Title: Talk 10 - Constraints on physical computers in holographic spacetimes

Speakers: Alex May

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Abstract: Within the setting of the AdS/CFT correspondence, we ask about the power of computers in the presence of gravity. We show that there are computations on n qubits which cannot be implemented inside of black holes with entropy less than $O(2^n)$. To establish our claim, we argue computations happening inside the black hole must be implementable in a programmable quantum processor, so long as the inputs and description of the unitary to be run are not too large. We then prove a bound on quantum processors which shows many unitaries cannot be implemented inside the black hole, and further show some of these have short descriptions and act on small systems. These unitaries with short descriptions must be computationally forbidden from happening inside the black hole.

Constraints on physical computers in holographic spacetimes

Alex May

Perimeter Institute

Joint with David Pérez-Garcia and Aleksander Kubicki

Models of computation

- In complexity theory, we
  1) Define a model of computation
  2) Study resources needed to solve problems in that model

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Models of computation

- In complexity theory, we

  1) Define a model of computation

  2) Study resources needed to solve problems in that model

- Models of computation are often chosen to try and capture the power of physical computers.

Turing machine ~ classical physics

Quantum circuits ~ quantum physics

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Physical computers

- Rather than start by choosing a model, we want

laws of physics → constraints on computation

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What are the physical limits of computation?

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Strategy

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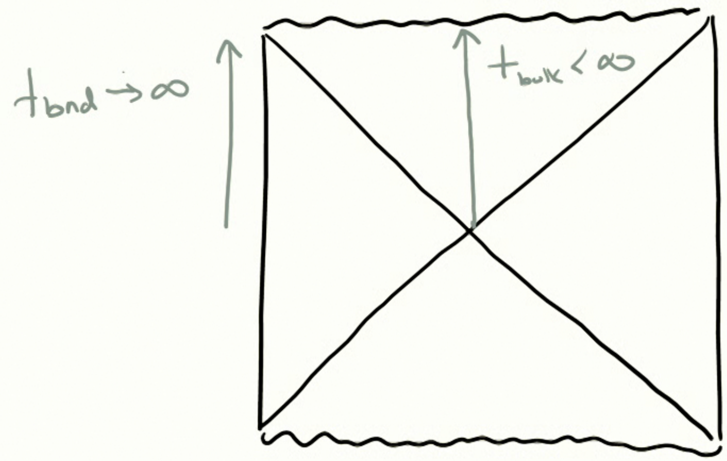
Strategy

- Lloyd (2000) studied limits of computers in QG directly, from a "bulk" perspective
  - ↳ "Lloyds bound" assumes quantum circuit model, not clear this is justified in the bulk
- Our strategy will be to go to AdS/CFT, and look for constraints on bulk computation using boundary

(speculative) second motivation

- Consider a spacetime with a (spacelike) singularity
- How does the CFT, where time doesn't end, record the bulk singularity?



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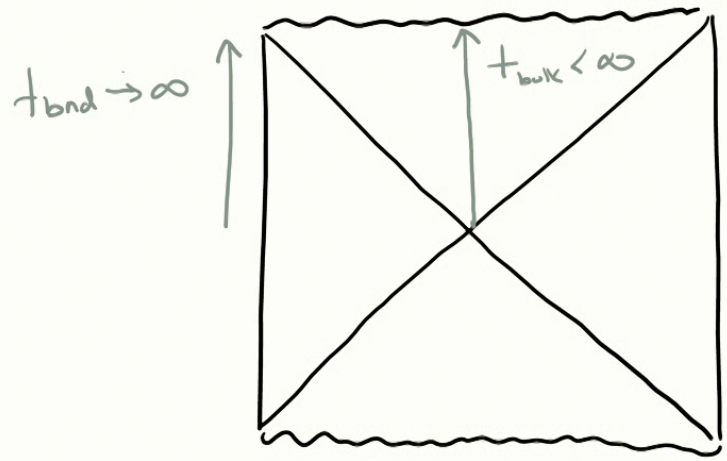

Idea: CFT constrains complexity of computation happening inside the black hole.

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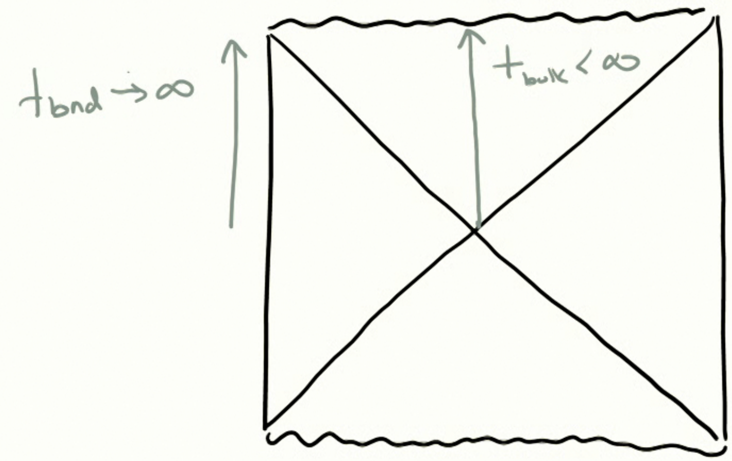

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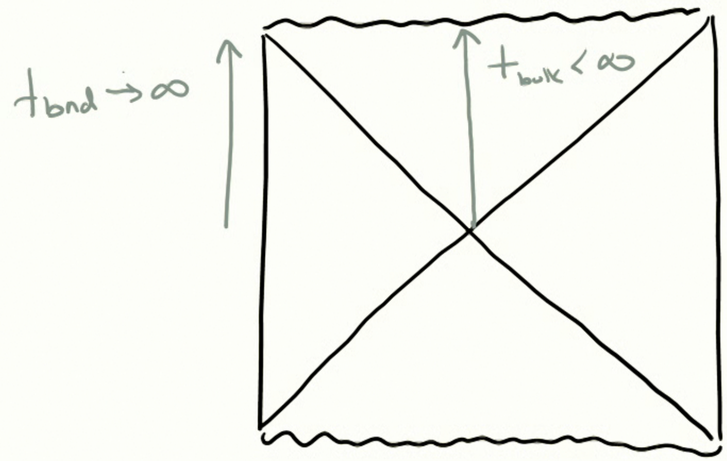

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Quantum processors

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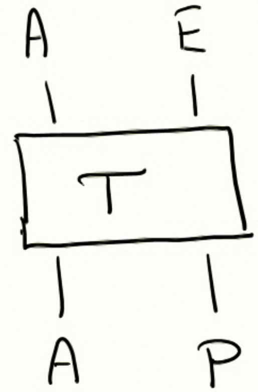

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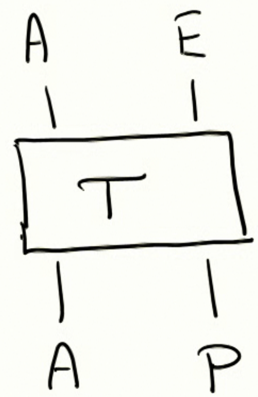

$$T\left(|\psi\rangle_A |\varphi_U\rangle_P\right) = \left(U|\psi\rangle_A\right) |\varphi'_U\rangle_E$$

- When we fix d_A, d_P we refer to T as a quantum processor.

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Bounds on quantum processors

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- We can prove that

$$\mathbb{E}_{\varepsilon}\, p(T, \varepsilon) \leq \frac{n_P}{2^{n_A}}$$

$p(T, \varepsilon) =$ "overlap of correct and actual output, optimized over all program states $|\varphi_U\rangle$"

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Set-up : computation inside a black hole

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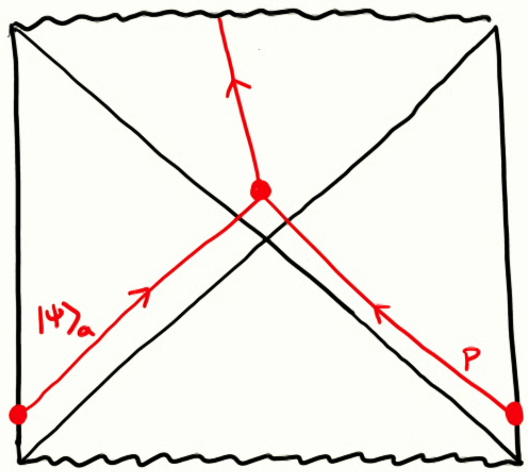
Set-up

- Try to implement $|\psi\rangle_a \to U^\dagger|\psi\rangle_a$ inside of a two-sided black hole.



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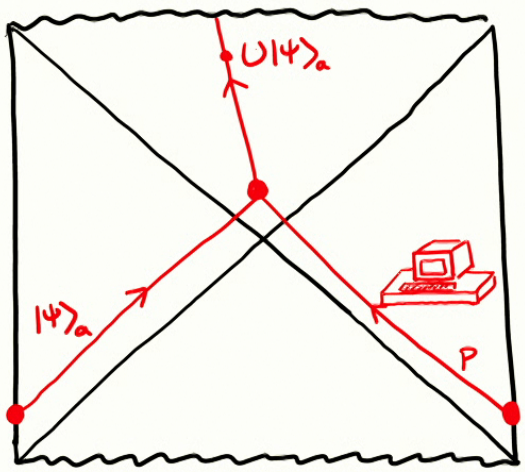

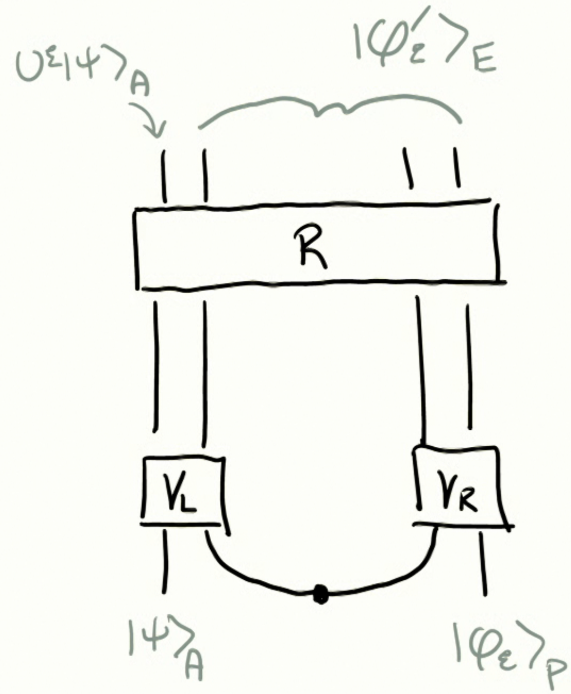

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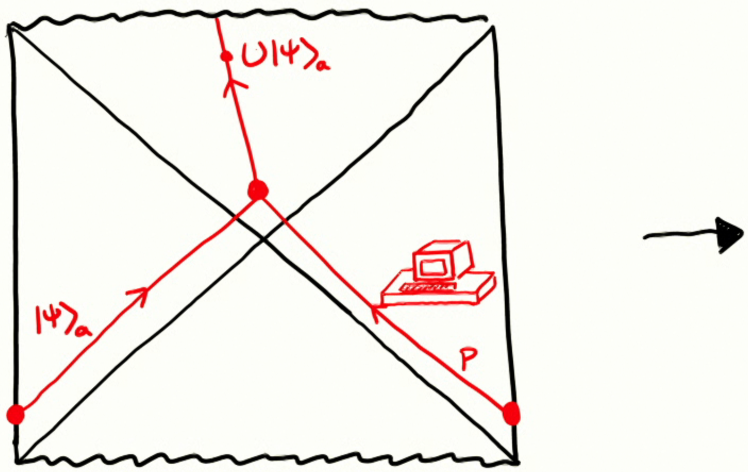

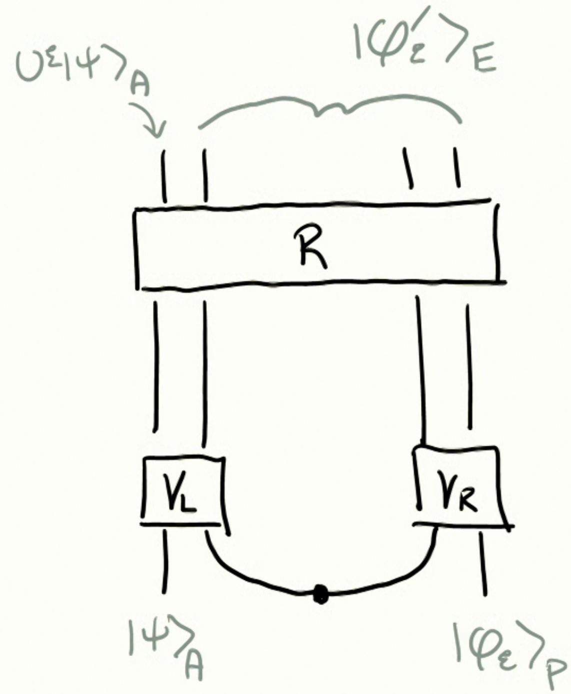

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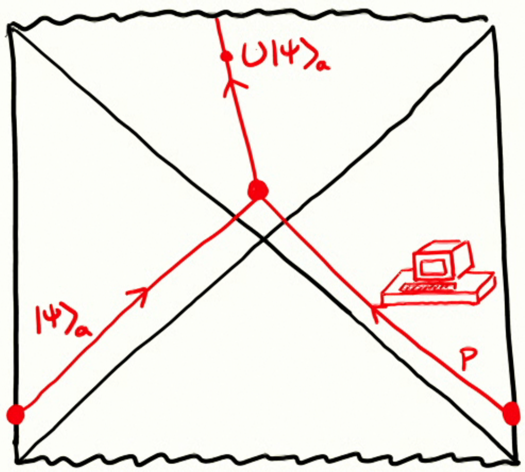

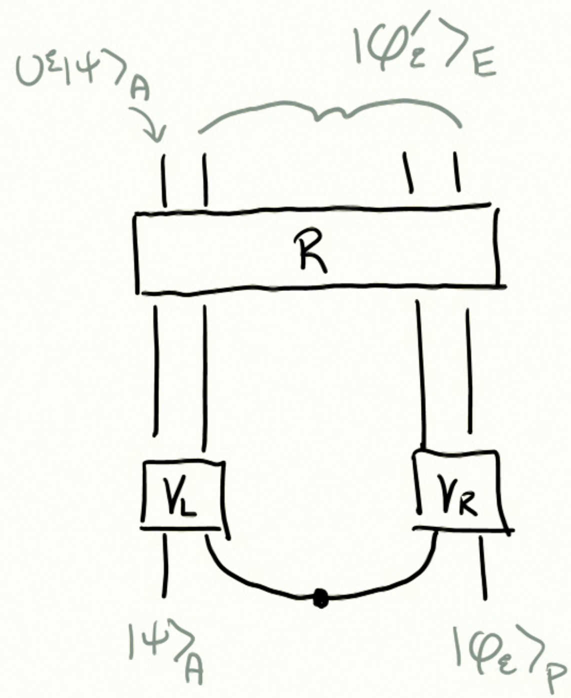

- A single fixed R works when

$$n_A, n_P \ll S_{bh}$$

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Processor bounds on black holes

- Consider the regime

$$n_A, n_p \ll S_{bh} \qquad \log(S_{bh}) \leq n_A$$

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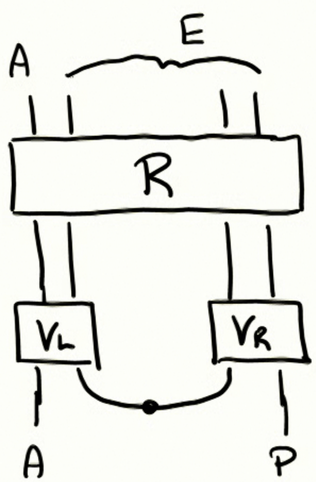

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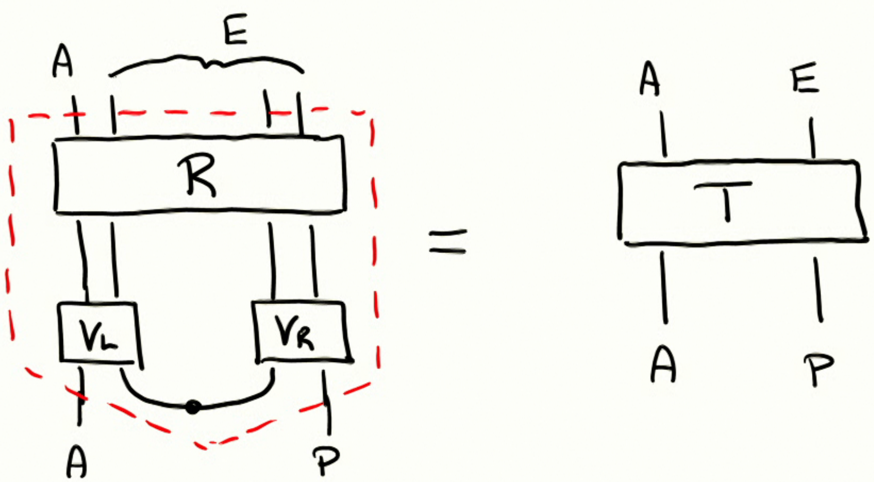

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Computationally forbidden unitaries

- So far this isn't very interesting:
  - Have that a typical unitary from $\{U_A^{\varepsilon}\}$ is "forbidden"
  - A typical $U_A^{\varepsilon} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \ldots \varepsilon_{2^{n_A}})$ requires $2^{n_A} > S_{bh}$ bits to describe, so forbidden information theoretically

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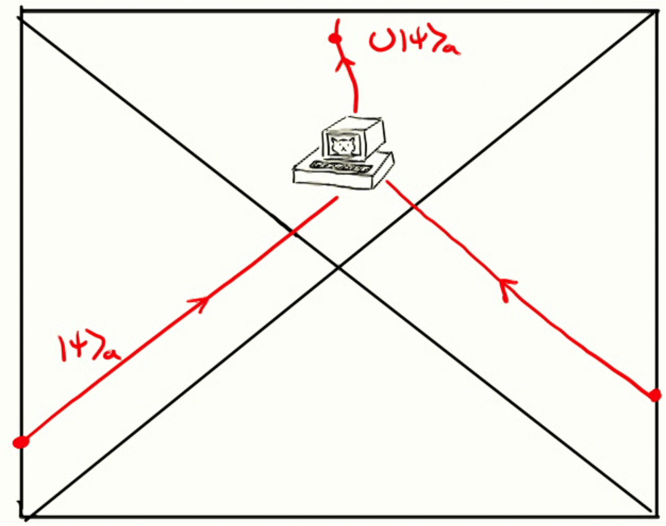
↳ Only requires $\log S_{bh}$ bits to specify!

Recap

- Throw (short) description of U_{ε_0} into black hole

- Universal computer can, given enough resources, apply U_{ε_0} to a subsystem

- Our bulk computer cannot apply U_{ε_0}, no matter how it is built or operates



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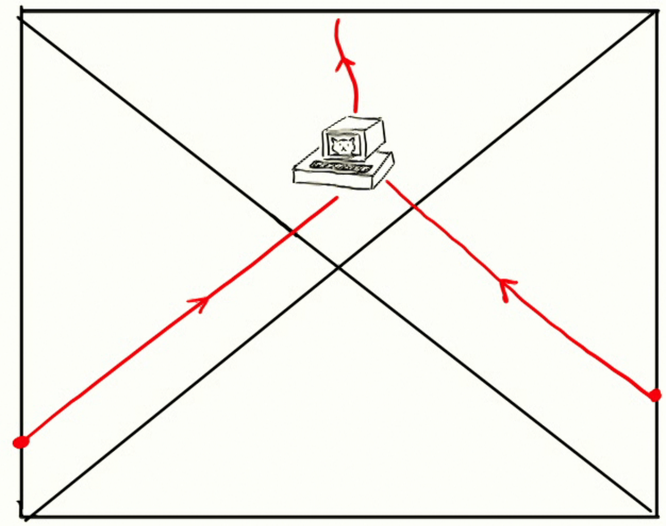

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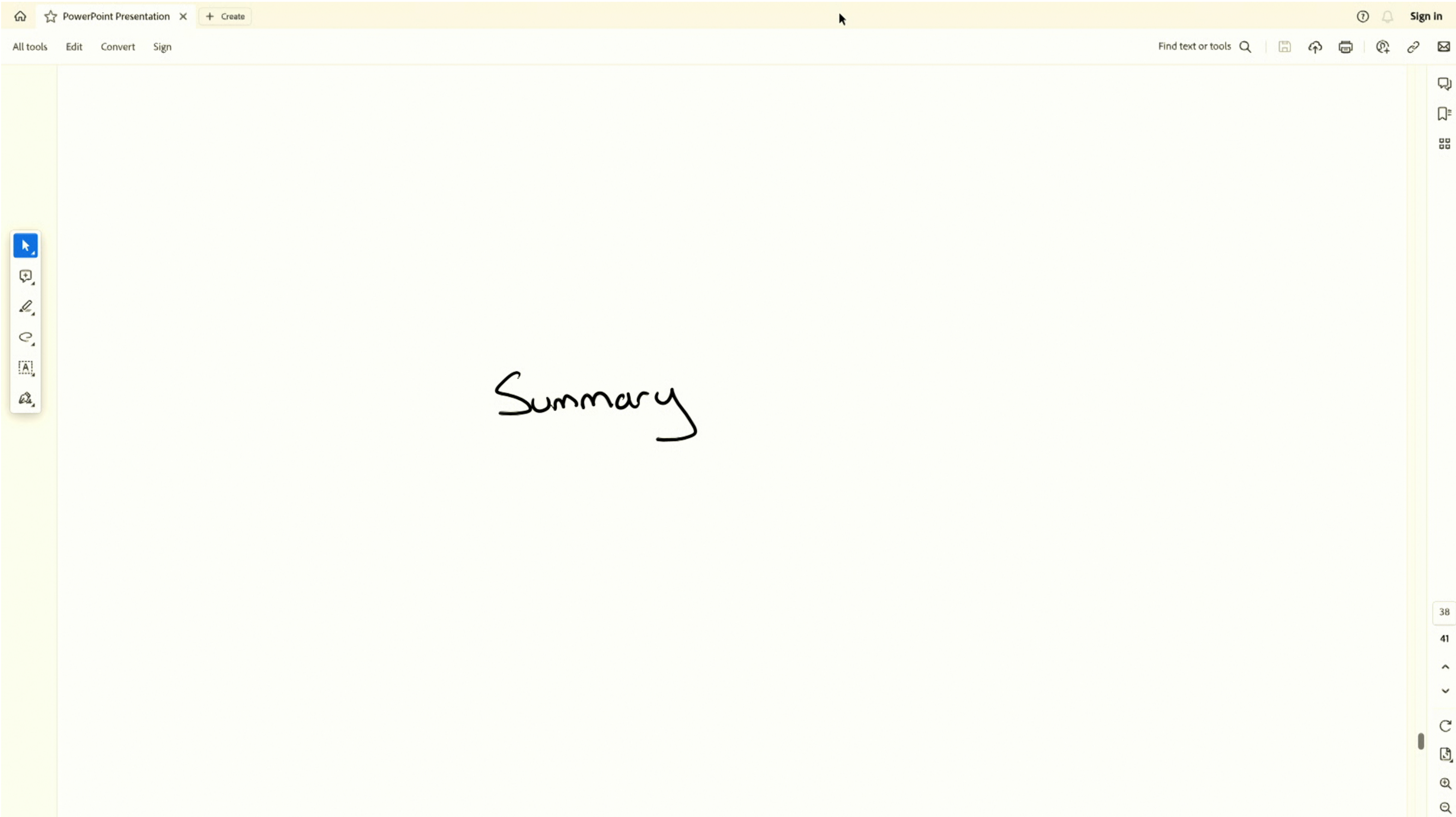
  → If we apply processor bound directly in bulk get constraint on specific computer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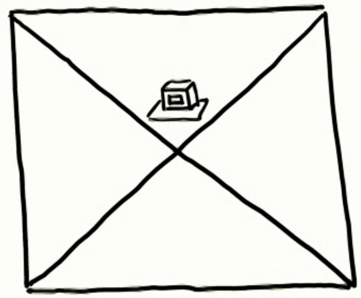
Summary of our construction

Inside view:

A universal computer, can run any program



Outside view:

A programmable processor, limited by size of its program space.

- Computational restrictions on bulk computer are necessary to resolve this tension.

↳ How do we understand these constraints in the bulk? Are they related to appearance of singularity?