Title: Research Talk 15 - dS_2 supergravity

Speakers:

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Abstract: I will discuss the interplay between 2d de Sitter and supersymmetry in two concrete supergravity models coupled to superconformal field theories. Upon fixing a supersymmetric analogue of the Weyl gauge these theories can be viewed as supersymmetric extensions of timelike Liouville theory with N=1 and N=2 supersymmetry respectively. Supersymmetric timelike Liouville is well behaved in the UV and combines supersymmetry with a positive cosmological constant and a de Sitter saddle. The theory is amenable to a variety of techniques including systematic loop expansions and localization methods.

# $dS_2$ supergravity

Strings 2023

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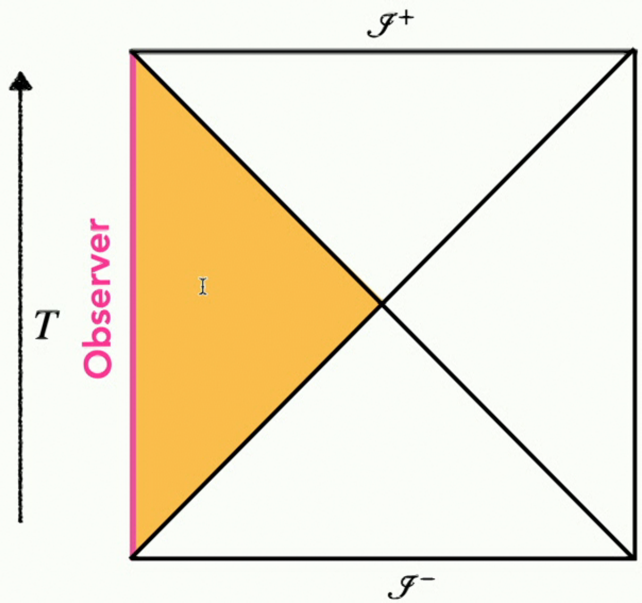
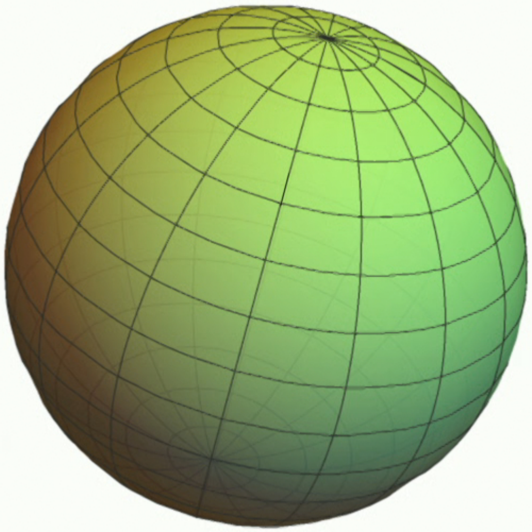
# de Sitter space

Our Universe is expanding at an accelerated rate driven by positive cc → asymptotically $dS_4$ Universe




Penrose diagram of $dS_4$ ⟶ Euclidean dS is the sphere



1

# Why $dS_2$?

- Irreps of $dS_2$ isometry group $SO(2,1)$ similar to those of $dS_4$ isometry group $SO(4,1)$
- $dS_2$ has a non-trivial Gibbons-Hawking de Sitter entropy/ non-trivial path integral
- $dS_2 \times S^2$ (=Nariai geometry) is a solution of 4d gravity with positive cc
- Recent progress on euclidean 2d BH and $AdS_2$
  [Saad-Shenker-Stanford,…,Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini,…]

# dS entropy?

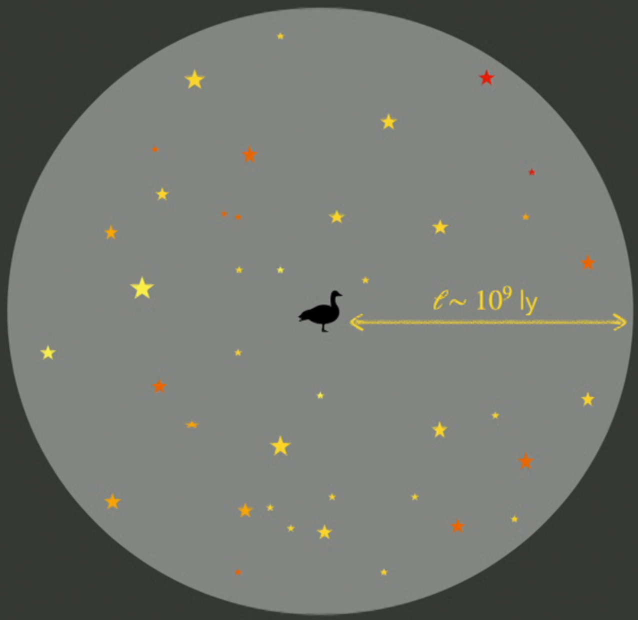
- Gibbons-Hawking conjecture $e^{S_{dS}} = \mathcal{Z}_{\text{grav}} = \sum_{\mathcal{M}\ compact} \int [\mathcal{D}g]\, e^{-S_{EH}[\Lambda, g, \mathcal{M}]} Z_{matter}[\mathcal{M}, g] \approx e^{\frac{A}{4G}} + \dots$

- For BH evidence that $S_{BH}$ is a counting problem
  [Bekenstein-Hawking,…, Strominger-Vafa,…]

- Nature of $S_{dS}$ ?

- SUSY leads to superstrings, AdS/CFT, BPS states for BH,…

- SUSY as a tool to understand dS entropy?




$$S_{dS} \approx \frac{A}{4G} + \dots \approx 10^{122}$$

3

# SUGRA & dS

d>2

d=2

SUSY extension of $\mathfrak{so}(d,1)$ + unitary rep. does not exist
[Pilch-van Nieuwenhuizen-Sohnius, Lukierski-Novicki, …]

→ Susy extension of $\mathfrak{so}(2,1)$ (isometry algebra of $AdS_2$ and $dS_2$) + unitary rep. exist
[Lukierski-Novicki,…]

Super-conformal field theory okay with dS
[Anous-Freedman-Maloney,…]

→ Same holds in 2d

No-go's for classical embedding in superstrings
[Gibbons, Maldacena-Nuñez…]

→ Consider 2d as a UV finite theory on its own

Non-linearly realised dS-SUGRA (Volkov-Akulov)
[Bergshoeff-Freedman-Kallosh-van Proeyen,…]

→ Not known in 2d

4

# Two concrete models

$\mathcal{N} = 1$ $dS_2$ supergravity: $\mathcal{N} = 1$ SUGRA + $\mathcal{N} = 1$ SCFT

$\mathcal{N} = 2$ $dS_2$ supergravity: $\mathcal{N} = 2$ SUGRA + $\mathcal{N} = 2$ SCFT

# $\mathcal{N}=1$ $dS_2$ supergravity

- 2d $\mathcal{N}=1$ gravity multiplet $(\mathrm{e}^a_\mu, \chi_\mu, A)$ : zweibein, spin 3/2 Majorana gravitino, real scalar

- $\mathcal{N}=1$ SUGRA + $\mathcal{N}=1$ SCFT on compact surface $\Sigma_h$ of genus $h$

$$\mathcal{Z}^{\mathcal{N}=1}_{\text{grav}} = \sum_{h=0}^{\infty} e^{\vartheta(2-2h)} \int [\mathcal{D}\mathrm{e}^a_\mu][\mathcal{D}\chi_\mu][\mathcal{D}A]\, e^{-\int_{\Sigma_h} d^2x\, \mathrm{e}\left(-4\mu A + \mu \bar{\chi}_\mu \gamma^{\mu\nu} \chi_\nu\right)} \times Z^{(h)}_{\text{SCFT}}[\mathrm{e}^a_\mu, \chi_\mu, A]$$

- $Z^{(h)}_{\text{SCFT}}[\mathrm{e}^a_\mu, \chi_\mu, A]$ is the genus $h$ partition function of a SCFT with central charge $c_m$

- Absence of gravitino kinetic term $\bar{\chi}_\mu \gamma^{\mu\nu\rho} D_\nu \chi_\rho$ in 2d

- Focus on $h=0$ in this talk

# super-Weyl gauge

- super-Weyl gauge: $\mathrm{e}_\mu^a = e^{b\varphi}\,\tilde{\mathrm{e}}_\mu^a\,,\qquad \chi_\mu = e^{\frac{1}{2}b\varphi}\gamma_\mu\psi\,,\qquad A = e^{-b\varphi}F$
- $\tilde{\mathrm{e}}_\mu^a$ background metric on round $S^2$ with radius $r$
- Chiral multiplet $(\varphi, \psi, F)$: real scalar, Majorana spin 1/2 fermion, real scalar $F$
- $Z_{\text{SCFT}}^{(0)}$ is determined by superconformal anomaly ($\mathcal{N} = 1$ analogue of Polyakov action)
- SUGRA with positive cc sector leads to $\mathcal{N} = 1$ super-Liouville [Distler-Hlousek-Kawai,…]

$$\mathcal{Z}_{\text{grav},(0)}^{\mathcal{N}=1} = e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{(c_m + c_{\mathfrak{bc}} + c_{\beta\gamma})/3} \times \int \frac{[\mathcal{D}\varphi][\mathcal{D}\psi]}{\text{vol}_{OSp(1|2;\mathbb{C})}} e^{-\mathcal{S}_L^{\mathcal{N}=1}}$$

$$\mathcal{S}_L^{\mathcal{N}=1} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{Q\varphi}{r^2} + \Lambda\, e^{2b\varphi} - \frac{i}{2}\overline{\psi}\not{D}\psi + \frac{i}{2}\sqrt{\Lambda}\, b e^{b\varphi}\overline{\psi}\psi\right)$$

- $OSp(1\,|\,2,\mathbb{C})$ residual gauge group on $S^2$

# $\mathcal{N} = 1$ super-Liouville

$$S_L^{\mathcal{N}=1} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{i}{2}\overline{\psi}\not{D}\psi + \Lambda e^{2b\varphi} + \frac{Q\varphi}{r^2} + \frac{i}{2}\sqrt{\Lambda}\, be^{b\varphi}\overline{\psi}\psi\right)$$

$\mathcal{N} = 1$ super-Liouville is a 2d SCFT with $Q = b + b^{-1}$ and $c_L^{\mathcal{N}=1} = 3/2 + 3Q^2$

Anomaly cancellation: $c_L^{\mathcal{N}=1} + c_m + c_{\mathfrak{bc}} + c_{\beta\gamma} = 0$


Spacelike super-Liouville:

- Positive cc
- For $c_m \to -\infty$ complex $S^2$ saddle
- For SCFT = super minimal model, non-perturbative completion known [Seiberg-Shih,…]

Timelike super-Liouville:

- Positive cc
- For $c_m \to \infty$ **real** $S^2$ saddle

# $\mathcal{N} = 1$ super-Liouville

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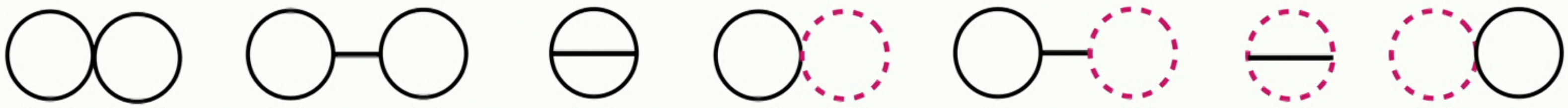
7

# Non-Gaussian fluctuations

- Systematic higher loop on top of $dS_2$ saddle



- Two-loop on $S^2$: all UV divergences cancel

$$\boxed{\mathcal{Z}^{\mathcal{N}=1}_{\text{grav},(0)} = e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{\frac{c_m + c_{\mathfrak{bc}} + c_{\beta\gamma}}{3}} \times \int \frac{[\mathcal{D}\varphi][\mathcal{D}\psi]}{\text{vol}_{OSp(1|2;\mathbb{C})}} e^{-\mathcal{S}^{\mathcal{N}=1}_{tL}} = e^{2\vartheta} \times \left(\frac{1}{\Lambda\, \ell_{\text{uv}}^2}\right)^{\left(\frac{c_m}{6} - \frac{7}{4} + \ldots\right)} \times f_0(c_m)}$$

- $\log \mathcal{Z}^{\mathcal{N}=1}_{\text{grav},0}$ has structure of 2d entanglement entropy
[Cardy-Calabrese, Casini-Huerta-Myers, Holzhey-Larsen-Wilczek,…]

# $\mathcal{N} = 2$ timelike super-Liouville

In super-Weyl gauge we obtain a theory of a chiral + anti chiral multiplet [Antoniadis-Bachas-Kounnas,…]

$$\mathcal{Z}^{\mathcal{N}=2}_{\text{grav},(0)} \approx e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{(c_m + c_{\mathfrak{bc}} + 2c_{\beta\gamma} + c_{U(1)})/3} \times \int \frac{[\mathscr{D}\Phi][\mathscr{D}\widetilde{\Phi}]}{\text{vol}_{OSp(2|2;\mathbb{C})}} e^{-\mathcal{S}^{\mathcal{N}=2}_{tL}}$$

$$\mathcal{S}^{\mathcal{N}=2}_{tL} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left( -\partial_\mu \widetilde{\varphi}\,\partial^\mu \varphi + i\,\overline{\widetilde{\psi}}\,\not{D}\psi + \beta^2|\lambda|^2 e^{\beta(\varphi+\widetilde{\varphi})} - \frac{1}{\beta r^2}\left(\varphi + \widetilde{\varphi}\right) + \frac{\beta^2}{2}\left(\lambda e^{\beta\varphi}\,\overline{\psi}\psi + \lambda^* e^{\beta\widetilde{\varphi}}\,\overline{\widetilde{\psi}}\,\widetilde{\psi}\right)\right), \quad \lambda \in \mathbb{C}$$

Timelike super-Liouville is a supersymmetric non-unitary SCFT with $c_{tL} = 3 - 6q^2$ , $\quad q = 1/\beta$

Vanishing conformal anomaly: $q = \dfrac{1}{\beta} = \sqrt{\dfrac{c_m - 3}{6}}$

$c_m = 3$

Spacelike super-Liouville

Timelike super-Liouville

13

# Gravity path integral

With regards to the $\mathcal{N} = 2$ SUGRA… … it has a positive cc

… the EOM admit semiclassical $dS_2$ vacua for $c_m \to \infty$

… is well behaved in the UV

We can also study one- and two-loop contributions leading to

$$\boxed{S_{dS} = \log \mathcal{Z}^{\mathcal{N}=2}_{\text{grav},(0)} = 2\theta - \left(\frac{c_m}{6} - \frac{1}{2}\right) \log\left(|\lambda|^2 \ell_{\text{uv}}^2\right) + f_0(c_m)\,, \quad f_0(c_m) \quad \text{are non-trivial}}$$

Structure of 2d entanglement entropy [Cardy-Calabrese, Casini-Huerta-Myers, Holzhey-Larsen-Wilczek,…]

# SUSY localization

- $\mathcal{N} = 2$ theory on $S^2$ amenable to SUSY localization
  [Benini-Cremonesi, Doroud-Gomis-Le Floch-Lee,…]

- Combine Gibbons-Hawking proposal with SUSY localization?
  [Anninos-Galante-BM,…]

- Localization leads to an effective reduction in the dof that are integrated over

- Finiteness of dS Hilbert space?
  [Banks, Fischler, Bousso, Parikh-Verlinde, Susskind,…]

# Localization for Liouville?

- Add a $\mathcal{Q}$-exact term $t\mathcal{Q}V$ with $\mathcal{Q}V|_{bos} > 0, t \in \mathbb{R}$

$$Z(t) = \int_{\mathscr{E}} [\mathscr{D}\Phi]\, e^{-S-t\mathcal{Q}V} \longrightarrow Z'(t) = -\int_{\mathscr{E}} [\mathscr{D}\Phi]\, \mathcal{Q}\left(V e^{-S-t\mathcal{Q}V}\right) \begin{cases} = 0 & \text{in most cases} \\ \neq 0 & \text{in the presence of bdy terms in field space} \end{cases}$$

- Independence of $t$ allows to evaluate for $t \to \infty$ on BPS solutions of $\mathcal{Q}V = 0$

- Liouville action is $\mathcal{Q}$-exact and hence naive localization predicts trivial partition function

- $\mathcal{N} = 2$ Liouville localizes onto boundary terms in field space
  [Hori-Kapustin,…]

- Compatible with idea of Polchinski for non-supersymmetric Liouville (strings 1990)

# Summary

$\mathcal{N} = 1$ SUGRA + $\mathcal{N} = 1$ SCFT

$\downarrow$ super-Weyl gauge

$\mathcal{N} = 1$ Liouville + $\mathcal{N} = 1$ SCFT

$\mathcal{N} = 2$ SUGRA + $\mathcal{N} = 2$ SCFT

$\downarrow$ super-Weyl gauge

$\mathcal{N} = 2$ Liouville + $\mathcal{N} = 2$ SCFT

spacelike regime

timelike regime

Non-perturbative completion known

positive cc

For $c_m \to \infty$ $\mathbf{dS}_2$ saddle

Localization onto bdy terms in field space

systematic loop expansion

**THANK YOU!**