Title: Talk 71 - Topological Toy Models for the Emergence of Spacetime.

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Abstract: We study entanglement entropy in (2+1)-dimensional gravity as a window into larger open questions regarding entanglement entropy in gravity. (2+1)-dimensional gravity can be rewritten as a topological field theory, which makes it a more tractable model to study. In these topological theories, there remain key questions which we seek to answer in this work, such as the questions 1) What is the entropy of the physical algebra of observables in a subregion, 2) How do we define a factorization map such that the entropy of the resulting factors agrees with this algebraic entropy, and 3) Can we use these insights to build a tensor network that exhibits non-commuting areas? We investigate non-Abelian toric codes / Levin-Wen models as a toy model for black hole entropy in Chern Simons theory. These differ from the usual model in that the stabilizers are implemented as constraints. By enforcing constraints for both Gauss' Law and the flatness of the gauge field, we obtain a choice of algebra that contains only topological operators. The desirable properties of this model are twofold: first, we produce the finiteness of black hole entropy described in previous literature while providing a natural algebraic motivation for this result. Secondly, we obtain non-commuting area operators on a toy model with the topology of a torus.

Topological Toy Models for the Emergence of Spacetime

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Outline

Introduction

Lattice Model

Center Operators

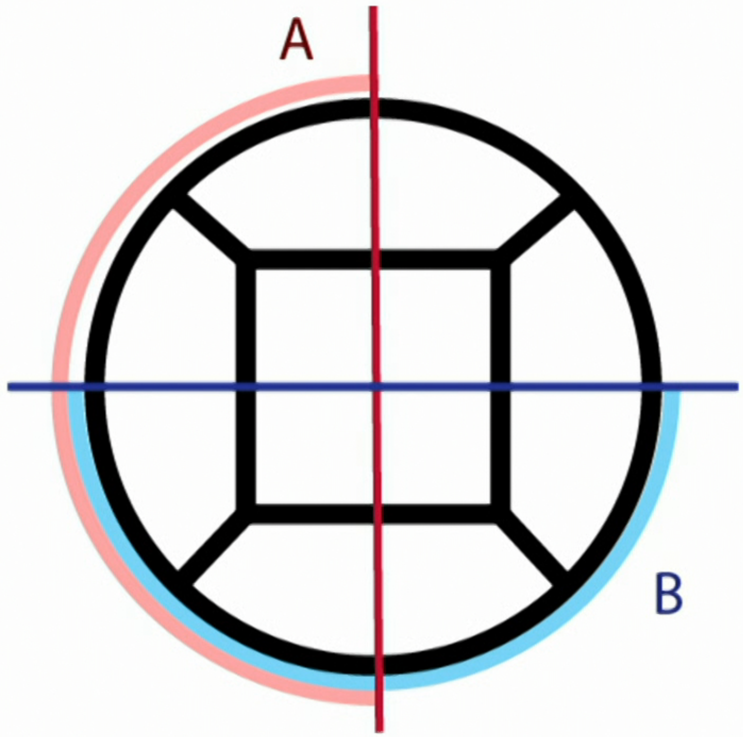
Next Steps

Motivation

- We study (2+1)D gravity, which can be rewritten as a topological field theory (up to caveats)
- Questions of interest:
  - Can we build a tensor network with non-commuting areas?
  - What is the entropy of the physical algebra of observables in a subregion?
  - How do we define a factorization map producing this entropy?

Non-Commuting Areas

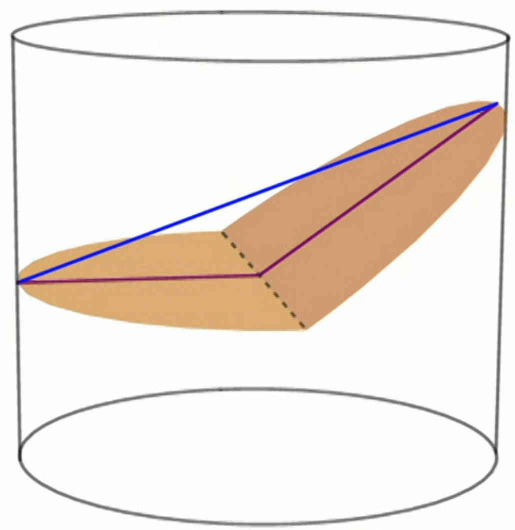
- Tensor networks can't accurately model overlapping area cuts. (Bao, Penington, Sorce, Wall '18)
- Their entanglement structure is too simple



- Tensor networks model a fixed area slice because they have a flat entanglement spectrum (Akers, Rath '18; Dong, Harlow, Marolf '18)
- In GR, area and boost angle are canonically conjugate

$$\Delta E \Delta t \geq 1/2$$

where $\Delta E \sim A/4G_N$ and Δt is boost angle.



(Bao, Penington, Sorce, Wall '18)

Entanglement Entropy

- We want to interpret Bekenstein-Hawking entropy

$$S_{BH} = \frac{\text{Area}}{4G_N}$$

  as entanglement entropy

$$S_A = -\text{Tr}\rho_A \log \rho_A$$

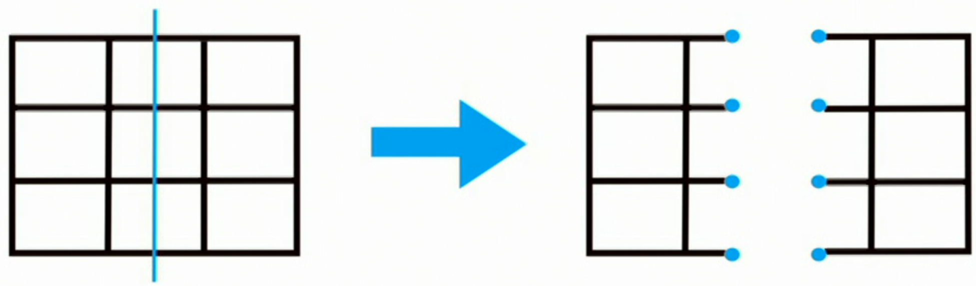
- Hilbert space does not factorize as

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

  so we need to make a choice of factorization map

Edge Modes

- In lattice gauge theory, factorize $\mathcal{H}$ by inserting edge modes (Donnelly '11)



  that transform under the gauge group on the boundary.
- S_{BH} as entanglement entropy of edge modes? (Donnelly '14; Lin '17, '18; Freidel, Donnelly '18; ...)

- Let's apply this factorization map in (2+1)D, where pure gravity is a parity-invariant Chern-Simons theory (Witten '89; Elitzur, Moore, Schwimmer, Seiberg '89,...)

$$S = \frac{k}{4\pi}\int_M \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) - \frac{k}{4\pi}\int_M \mathrm{tr}\left(\bar{A}\wedge d\bar{A} + \frac{2}{3}\bar{A}\wedge \bar{A}\wedge \bar{A}\right)$$

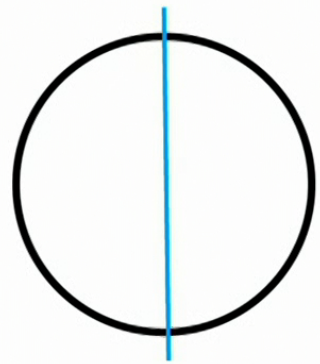
up to caveats (Collier, Eberhardt, Zhang '23)

- We will work with pure gravity without bulk matter

- Problem: In (2+1)D, obtain

$$S_{EE} \sim \ell/\epsilon - S_{TEE} + \sum_r p_r \underbrace{\log d_r}_{\text{anyon EE}} - \sum_r p_r \log p_r \ldots$$

(Wong '17; Belin, Iqbal, Kruthoff '19)



- This doesn't work for gravity: first term diverges with length of boundary, second term is negative

Finite EE

- Using axioms of TQFT, (Mertens, Simón, Wong '22; Wong '22) (McGough, Verlinde '13; Castro, Iqbal, Llabrés '20)

$$S_{EE} \sim \cancel{\ell/\epsilon} - \cancel{S_{TEE}} + \sum_r p_r \underbrace{\log d_r}_{\text{anyon EE}} - \sum_r p_r \log p_r \ldots$$

- Can we give a concrete definition of this factorization map (in terms of algebra or boundary conditions)?
- We will work on the lattice, using a simpler toy model

Lattice Models

- In addition to imposing lattice gauge theory Gauss law constraints, also impose flatness constraints (Delcamp, Dittrich, Riello '16)
- Additional constraints lead to multipartite entanglement structure

Non-Commuting Areas

Can we use new factorization map to build tensor networks with non-commuting areas?

Quantum Double

- Quantum double model (Kitaev '97)
- Given a lattice where edges are labeled by group elements $g \in G$, quantum double model corresponds to the Hamiltonian

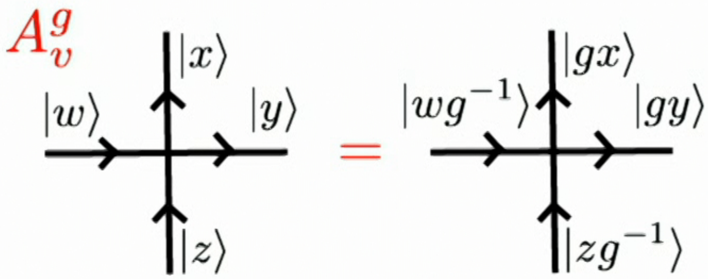
$$H = -\sum_{\text{vertices } v} A_v - \sum_{\text{plaquettes } p} B_p$$

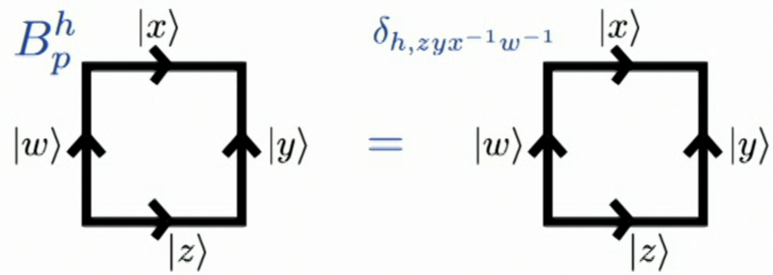
- Operators A_v enforce gauge invariance, B_p enforce flatness
- Rather than taking this to be the Hamiltonian, we will impose the constraints

$$A_v = 1, \qquad B_p = 1$$

on the physical Hilbert space

Defining





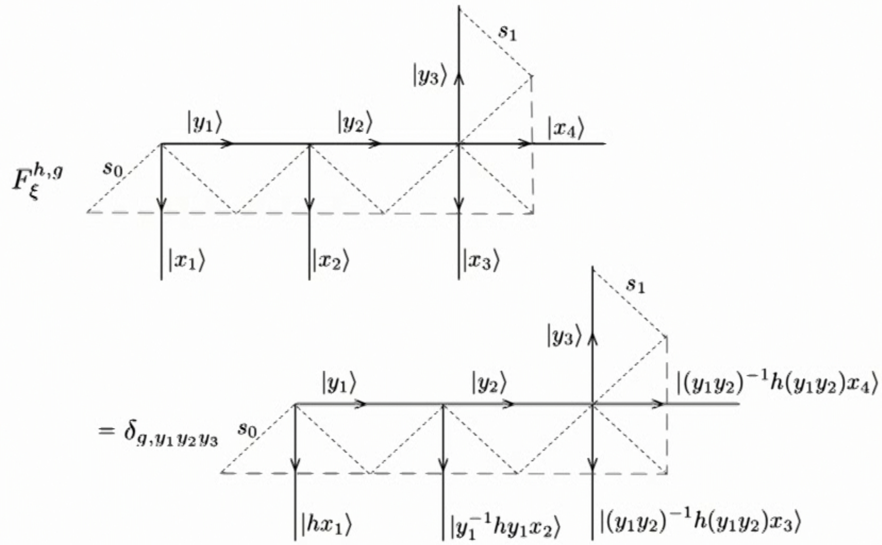
we then define electric and magnetic constraints

$$A_v = \frac{1}{|G|} \sum_{g \in G} A_v^g$$

$$B_p = B_p^e$$

Ribbon Operators

- Create anyonic excitations using ribbon operator carrying electric and magnetic pieces:

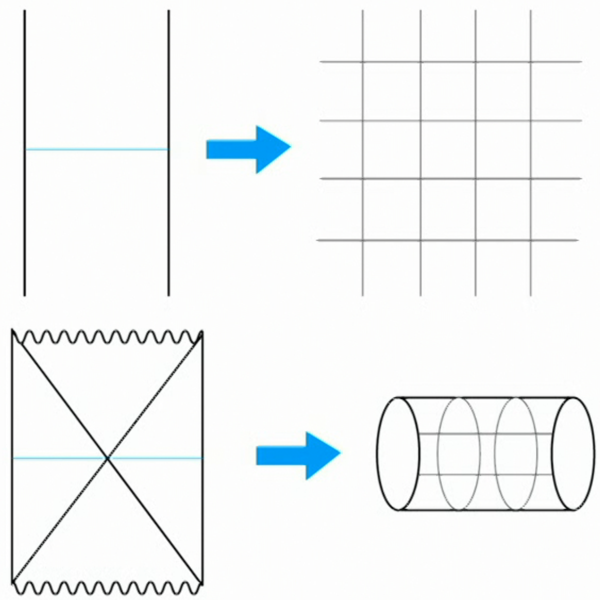


(Kitaev '97)

- Ribbons commute with constraints except at endpoints.

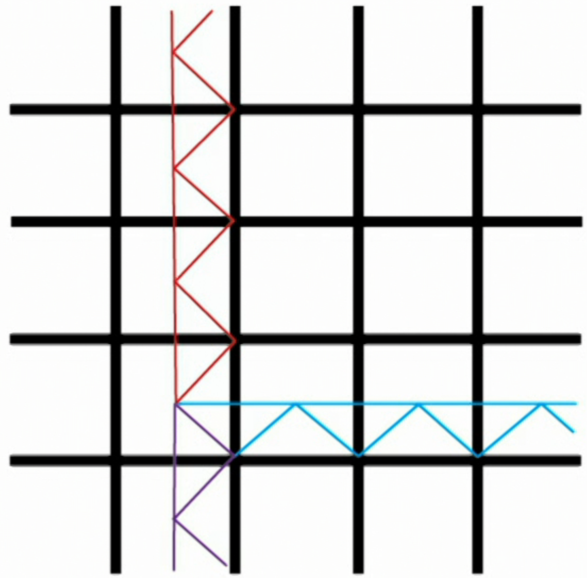
Ribbon Operators

- In our model, we only consider ribbons ending on the physical boundary.
- Topology of lattice = topology of Cauchy slice (disk = global AdS, cylinder = two-sided black hole)



Center Operators

- Area operators are center operators for either side of cut (Harlow '16)
- To find the center operator, ensure that it commutes with
  - ribbons sharing one endpoint.
  - other operators on the same ribbon.



- Obtain center operators

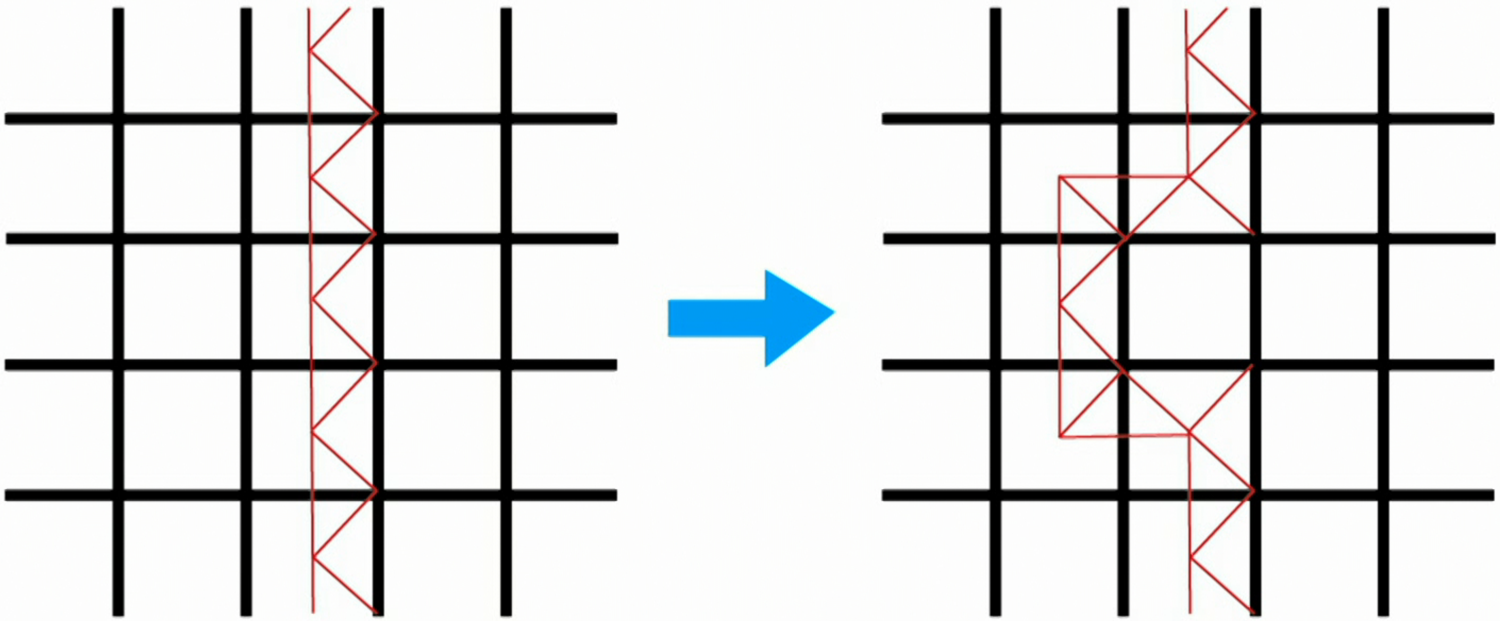
$$F_c^{[k]} = \sum_{k \in [k], \ell \in G} F_c^{k,\ell}$$

where $[k]$ is conjugacy class of k.

- Note that $F_c^{[k]}$ has no projection on the magnetic part.

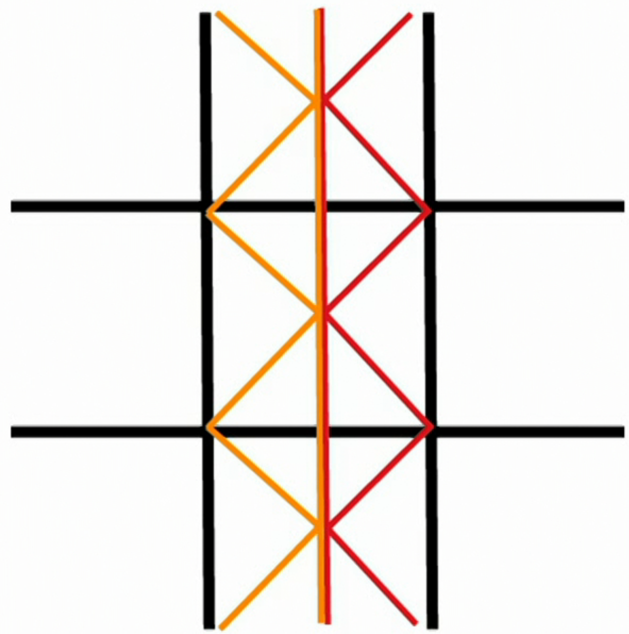
Can check that

- These are the only operators in the center
- Using constraints, for two ribbons intersecting multiple times, can reduce to one intersection

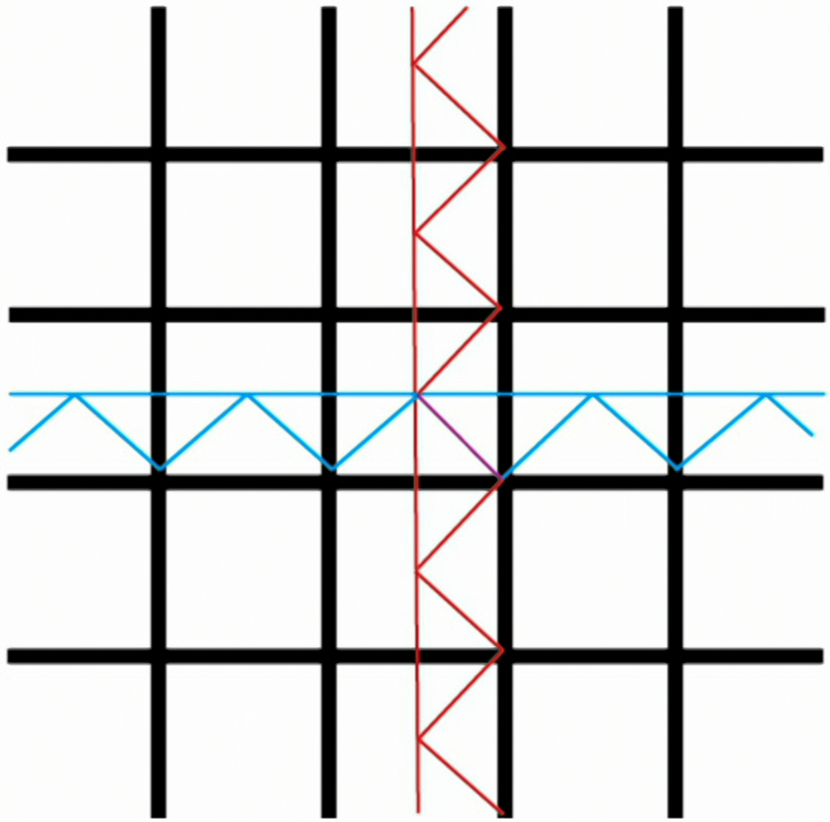


Can check that

- Using constraints, can move the center to either side of the cut.



Non-commutation of intersecting cuts due to non-commutation of ribbons:



Can check that

- Fourier transform

$$F_c^r = \frac{d_r}{|G|} \sum_k \chi_r(k)^* F_c^{[k]}$$

  is a projector,

$$F_c^r F_c^{r'} = \delta_{rr'} F_c^r,$$

  where r labels the represention corresponding to the entire cut

- These projectors fully generate the center

$$\sum_r F_c^r = \mathbb{I}$$

Area Operator

Define area operator

$$\mathcal{L}_A = \sum_r (\log d_r)\ F_c^r$$

- This yields the desired entropy.
- It comes from a natural extended Hilbert space prescription (Delcamp, Dittrich, Riello '16)

Next Steps

- Can we connect the commutator $[\mathcal{L}_{A_1}, \mathcal{L}_{A_2}]$ to gravity quantities?
- How do we add matter to this model?
- Can we generalize this to Levin-Wen models (which describe a more general class of TQFTs) and 3D gravity?
- How do we consider multiple intervals and multipartite entanglement?
- What can we learn about tensor networks in higher dimensions? (Ciambelli, Leigh, Pai '21; Chen, Czech, Frenkel, Qi, Soni '23)
- What is the connection to LQG?

What is the corresponding boundary condition in the continuum?

- Vary Chern-Simons action

$$S = \frac{k}{4\pi} \int_M \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

$$\delta S = \frac{k}{4\pi} \int_{\partial M} \mathrm{tr}\,(\delta A \wedge A) + \frac{k}{2\pi} \int_M \mathrm{tr}\,(\delta A \wedge F)$$

- Typical boundary condition sets $A_0 = 0$.
- Our candidate boundary condition: $A_\phi = \mathrm{const}$, $\int d\phi\, A_0 = 0$.