Title: Research Talk 24 - A fake explanation of sub-maximal chaos

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A fake explanation of sub-maximal chaos

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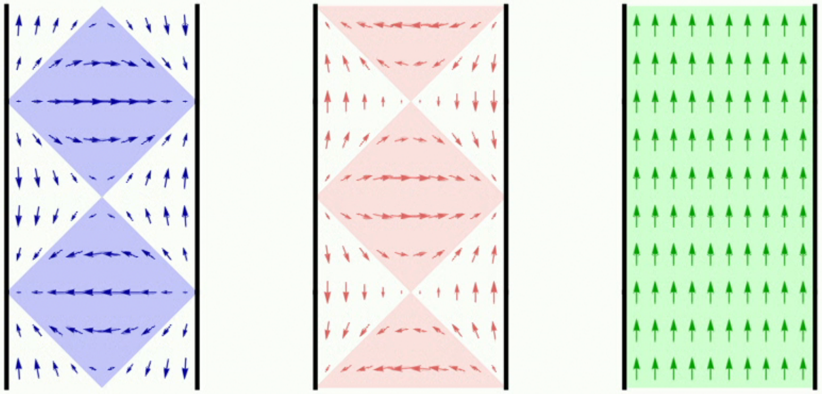
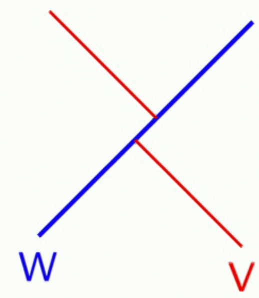
(chaos) (size) (symmetries) (traversable wormholes) (chord number) (q-deformations)

see also: 2208.07032 [HL], 1811.0258 [Berkooz, Isachenkov, Narovlansky, Torrents], 2108.04841 [Harlow & Wu], 1904.12820 [HL, Maldacena & Zhao]

For near-extremal black holes, $\mathfrak{sl}_2$ of the $NAdS_2$ throat:



To understand why P^- is relevant, consider $2 \to 2$ gravitational scattering [Dray & 't Hooft; Shenker & Stanford; Gao-Jafferis-Wall; Lam *et al.*, ...]:



Shapiro time delay $\Rightarrow$ $\mathrm{P}^{\pm}$ symmetry

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In string theory, inelastic effects reduce the exponent [Shenker & Stanford]:

$$\text{chaos exponent} = \frac{2\pi v}{\beta}, \quad v = 1 - \# \left(\frac{\ell_s}{\ell_{\text{AdS}}} \right)^2 + \cdots$$

Wish list: v in $\mathcal{N} = 4$ SYM at finite $(\lambda_{\text{'t Hooft}}, \beta)$.

A simpler model with a known chaos exponent is "large p SYK" [Maldacena & Stanford]:

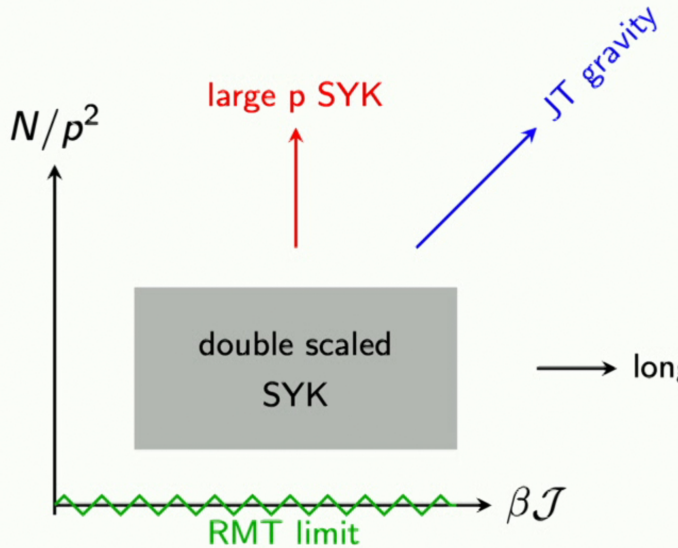
$$\text{chaos exponent} = \frac{2\pi v}{\beta}, \qquad \frac{\pi v}{\beta \mathcal{J}} = \cos\left(\frac{\pi v}{2}\right).$$

Strategy

1. scenic detour through double scaled SYK [Berkooz *et al.*, $\cdots$], where the "bulk Hilbert space" is known [HL].
2. Find the symmetries (q-deformed algebra).
3. Go back to large p SYK $\Rightarrow$ fake geometry.

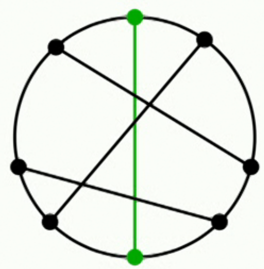
Fake geometry $\sim$ subtlety with the continuum limit.

scaling limits of SYK

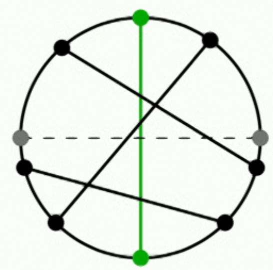


Chord rules

1. Draw Wick contractions[1] "chords" between like operators
2. Intersections between chords get a factor of $q = e^{-\lambda}$ (Hamiltonian) or $r = e^{-\lambda\Delta}$ (matter, $\Delta = p'/p$).
3. Sum over chord diagrams

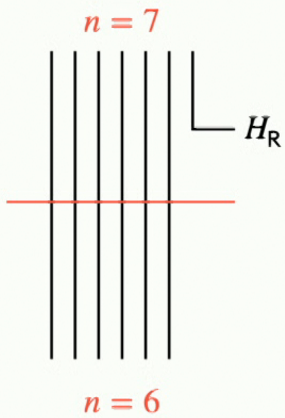
$\mathrm{tr}\left(H^3 M H^3 M\right) \supset$  $= \quad q^2 r^3$

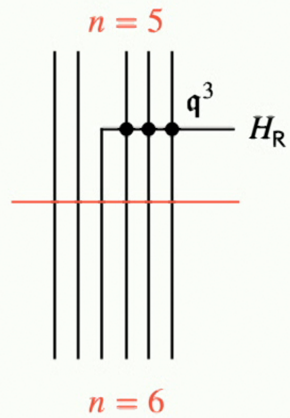
[1] Double intersections are forbidden



$$\Rightarrow \qquad |\,|\wr|\,\rangle = |1, 1\rangle$$

Two processes that happen when we act with H_R:





$$H = \mathfrak{a}^\dagger + \mathfrak{a}, \quad \mathfrak{a} = \alpha[n], \quad \alpha\,|n\rangle = |n-1\rangle$$

$$[n] = q^0 + q^1 + \cdots + q^{n-1} = \frac{1-q^n}{1-q}$$

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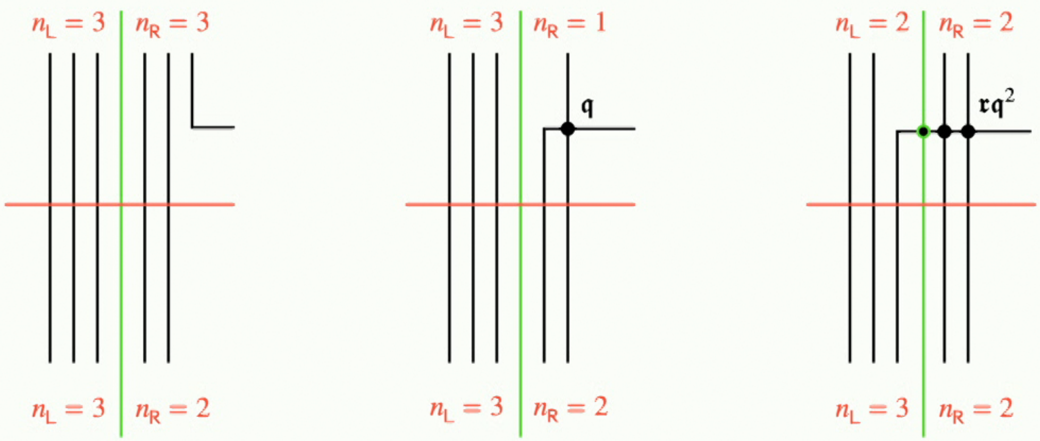
Including matter

With 1 matter chord, states are labeled by $|n_\mathrm{L}, n_\mathrm{R}\rangle$.



$$H_\mathrm{R} = \mathfrak{a}_\mathrm{R}^\dagger + \mathfrak{a}_\mathrm{R}, \quad \mathfrak{a}_\mathrm{R} = \alpha_\mathrm{R}[n_\mathrm{R}] + \alpha_\mathrm{L} q^\Delta q^{n_\mathrm{R}}[n_\mathrm{L}]$$

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Chord algebra is generated by:

$$\{H_L, H_R, \bar{n}\}$$

$\bar{n}$ counts the number of chords (weighted by Δ). Discrete analog of the length of the Einstein-Rosen bridge.

Microscopic interpretation of chord number

Operator size [Roberts-Stanford-Streicher, Qi-Streicher] is measured by the 2-sided operator:

$$\text{size} = \frac{1}{2}\sum_{\alpha=1}^{N}\left(1 + \mathrm{i}\psi_\alpha^{\mathsf{L}}\psi_\alpha^{\mathsf{R}}\right).$$

Let $\bar{n}$ be the total chord number, weighted by dimension:

$$\boxed{\bar{n} = \text{size}/p}$$

Each H chord is associated with an operator of size p.

q-deformed commutator:

$$[A, B]_q \equiv AB - qBA$$

Writing $H_L = \mathfrak{a}_L^\dagger + \mathfrak{a}_L$ and $H_R = \mathfrak{a}_R^\dagger + \mathfrak{a}_R$,

$$[\mathfrak{a}_L, \mathfrak{a}_R] = [\mathfrak{a}_L^\dagger, \mathfrak{a}_R^\dagger] = 0, \quad (1)$$

$$[\bar{n}, \mathfrak{a}_{\mathrm{L/R}}^\dagger] = \mathfrak{a}_{\mathrm{L/R}}^\dagger, \quad [\bar{n}, \mathfrak{a}_{\mathrm{L/R}}] = -\mathfrak{a}_{\mathrm{L/R}} \quad (2)$$

$$[\mathfrak{a}_L, \mathfrak{a}_R^\dagger] = [\mathfrak{a}_R, \mathfrak{a}_L^\dagger] = q^{\bar{n}} \quad (3)$$

$$[\mathfrak{a}_{\mathrm{L/R}}, \mathfrak{a}_{\mathrm{L/R}}^\dagger]_q = 1. \quad (4)$$

Chord algebra $\mathcal{A}_{\text{chord}}$ is also a *bi-algebra*.

Algebra: a vector space equipped with an associative product $\mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$.

Bi-algebra: an algebra with a *coproduct*: $D : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$ that is compatible with multiplication $D(a \cdot b) = D(a) \cdot D(b)$.

$$D(\mathfrak{a}_L^\dagger) = \mathfrak{a}_L^\dagger \otimes 1, \tag{5}$$

$$D(\mathfrak{a}_L) = \mathfrak{a}_L \otimes 1 + q^\Delta q^{\bar{n}} \otimes \mathfrak{a}_L, \tag{6}$$

$$D(\bar{n}) = \bar{n} \otimes 1 + 1 \otimes \bar{n} + \Delta(1 \otimes 1) \tag{7}$$

This coproduct has a simple physical interpretation:

$$\delta : |{\mid}{\wr}{\mid}{\mid}\rangle \otimes |{\mid}{\wr}{\mid}\rangle \to |{\mid}{\wr}{\mid}{\mid}{\wr}{\mid}{\wr}{\mid}\rangle$$

δ joins two wormholes; δ^{-1} splits:

$$\delta^{-1} : |{\mid}{\wr}{\mid}{\mid}{\wr}{\mid}{\wr}{\mid}\rangle \to |{\mid}{\wr}{\mid}{\mid}\rangle \otimes |{\mid}{\wr}{\mid}\rangle$$

Coproduct:

$$D(a) = \delta^{-1} \cdot a \cdot \delta$$

Related to the factorisation problem [..., Harlow and Jafferis, ...].

What are the chord algebra irreps?

1. Short irrep, consisting of the "empty wormhole" states:

$$|n\rangle\,, \quad n \in \mathbb{Z}_{\geq 0}$$

2. 1-particle irrep (highest weight irrep $\mathfrak{a}_L |\wr\rangle = \mathfrak{a}_R |\wr\rangle = 0$.):

$$|n_L, n_R\rangle = |{\mid\mid} \cdots {\mid} \wr {\mid\mid} \cdots {\mid}\rangle$$

The Hilbert space decomposes into a sum over these irreps. What about multi-particle states?

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We define 4-pt "chord blocks" $=$ projector onto a particular irrep. Analogous to Virasoro blocks.

These blocks enjoy crossing symmetry, thanks to co-associativity of the co-product:

$$(D \otimes 1) \cdot D = (1 \otimes D) \cdot D$$

[Gives the Hilbert space interpretation of expressions in Jafferis, Kolchmeyer, Mukhametzhanov & Sonner]

Let's come back to the symmetries of large p SYK at finite temp.

Look for a subalgebra $\subset \mathcal{A}_{\text{chord}}$ that commutes with $\bar{n}$:

$$J_{ij} = \mathfrak{a}_i^\dagger \mathfrak{a}_j - [\bar{n}], \qquad i, j \in \{L, R\}$$

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Define $c = q^{\bar{n}/2}$. Take $\lambda \to 0$, find an $\mathfrak{sl}_2$:

$$\mathrm{E} \doteq -\frac{1}{2c}\left(J_{LL} + J_{RR}\right), \quad \mathrm{B} \doteq \frac{1}{2c\sqrt{1-c^2}}\left(J_{LL} - J_{RR}\right)$$
$$\mathrm{P} \doteq \frac{\mathrm{i}}{2\sqrt{1-c^2}}\left(J_{LR} - J_{RL}\right)$$

Valid at all temperatures

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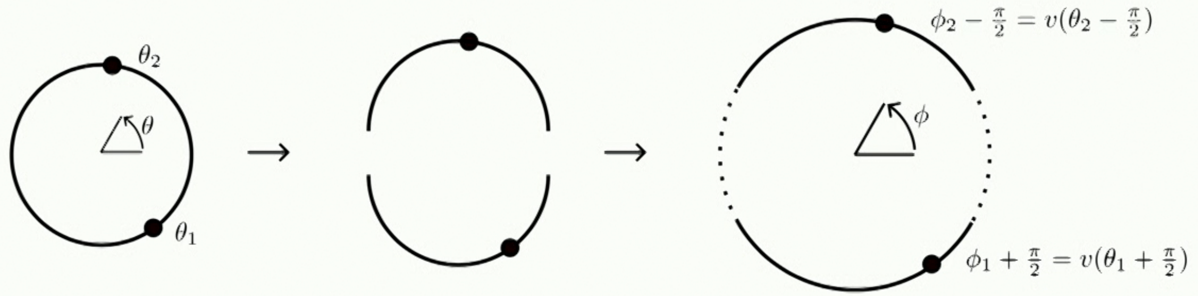
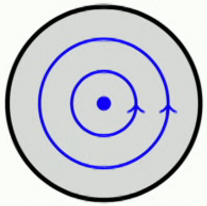
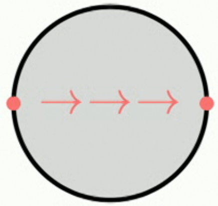
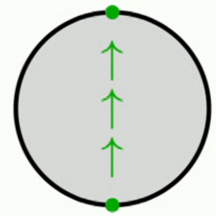


$$\langle \mathcal{O}(\theta_2) \mathsf{B} \mathcal{O}(\theta_1) \rangle = \partial_{\phi_1} \langle \mathcal{O}(\theta_2) \mathcal{O}(\theta_1) \rangle$$
$$\langle \mathcal{O}(\theta_2) \mathsf{E} \mathcal{O}(\theta_1) \rangle = (\cos(\phi_1) \partial_{\phi_1} - \Delta \sin \phi_1) \langle \mathcal{O}(\theta_2) \mathcal{O}(\theta_1) \rangle$$
$$\langle \mathcal{O}(\theta_2) \mathsf{P} \mathcal{O}(\theta_1) \rangle = \mathrm{i}(\sin(\phi_1) \partial_{\phi_1} + \Delta \cos \phi_1) \langle \mathcal{O}(\theta_2) \mathcal{O}(\theta_1) \rangle$$

$\mathsf{B} =$  $-\mathrm{i}\mathsf{P} =$  $\mathsf{E} =$ 

$$\mathrm{i}[\mathsf{B}, \mathsf{P}^{\pm}] = \mp \mathsf{P}^{\pm}, \quad \mathsf{P}^{\pm} = \mathsf{E} \pm \mathsf{P}, \quad \mathsf{B} \approx \frac{\beta}{2\pi v}(H_R - H_L)$$

$$e^{\mathrm{i}(H_R - H_L)t}\mathsf{P}^{-}e^{-\mathrm{i}(H_R - H_L)t} = e^{\frac{2\pi v}{\beta}t}\mathsf{P}^{-}$$

Ward identites: *finite temp* 2-pt function is conformally covariant on the fake circle:

$$\langle \mathcal{O}(\theta_2)\mathcal{O}(\theta_1)\rangle \propto \frac{1}{\sin^{2\Delta}(\frac{\phi_2-\phi_1}{2})} \quad \checkmark$$

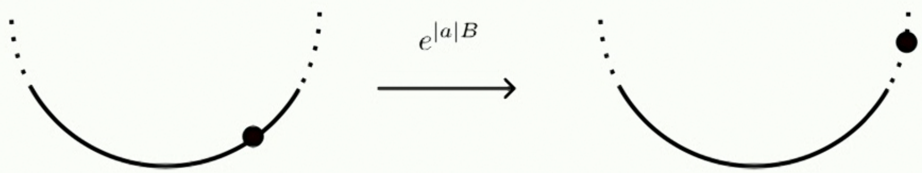
4-pt function $\Rightarrow$ commutator OTOC is only a function of the *fake* cross ratio.

$$\frac{\langle [W_2, V_4][V_3, W_1]\rangle}{\langle W_2 W_1\rangle\langle V_4 V_3\rangle} = 2\lambda\Delta_W\Delta_V\frac{1+\chi}{1-\chi} = \sum_k c_k^2 \mathcal{F}_{\Delta_V,\Delta_W,k}(\chi),$$

$$\chi = \frac{\sin\frac{\phi_{13}}{2}\sin\frac{\phi_{42}}{2}}{\sin\frac{\phi_{14}}{2}\sin\frac{\phi_{32}}{2}}$$

$c_k^2 \Rightarrow$ Clebsch-Gordan problem described earlier.

If ϕ starts out in the physical region, can act with a symmetry generator to leave the physical region:



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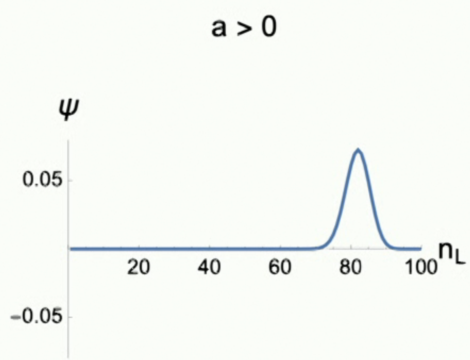
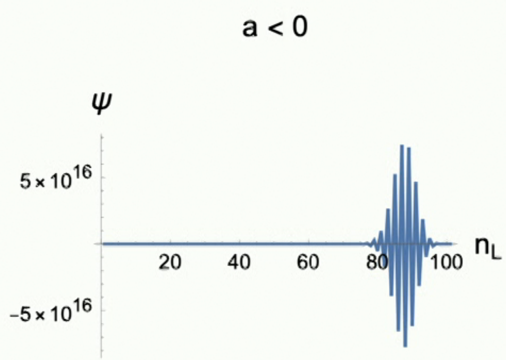
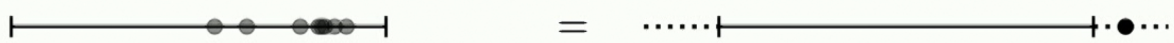
What do the states look like in the chord Hilbert space?





Note that the y-axis rescaled by 10^{18} between the two figures.

Two pictures:

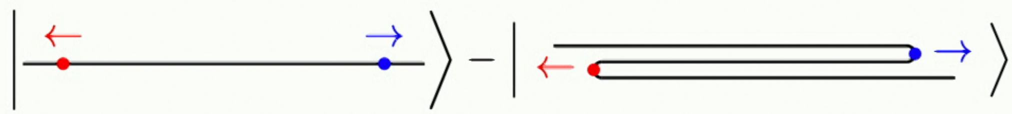


We checked (numerically and analytically) that such states reproduce the fake disk predictions as $\lambda \to 0$.

Fake disk: makes sub-max chaos manifest

Fakeness is a geometric representation of certain "subtle" states, $\sim$ chiral fermion problem on the lattice.

$$|\text{out}\rangle - S\,|\text{in}\rangle =$$



Question: general lessons for sub-max chaos?

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