Title: Research Talk 6 - Soft theorems: symmetry and geometry

Speakers: Julio Parra Martinez

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Soft theorems: Symmetry & Geometry

Julio Parra-Martinez

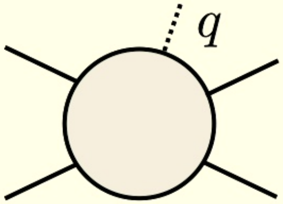
Based on work with:

Berean-Dutcher, Cheung, Derda, Helset

@ Strings 2023, Perimeter Insitute, July 2023

Soft theorems

- The behavior of scattering amplitudes when the momentum of a particle is small is often universal



$$\lim_{q\to 0} A_{n+1} = \mathcal{S} A_n$$

- Earliest example: Soft photon theorem [Low; Burnett, Kroll; Weinberg]

$$\lim_{q_\gamma \to 0} A_{n+1} = \sum_a \frac{q_a}{q \cdot p_a} \left[\underset{\text{“leading”}}{\epsilon \cdot p_a} + \underset{\text{“subleading”}}{\epsilon \cdot J_a \cdot q} \right] A_n$$

similar for soft gluons, gravitons, pions

Perspectives on soft theorems

Symmetry

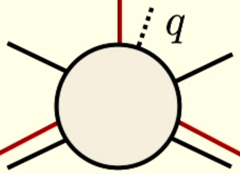
“Soft theorems are a consequence of symmetry”

e.g. asymptotic (photons, gravitons), spontaneously broken (pions)

Effective field theory

“Soft theorems are EFT of long-wavelenght modes”

e.g. factorization, ultrasoft decoupling in SCET



Geometry

This talk

Soft Nambu-Goldstone Bosons

- Spontaneous symmetry breaking implies existence of NGB

$$\pi^a \to \pi^a + v^a + \mathcal{O}(\pi^2) \qquad\qquad j^a_\mu = \partial_\mu \pi^a + \mathcal{O}(\pi^2)$$

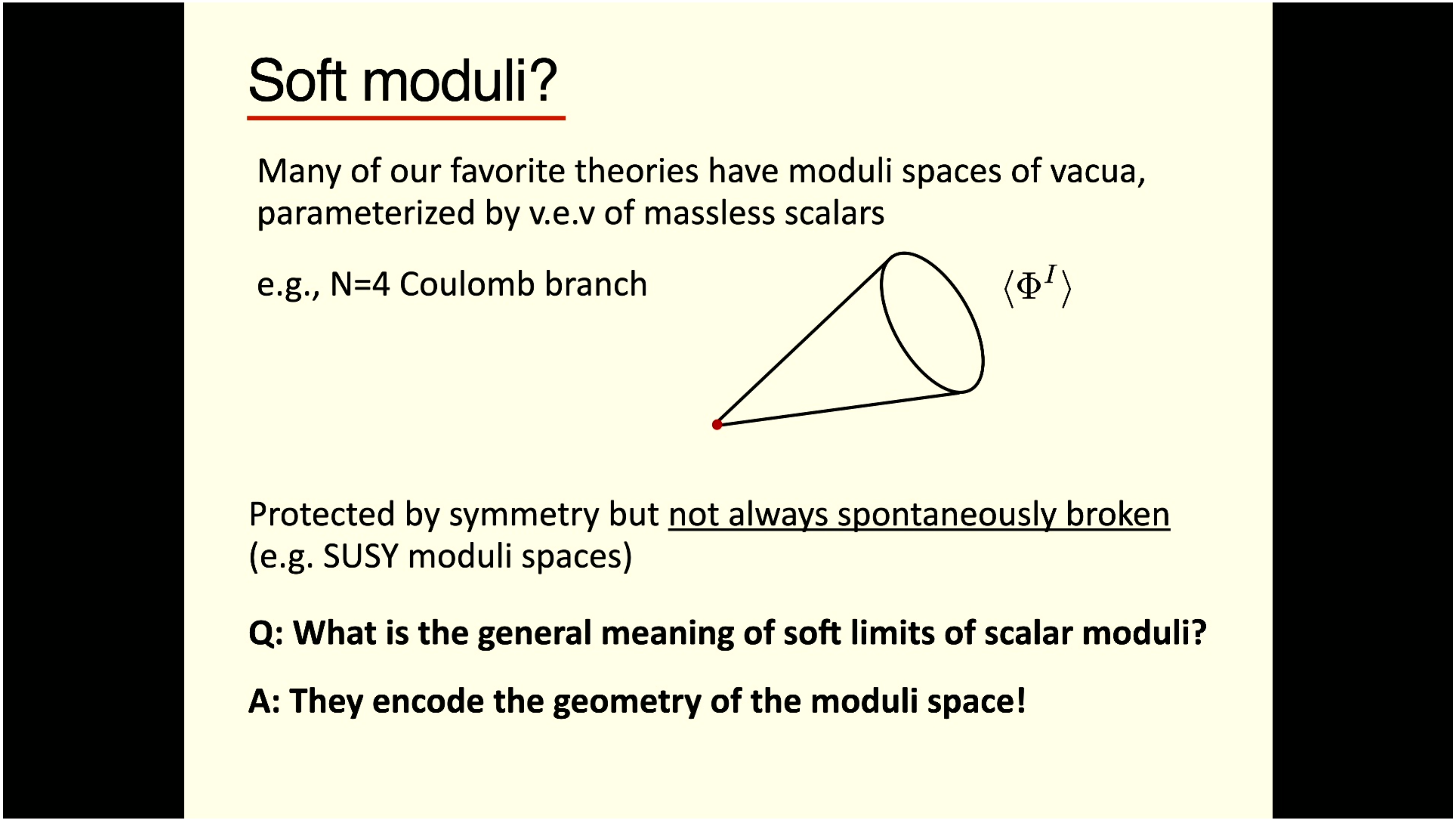
- Also current algebra constrains their dynamics via soft theorems

$$\partial \cdot j^a(x) j^b_\mu(y) \sim f^{abc}\delta(x-y) j^c_\mu(y)$$

⬇

$$\lim_{q_\pi \to 0} A_{n+1} = 0 \qquad \text{“Adler zero”}$$

$$\lim_{q_{\pi^a}, q_{\pi^b} \to 0} A_{n+2}^{i_1 \cdots i_n i_a i_b} = \frac{1}{2} \sum_{c \neq a,b} \frac{s_{ac} - s_{bc}}{s_{ac} + s_{bc}} [\mathcal{X}^a, \mathcal{X}^b]^{i_c}{}_{j_c} A_n^{i_1 \cdots j_c \cdots i_n}$$



Soft moduli?

Many of our favorite theories have moduli spaces of vacua, parameterized by v.e.v of massless scalars

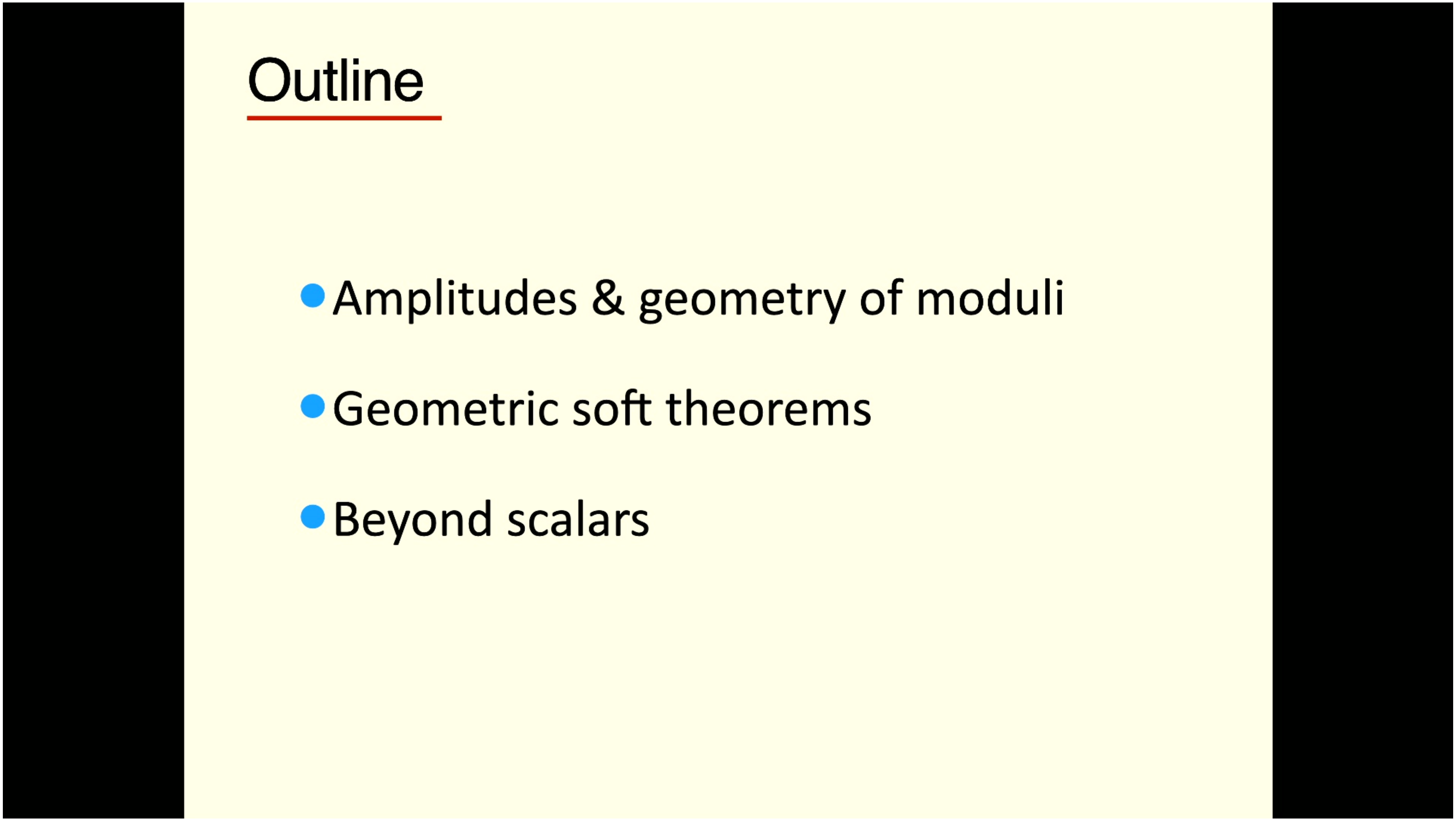
e.g., N=4 Coulomb branch

$\langle \Phi^I \rangle$

Protected by symmetry but <u>not always spontaneously broken</u> (e.g. SUSY moduli spaces)

Q: What is the general meaning of soft limits of scalar moduli?

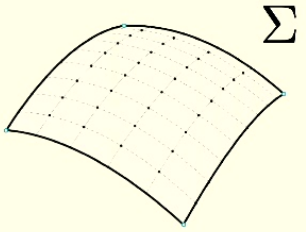
A: They encode the geometry of the moduli space!



Outline

- Amplitudes & geometry of moduli
- Geometric soft theorems
- Beyond scalars

Review: Geometry of fields

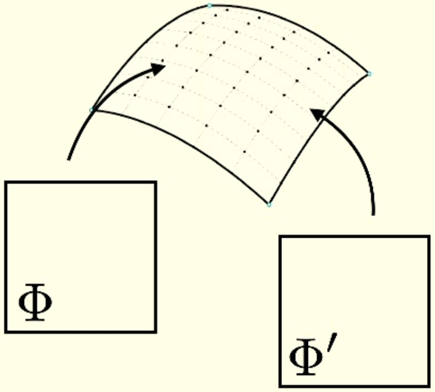


- Scalar fields take values in a target space manifold
- Lagrangian can be organized by derivative order

$$\tfrac{1}{2}g_{IJ}(\Phi)\partial_\mu\Phi^I\partial^\mu\Phi^J - V(\Phi) + \tfrac{1}{4}\lambda_{IJKL}(\Phi)\partial_\mu\Phi^I\partial^\mu\Phi^J\partial_\nu\Phi^K\partial^\nu\Phi^L + \cdots ,$$

- Field redefinitions = changes of coordinates $\Phi^I = \Phi^I(\Phi')$

$$\partial_\mu\Phi^I \quad \to \quad \frac{\partial\Phi'^I}{\partial\Phi^J}\partial_\mu\Phi^J$$



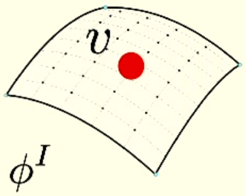
- Couplings are tensors in the target space

  e.g. two-derivative = metric

$$g_{IJ}(\Phi) \quad \to \quad \frac{\partial\Phi^K}{\partial\Phi'^I}\frac{\partial\Phi^L}{\partial\Phi'^J}g_{KL}(\Phi')$$

Familiar from world-sheet, but more general for EFT of moduli in D>2.

Geometry of amplitudes

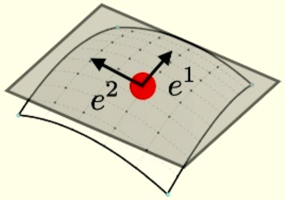


- Amplitudes defined by expanding around VEV $\Phi^I = v^I + \phi^I$
- Do not depend on field basis $\phi \to \phi + \epsilon f(\phi)$

$$S(\phi) \to S(\phi) + \frac{\delta S}{\delta \phi}\epsilon f(\phi) + \cdots$$

equations of motion

$$\propto p^2 - m^2 + \cdots$$



but on a choice of frame $\langle p^i | \phi^J(x) | 0 \rangle = e^{iJ}(v) e^{ip\cdot x}$

wavefunction ren.

- Must be a function of geometric invariants on $T\Sigma$!

e.g. curvature of metric connection on $T\Sigma$ $R^{ijkl}(v)$

[Volkov; Dixon, Kaplunovsky, Louis] $\nabla^m R^{ijkl}(v)$

Examples at tree level

- **Two-derivative** $\frac{1}{2}g_{IJ}(\Phi)\partial_\mu\Phi^I\partial^\mu\Phi^J$

$$\begin{aligned}
A_4^{i_1i_2i_3i_4} &= R^{i_1i_3i_2i_4}s_{34} + R^{i_1i_2i_3i_4}s_{24},\\
A_5^{i_1i_2i_3i_4i_5} &= \nabla^{i_3}R^{i_1i_4i_2i_5}s_{45} + \nabla^{i_4}R^{i_1i_3i_2i_5}s_{35} + \nabla^{i_4}R^{i_1i_2i_3i_5}s_{25}\\
&\quad + \nabla^{i_5}R^{i_1i_3i_2i_4}s_{34} + \nabla^{i_5}R^{i_1i_2i_3i_4}(s_{24}+s_{45}),\\
A_6^{i_1i_2i_3i_4i_5i_6} &= -\tfrac{1}{72}\left(R^{i_1i_3i_2j}s_{12} + R^{i_1i_2i_3j}s_{13}\right)\frac{1}{s_{123}}\left(R_j{}^{i_6i_5i_4}s_{46} + R_j{}^{i_5i_6i_4}s_{45}\right)\\
&\quad + \tfrac{1}{108}\left(R^{i_1i_3i_2j}(s_{12}-\tfrac{1}{6}s_{123}) + R^{i_1i_2i_3j}(s_{13}-\tfrac{1}{6}s_{123})\right)\left(R_j{}^{i_6i_5i_4} + R_j{}^{i_5i_6i_4}\right)\\
&\quad + \tfrac{1}{90}R^{i_1i_6i_5j}R_j{}^{i_2i_3i_4}s_{13} + \tfrac{1}{80}\nabla^{i_6}\nabla^{i_5}R^{i_1i_2i_3i_4}s_{13} + \text{perm}\,.
\end{aligned}$$

- **Four-derivative** $\lambda_{IJKL}(\Phi)\partial_\mu\Phi^I\partial^\mu\Phi^J\partial_\nu\Phi^K\partial^\nu\Phi^L$

$$\begin{aligned}
A_{4,\lambda}^{i_1i_2i_3i_4} =\ & \tfrac{1}{2}s_{12}s_{34}\lambda^{i_1i_2i_3i_4} + \tfrac{1}{2}s_{13}s_{24}\lambda^{i_1i_3i_2i_4} + \tfrac{1}{2}s_{23}s_{14}\lambda^{i_2i_3i_1i_4}\,,\\
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\end{aligned}$$

New soft scalar theorem

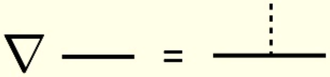
$$\lim_{q\to 0} A_{n+1}^{i_1\cdots i_n i} = \nabla^i A_n^{i_1\cdots i_n} + \sum_{a=1}^{n} \frac{\nabla^i V^{i_a}{}_{j_a}}{(p_a+q)^2 - m_{j_a}^2}\left(1 + q^\mu \frac{\partial}{\partial p_a^\mu}\right) A_n^{i_1\cdots j_a\cdots i_n}$$

subleading

leading

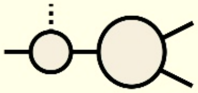
Intuition:

$$\lim_{q\to 0} A_{n+1} \sim \left(\nabla + \frac{\nabla m^2}{p^2 - m^2}\right) A_n$$



“Derivative w.r.t. VEV”

“on-shell connection”



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&\quad + \nabla^{i_5} R^{i_1 i_3 i_2 i_4} s_{34} + \nabla^{i_5} R^{i_1 i_2 i_3 i_4} (s_{24} + s_{45}), \\
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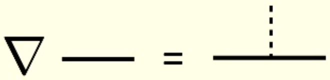

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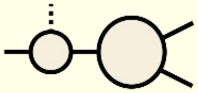

Examples

- Two derivatives

$$
\begin{aligned}
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&\quad + \nabla^{i_5} R^{i_1 i_3 i_2 i_4} s_{34} + \nabla^{i_5} R^{i_1 i_2 i_3 i_4} (s_{24} + \cancelto{0}{s_{45}}) \\
&= \nabla^{i_5} (R^{i_1 i_3 i_2 i_4} s_{34} + R^{i_1 i_2 i_3 i_4} s_{24})
\end{aligned}
$$

- Four derivatives

$$
\begin{aligned}
A_{4,\lambda}^{i_1 i_2 i_3 i_4} &= \tfrac{1}{2} s_{12} s_{34} \lambda^{i_1 i_2 i_3 i_4} + \tfrac{1}{2} s_{13} s_{24} \lambda^{i_1 i_3 i_2 i_4} + \tfrac{1}{2} s_{23} s_{14} \lambda^{i_2 i_3 i_1 i_4}, \\
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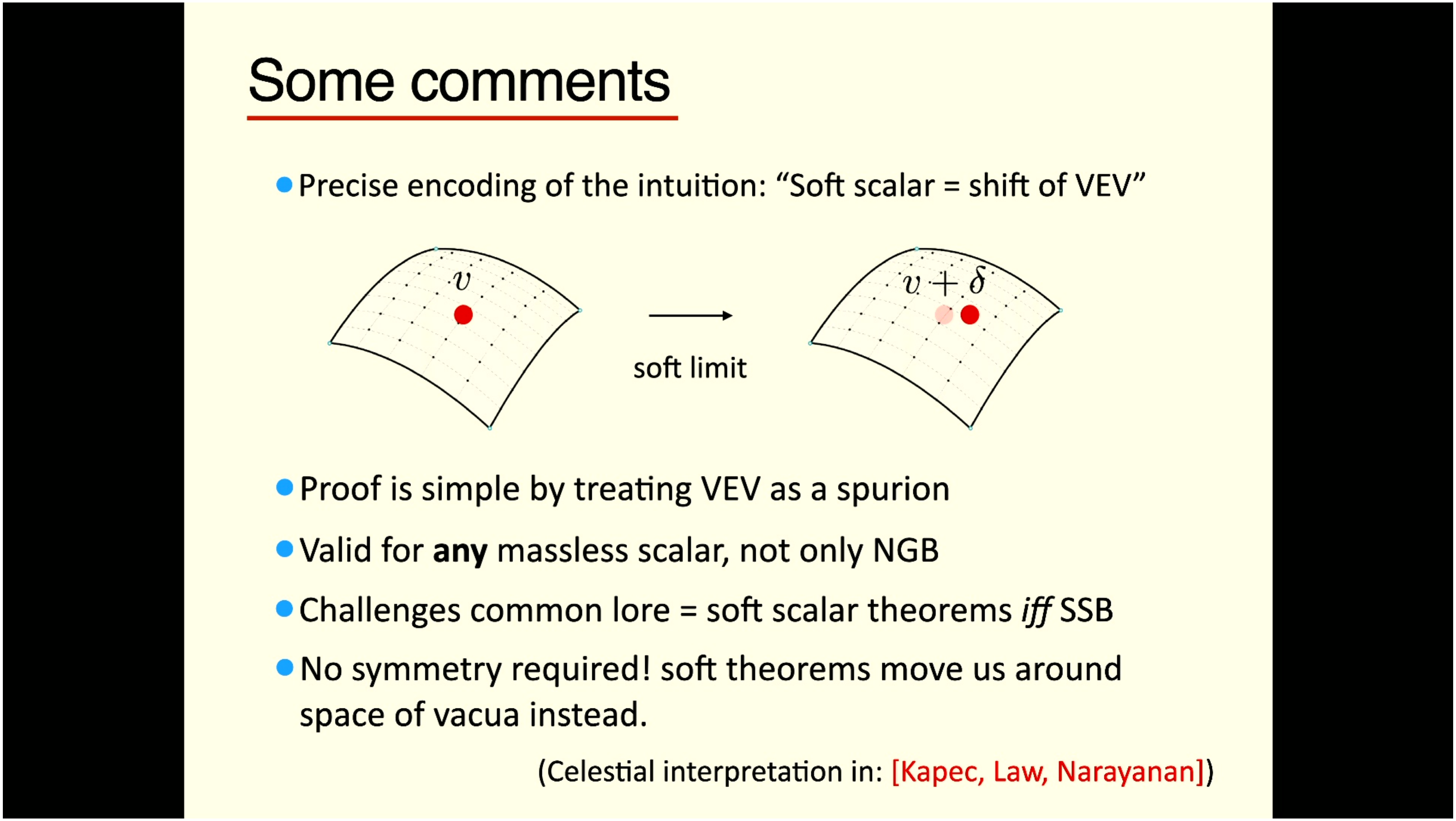
- Four derivatives

$$\begin{aligned} A_{4,\lambda}^{i_1 i_2 i_3 i_4} &= \tfrac{1}{2} s_{12} s_{34} \lambda^{i_1 i_2 i_3 i_4} + \tfrac{1}{2} s_{13} s_{24} \lambda^{i_1 i_3 i_2 i_4} + \tfrac{1}{2} s_{23} s_{14} \lambda^{i_2 i_3 i_1 i_4}, \\ A_{5,\lambda}^{i_1 i_2 i_3 i_4 i_5} &= \frac{1}{2} s_{12} s_{34} \nabla^{i_5} \lambda^{i_1 i_2 i_3 i_4} + \frac{1}{2} s_{13} s_{24} \nabla^{i_5} \lambda^{i_1 i_3 i_2 i_4} + \frac{1}{2} s_{23} s_{14} \nabla^{i_5} \lambda^{i_2 i_3 i_1 i_4} \\ &+ \frac{1}{2} s_{23} s_{45} \nabla^{i_1} \lambda^{i_2 i_3 i_4 i_5} + \frac{1}{2} s_{24} s_{35} \nabla^{i_1} \lambda^{i_2 i_4 i_3 i_5} + \frac{1}{2} s_{34} s_{25} \nabla^{i_1} \lambda^{i_3 i_4 i_2 i_5} \\ &+ \frac{1}{2} s_{13} s_{45} \nabla^{i_2} \lambda^{i_1 i_3 i_4 i_5} + \frac{1}{2} s_{14} s_{35} \nabla^{i_2} \lambda^{i_1 i_4 i_3 i_5} + \frac{1}{2} s_{34} s_{15} \nabla^{i_2} \lambda^{i_3 i_4 i_1 i_5} \\ &+ \frac{1}{2} s_{12} s_{45} \nabla^{i_3} \lambda^{i_1 i_2 i_4 i_5} + \frac{1}{2} s_{14} s_{25} \nabla^{i_3} \lambda^{i_1 i_4 i_2 i_5} + \frac{1}{2} s_{24} s_{15} \nabla^{i_3} \lambda^{i_2 i_4 i_1 i_5} \\ &+ \frac{1}{2} s_{12} s_{35} \nabla^{i_4} \lambda^{i_1 i_2 i_3 i_5} + \frac{1}{2} s_{13} s_{25} \nabla^{i_4} \lambda^{i_1 i_3 i_2 i_5} + \frac{1}{2} s_{23} s_{15} \nabla^{i_4} \lambda^{i_2 i_3 i_1 i_5} \end{aligned}$$



Some comments

- Precise encoding of the intuition: “Soft scalar = shift of VEV”
- Proof is simple by treating VEV as a spurion
- Valid for **any** massless scalar, not only NGB
- Challenges common lore = soft scalar theorems *iff* SSB
- No symmetry required! soft theorems move us around space of vacua instead.

(Celestial interpretation in: [Kapec, Law, Narayanan])

Double soft measures curvature

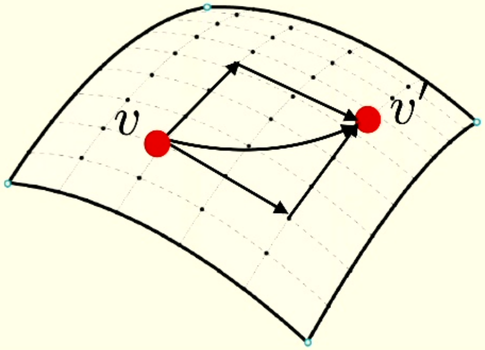
- Consecutive double soft

$$\left[\lim_{q_a\to 0},\lim_{q_b\to 0}\right] A_{n+2}^{i_1\cdots i_n i_a i_b} = [\nabla^{i_a},\nabla^{i_b}]A_n^{i_1\cdots i_n} = \sum_{c\neq a,b} R^{i_a i_b i_c}{}_{j_c} A_n^{i_1\cdots j_c\cdots i_n}$$

- Simultaneous double soft

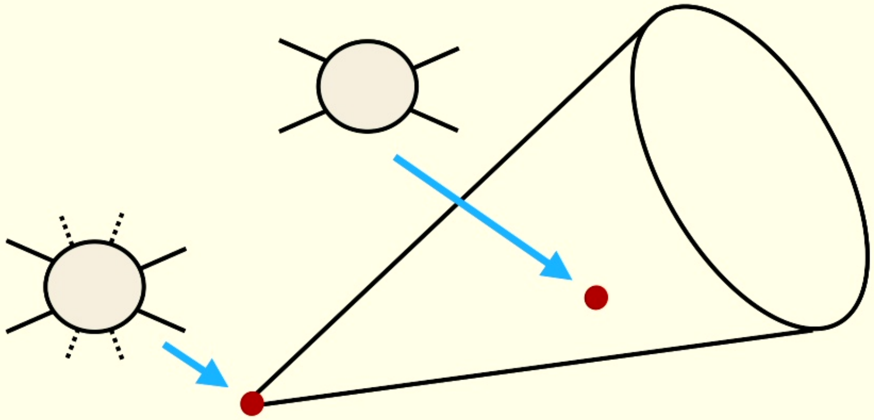
$$\lim_{q_a,q_b\to 0} A_{n+2}^{i_1\cdots i_n i_a i_b} = \frac{1}{2}\sum_{c\neq a,b}\frac{s_{ac}-s_{bc}}{s_{ac}+s_{bc}} R^{i_a i_b i_c}{}_{j_c} A_n^{i_1\cdots j_c\cdots i_n} + \nabla^{(i_a}\nabla^{i_b)} A_n^{i_1\cdots i_n}$$

- Difference: which path in field space



Exploring moduli space

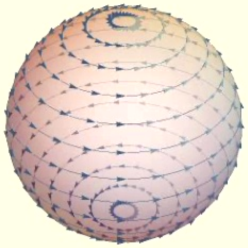
- Geometric (multi)soft theorem lets us move around moduli space



- Requires knowledge of all amplitudes at given point in moduli space
- Converse enables soft recursion relations (ask me later)

Geometry of symmetry

- Symmetry = Killing vector



$$\Phi^I \quad \rightarrow \quad \Phi^I + \mathcal{K}^I(\Phi)$$

$$g_{IJ}(\Phi) \quad \rightarrow \quad g_{IJ}(\Phi) + \mathcal{L}_{\mathcal{K}} g_{IJ}(\Phi)$$

- Ward identity

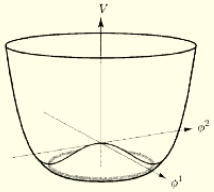
$$\mathcal{L}_{\mathcal{K}} A_n^{i_1 \cdots i_n} = \mathcal{K}_i \nabla^i A_n^{i_1 \cdots i_n} - \sum_{a=1}^{n} \nabla_{j_a} \mathcal{K}^{i_a} A_n^{i_1 \cdots j_a \cdots i_n} = 0$$

- Makes soft theorem multiplicative

$$\lim_{q \to 0} \mathcal{K}_i A_{n+1}^{i_1 \cdots i_n i} = \sum_{a=1}^{n} \nabla_{j_a} \mathcal{K}^{i_a} A_n^{i_1 \cdots j_a \cdots i_n}$$

Example: non-symmetric NBG

[Kampf, Novotny, Shifman, Trnka; Cheung, Helset, **JPM**]



- Spontaneously broken global symmetry

- Coset space G/H

$$
\begin{aligned}
[\mathcal{T}_a, \mathcal{T}_b] &= f_{ab}{}^{c}\mathcal{T}_c \,, \\
[\mathcal{T}_a, \mathcal{X}_i] &= f_{ai}{}^{j}\mathcal{X}_j \,, \\
[\mathcal{X}_i, \mathcal{X}_j] &= f_{ij}{}^{a}\mathcal{T}_a + f_{ij}{}^{k}\mathcal{X}_k
\end{aligned}
$$

- Soft theorem

$$\lim_{q\to 0} A_{n+1}^{i_1\ldots i_n i} = -\frac{1}{2}\sum_{a=1}^{n} f_{j_a}{}^{i_a i} A_n^{i_1\cdots j_a\cdots i_n}$$

$\nwarrow$ $\nabla\mathcal{X}$

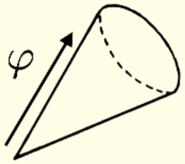
- Symmetric coset ($\mathcal{X} \to -\mathcal{X}$) = Adler zero

$$\lim_{q\to 0} A_{n+1}^{i_1\ldots i_n i} = 0$$

Example: soft dilaton

[Callan]

- Spontaneously broken spacetime symmetry - scale invariance
- Dilaton (NGB) = flat direction in moduli space $\nabla_\varphi = \partial_\varphi$



- New proof of soft dilaton theorem

$$\left(f_\varphi \partial_{\langle\varphi\rangle} + \sum_{a=1}^{n} p_a^\mu \frac{\partial}{\partial p_a^\mu} \right) A = (D - n\Delta) A$$

$$\lim_{q \to 0} A_{n+1} = \partial_{\langle\varphi\rangle} A_n = \frac{1}{f_\varphi} \left(D - n\Delta - \sum_{a=1}^{n} p_a^\mu \frac{\partial}{\partial p_a^\mu} \right) A_n$$

also works with masses!

Beyond scalars

Review: Coupling to matter

- Fermions specify vector bundle over moduli space $\mathcal{V}_f \to \Sigma$

$$h_{PQ}(\Phi)\bar{\psi}^P \gamma^\mu \overleftrightarrow{\partial}_\mu \psi^Q + \Omega_{PQI}(\Phi)\bar{\psi}^P \gamma^\mu \psi^Q \partial_\mu \Phi^I$$

↑ fiber metric on $\mathcal{V}_f$

↑ connection on $\mathcal{V}_f$

- Similarly, vectors live in $\mathcal{V}_v \to \Sigma$ with a metric $\frac{1}{2}h'_{AB}(\Phi)F^A \wedge \star F^B$
- Supersymmetry, among other things, can imply $\mathcal{V}_f \sim \mathcal{V}_v \sim T\Sigma$

Do soft moduli realize this geometry?

Soft scalars with matter

- Soft theorem works!

$$\lim_{q\to 0} A_{n+1} = \bar{\nabla} A_n = (\nabla + \Omega + \nabla h)\, A_n$$

$\uparrow$

connection on $T\Sigma \oplus \mathcal{V}_f \oplus \mathcal{V}_v$

- Charged scalars more involved: gives a version of Goldstone boson equivalence theorem.

- E.g. dipole coupling in D=4 $D_{PQA}(\Phi)(\bar{\psi}^P \sigma^{\mu\nu} \psi^Q) W^A_{\mu\nu}$

$$\begin{aligned} A_3^{pqa} &= \langle 13\rangle\langle 23\rangle D^{pqa}, \\ A_4^{pqai_4} &= \langle 13\rangle\langle 23\rangle \bar{\nabla}^{i_i} D^{pqa}, \end{aligned}$$

Photon as NG boson

- Modern perspective: photon NG boson for $U(1)^{(1)}_e \times U(1)^{(1)}_m$

  e.g. electric 1-form symmetry

$$A_\mu \to A_\mu + \bar{A}_\mu \qquad \bar{A}_\mu \quad \text{Flat connection}$$

- Just like for pions, conserved current interpolates photon

$$J^{(1)}_{\mu\nu} = \partial_{[\mu} A_{\nu]} + \mathcal{O}(A^2)$$

- Symmetry is broken by charged matter

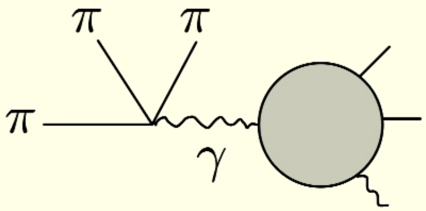
Soft photon

- In the absence of charged matter there is indeed an "Adler zero"

$$\lim_{q_\gamma \to 0} A_{n+1} = 0$$

- A bit boring, because higher-form symmetries abelian, and often only emergent
- Double soft gets interesting for two-group

$$\partial \cdot j^a(x) j^b_\mu(y) \sim f^{abc}\delta(x-y) j^c_\mu(y) + f^{ab\gamma}\partial^\mu \delta(x-y) J^{(1)}_{\mu\nu}(y)$$



Soft photon

- In the presence of charged matter

$$\lim_{q_\gamma \to 0} A_{n+1} = \sum_a \frac{q_a}{q \cdot p_a} \left[\epsilon \cdot p_a + \epsilon \cdot J_a \cdot q\right] A_n$$

- Both masslessness of photon and usual soft photon theorem stems from robustness of one-form symmetry

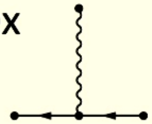
$$\delta\mathcal{L} = \bar{A}_\mu j^\mu \qquad\qquad \partial^\mu J^{(1)}_{\mu\nu} = j_\nu$$

- Intuition: Space of vacua still parameterized by $\bar{A}_\mu$

Geometric soft photon

- Geometric proof with $\bar{A}_\mu$ as a spurion yields

$$\begin{tikzcd}
 & \lim\limits_{q\to 0} A_{n+1} \sim \left(\nabla + \bar{\Gamma}\right) A_n & \\
\epsilon^\mu \dfrac{\delta}{\delta \bar{A}^\mu} \overset{D=\partial+q\bar{A}}{\overset{\downarrow}{=}} -\sum\limits_a q_a \epsilon \cdot \dfrac{\partial}{\partial p_a} \arrow[ur] \arrow[dr] & & q_a \frac{\epsilon \cdot p_a}{q \cdot p_a}\left(1 + p_a \cdot \frac{\partial}{\partial q}\right) \arrow[ul, "\text{Three-point vertex}"'] \arrow[dl] \\
 & \sum\limits_a \frac{q_a}{q \cdot p_a}\left[\epsilon \cdot p_a + \epsilon \cdot J_a \cdot q\right] \checkmark &
\end{tikzcd}$$

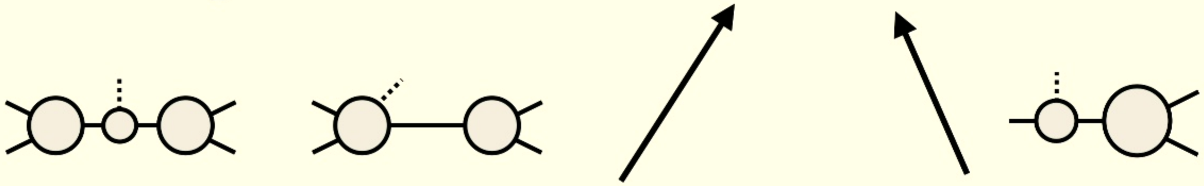


Note: space of vacua flat in this case

A general soft theorem?

Generalize the intuition:

$$\lim_{q\to 0} A_{n+1} \sim \left(\nabla + \bar{\Gamma}\right) A_n$$



“Soft particle = derivative w.r.t. flat background”

3pt-vertex “on-shell connection”

Does this work?

Conclusions

- Geometric perspective gives new and general soft theorems for scalar moduli beyond NG bosons
- Hints of general organizing principle for general soft theorems. Can we make this precise? Find new ones? Soft fermions beyond goldstino?
- Could enable new on-shell perspective of non-renormalization theorems
- Geometry perspective useful for SMEFT, soft recursion?
  [Helset, Trott, Alonso, Manohar, Jenkins, ….]
- Important question remains: Systematic way to move a finite distance in moduli space? Infinite? Massive amplitudes from massless amplitudes?

Thank you!