Title: Research Talk 22 - Infrared Aspects of QFT and Quantum Gravity: Scattering and Coherence

Speakers:

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Abstract: A long-standing problem in QFT and quantum gravity is the construction of an "IR-finite" S-matrix. Infrared divergences in scattering theory are intimately tied to the "memory effect" and the existence of an infinite number of "large gauge charges". A suitable "IR finite" S-matrix requires the inclusion of states with memory (which do not lie in the standard Fock space). For QED such a construction was achieved by Faddeev and Kulish by appropriately "dressing" charged particles with memory. However, we show that this construction fails in the case of massless QED, Yang-Mills theories, linearized quantum gravity with massless/massive sources, and in full quantum gravity. In the case of quantum gravity, we prove that the only "Faddeev-Kulish" state is the vacuum state. We also show that non-Faddeev Kulish representations are also unsatisfactory. Thus, in general, it appears there is no preferred Hilbert space for scattering in QFT and quantum gravity. Nevertheless we show how scattering can be formulated in a manner that manifestly IR-finite without any "ad-hoc" restrictions or dressing on the states. Finally, we investigate the consequences of the superselection due to the "large gauge charges" and illustrate that, in QED, nearly all scattering states are completely decohered in the bulk.

Infrared Aspects of QFT and Quantum Gravity: Scattering and Coherence

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K.Prabhu, G.S., & R.M. Wald, Phys. Rev. D 106, 066005 (2022) [arXiv:2203.14334]

G.S., D. Danielson, & R.M. Wald (in prep.)

K. Prabhu & G.S. (in prep.)

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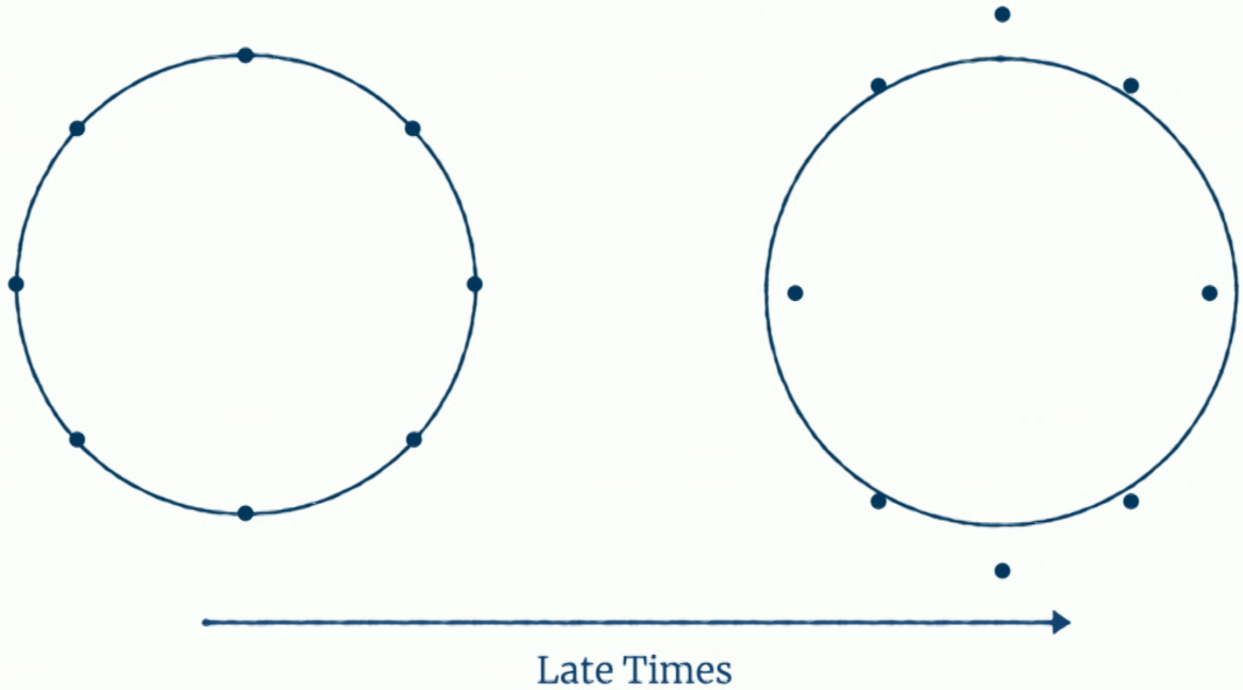
Gravitational and Electromagnetic Memory Effects

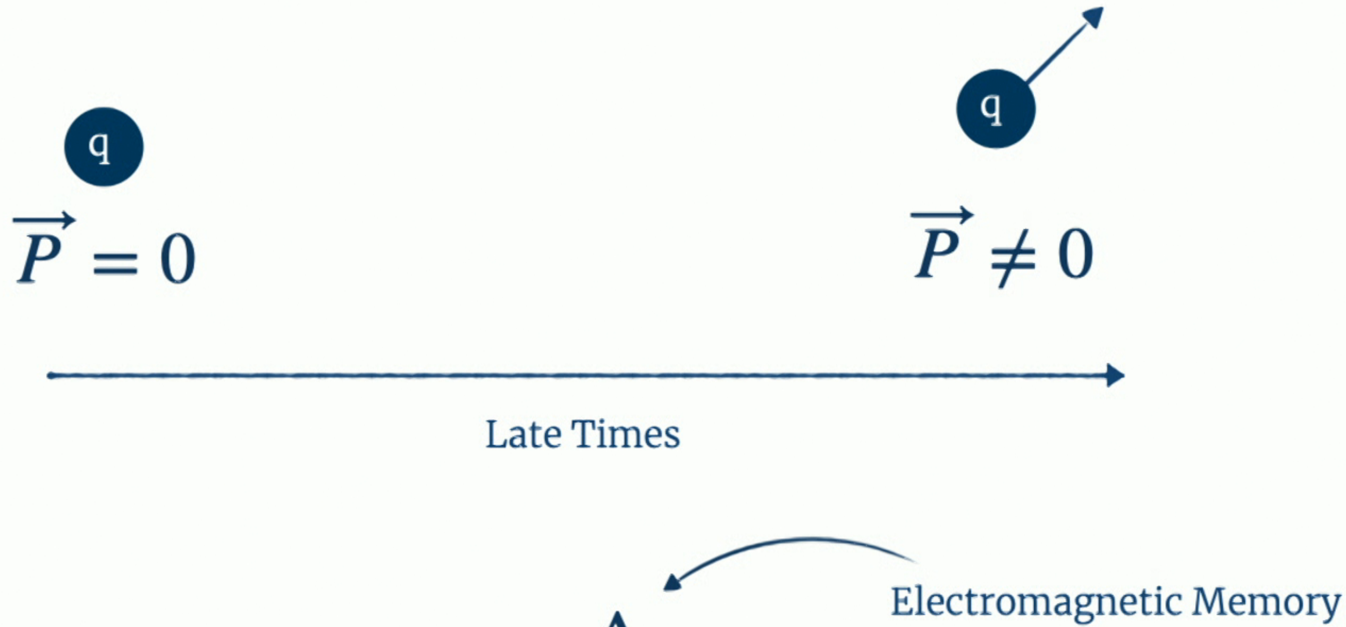


relative displacement

Gravitational Memory

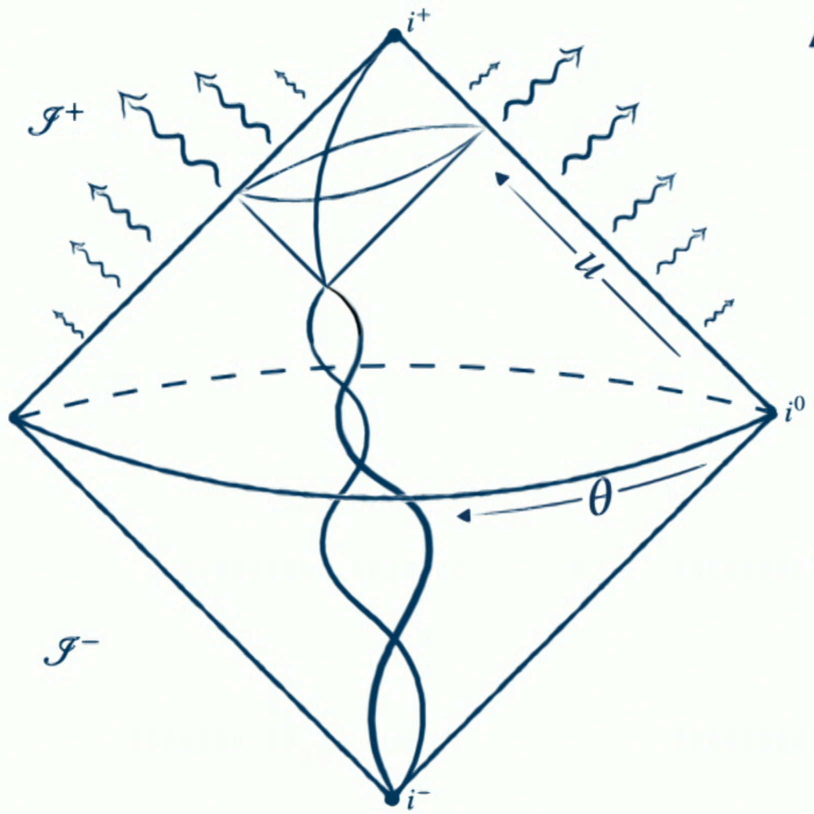
$$\Delta\xi_a = \frac{\Delta_{ab}\xi^b_{(0)}}{r}$$

Gravitational and Electromagnetic Memory Effects

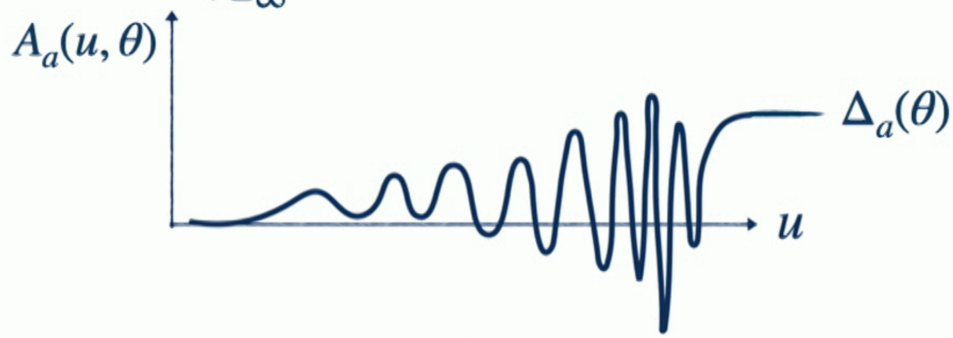


$$\Delta P_a = \frac{\Delta_a}{r}$$

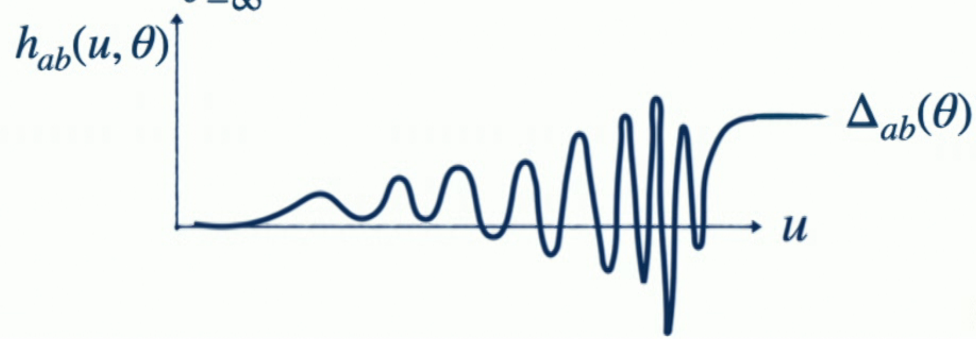
Classical Scattering, Radiation and Memory



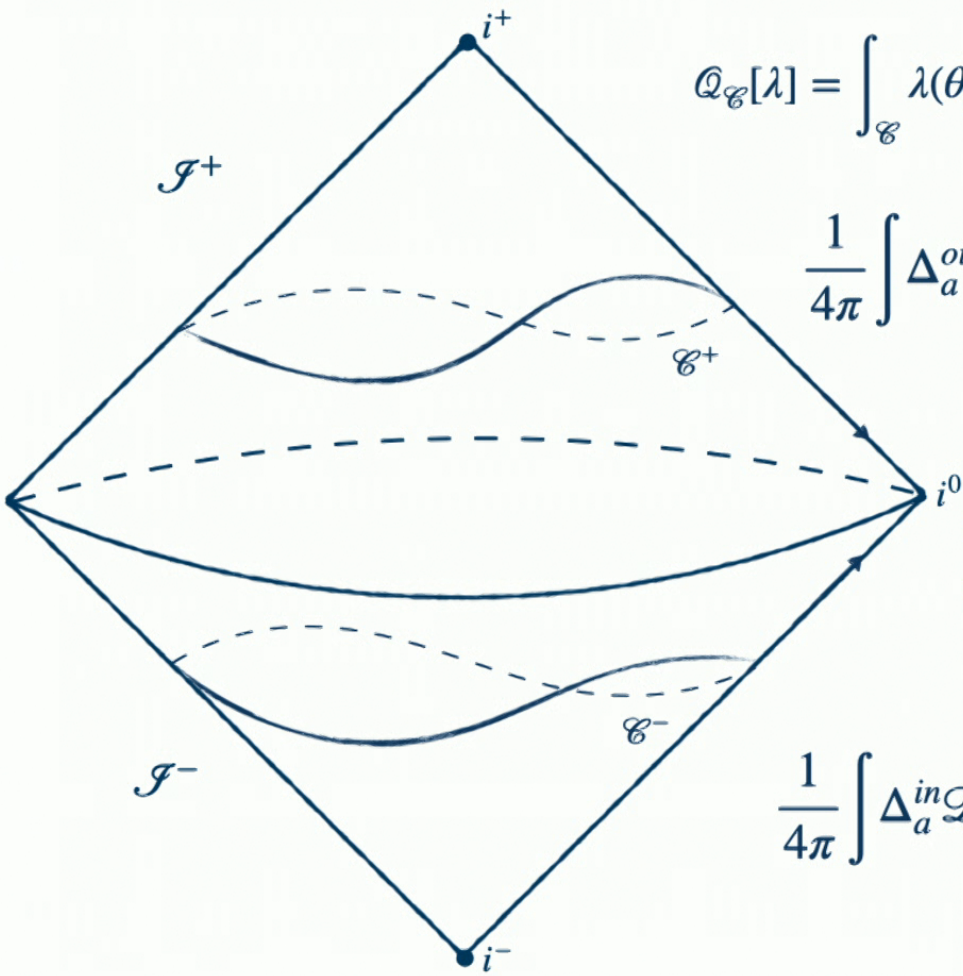
$$\Delta_a(\theta) = -\int_{-\infty}^{\infty} du\ E_a(u,\theta) \qquad E_a(u,\theta) = \partial_u A_a$$



$$\Delta_{ab}(\theta) = \int_{-\infty}^{\infty} du\ N_{ab}(u,\theta) \qquad N_{ab}(u,\theta) = \partial_u h_{ab}$$



Memory and Charges



$$\mathcal{Q}_{\mathscr{C}}[\lambda] = \int_{\mathscr{C}} \lambda(\theta) F^{(2)}_{ur}(u,\theta)\, d\Omega$$

massless charge-current flux

$$\frac{1}{4\pi}\int \Delta^{out}_a \mathscr{D}^a \lambda\, d\Omega = \mathcal{Q}_{i^+}[\lambda] - \mathcal{Q}_{i^0}[\lambda] + \int_{\mathscr{I}^+} J_{out}\lambda$$

$$\lim_{\mathscr{C}^+\to i^0} \mathcal{Q}_{\mathscr{C}^+}[\lambda] = \lim_{\mathscr{C}^-\to i^0} \mathcal{Q}_{\mathscr{C}^-}[\tilde{\lambda}]$$

where $\tilde{\lambda}(\theta) = \lambda(-\theta)$

$$\frac{1}{4\pi}\int \Delta^{in}_a \mathscr{D}^a \lambda\, d\Omega = \mathcal{Q}_{i^-}[\lambda] - \mathcal{Q}_{i^0}[\lambda] + \int_{\mathscr{I}^-} J_{in}\lambda$$

[Campiglia et al. 2017], [Prabhu, 2018] ...

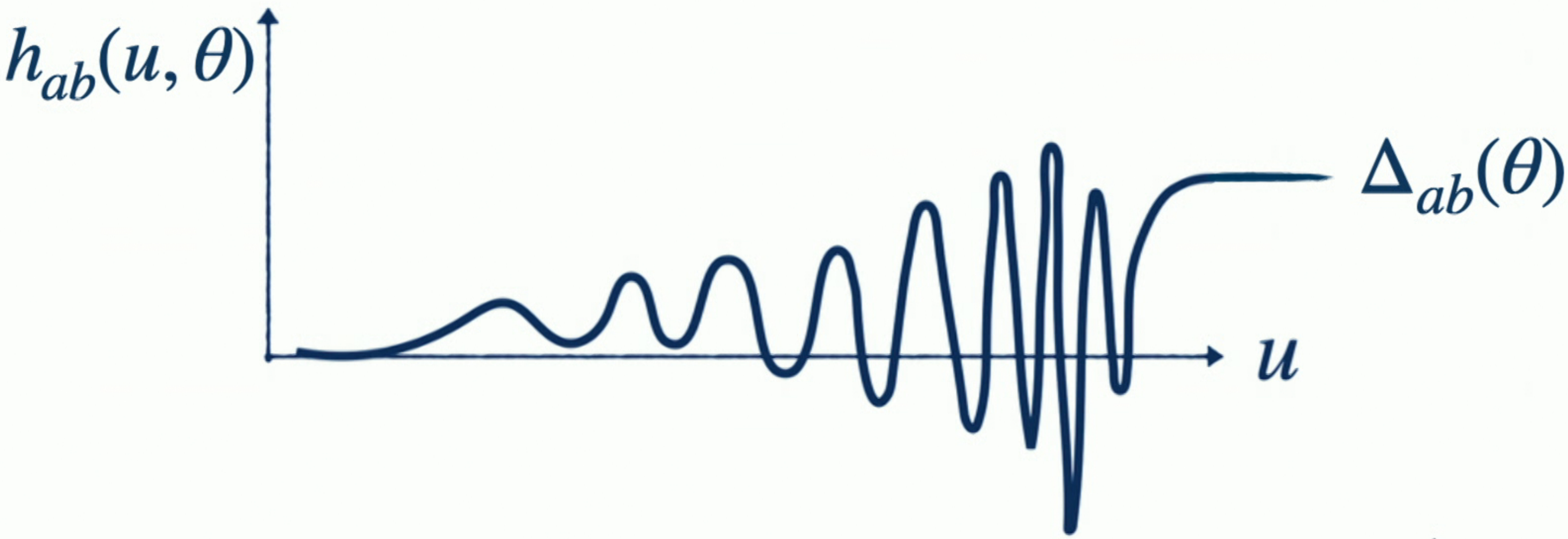
Fock Quantization

- The radiative degrees of freedom at $\mathscr{I}$ of gravity and EM fields can be quantized even in the absence of a "bulk" theory of quantum gravity. [Ashtekar, '87]
- The standard Fock space $\mathscr{F}_0^{\mathscr{I}}$ is constructed from the "one-particle" Hilbert space $\mathscr{H}_0^{\mathscr{I}}$ of gravitons:

$$||h||^2 = 16\pi \int_0^\infty \int_{\mathbb{S}^2} d\omega d\Omega \; \omega |\tilde{h}_{ab}(\omega, \theta)|^2$$

where $\tilde{h}_{ab}(\omega, \theta)$ is Fourier transform of $h_{ab}(u, \theta)$.

Fock Quantization



$$\Delta_{ab} \neq 0 \implies \Delta h_{ab} \neq 0 \implies \tilde{h}_{ab}(\omega,\theta) \sim \frac{1}{\omega}$$

$$\implies ||h||^2 \sim \int_0^\infty \frac{d\omega}{\omega}$$

Fock Quantization

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where $\tilde{h}_{ab}(\omega, \theta)$ is Fourier transform of $h_{ab}(u, \theta)$.

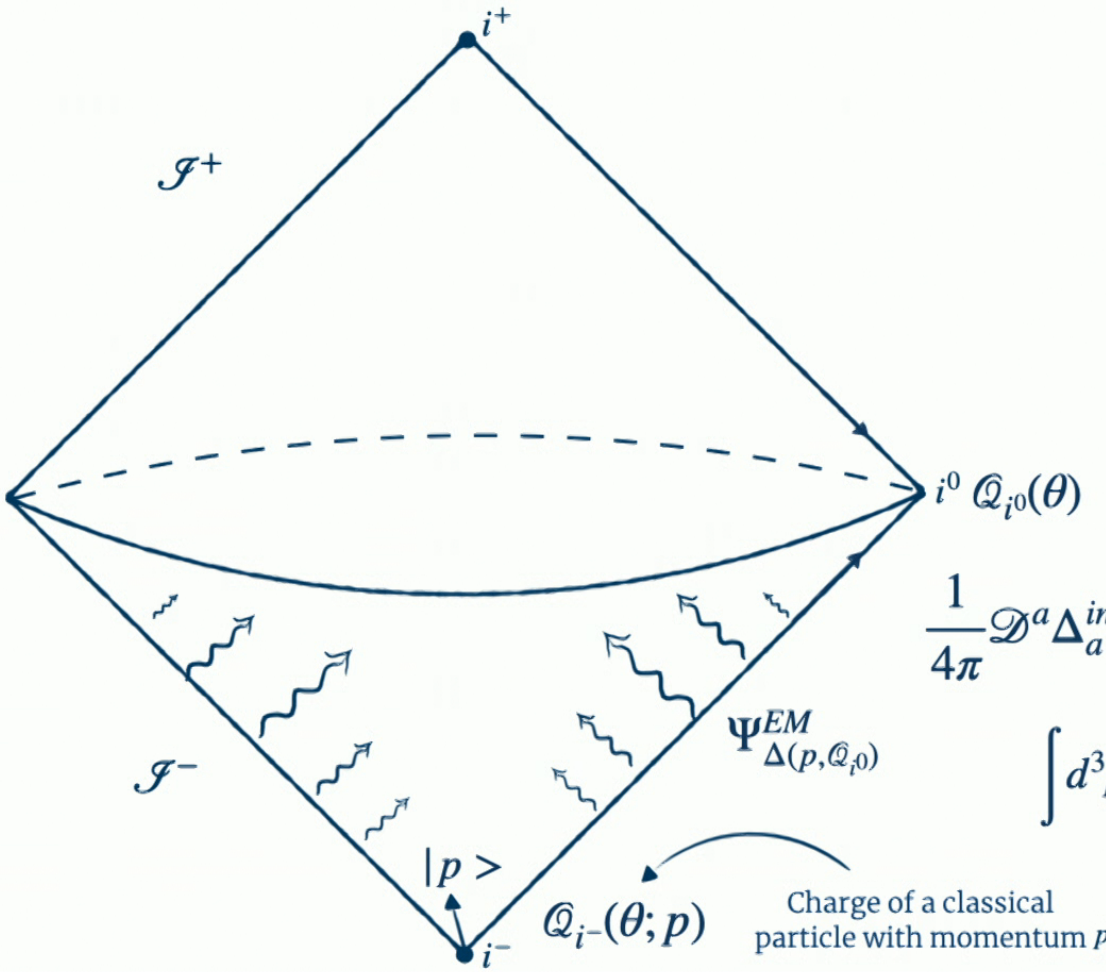
- The Fock space $\mathscr{F}_0^{\mathscr{I}}$ does not contain *any* states with memory. States with memory Δ are elements of a *different* Fock space $\mathscr{F}_\Delta^{\mathscr{I}}$ which is unitarily inequivalent to $\mathscr{F}_0^{\mathscr{I}}$. This is the source of all IR divergences.
- There are an *uncountably infinite* number of "in/out" Fock spaces labeled by all possibles memories $\Delta^{\mathrm{in/out}}$. Memory is not conserved. To go beyond "inclusive cross sections" and have a well-defined S-matrix one needs to include states with memory.

Massive QED - Faddeev-Kulish Hilbert Space

$$\mathcal{Q}_{i^0}(\lambda) = \mathcal{Q}_{i^-}(\lambda) - \frac{1}{4\pi}\int_{\mathbb{S}^2} \Delta_a^{\text{in}} \mathscr{D}^a \lambda$$

- Key Idea: The charge at spatial infinity is conserved. Therefore "in" Hilbert space of eigenstates of the charge $\boldsymbol{\mathcal{Q}}_{i^0}(\lambda)$ with eigenvalue $\mathcal{Q}_{i^0}(\lambda)$ will will map to an "out" Hilbert space of eigenstates with eigenvalue $\mathcal{Q}_{i^0}(\tilde{\lambda})$ [Faddeev & Kulish, 70']...

Massive QED - Faddeev-Kulish Hilbert Space



$$\frac{1}{4\pi}\mathscr{D}^a \Delta_a^{in}(\theta; p, \mathscr{Q}_{i^0}) = \mathscr{Q}_{i^-}(\theta; p) - \mathscr{Q}_{i^0}(\theta)$$

$$\int d^3p \; w(p)\, |p> \otimes \Psi^{EM}_{\Delta(p,\mathscr{Q}_{i^0})}$$

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- Charge eigenstates are states where the "in" electromagnetic memory is correlated with the incoming electrons. This yields a Hilbert space $\mathscr{H}_{\mathcal{Q}_{i^0}}$ of dressed electrons

$$\int_{\mathcal{H}} d^3p \; w(\boldsymbol{p}) \, |\boldsymbol{p}\rangle \otimes \Psi^{\text{EM}}_{\Delta(\boldsymbol{p}, \mathcal{Q}_{i^0})}$$

- $\mathscr{H}_{\mathcal{Q}_{i^0}}$ consists physically reasonable states and yields an IR finite S-matrix.

- This construction fails in *all* other theories including quantum gravity.

Vacuum Gravity - Failure of Faddeev-Kulish Hilbert Space

$$\mathcal{Q}^{\mathrm{GR}}_{i^0}(f) = -\frac{1}{8\pi}\int_{\mathbb{S}^2} \Delta^{\mathrm{in}}_{ab}\mathscr{D}^a\mathscr{D}^b f(\theta) + \int_{\mathscr{I}^-} f(\theta) N^2$$

- The analogous construction in GR is to attempt to correlate the incoming energy flux of the incoming gravitational radiation with the incoming memory.

Theorem

The unique eigenstate of $\mathcal{Q}^{GR}_{i^0}(f)$ is the vacuum with vanishing eigenvalue.

- Intuition: Memory and Energy flux are not independent! In gravity, the gravitational radiation "sources" (i.e. via energy flux) its own memory. Matching the memory to the energy flux introduces more radiation! This introduces more energy flux and so on...

There does not appear to be *any* "preferred" Hilbert space for scattering in QG ("Non-Faddeev-Kulish" representations also fail)

Algebraic Scattering Theory

- The *correlation functions* of all states that arise in scattering theory are perfectly well-defined, they simply do not fit into a single Hilbert space.
- Given any state $|\Psi\rangle$ in a Hilbert space $\mathscr{H}$ one can express that state as a list of correlation functions of operators in an Algebra $\mathscr{A}$. For example,

$$\langle\phi(x)\rangle_\Psi\,, \langle\phi(x_1)\phi(x_2)\rangle_\Psi\,, \dots, \langle\phi(x_1)\dots\phi(x_n)\rangle_\Psi\,, \dots$$

  Conversely, given a list of correlation functions on $\mathscr{A}$ (satisfying commutation relations, positivity, ...) one can construct (by GNS) a Hilbert space where this list of correlation functions is packaged as a vector $|\Psi\rangle \in \mathscr{H}$. Thus viewing a state as a list of correlation functions on $\mathscr{A}$ or as a vector in a Hilbert space are essentially equivalent. [Witten, 2022],[Hollands & Wald, 2014]
- However, by considering states as lists of correlation functions one is now freed from choosing in advance a particular Hilbert space!

Algebraic Scattering Theory

- Given a set of correlation functions Ψ_{in} on the "in" algebra $\mathscr{A}_{\text{in}}$ (i.e. thus specifying the "in" state) then what is the expected value of any "out" observable in $\mathscr{A}_{\text{out}}$ (which would then specify the "out" state)?

$$\langle a_{\text{out}} \rangle_{\Psi_{\text{in}}} \text{ for any } a_{\text{out}} \in \mathscr{A}_{\text{out}}.$$

- To compute $\langle a_{\text{out}} \rangle_{\Psi_{\text{in}}}$ we can use the Heisenberg equations of motion to define an (invertible) map between the "in" and "out" Algebras [Källén, '49]

$$S : \mathscr{A}_{\text{out}} \to \mathscr{A}_{\text{in}} \implies \langle a_{\text{out}} \rangle_{\Psi_{\text{in}}} = \langle S[a_{\text{out}}] \rangle_{\Psi_{\text{in}}}$$

- This construction does not pre-suppose what Hilbert space the "out" state lives in and is therefore manifestly IR finite.
- The (perturbative) formulation of algebraic scattering theory for a massive scalar field coupled to a massless scalar field can be straightforwardly constructed and one can compute the correlation functions of any "out" observables (fields, memory, charges, ...) to any order in perturbation theory [G.S., K. Prabhu, in prep.].

Bad Things Happen to "Good" Scattering Data

- In any gauge theory, the charges $\mathcal{Q}_{i^0}(\lambda)$ have serious implications for coherence. This ultimately comes from the charges "superselect". Any local gauge invariant observable $\mathcal{O}$ commutes with all of the charges

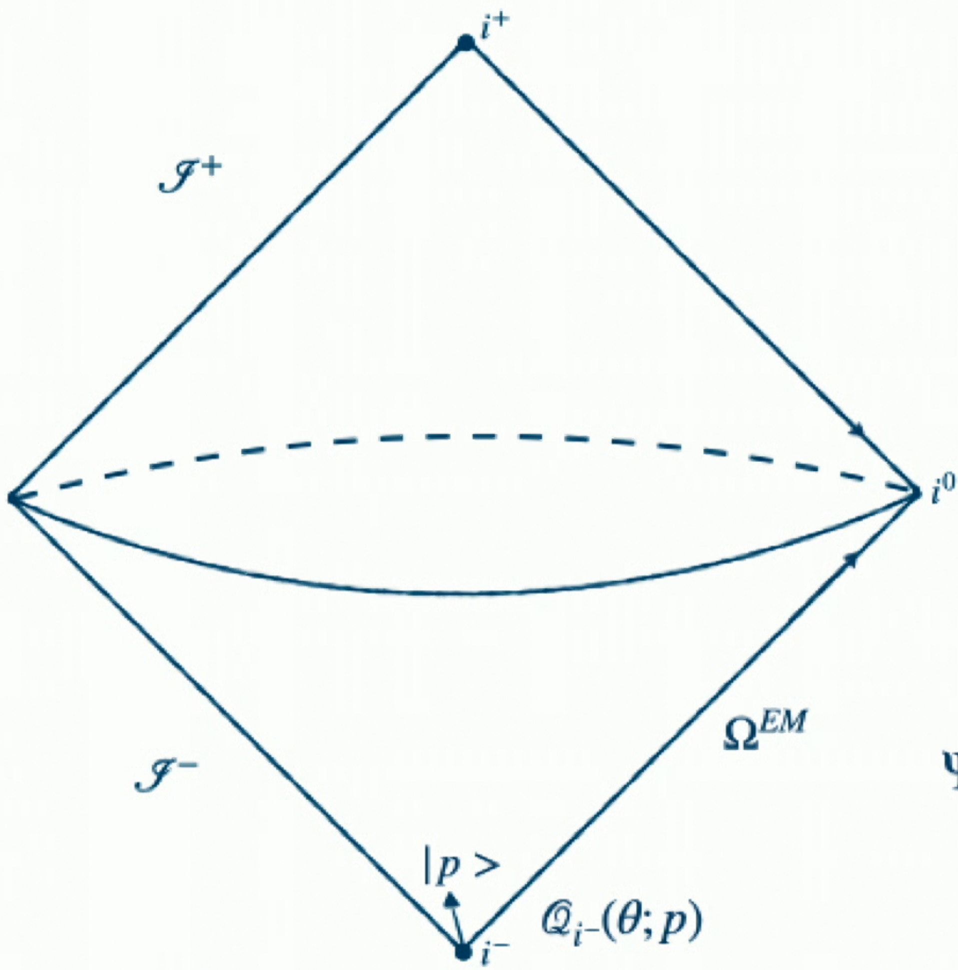
$$[\mathcal{Q}_{i^0}(\lambda), \mathcal{O}] = 0 \text{ for all } \lambda(\theta)$$

- A more familiar case of superselection is the total electric charge $\mathcal{Q}(1)$. Given states Ψ_{q_1} and Ψ_{q_2} with total charge q_1 and q_2, a standard argument shows that the matrix element $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle$ for any local gauge invariant observable $\boldsymbol{O}$ must vanish

$$\langle \Psi_{q_1} | [\mathcal{Q}(1), \boldsymbol{O}] | \Psi_{q_2} \rangle = (q_1 - q_2) \langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$$

Therefore, if $q_1 \neq q_2$ then $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$. In other words, for any local gauge invariant observable $\boldsymbol{O}$, a superposition of Ψ_{q_1} and Ψ_{q_2} is an *incoherent* superposition — these states cannot interfere.

Bad Things Happen to “Good” Scattering Data



$$\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta; p)$$

$$\Psi_f = \int d^3p\, f(p)\, |p> \otimes \Omega^{EM}$$

Bad Things Happen to "Good" Scattering Data

- An incoming electron Ψ_f has definite total electric charge but is a superposition of incoming momenta and therefore superposition of large gauge charges.

- By the same kind of argument that we just used for the total charge, any local gauge invariant observable $\boldsymbol{O}$ *cannot see interference* between the different momentum modes of Ψ_f.

Theorem

The expected value $\langle \boldsymbol{O}(x) \rangle_{\Psi_f}$ is spacetime translation invariant for any gauge invariant observable $\boldsymbol{O}$ *[D. Danielson,* **G.S.** *& R. M. Wald, in prep.]*

Ψ_f does not correspond to a localised electron and is *not* a physical state!

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- Physical states can be obtained by starting with a localised electron in the bulk.

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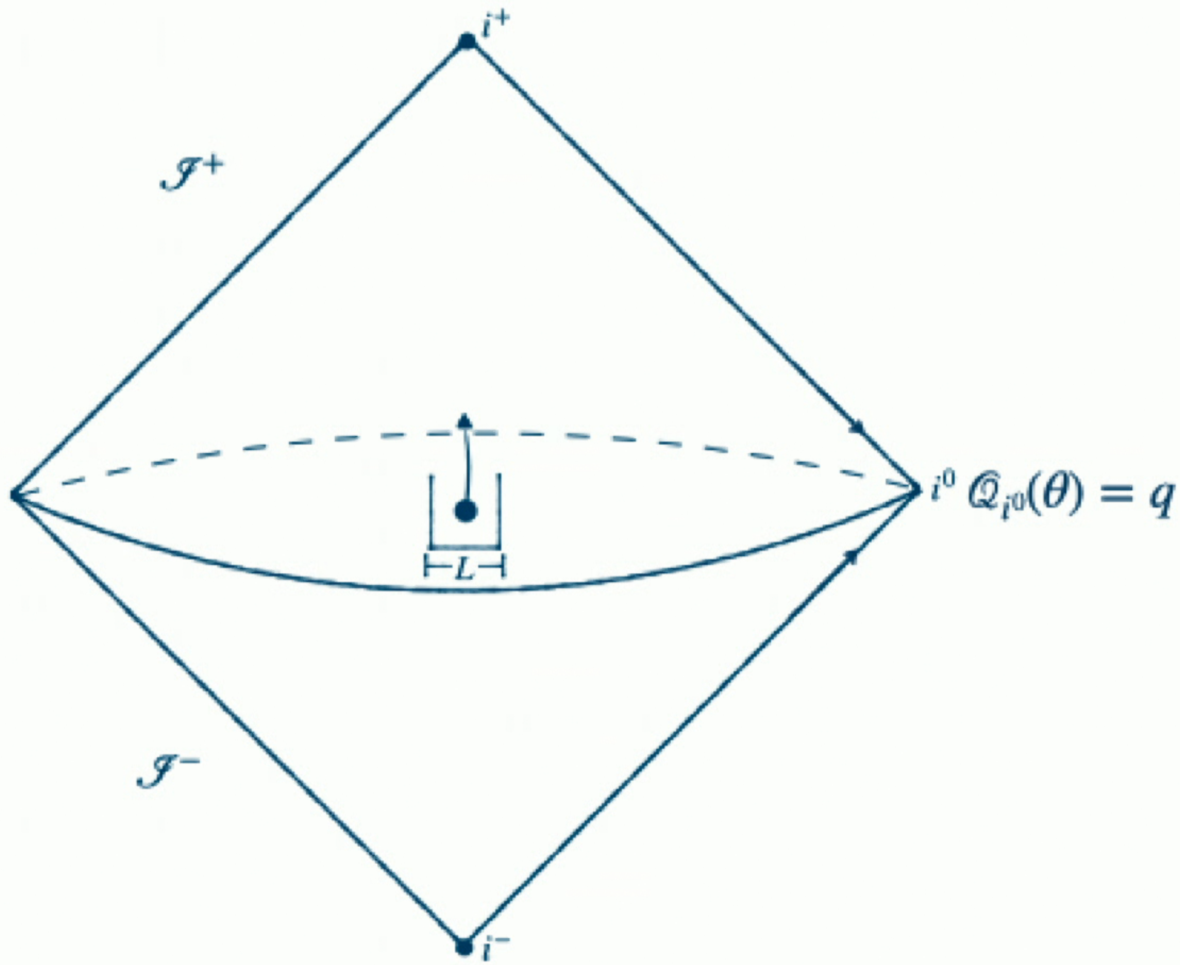

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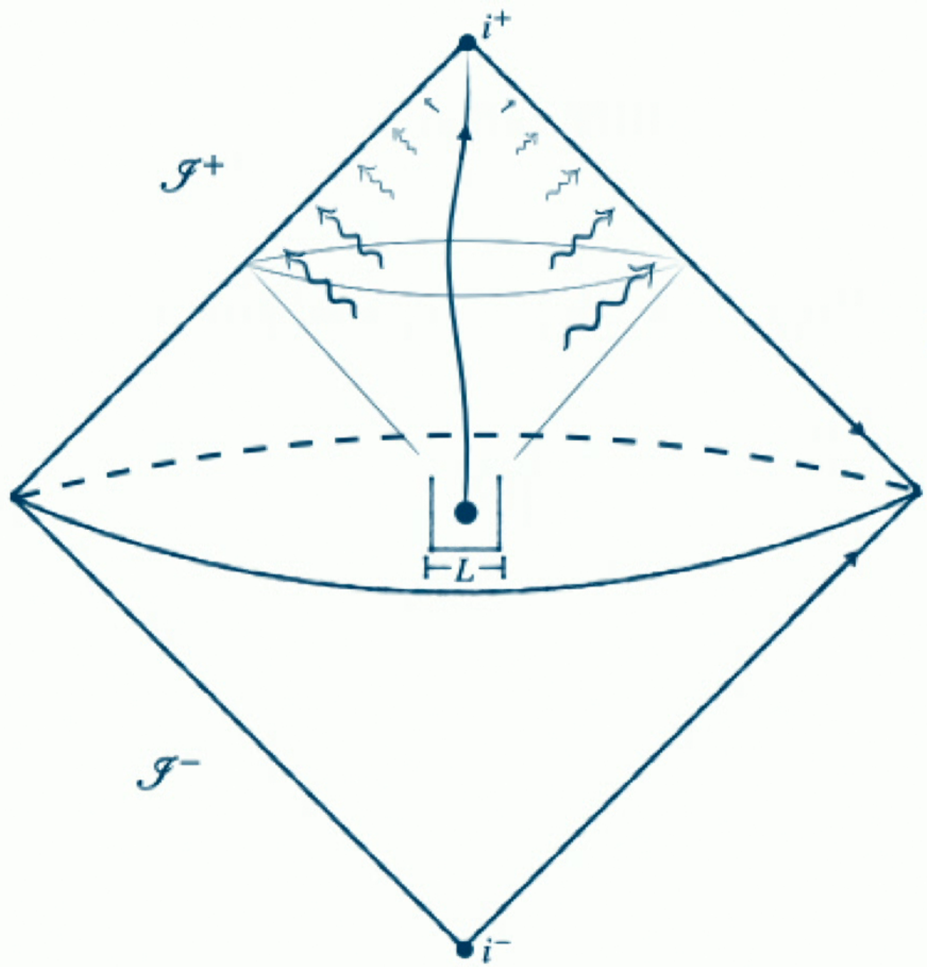

$$\Psi_{FK} = \int d^3p \, f(p) \,|\, p > \otimes \, \Psi^{EM}_{\Delta(q,p)}$$

$$\frac{1}{4\pi} \mathscr{D}^a \Delta_a^{out}(\theta; p, q) = \mathscr{Q}_{i^-}(\theta; p) - q$$

$$\mathscr{Q}_{i^0}(\theta) = q$$

Summary

- IR divergences arise from sticking a state in a Hilbert space to which it doesn't belong.
- In massive QED the Faddeev-Kulish representation is a preferred representation but, as opposed to a "proof of principle" it is actually a "fluke"!
- Non-Kulish-Faddeev representations don't work
- A well-defined (IR-finite) scattering theory can be constructed by simply evolving "in" correlation functions to "out" correlation functions.
- Due to the infrared properties of the theory, the space of asymptotic states in QED (or Yang Mills) which correspond to physical "bulk" states are highly fine-tuned and there are many states that are "junk"!