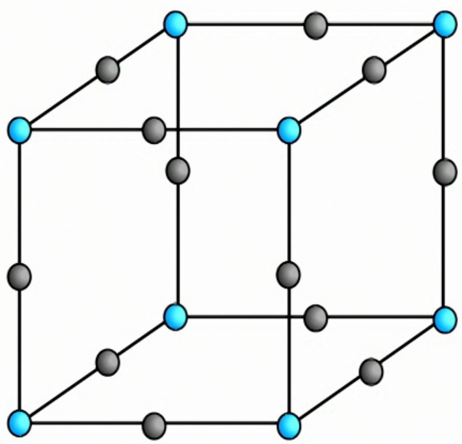
$$|\mathrm{gCS}_{(d,n)}\rangle := \mathcal{U}_{CZ}|+\rangle^{\breve{\Delta}_n}|+\rangle^{\breve{\Delta}_{n-1}}$$

$$\mathcal{U}_{CZ} = \prod_{\breve{\sigma}_n \in \breve{\Delta}_n} \left(\prod_{\breve{\sigma}_{n-1} \subset \partial \breve{\sigma}_n} CZ_{\breve{\sigma}_{n-1},\breve{\sigma}_n} \right).$$

$(d,n) = (3,1)$

0-cell $\breve{\sigma}_0$

1-cell $\breve{\sigma}_1$

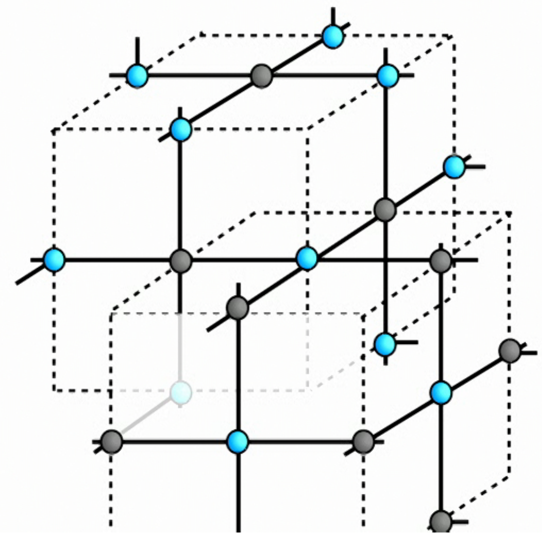


$(d,n) = (3,2)$

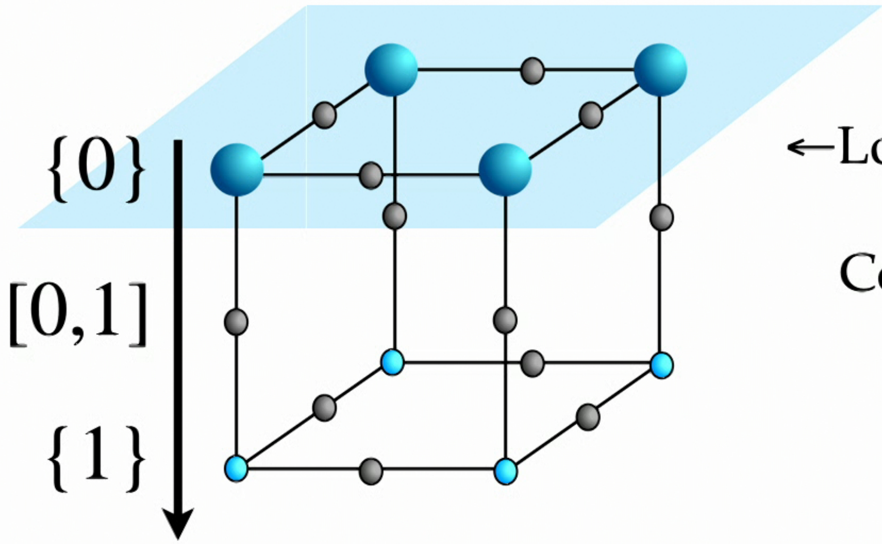
[Raussendorf Bravyi Harrington (2007)]

1-cell $\breve{\sigma}_1$

2-cell $\breve{\sigma}_2$



MBQS: simulating $M_{(3,1)}$ on gCS$_{(3,1)}$

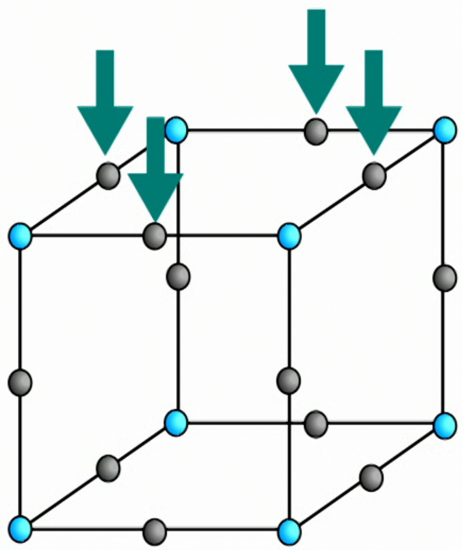


x_3-direction
="time" in the simulated world

←Load a 2d initial state $|\psi(0)\rangle_{\text{bdry}}$ at $x_3 = 0$.

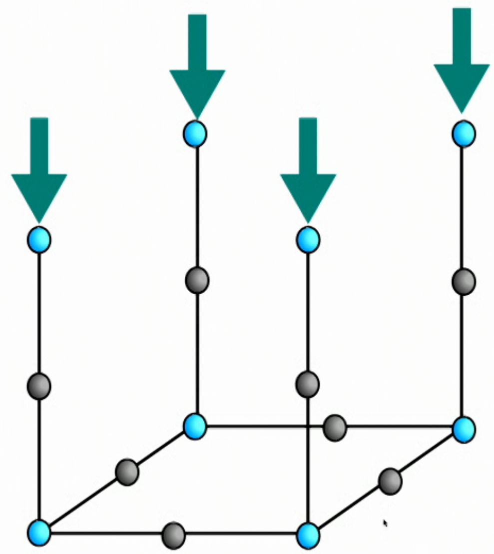
Couple it to the rest of the resource state.

MBQS: simulating $M_{(3,1)}$ on $\mathrm{gCS}_{(3,1)}$



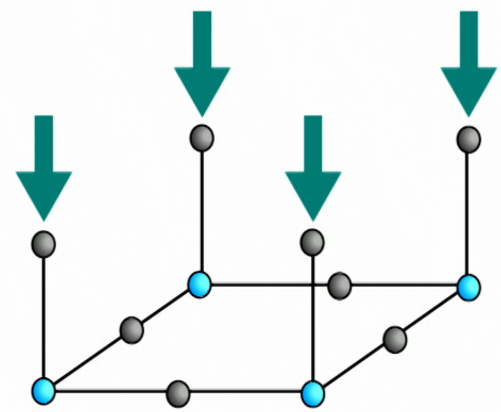
$\breve{\sigma}_1 = \sigma_1 \times \{j\}$

$$\prod_{\sigma_1} e^{-i\xi_1 Z(\partial\sigma_1)}$$



$\breve{\sigma}_0 = \sigma_0 \times \{j\}$

teleported to $[j, j+1]$

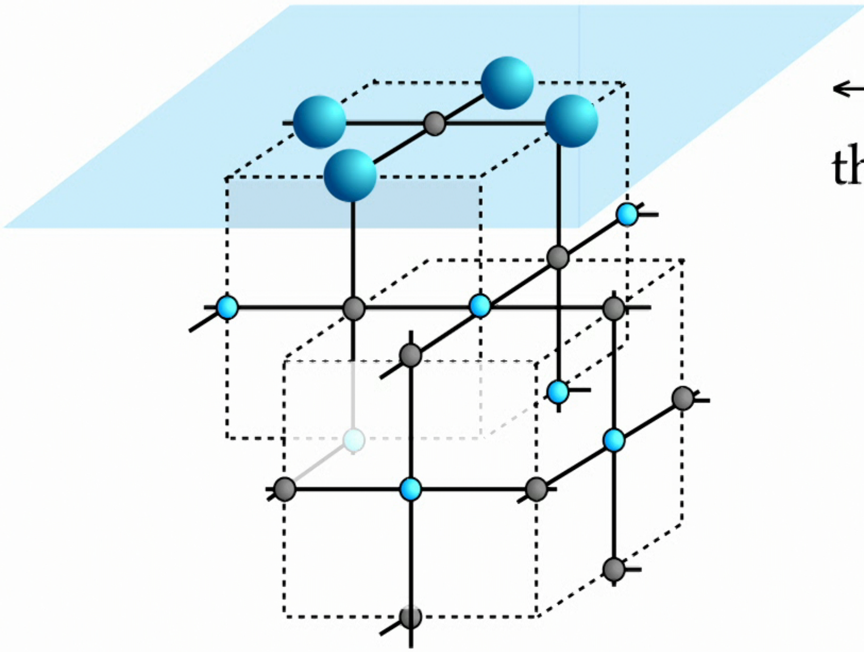


$\breve{\sigma}_1 = \sigma_0 \times [j, j+1]$

$$\prod_{\sigma_0} e^{-i\xi_3 X(\sigma_0)}$$

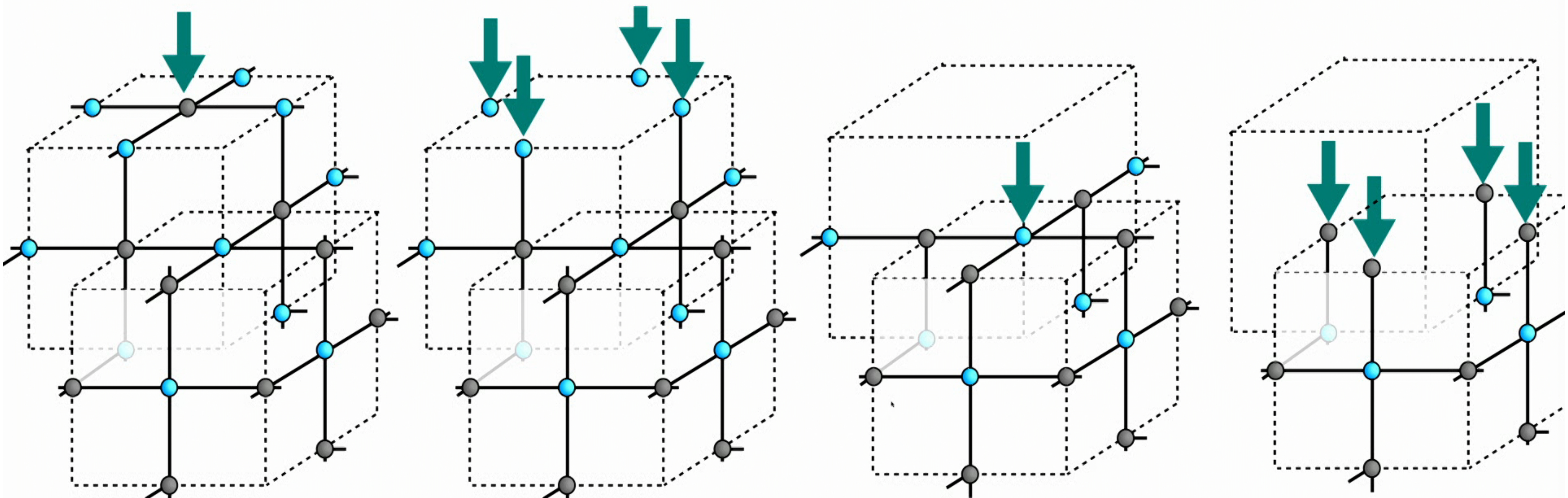
teleported to $\{j+1\}$

MBQS: simulating $M_{(3,2)}$ on $\text{gCS}_{(3,2)}$



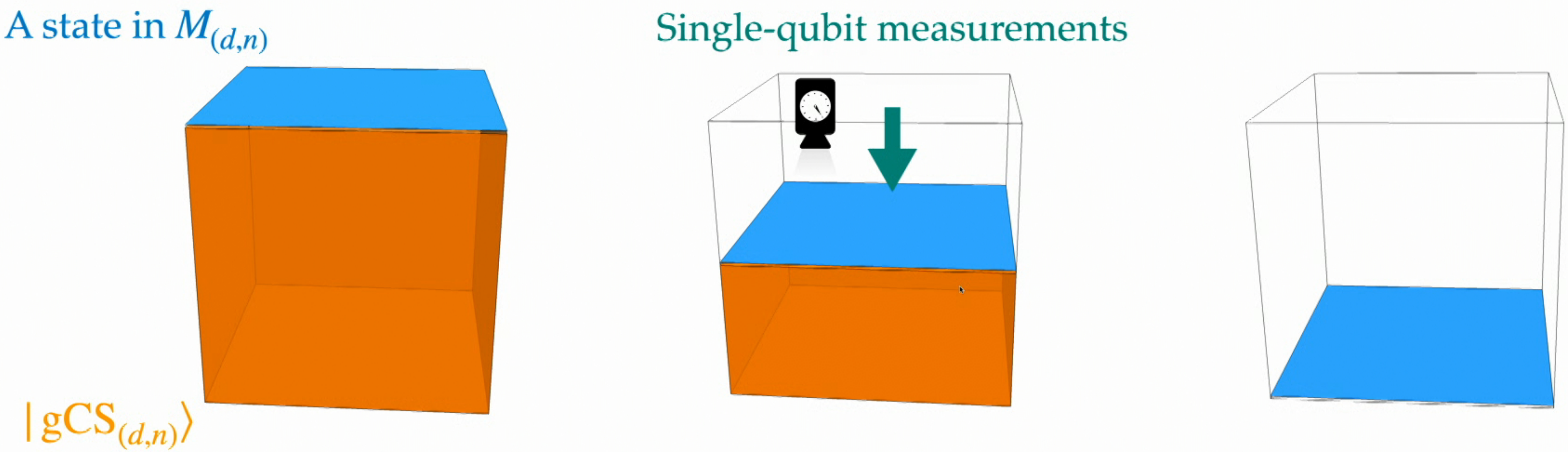
←Load a 2d initial state $|\psi(0)\rangle_{\text{bdry}}$ of the gauge theory

MBQS: simulating $M_{(3,2)}$ on $\mathrm{gCS}_{(3,2)}$



$\breve{\sigma}_2 = \sigma_2 \times \{j\}$	$\breve{\sigma}_1 = \sigma_1 \times \{j\}$	$\breve{\sigma}_1 = \sigma_0 \times [j, j+1]$	$\breve{\sigma}_2 = \sigma_1 \times [j, j+1]$
$\prod_{\sigma_2} e^{-i\xi_1 Z(\partial \sigma_2)}$	teleported to $[j, j+1]$	Gauss law check. (details omitted)	$\prod_{\sigma_1} e^{-i\xi_4 X(\sigma_1)}$ teleported to $\{j+1\}$

MBQS: simulating $M_{(d,n)}$ on $\mathrm{gCS}_{(d,n)}$

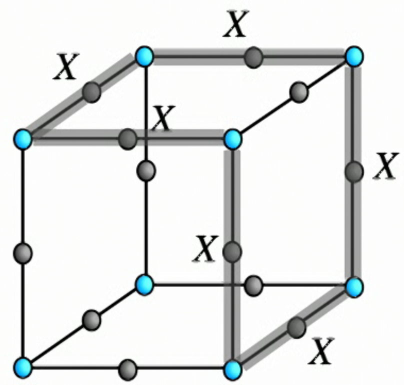


Aspects of symmetries in MBQS
SPT and holographic interplay

Higher-form symmetries in gCS

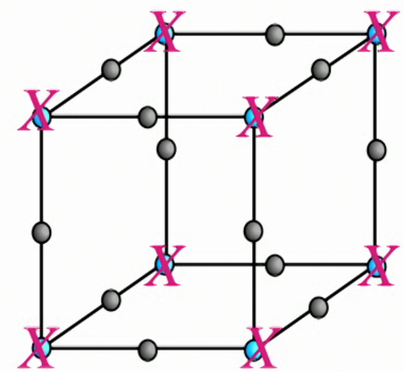
$(d, n) = (3,1)$

$(d - n) = 2$-form symmetry

$X(\breve{z}_1)$ 

$\partial \breve{z}_1 = 0$

$(n - 1) = 0$-form symmetry

$X(\breve{z}_3^*)$ 

$\partial^* \breve{z}_3^* = 0$

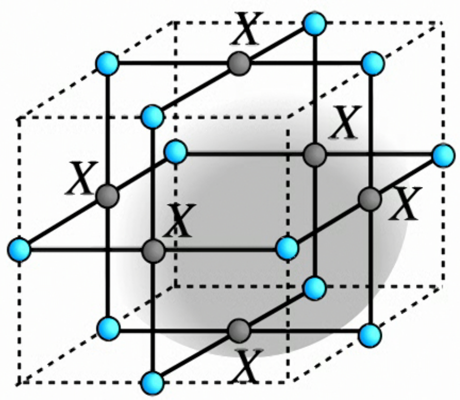
Higher-form symmetries in gCS

$(d, n) = (3,2)$

$(d - n) = 1$-form symmetry

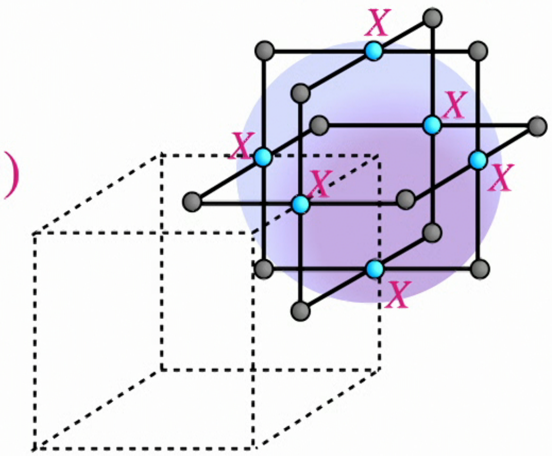
$(n - 1) = 1$-form symmetry

$X(\breve{z}_1)$



$\partial \breve{z}_1 = 0$

$X(\breve{z}_2^*)$



$\partial^* \breve{z}_2^* = 0$

Higher-form symmetries in gCS

$(d-n)$-form and $(n-1)$-form symmetry:

$$|\,\mathrm{gCS}\rangle = X(\breve{z}_n)\,|\,\mathrm{gCS}\rangle = X(\breve{z}^*_{d-n+1})\,|\,\mathrm{gCS}\rangle$$

with $M_{d-n} = \{\breve{z}_n \,|\, \partial \breve{z}_n = 0\}$, $M'_{n-1} = \{\breve{z}^*_{d-n+1} \,|\, \partial^* \breve{z}^*_{d-n+1} = 0\}$.

SPT order in gCS

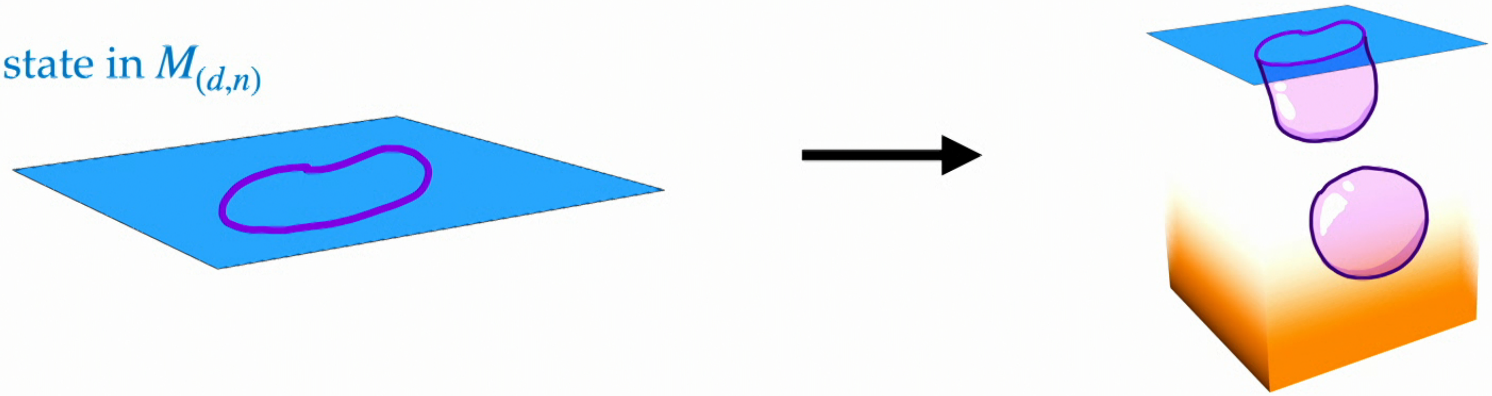
Result

$\mathrm{gCS}_{(d,n)}$ has an SPT order protected by $(d-n)$-form and $(n-1)$-form $\mathbb{Z}_2$

- Two symmetry generators act projectively at the boundaries of the lattice → SPT. See Yoshida (2016) and Roberts-Kubica-Yoshida-Bartlett (2017).
- The simulated state as an edge state of an SPT. See Miyake (2010) for 1d examples.

Bulk/boundary symmetries in MBQS

A state in $M_{(d,n)}$



Boundary symmetry generator $X(z^*_{d-n})$	Bulk symmetry generator $X(\breve{z}^*_{d-n+1})$ with $\partial^* \breve{z}^*_{d-n+1} = 0$ or $= z^*_{d-n}$.

(3,1) Ising 0-form symmetry $X(z_2^*) = \prod_{v \in V} X_v$ $\longrightarrow$ 0-form symmetry $X(\breve{z}_3^*) = \prod_{\breve{v} \in \breve{V}} X_{\breve{v}}$

(3,2) gauge Electric 1-form symmetry $X(z_1^*)$ $\longrightarrow$ 1-form symmetry $X(\breve{z}_2^*)$

Bulk/boundary symmetries in MBQS

Consider a d-dimensional Hamiltonian

$$H = -\sum Z(\partial\breve{\sigma}_n)\ ,$$

which is symmetric under the transformation with the **global** $(n-1)$-form, $X(\breve{z}^*_{d-n+1})$.

Cluster state gCS:

It is described by the local stabilizer conditions:

$$X(\breve{\sigma}_n)Z(\partial\breve{\sigma}_n)\,|\,\mathrm{gCS}_{(d,n)}\rangle = X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1})\,|\,\mathrm{gCS}_{(d,n)}\rangle = |\,\mathrm{gCS}_{(d,n)}\rangle\ .$$

It can be seen as the ground state of the **gauged version** of the above Hamiltonian,

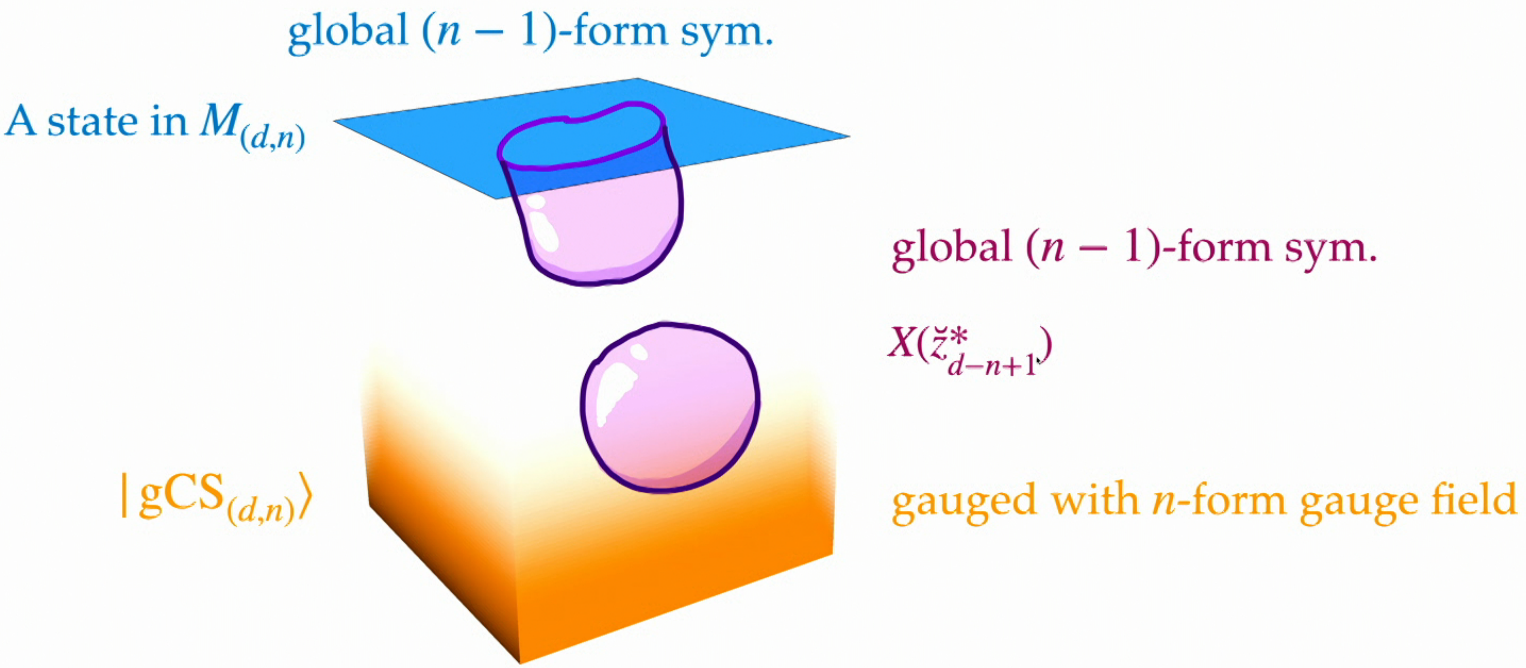
$$H_{\text{gauged}} = -\sum X(\breve{\sigma}_n)Z(\partial\breve{\sigma}_n)\ ,$$

with the local gauge constraint $X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1}) = 1$ $(\forall\breve{\sigma}_{n-1})$.

(The global symmetry $X(\breve{z}^*_{d-n+1})$ is a product of local stabilizers $X(\breve{\sigma}_{n-1})Z(\partial^*\breve{\sigma}_{n-1})$.)

Bulk/boundary symmetries in MBQS

In other words, the boundary global symmetry is promoted to the bulk(+boundary) global symmetry $X(\breve{z}^*_{d-n+1})|\psi_C\rangle = |\psi_C\rangle$, and it is gauged in the cluster state.

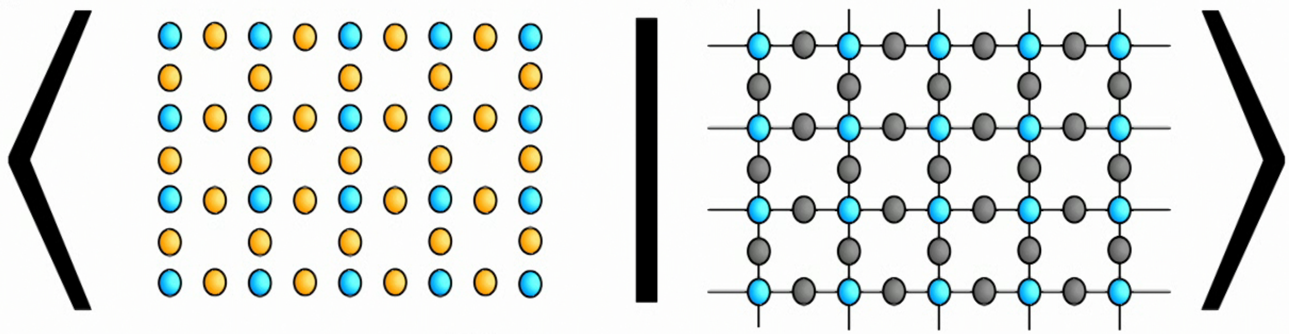


"Holographic interplay"

Comments

Our MBQS measurement pattern is related to the *overlap formula* below:

$$Z_{(2,1)} = \mathcal{N} \times$$



2d ***classical*** Ising partition function

$\langle 0 | e^{-KX}$ (orange)

$\langle + |$ (blue)

(K : real !)

$\mathrm{gCS}_{(2,1)}$

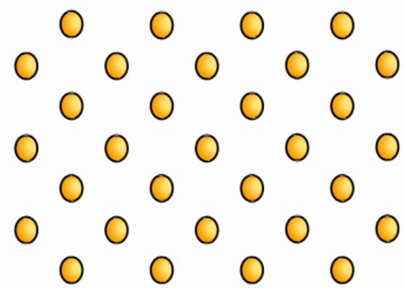
Resource state for (1+1)d transverse-field Ising model

It is a classical-quantum correspondence (VDB correspondence) [Van den Nest-Dur-Briegel (2008)] relating a 2d quantum state and a 2d classical statistical model. See also [Lee-Ji-Bi-Fisher (2022)].

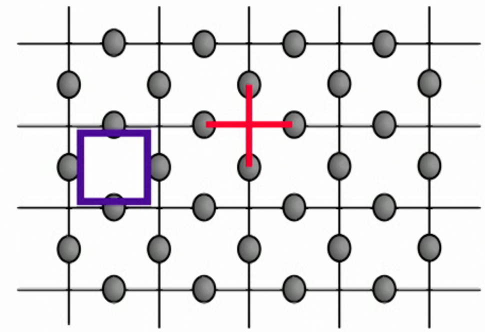
Comments

Rewriting it further,

$$Z_{(2,1)} = \mathcal{N} \times \Big\langle$$



$$\Big|$$



$$\Big\rangle$$

2d ***classical*** Ising partition function

$\langle 0 | e^{-KX}$

(K : real !)

Toric code

= partially "measuring" out $\mathrm{gCS}_{(2,1)}$

A map from TQFT_{d+1} state to a d-dim classical spin system. We speculate that physical interpretations is possible through

- Strange correlator [M. Bal et al. (2018) etc.]
- $\mathrm{TQFT}_{d+1}/\mathrm{CFT}_d$ correspondence [Chen-Zhang-Ji-Shen-Wang-Zeng-Hung (2022) etc.].

Summary

- Entanglement structure of $gCS_{(d,n)} \rightleftarrows$ Spacetime structure of $M_{(d,n)}$.
- Single-qubit measurement on $gCS_{(d,n)} \rightleftarrows$ Hamiltonian quantum simulation of $M_{(d,n)}$.
- Overlap between a product state and $gCS_{(d,n)} \rightleftarrows$ Partition function of $M_{(d,n)}$.

  (Strange correlator, TQFT/CFT correspondence?)

- The gCS possesses $(n-1)$- and $(d-n)$-form global symmetries.
  1. A state of $M_{(d,n)}$ as an edge state of an SPT
  2. Boundary $(n-1)$-from symmetry is promoted to bulk $(n-1)$-from symmetry, which is gauged in gCS.