Title: Talk 106 - Holographic Codes from Hyperinvariant Tensor Networks

Speakers: Alexander Jahn

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Abstract: Holographic quantum-error correcting codes are models of bulk/boundary dualities such as the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, where a higher-dimensional bulk geometry is associated with the code's logical degrees of freedom. Previous discrete holographic codes based on tensor networks have reproduced the general code properties expected from continuum AdS/CFT, such as complementary recovery. However, the boundary states of such tensor networks typically do not exhibit the expected correlation functions of CFT boundary states.

In this work, we show that a new class of exact holographic codes, extending the previously proposed hyperinvariant tensor networks into quantum codes, produce the correct boundary correlation functions. This approach yields a dictionary between logical states in the bulk and the critical renormalization group flow of boundary states. Furthermore, these codes exhibit a state-dependent breakdown of complementary recovery as expected from AdS/CFT under small quantum gravity corrections.

Holographic Codes from Hyperinvariant Tensor Networks

(arXiv:2304.02732)

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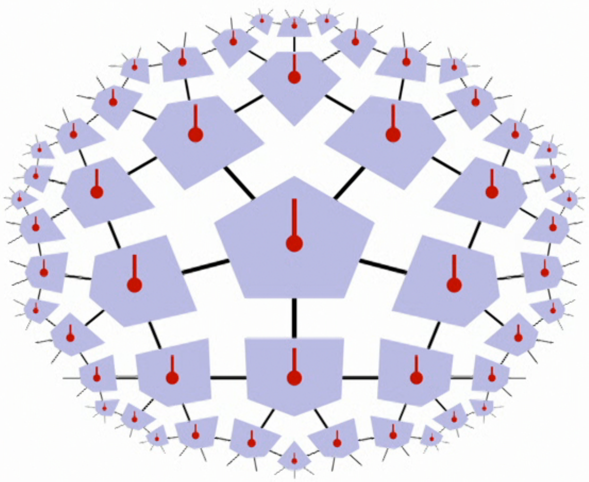
Perimeter Institute, Aug 1, 2023

HaPPY codes

Eight years ago, it all started with a toy model of **holographic quantum error correction**.



[Pastawski/Yoshida/Harlow/Preskill '15]

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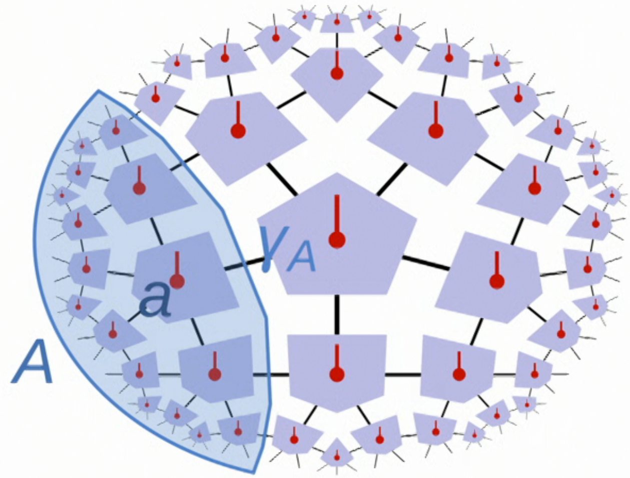

[Pastawski/Yoshida/Harlow/Preskill '15]

Features:

1. Map between **bulk** and **boundary** indices
2. *Perfect tensors* representing exact QEC codes
3. Uniform hyperbolic lattice
4. Reproduces Ryu-Takayanagi formula and entanglement wedge reconstruction

HaPPY codes

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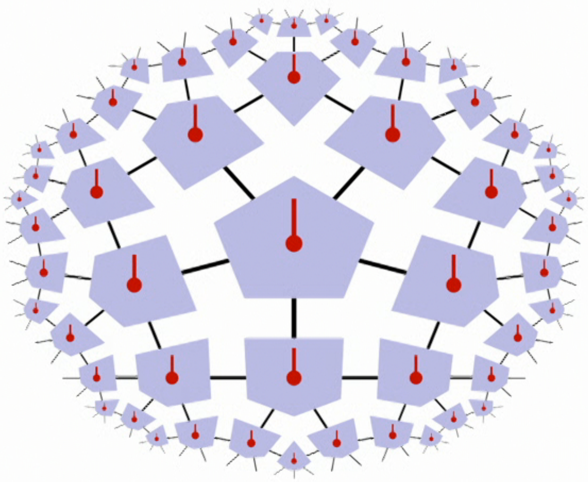


[Pastawski/Yoshida/Harlow/Preskill '15]

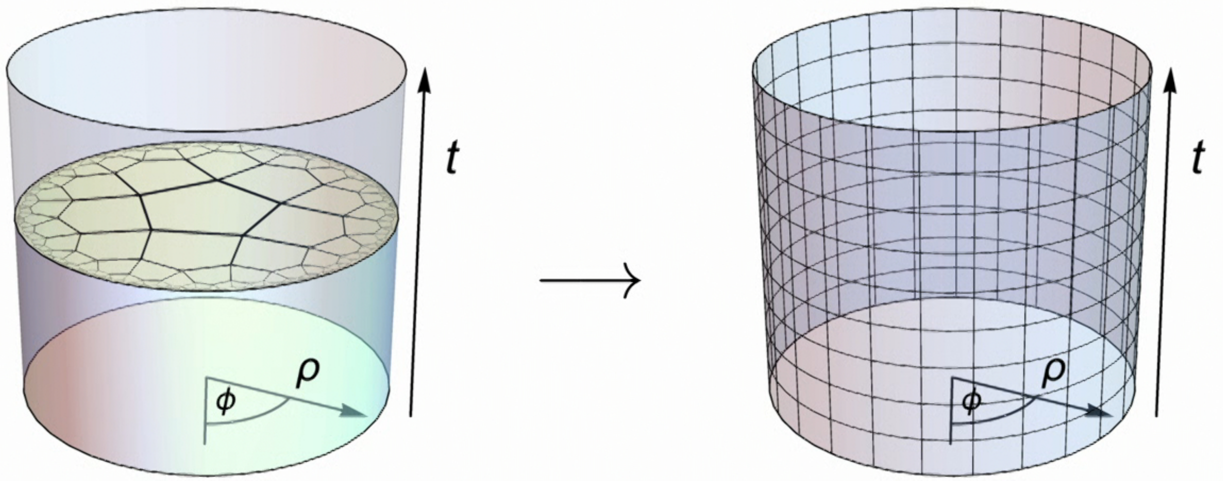
Problems:

1. Limited **finite N effects**: No state dependence, trivial area operator, flat Rényi spectrum
2. Very **sparse 2pt functions**: No CFT-like correlation decay, no local boundary Hamiltonian

Can we build a **new tensor network code** that keeps the HaPPY features but resolves these problem?

Why holographic codes?

AdS/CFT proposes a bulk/boundary map:

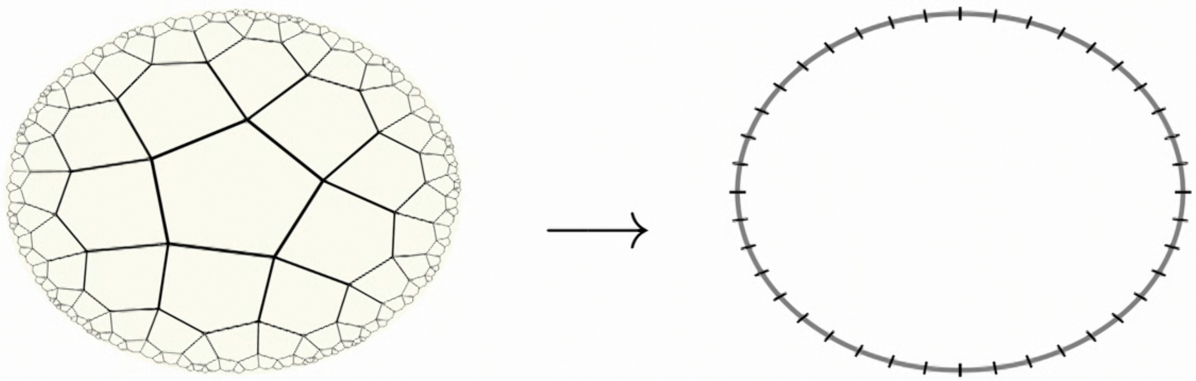


AdS bulk

AdS boundary = CFT spacetime

Why holographic codes?

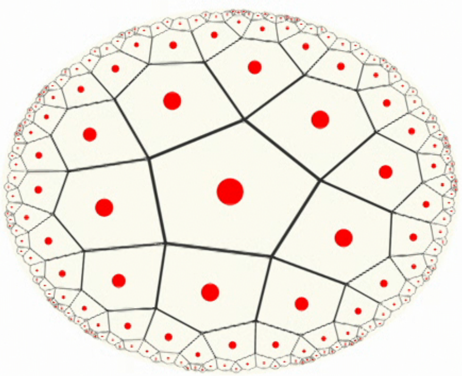
Time-slice picture (in 2+1d):



AdS time-slice   CFT state

Why holographic codes?

Time-slice picture (in 2+1d):



Semiclassical bulk qubits

$\longrightarrow$

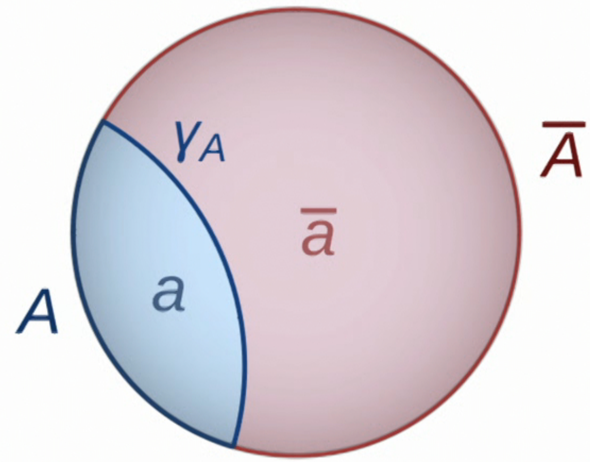
CFT boundary qubits

Holography implies an *encoding map* $V : \mathcal{H}_{\text{bulk}} \to \mathcal{H}_{\text{boundary}}$ that defines a quantum code.

Features of holographic codes

Holography constrains code properties:

1. **Complementary recovery**:
   Boundary bipartition into $A, \bar{A}$ recovers full bulk: $\mathcal{M}'_A = \mathcal{M}_{\bar{A}}$
2. **Superselection sectors**:
   $V : \bigoplus_\alpha (\mathcal{H}_{a^\alpha} \otimes \mathcal{H}_{\bar{a}^\alpha}) \to \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
3. **Ryu-Takayanagi formula**:
   $S_A = \frac{\langle \text{area}(\gamma_A) \rangle}{4G} + S_a$



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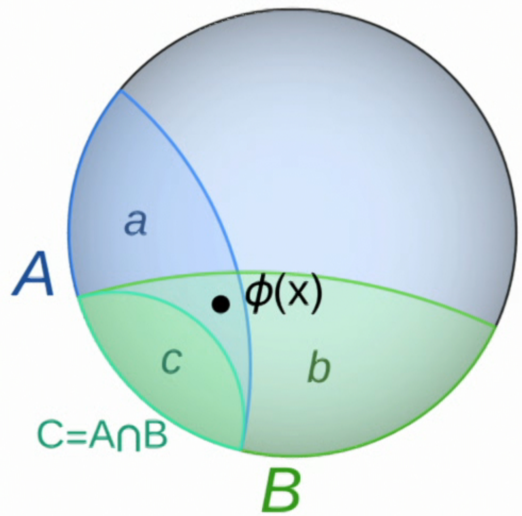

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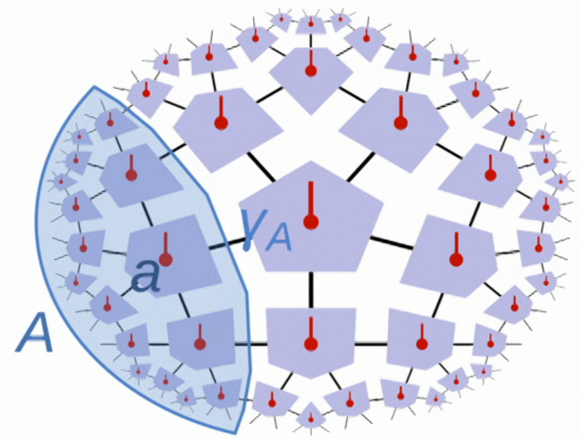
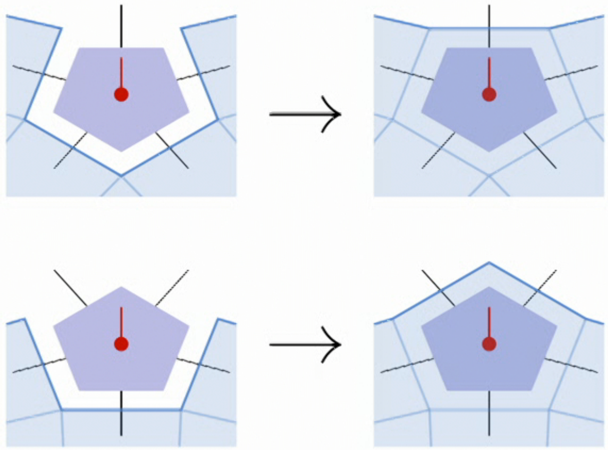


How do HaPPY codes reproduce these features?

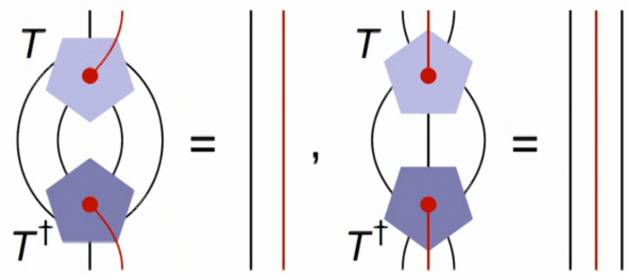
Greedy wedge reconstruction

HaPPY codes implement discrete bulk reconstruction via a **greedy algorithm**, following from the perfect tensor condition.

Algorithm steps:



Isometry conditions:



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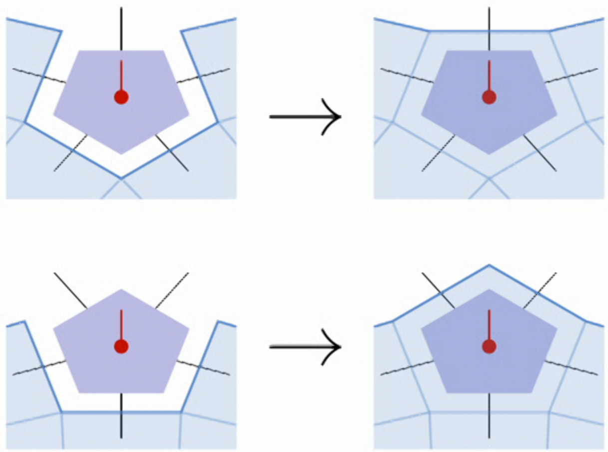
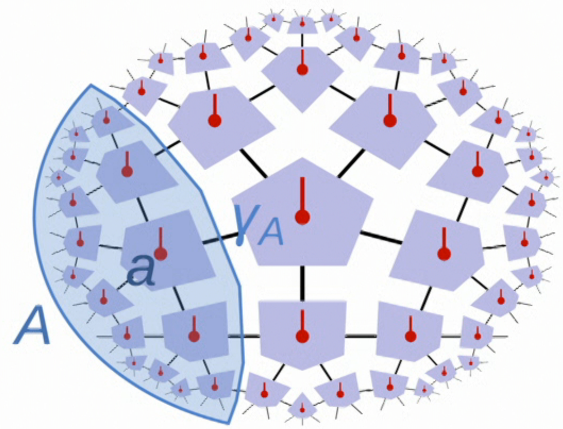


Wedge reconstruction:



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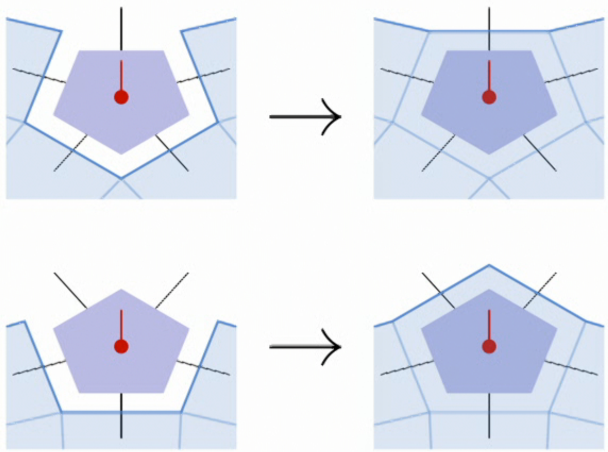

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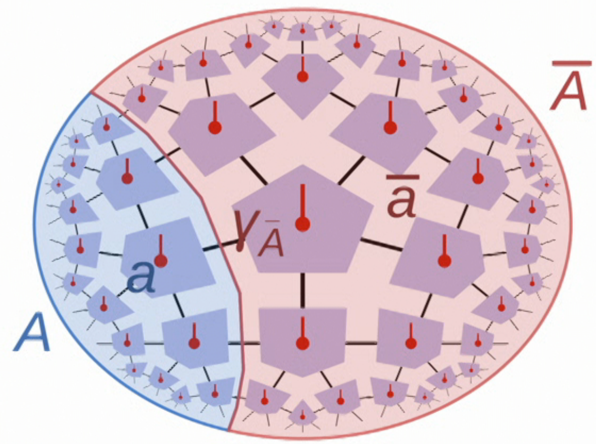

HaPPY codes describe *fixed area states* [Dong/Harlow/Marolf '18] in the bulk, with **state-independent RT surfaces**.

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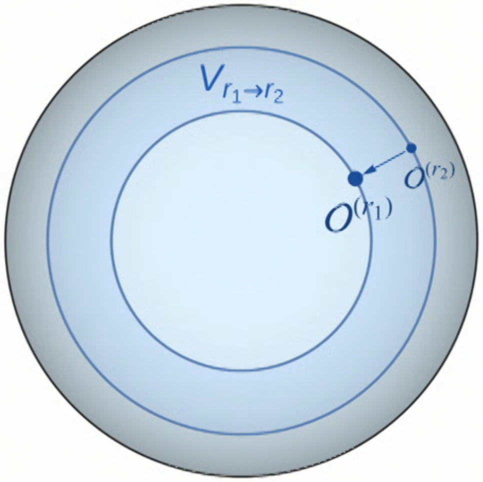
RG flow and correlation functions

An annulus region of an AdS time-slice is associated with a CFT **renormalization group (RG) flow** between cutoffs:



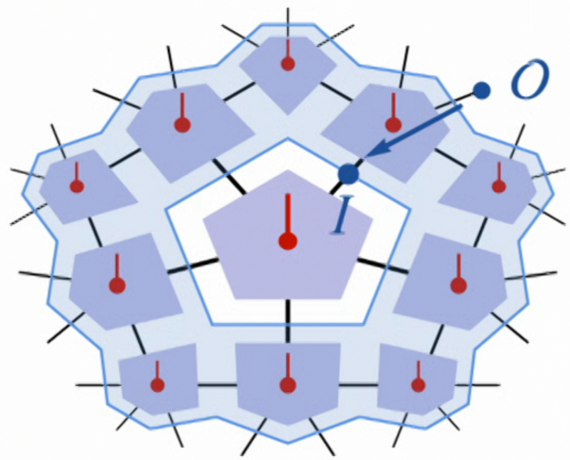
The isometry $V_{r_1 \to r_2}$ induces a **coarse-graining transformation**

$$\mathcal{O}^{(r_2)} \to \mathcal{O}^{(r_1)} = V^\dagger \mathcal{O}^{(r_2)} V$$

An operator $\mathcal{O}^{(r_1)} = \lambda^\Delta \, \mathcal{O}^{(r_2)}$ is a **CFT primary**.

RG flow and correlation functions

Problem for HaPPY codes:
Small operators are **correctable errors**!



Post-HaPPY codes

Wishlist for better holographic model:

1. State-dependent bulk reconstruction
2. Non-trivial boundary theory

Without breaking:

3. Complementary recovery
4. Bulk uniformity
5. Exact code construction

[Cao/Pollack/Wang '21] showed that this is **impossible to achieve** by just modifying the local tensors in the HaPPY models.

Claim: There exists a more general class of tensor network codes that fulfill our wishlist!

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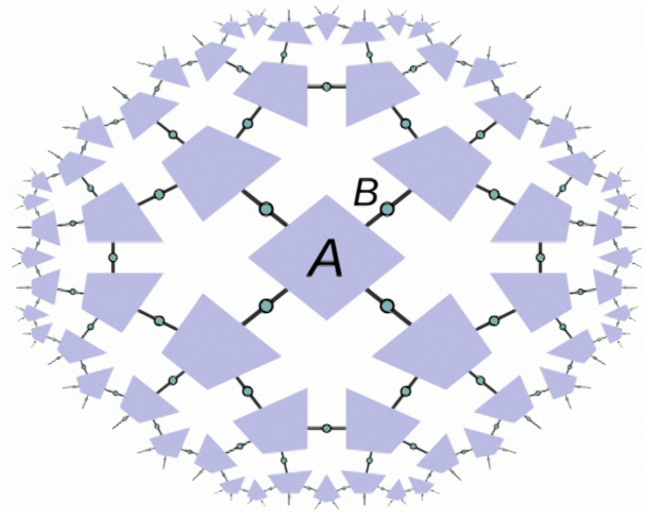
Hyperinvariant codes

Generalization: *Hyperinvariant tensor network* (HTN) [Evenbly '17] with **symmetric unitaries** on each edge.

Properties:

1. Uniform hyberbolic $\{p, q\}$ lattice
2. **Two** types of tensors: q-leg *vertex tensors* A, 2-leg *edge tensors* B



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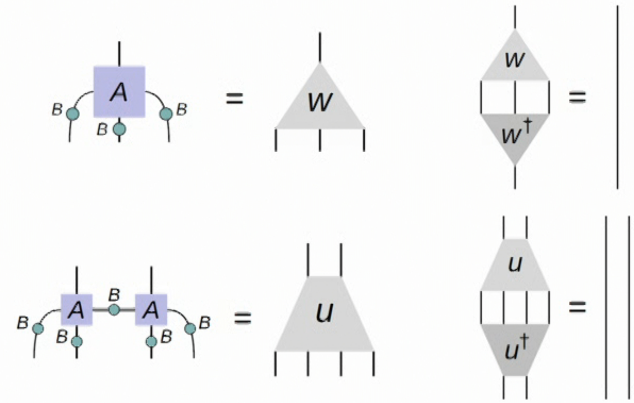

This ansatz leads to polynomially decaying correlations.

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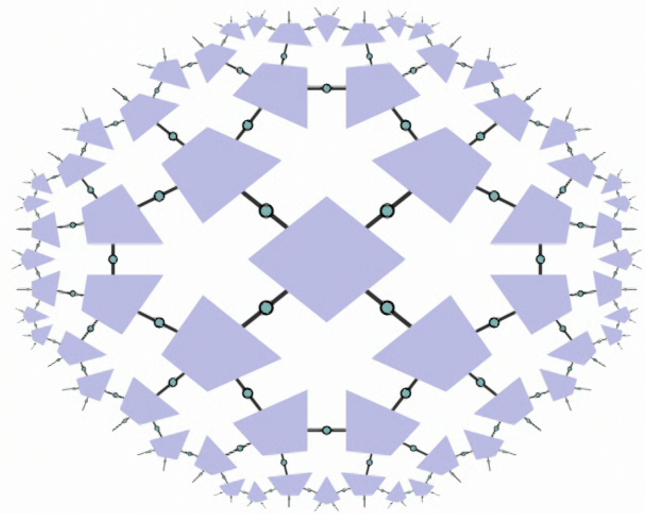
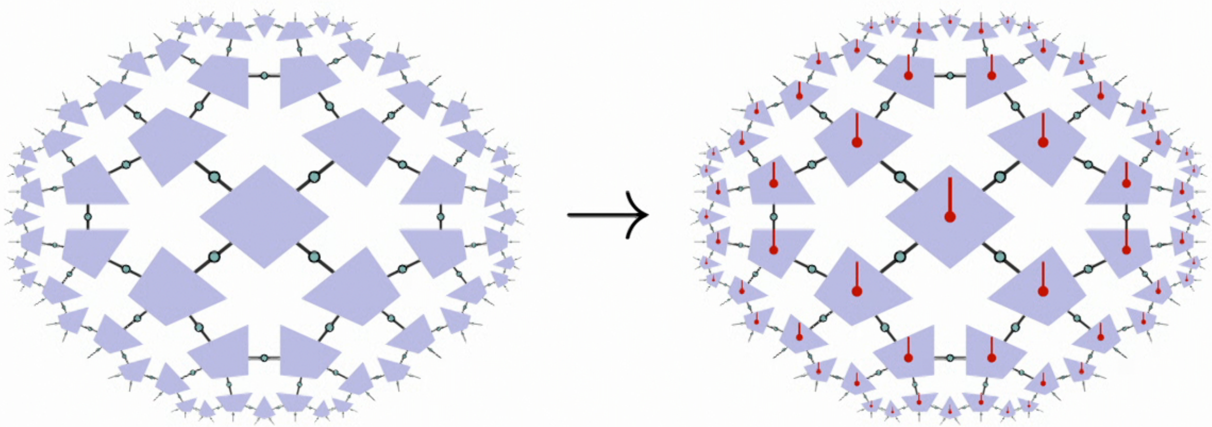


This ansatz leads to polynomially decaying correlations.

Problem: No bulk legs = Not a code!

Hyperinvariant codes

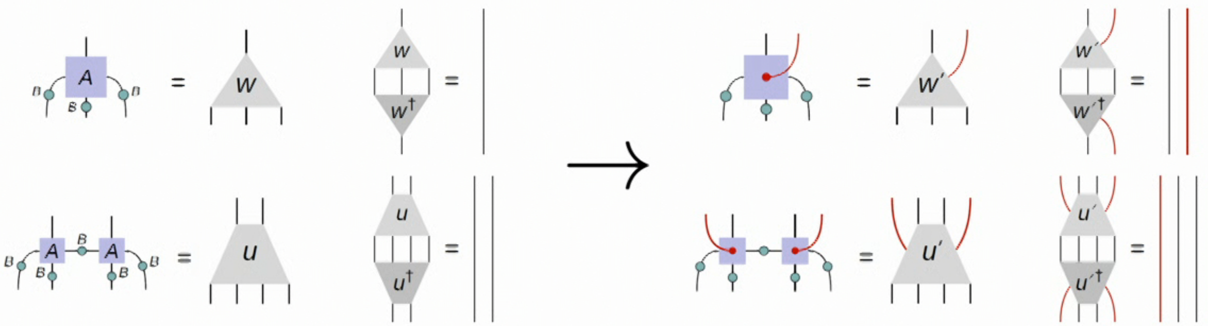
To produce a code, we need bulk legs on the vertex tensors.



Hyperinvariant codes

To produce a code, we need bulk legs on the vertex tensors.

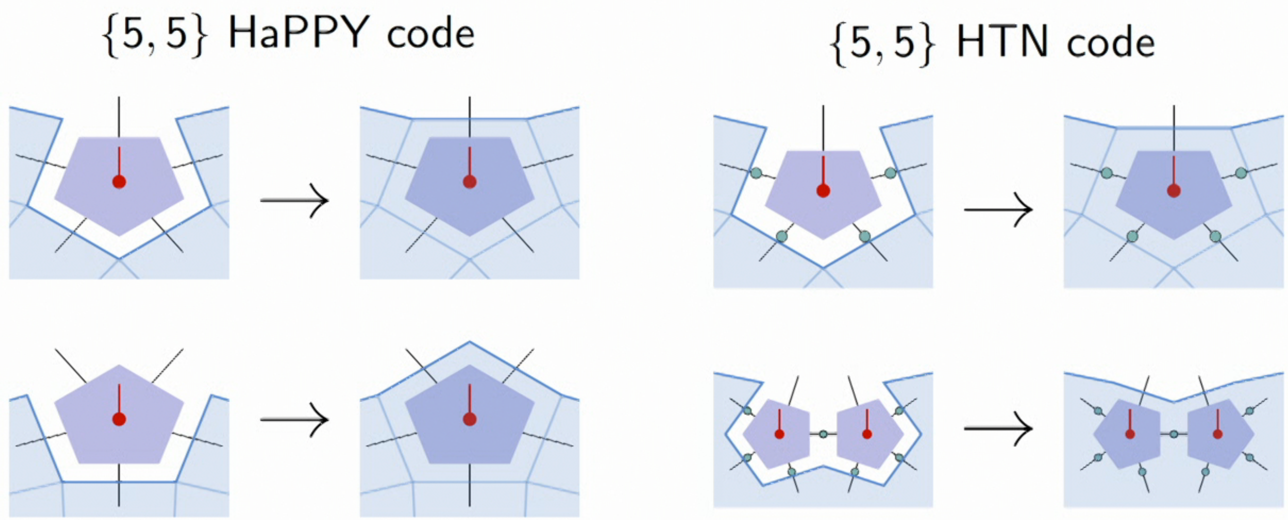
The single- and multi-tensor conditions change accordingly:



One solution: A from $[[4,1,2]]$ code and B as **Hadamard** unitaries.
These are **error-detecting** rather than error-correcting codes.

Hyperinvariant codes: Entanglement wedges

HaPPY and HTN codes have **different greedy algorithms**.



Hyperinvariant codes: Entanglement wedges

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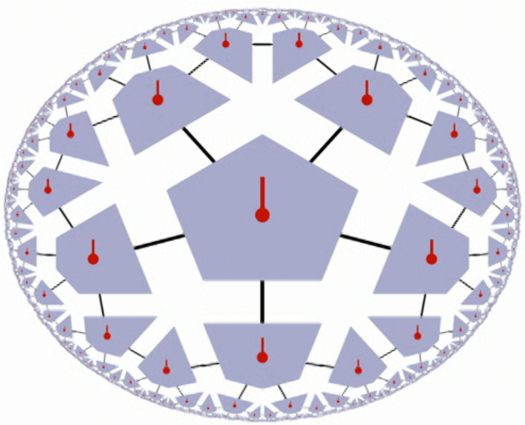
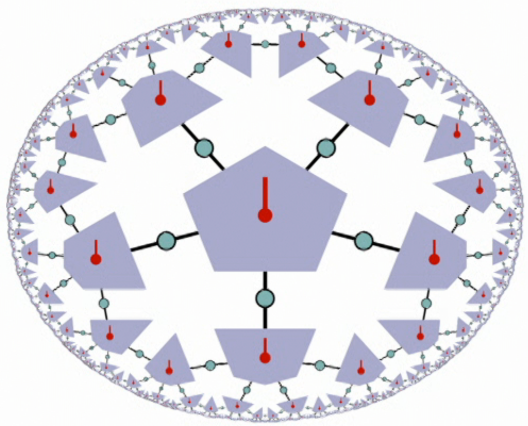
As a result, bulk recovery changes:

$\{5,5\}$ HaPPY code



$\{5,5\}$ HTN code



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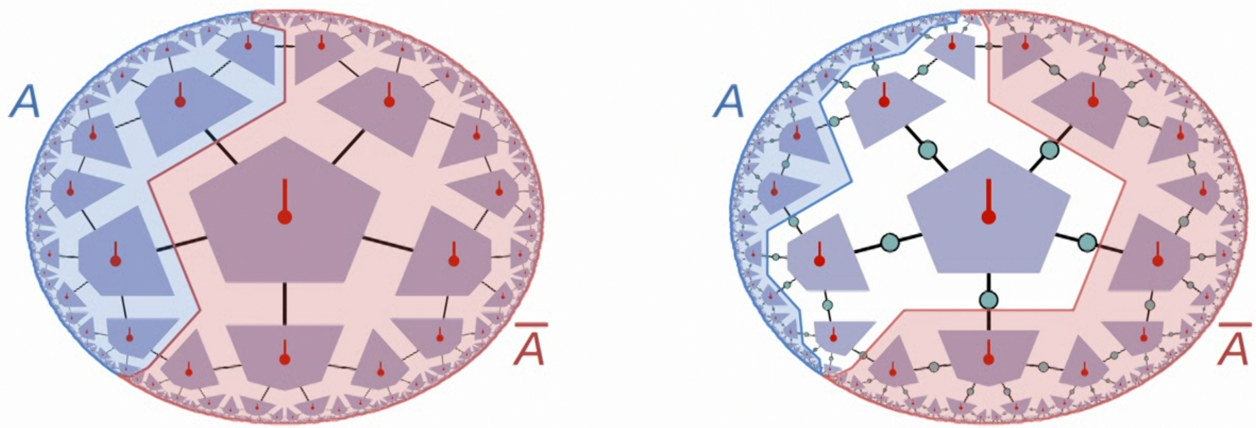
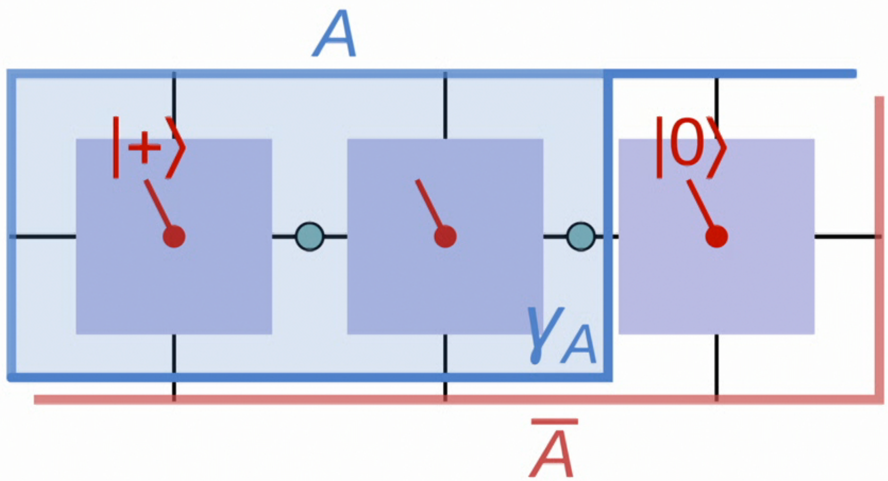


In HaPPY codes, bulk reconstruction is always fixed;
In HTN codes, **a strip of bulk qubits** has state-dependent reconstruction.

Hyperinvariant codes: State dependence

Unlike HaPPY codes, HTN codes have **state-dependent Ryu-Takayanagi surfaces** as in finite N holography.

Example of bulk residual region for $\{5, 4\}$ qubit HTN code:



Depending on the bulk state, **bulk reconstruction changes**.

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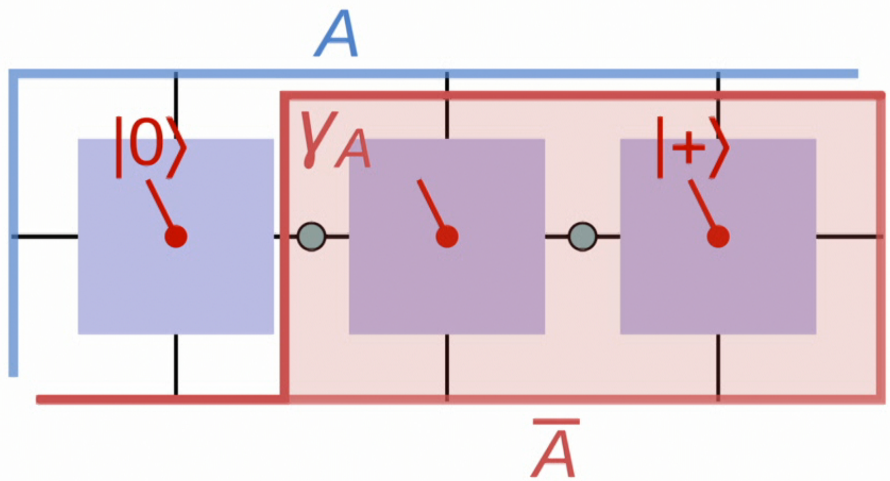
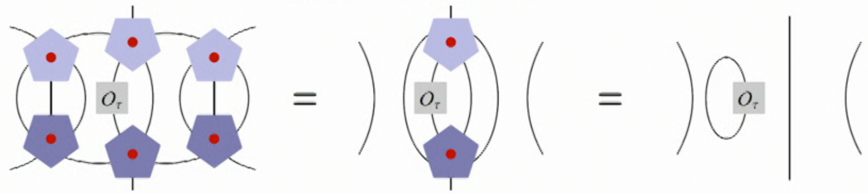


Depending on the bulk state, **bulk reconstruction changes**.
Restriction: area(γ_A) does not change!

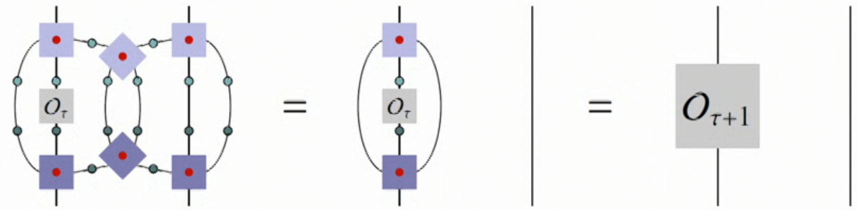
Hyperinvariant codes: Correlation functions

HTN codes allow for non-trivial **scaling superoperators**, so we can identify effective CFT primaries.

{4,5} HaPPY code



{5,4} HTN code



Unlike the MERA tensor network, the spectrum of primaries is **parametrized by the logical bulk state**.

Discussion

Main result

Hyperinvariant tensor network (HTN) codes extend HaPPY codes with features of **state-dependent bulk reconstruction** and **non-trivial operator spectra**.

Limitations

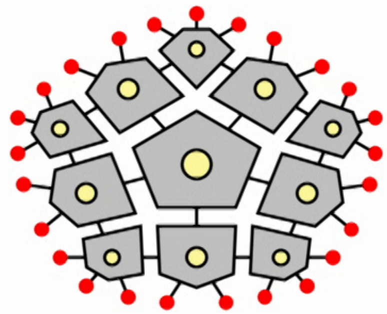
HTN codes are **exact stabilizer codes** and thus have trivial area operators [Cao '23]. A complete model of finite N holography is necessarily an approximate code.

Outlook

- What CFT models are implementable by HTN codes?
- What are their practical QEC properties?
- How does dynamical (quantum) gravity appear?

Thank you for your attention!

My new research group in Berlin is funded by the Einstein Research Unit *Perspectives of a quantum digital transformation*.



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