Title: Talk 16 - Toward random tensor networks and holographic codes in CFT

Speakers: Jeevan Chandra Namburi

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Abstract: In holographic CFTs satisfying eigenstate thermalization, there is a regime where the operator product expansion can be approximated by a random tensor network. The geometry of the tensor network corresponds to a spatial slice in the holographic dual, with the tensors discretizing the radial direction. In spherically symmetric states in any dimension and more general states in 2d CFT, this leads to a holographic error-correcting code, defined in terms of OPE data, that can be systematically corrected beyond the random tensor approximation. The code is shown to be isometric for light operators outside the horizon, and non-isometric inside, as expected from general arguments about bulk reconstruction. The transition at the horizon occurs due to a subtle breakdown of the Virasoro identity block approximation in states with a complex interior.

Toward random tensor networks and holographic codes in CFT

Jeevan Chandra

Cornell Physics

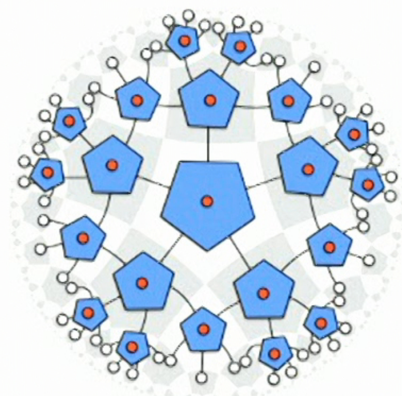
based on arXiv 2302.02446 with Tom Hartman,
arXiv 2305.07183

August 1, 2023

It from Qubit, Perimeter Institute

Motivation: Holographic tensor networks

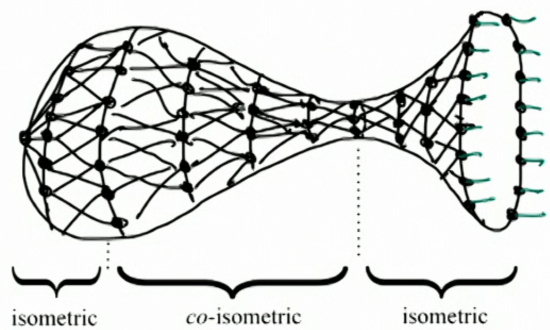
1. Tensor networks provide a succinct and computationally useful graphical description of many-body quantum states.
2. Tensor networks define error-correcting codes which serve as toy models of holography in a setup where the bulk geometry is discretized. Eg: MERA, the HaPPY code (below) and the Random Tensor Network model



3. These models display a striking interplay between quantum information theory and quantum gravity and have played an important role in understanding the black hole information paradox.

Motivation: Non-isometric codes

1. The black hole interior is relatively less understood from the point of view of bulk reconstruction.
2. Promising direction: Reconstructing the black hole interior using the framework of "non-isometric codes". [Akers et al 22'; Verlinde, Verlinde 12'].
3. Spatial slice of black hole (one-sided python) as a holographic code:



4. Eg: Evaporating black hole at late times has a lot of EFT degrees of freedom in the interior. $\dim(\mathcal{H}_{\mathsf{EFT}}) \gg e^{S_{BH}} \implies$ holographic code is necessarily non-isometric.

Motivation: CFT tensor networks

How are non-isometric codes realized in quantum gravity?

We address this question in our work (primarily in AdS_3/CFT_2) [JC,Hartman 23']

1. We constructed tensor network models for black holes directly using the OPE data of the dual CFT.
2. In the regime where ETH applies, the CFT tensor networks describe random tensor network models.
3. Interestingly, the same ETH regime reproduces wormhole amplitudes from CFT. [JC,Collier,Hartman,Maloney 22']
4. Our models describe holographic codes that transition from being isometric outside the black hole to non-isometric inside.
5. This is a step toward constructing a model of the black hole interior using CFTs.

Outline

In the talk, I shall

1. Provide a prescription for turning pure states in 2d CFT into random tensor networks.
2. Describe 3d black hole geometries discretized by these networks.
3. Add probe matter to the black hole background. Show that the holographic code described by the probe OPE undergoes an isometric transition across the horizon.
4. Show that the tensor networks exhibit a toy version of the island rule formula.
5. Describe Euclidean wormholes by coarse graining the tensor networks.

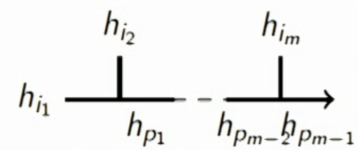
Tensor networks from 2d CFT

1. Consider a CFT state created by Virasoro primary operators

$$|\Psi\rangle = \mathcal{O}_{i_m}(-\tau_m, \phi_m) \dots \mathcal{O}_{i_2}(-\tau_2, \phi_2)\mathcal{O}_{i_1}(-\tau_1, \phi_1)|0\rangle$$

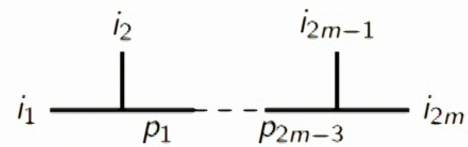
2. Expand the OPE in some channel,

$$|\Psi\rangle = \sum_{p_1,\dots,p_{m-1}} c_{i_1 i_2 p_1} \dots c_{i_m p_{m-2} p_{m-1}} \left| \mathcal{B} \left[\quad \right] \right|^2 |p_{m-1}\rangle$$



3. The norm is $2m$-point correlator expanded in conformal blocks,

$$\langle\Psi|\Psi\rangle = \sum_{p_1,\dots,p_{2m-3}} c_{i_1 i_2 p_1} \cdots c_{i_{2m} i_{2m-1} p_{2m-3}} \left| \quad \right|^2$$



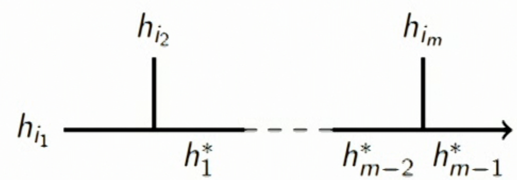
Tensor networks from 2d CFT

1. When the norm is dominated by a saddlepoint at large-c, we truncate the sum to a microcanonical window around the saddlepoint weights,

$$|\Psi\rangle_* = |\mathcal{B}|_*^2 \sum_{p_k \in \mathcal{H}_k} c_{i_1 i_2 p_1} \dots c_{i_m p_{m-2} p_{m-1}} |p_{m-1}\rangle$$

2. OPE block only depends on weights of primaries, so can be pulled out of the sum.

$$|\mathcal{B}|_*^2 = \left| \mathcal{B} \left[\quad \cdots \quad \right] \right|^2 .$$

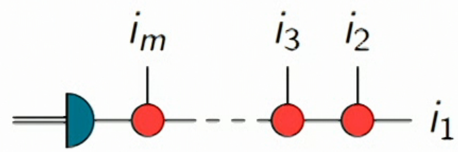


3. $\mathcal{H}_k$: Auxiliary space of Virasoro primaries around saddlepoint weight $(h_k^*, \overline{h}_k^*)$,

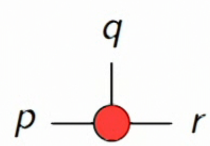
$$|\mathcal{H}_k| = e^{S_0(h_k^*, \overline{h}_k^*)} , \quad S_0(h, \overline{h}) = 2\pi \sqrt{\frac{c}{6}(h - \frac{c}{24})} + 2\pi \sqrt{\frac{c}{6}(\overline{h} - \frac{c}{24})}$$

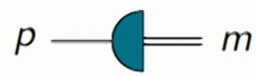
Tensor networks from 2d CFT

1. Semiclassical CFT state as a 1d MPS tensor network,

$|\Psi\rangle_* =$ 

2. Building blocks:

$c_{pqr} =$ 

$\langle m||\mathcal{B}|^2_*|p\rangle =$ 

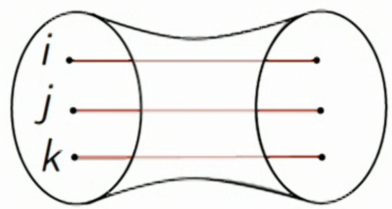
3. OPE tensor legs label finite dimensional $\mathcal{H}_{\text{prim}}$ (single line) and $|\mathcal{B}|^2_* : \mathcal{H}_{\text{prim}} \to \mathcal{H}_{\text{CFT}}$ (double line).
4. **OPE tensor contractions define a "primary state" which is dressed by Virasoro descendents (boundary gravitons).**

Random tensor approximation

1. 2d CFTs obey a version of ETH which incorporates Virasoro symmetry (Virasoro ETH), [Collier et al 19'].

$$\overline{c_{ijk}c^*_{lmn}} = C_0(h_i, h_j, h_k)C_0(\overline{h}_i, \overline{h}_j, \overline{h}_k)\,(\delta_{il}\delta_{jm}\delta_{kn} \pm \text{permutations}) + \ldots$$

2. ETH statistics match with the partition functions of Euclidean wormholes in 3d gravity [JC, Collier, Hartman, Maloney 22']. For example,

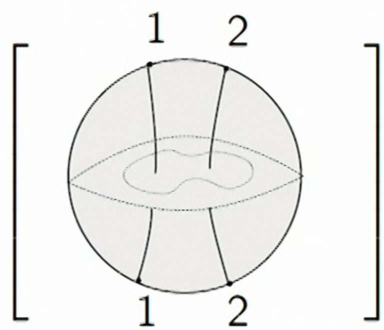
$Z_{\text{grav}}\Big[$ 

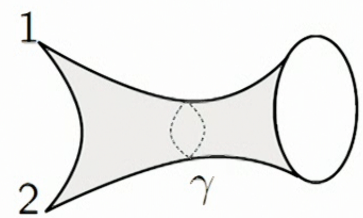
$\Big] = |C_0(h_i, h_j, h_k)|^2$

3. **Tensor network representation of CFT states + Virasoro ETH $\Longrightarrow$ A random tensor network model using 2d CFTs.**

Simplest pure state black hole: Geometry

1. Consider a 3d black hole formed by backreaction of heavy point particles dual to CFT state: $|\Psi\rangle = \mathcal{O}_2(-\tau_2, \phi_2)\mathcal{O}_1(-\tau_1, \phi_1)|0\rangle$

$\langle \Psi | \Psi \rangle \approx Z_{\text{grav}}$ 

Spatial slice : 

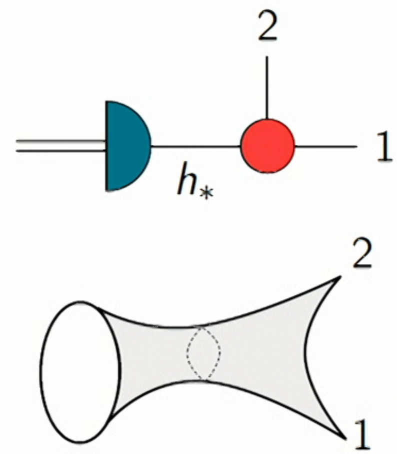
2. Area of the horizon matches with the Cardy entropy at the saddlepoint weight $(h_*, \bar{h}_*)$ of $\langle \Psi | \Psi \rangle$,

$$\frac{\text{Area}(\gamma)}{4G_N} = S_0(h_*, \bar{h}_*)$$

Simplest pure state black hole: Tensor network

1. Tensor network representation of $|\Psi\rangle = \mathcal{O}_2\mathcal{O}_1|0\rangle$ and its relation to the spatial slice:

$|\Psi\rangle_* =$

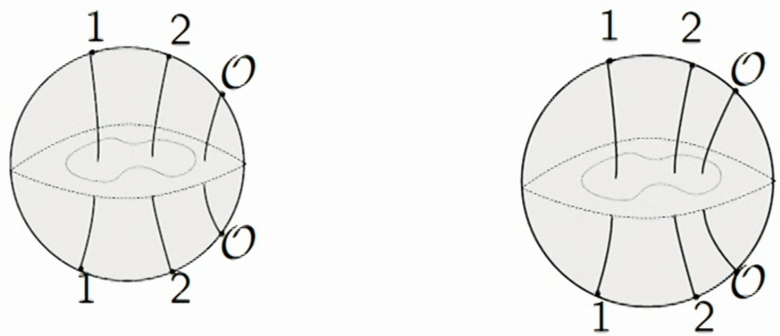


2. By construction, bond dimension $|\mathcal{H}| = e^{S_0(h_*, \bar{h}_*)}$ so that

$$\text{bond dimension} = e^{\frac{\text{Area(horizon)}}{4G_N}}$$

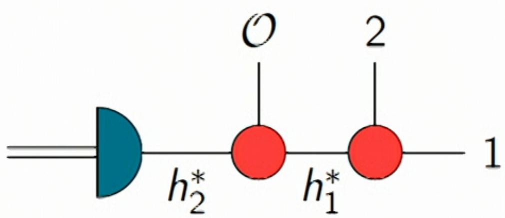
Probing the apparent horizon

1. Add a probe particle: Excited microstate is $|\tilde{\Psi}\rangle = \mathcal{O}\mathcal{O}_2\mathcal{O}_1|0\rangle$.
2. The geodesic probe could go inside/outside the horizon.



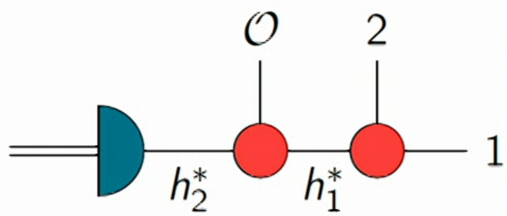
3. The corresponding tensor network is

$$|\tilde{\Psi}\rangle_* =$$



4. Bond dimensions: $|\mathcal{H}_{1,2}| = e^{S_{1,2}}$ with $S_1 = S_0(h_1^*, \overline{h}_1^*)$ and $S_2 = S_0(h_2^*, \overline{h}_2^*)$.

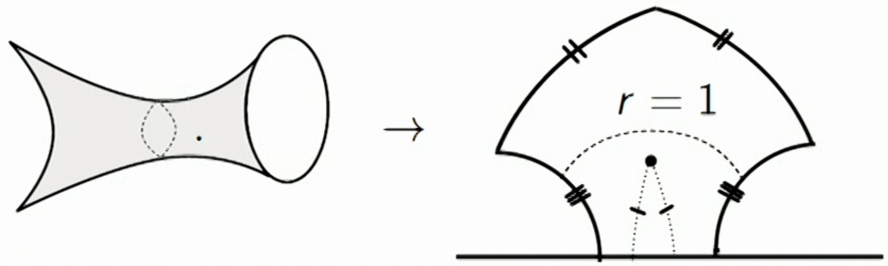
Probing the apparent horizon

$|\tilde{\Psi}\rangle_* =$ 

1. Probe OPE coefficient defines a random linear map $c_{\mathcal{O}pq} : \mathcal{H}_1 \to \mathcal{H}_2$ where $\mathcal{H}_{1,2}$ are spaces of Virasoro primaries around weights $h^*_{1,2}$.
2. Fact: Random linear maps between large dimensional vector spaces are isometric/ co-isometric depending on relative sizes. [Hayden et al 16']
3. Condition for OPE coefficient to define an isometric map:

$$|\mathcal{H}_2| > |\mathcal{H}_1| \leftrightarrow h_2^* > h_1^*$$

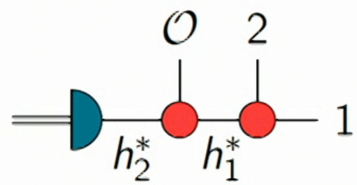
The isometric transition



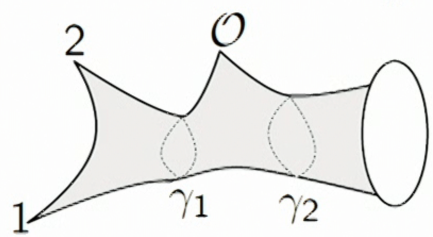
1. $\boxed{h_2^* - h_1^* = \frac{m\ell}{8\pi}\frac{1-r^2}{r\sin(\theta)}}$ m: Mass of probe; ℓ: Area of horizon. (r,θ): Location of the probe particle on the spatial slice.
2. $r = 1$: Horizon, $r > 1$: interior and $r < 1$: exterior. $(\sin(\theta) > 0)$
3. $h_2^* > h_1^*$ outside $(r < 1)$ while $h_2^* < h_1^*$ inside $(r > 1)$.
4. **The horizon is a unitary locus for geodesic probes.**

Isometric transition of heavy operators

1. $|\tilde{\Psi}\rangle = \mathcal{O}\mathcal{O}_2\mathcal{O}_1|0\rangle$. What happens to the isometric transition when $\mathcal{O}$ is heavy enough to create a new minimal surface?
2. Relation between the bond dimensions in the tensor network below?

$|\tilde{\Psi}\rangle_* =$ 

3. Spatial slice: $\frac{\text{Area}(\gamma_1)}{4G_N} = S_0(h_1^*, \overline{h}_1^*)$ and $\frac{\text{Area}(\gamma_2)}{4G_N} = S_0(h_2^*, \overline{h}_2^*)$



4. When $\text{Area}(\gamma_1) < \text{Area}(\gamma_2)$, $\mathcal{O}$ acts isometrically. **Heavy operators can act isometrically even if they are hidden behind a horizon.**

Isometric transition of heavy operators

1. Exact expression for the relation between saddle-point weights

$$2\cosh\left(\pi\sqrt{\frac{24h_1^*}{c}-1}\right)-2\cosh\left(\pi\sqrt{\frac{24h_2^*}{c}-1}\right)$$
$$=2\cosh(\frac{\ell}{2})(\cos(2\pi\eta)-1)+\sinh(\frac{\ell}{2})\sin(2\pi\eta)(r-\frac{1}{r})\csc(\theta)$$

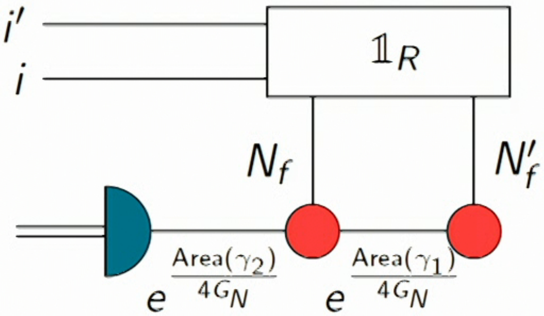
where $h_3=\frac{c}{6}\eta(1-\eta)$.

2. Importantly, the locus where the isometric transition occurs i.e where $h_1^*=h_2^*$ is inside the horizon.
3. **The unitary locus of a heavy operator is like a 'non-perturbative horizon'.**

A toy island rule formula

1. Entangle the flavoured microstate $|\Psi_{i,i'}\rangle = \mathcal{O}_i \mathcal{O}_{i'} \mathcal{O}|0\rangle$ with a "radiation" reservoir, (Assume $\mathcal{O}$ has no flavour indices)

$$|\tilde{\Psi}\rangle = \sum_{i,i'} |\Psi_{i,i'}\rangle \, |i\rangle_R \, |i'\rangle_R =$$



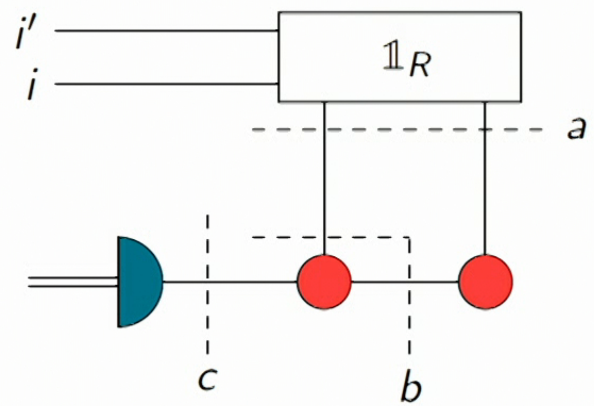
2. For random tensor networks, entropy of a subregion is calculated from the minimal number of cuts to remove the subregion from the network [Hayden et al 16'],

$$S(\rho_R) = \min_{\text{cuts}} \left(\sum_{\text{bonds} \in \text{cut}} \log(\text{bond dim}) \right)$$

A toy island rule formula

1. For the present setup,

$$S(\rho_R) = \min\left(\log(N_f N_f'), \log N_f + \frac{\text{Area}(\gamma_1)}{4G_N}, \frac{\text{Area}(\gamma_2)}{4G_N}\right)$$



2. Could be thought of as a toy model for evaporation of an 'old' black hole made 'young' again by throwing in heavy stuff.

A toy island rule formula

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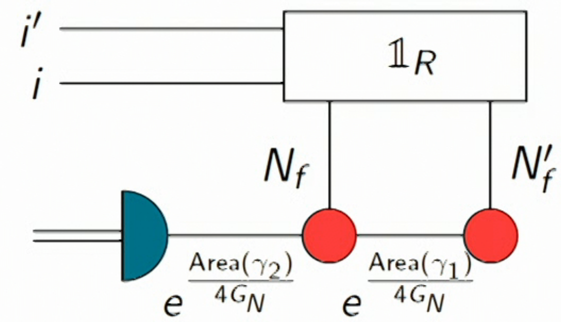
$$|\tilde{\Psi}\rangle = \sum_{i,i'} |\Psi_{i,i'}\rangle\, |i\rangle_R\, |i'\rangle_R =$$



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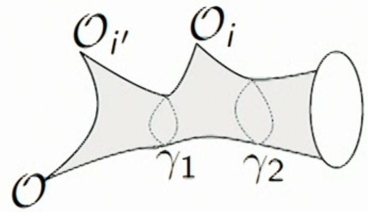
$$S(\rho_R) = \min_{\text{cuts}}\left(\sum_{\text{bonds}\in\text{cut}} \log(\text{bond dim})\right)$$

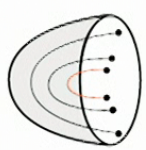
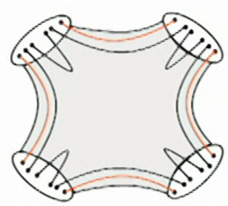
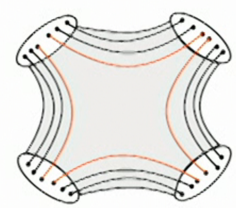
A toy island rule formula

1. Agrees with the expectation from replica wormholes constructed by branching around either of the minimal surfaces, [JC 23']



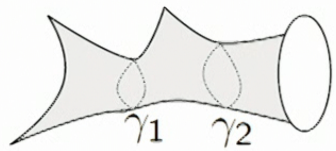
$$S(\rho_R) = \min\left(\log(N_f N_f'), \log N_f + \frac{\text{Area}(\gamma_1)}{4G_N}, \frac{\text{Area}(\gamma_2)}{4G_N}\right)$$

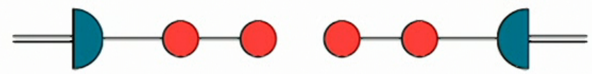
2. The three terms arise respectively from the following bulk geometries (contributing to $\text{Tr}(\rho_R^4)$),

$N_f N_f' \Big[$  $\Big]^4 + N_f (N_f')^4$  $+ (N_f N_f')^4$ 

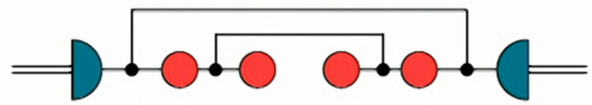
Coarse graining tensor networks and wormholes

1. These wormholes can be given a coarse grained interpretation [JC 23'; JC, Hartman 22'],

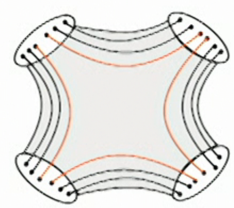


$\rho =$ 

2. Define a coarse grained state outside the horizon γ_2,

$\overline{\rho} =$ 

3. Renyis of $\overline{\rho}$ match with the partition functions of Replica wormholes obtained by branching around γ_2,

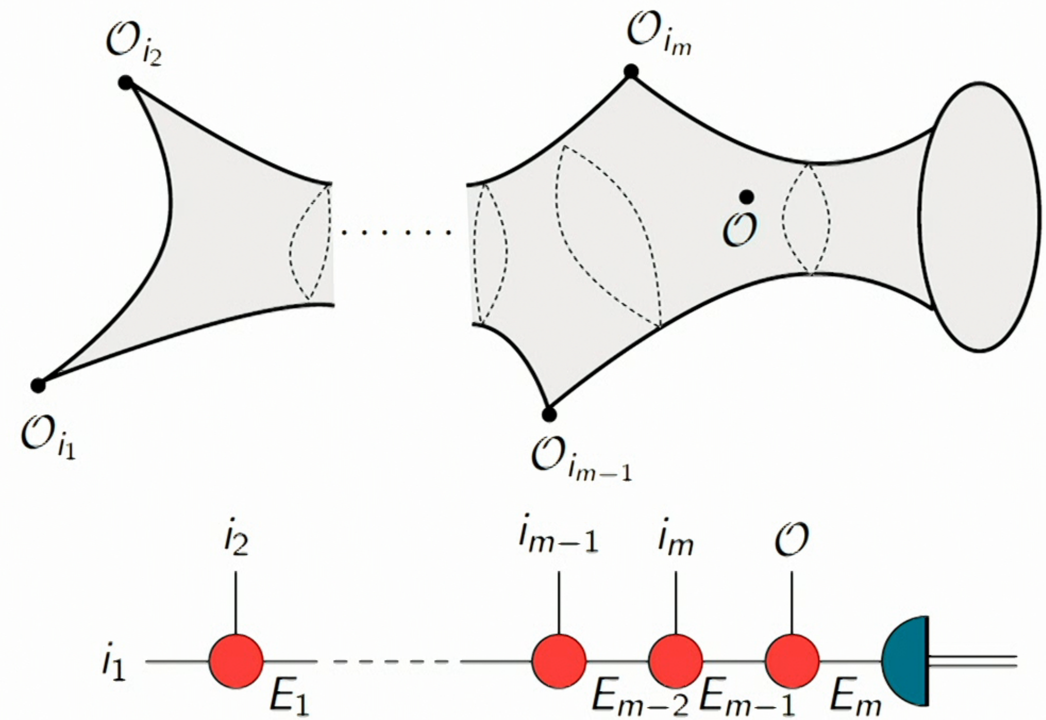
$\mathrm{Tr}(\overline{\rho}^4) = Z_{\mathrm{grav}}\Bigg[$  $\Bigg]$, $\qquad S(\overline{\rho}) = \dfrac{\mathrm{Area}(\gamma_2)}{4G_N}$

4. Relation to complexity coarse graining? [Engelhardt, Wall 18']

Summary

1. Constructed quantitative tensor networks for AdS_3/CFT_2 using OPE data defined on the same graph as a Virasoro conformal block.
2. Discretized 3d black hole geometries in the sense that the tensor network is the dual graph associated with a pair of pants decomposition of the time-symmetric spatial slice.
3. Areas of minimal surfaces in the bulk match with the bond dimensions of corresponding internal legs in the network.
4. In the random tensor approximation, probes undergo an isometric transition across a minimal surface (horizon) in the bulk.
5. Showed that the tensor networks display a toy version of the island rule formula.

Summary



$\mathcal{O}_{i_2}$
$\mathcal{O}_{i_m}$
$\mathcal{O}$
$\mathcal{O}_{i_1}$
$\mathcal{O}_{i_{m-1}}$
i_2
i_{m-1}
i_m
$\mathcal{O}$
i_1
E_1
E_{m-2}
E_{m-1}
E_m

Future directions

1. **Locality constraints on holographic codes**: Crossing symmetry requires non-Gaussian corrections to the ETH ansatz. Systematically correct the holographic codes beyond the Gaussian random approximation using crossing symmetry. Affects the deviation bounds.
2. **Finer grained tensor networks from CFTs**: Tree tensor networks (discretize transverse direction along the boundary) and loop tensor networks (discretize transverse direction along minimal surfaces) of [Bao et al 18'] from CFTs.
3. Dynamical (time-dependent) tensor networks from CFTs.