Title: Research Talk 5 - Crossing beyond scattering amplitudes

Speakers:

Collection: Strings 2023

Date: July 24, 2023 - 3:00 PM

URL: https://pirsa.org/23070018

Abstract: What can be measured asymptotically in quantum field theory? Among the answers to this question are scattering amplitudes, but also a whole compendium of inclusive measurements, such as expectation values of gravitational radiation and out-of-time-ordered amplitudes. We show that these asymptotic observables can be related to one another through new versions of crossing symmetry. Assuming analyticity, we propose generalized crossing relations and corresponding paths of analytic continuation. Throughout the talk, we show how to apply crossing in practice, using various tree- and loop-level examples.

Crossing Beyond Scattering Amplitudes

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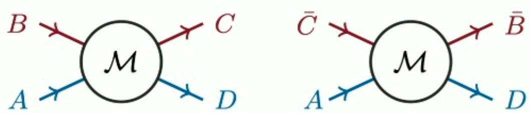
Institute for Advanced Study

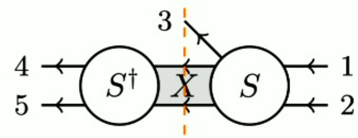
with Simon Caron-Huot, Mathieu Giroux and Sebastian Mizera

Outline

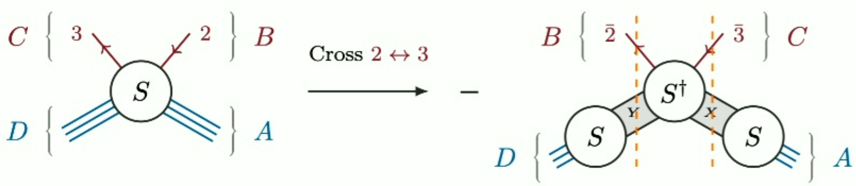
1. Introduction



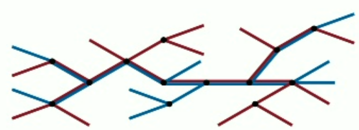
2. What can be measured asymptotically?

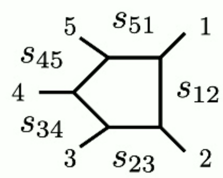


3. Crossing equation



4. Examples





Review on crossing symmetry

Amplitudes for $AB \to CD$ and $A\bar{C} \to \bar{B}D$ are boundary values of the **same analytic function**

$$\mathcal{M}_{AB\to CD} \underset{\substack{\text{Analytic}\\ \text{continuation}}}{\longleftrightarrow} \mathcal{M}_{A\bar{C}\to\bar{B}D}$$

Particles indistinguishable from antiparticles traveling back in time?

Review on crossing symmetry

Amplitudes for $AB \to CD$ and $A\bar{C} \to \bar{B}D$ are boundary values of the **same analytic function**

Not relabeling or cyclic invariance

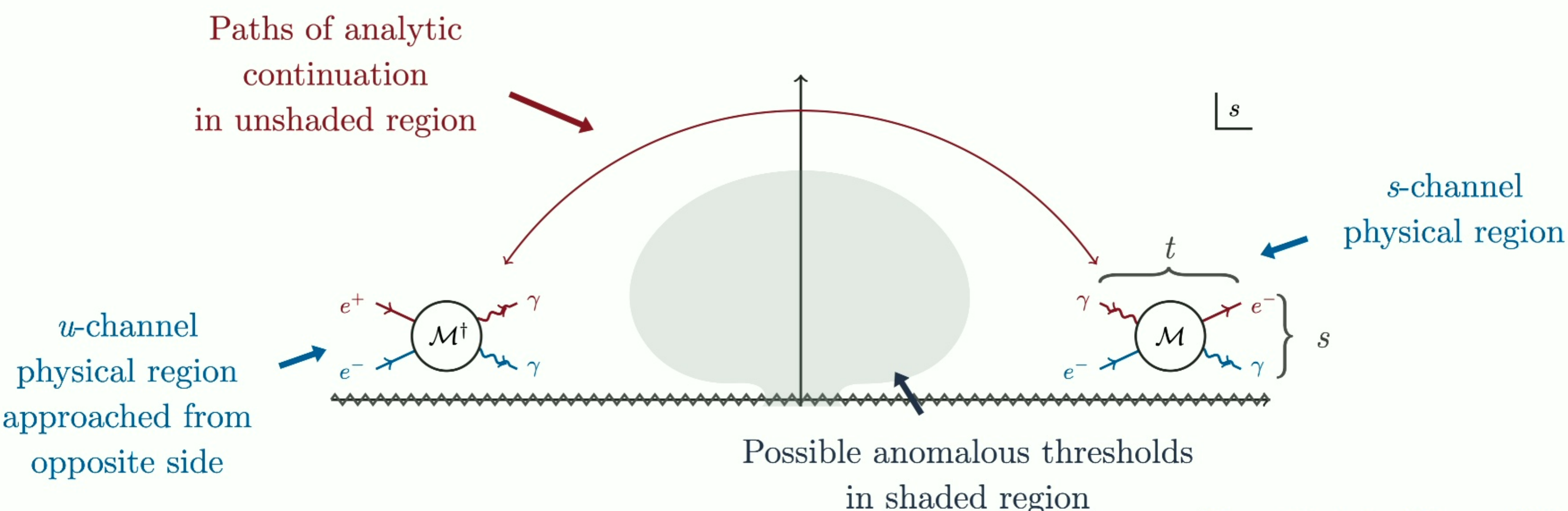
$$\mathcal{M}_{AB\to CD} \underset{\text{Analytic continuation}}{\longleftrightarrow} \mathcal{M}_{A\bar{C}\to\bar{B}D}$$

Particles indistinguishable from antiparticles traveling back in time?

Crossing symmetry would allow us to use results from previous computations:

Crossing symmetry in 2 to 2 scattering

Proven for the non-perturbative amplitude at fixed momentum transfer $t<0$ in theories with mass gap



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[Bros, Epstein, Glaser 1964, 1965]

CROSSING SYMMETRY IN 2 TO 2 SCATTERING

Proven for the non-perturbative amplitude at fixed momentum transfer $t<0$
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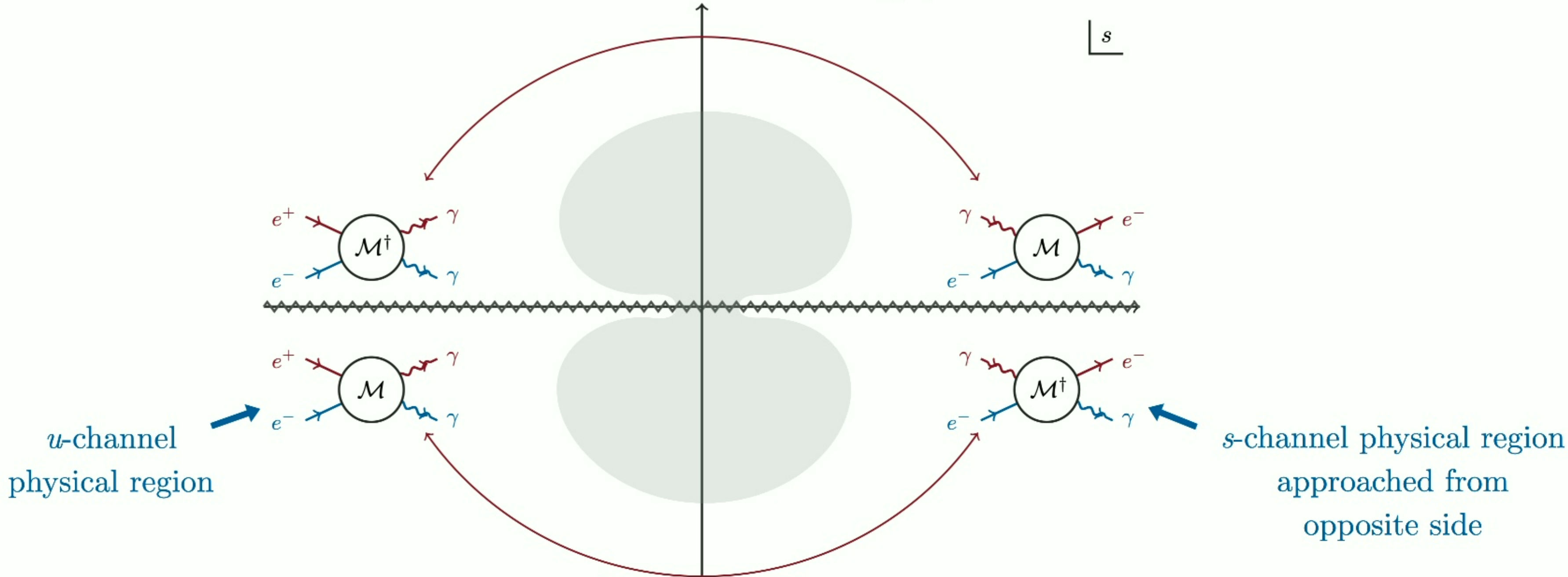

[Bros, Epstein, Glaser 1964, 1965]

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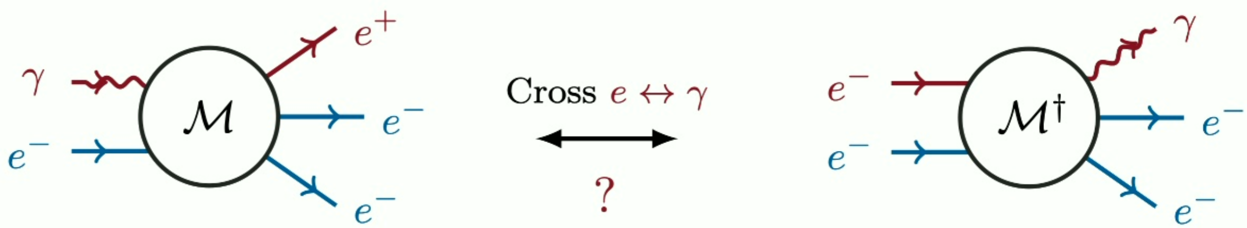
Crossing symmetry in 2 to 3 scattering

Same for the 5 pt amplitude, right?



Crossing symmetry in 2 to 3 scattering

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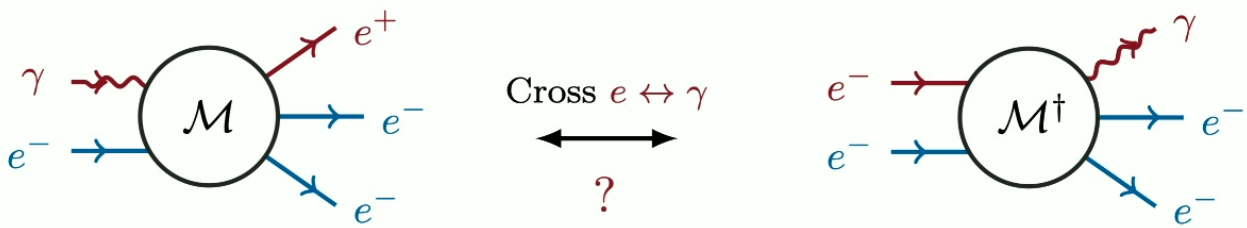
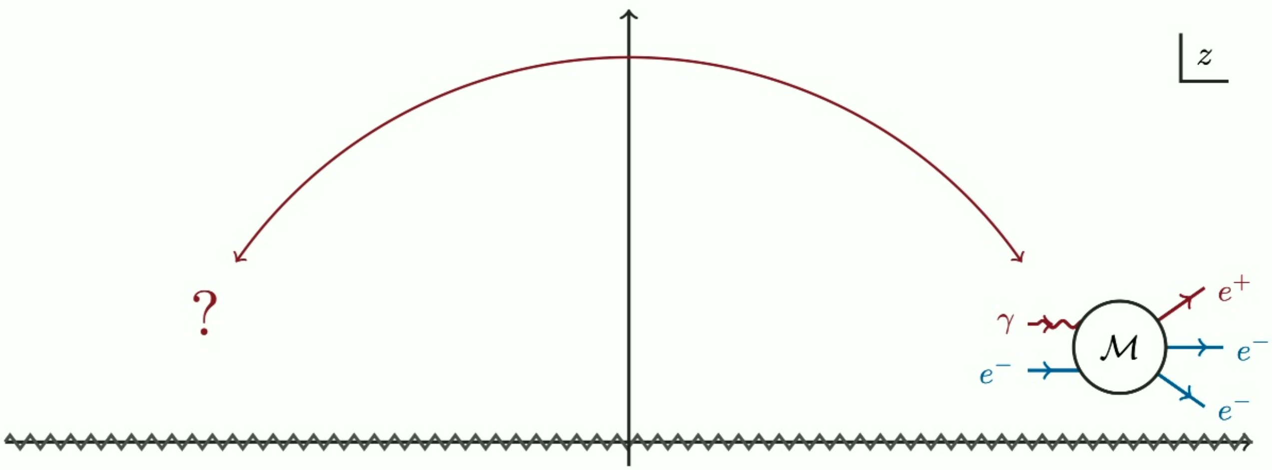


No.

Crossing symmetry in 2 to 3 scattering

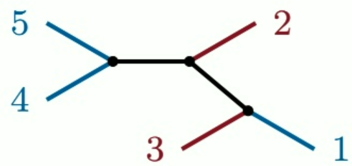
The central topic of this talk:

What is the result of analytically continuing scattering amplitudes?



Simple example at Tree level

$$\mathcal{M}_{543\leftarrow 21} =$$

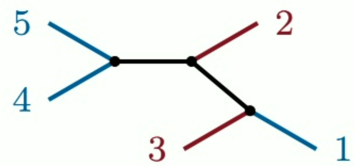


$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)}\,,$$

Simple example at Tree level

Distributions at tree level useful for understanding loop level

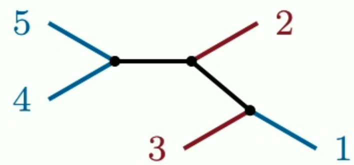
$$\mathcal{M}_{543\leftarrow 21} =$$



$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)},$$

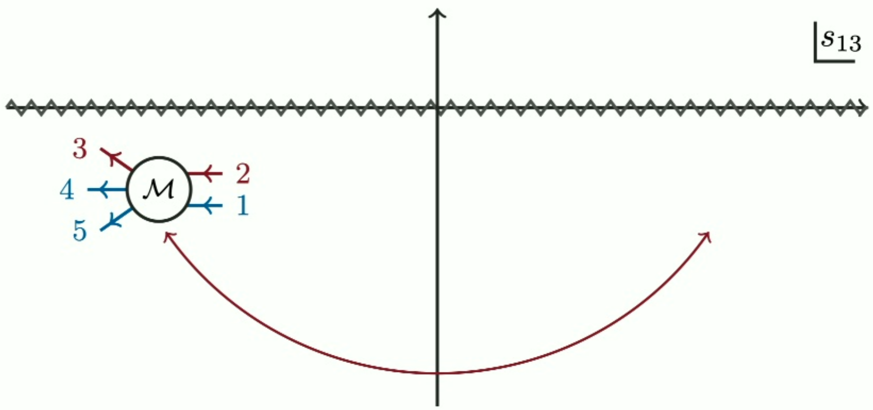
We will take time flowing to the left!

Simple example at Tree level

$$\mathcal{M}_{543\leftarrow 21} =$$ 

$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)},$$

Rotate s_{13} in the lower half plane at fixed s_{45}



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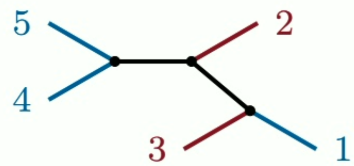
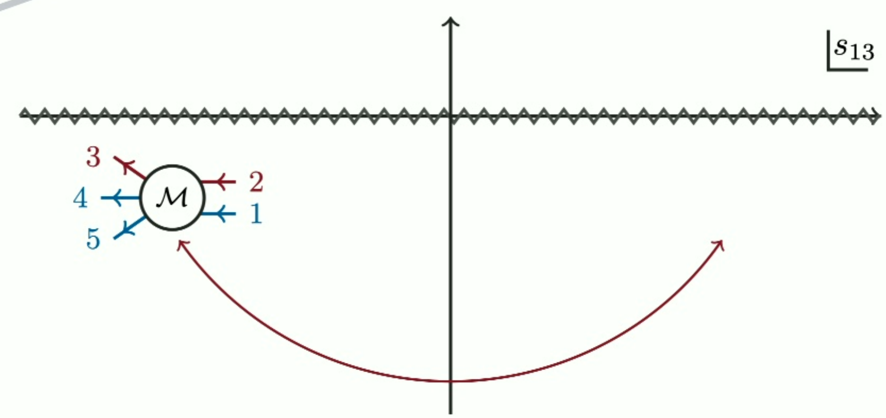


$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)}\,,$$

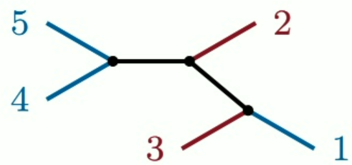
Rotate s_{13} in the lower half plane at fixed s_{45}

Exchanges
2 and 3



Simple example at Tree level

$$\mathcal{M}_{543\leftarrow 21} =$$

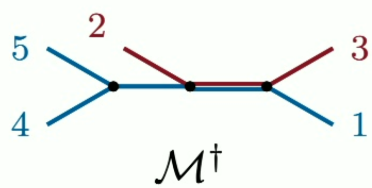


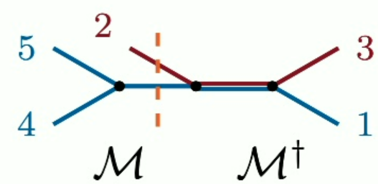
$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)},$$

Rotate s_{13} in the lower half plane at fixed s_{45}

$$[\mathcal{M}_{543\leftarrow 21}]_{s_{13}^{\circlearrowleft}} = \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}$$

$$= \underbrace{\frac{g^3}{(s_{45} - m_{45}^2 - i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}} - \underbrace{2\pi i \delta(s_{45} - m_{45}^2) \frac{g^3}{(s_{13} - m_{13}^2 - i\varepsilon)}}$$





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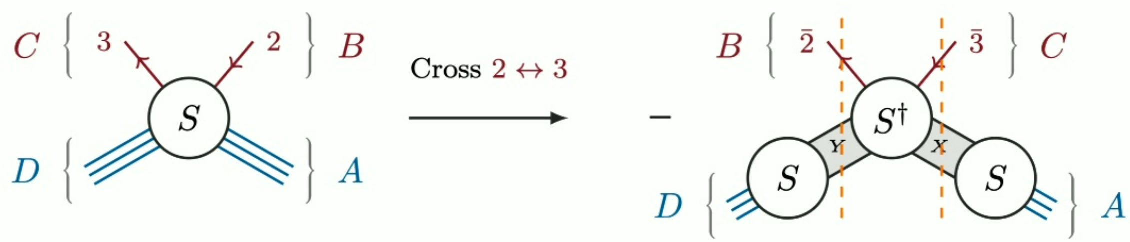
Takeaway point:

Analytic continuation *from* $\mathcal{M}$ *lands on something* ***new***

Here: Relate asymptotic observables

We will learn: Scattering amplitudes are part of a **larger family of observables**, related by analytic continuation



Crossing equation describes the result of analytic continuation

Previous progress on crossing

- Proposed for quantum field theory in 1954

[Gell-Mann, Goldberger, Thirring]

- Proven for non-perturbative 2→2 and 2→3 scalar amplitudes, assuming mass gap
  - *Proofs use* ***mass gap, causality, unitarity,*** *and* ***analytic extension*** *theorems*

[Bros, Epstein, Glaser 1964, 1965; Bros 1986]

- Recent progress in Chern-Simons theories and string theory for 2→2 amplitudes

[See e.g. Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama 2014; Lacroix, Erbin, Sen 2018; Mehta, Minwalla, Patel, Prakash, Sharma 2022; Gabai, Sandor, Yin 2022]

- Proven in the planar limit to any multiplicity using perturbation theory

[Mizera 2021]

Challenge: understand connection between crossing and physical principles

Outline

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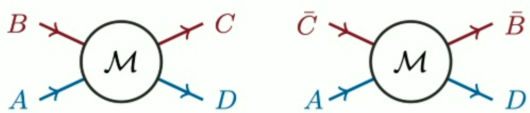
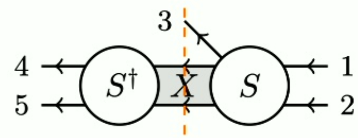
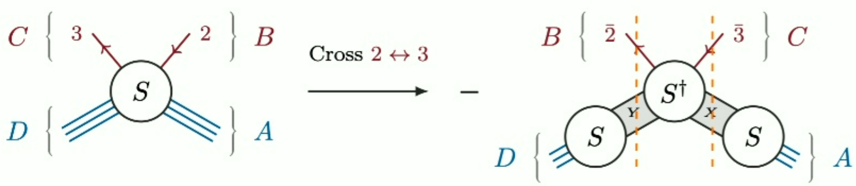


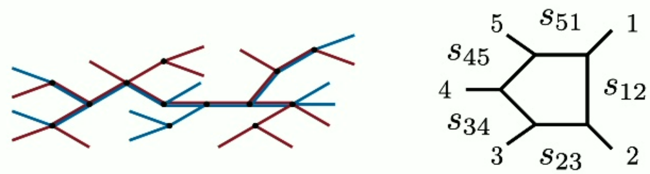
2. What can be measured asymptotically?



3. Crossing equation



4. Examples



Asymptotic algebra in quantum field theory

1. Algebra of asymptotic measurements in the far past and far future,

$$[a_1, a_2^\dagger] = \delta_{1,2}\, 2p_1^0 (2\pi)^{\mathrm{D}-1} \delta^{\mathrm{D}-1}(\vec{p}_1 - \vec{p}_2)$$

$$[b_1, b_2^\dagger] = \delta_{1,2}\, 2p_1^0 (2\pi)^{\mathrm{D}-1} \delta^{\mathrm{D}-1}(\vec{p}_1 - \vec{p}_2)$$

2. These operators act on equivalent Hilbert spaces and are related by a unitary evolution operator S:

$$b = S^\dagger a S, \quad b^\dagger = S^\dagger a^\dagger S; \qquad SS^\dagger = \mathbb{1}$$

3. There exists a time-invariant vacuum $|0\rangle$:

$$a_i|0\rangle = b_i|0\rangle = 0, \quad S|0\rangle = |0\rangle$$

4. Stability:

$$Sa_i^\dagger|0\rangle = a_i^\dagger|0\rangle, \qquad Sb_i^\dagger|0\rangle = b_i^\dagger|0\rangle,$$

Asymptotic algebra in quantum field theory

1. Algebra of asymptotic measurements in the far past and far future,

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$$a_{\text{out}} \quad [b_1, b_2^\dagger] = \delta_{1,2}\, 2p_1^0 (2\pi)^{\text{D}-1} \delta^{\text{D}-1}(\vec{p}_1 - \vec{p}_2)$$

Assume Bose/Fermi statistics, flat space, Poincaré invariance

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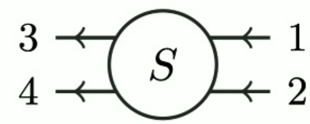
Using this algebra,

What can be measured asymptotically?

4 pt asymptotic measurements

$$\langle 0|b_4 b_3 a_2^\dagger a_1^\dagger|0\rangle = {}_{\text{in}}\langle 43|S|21\rangle_{\text{in}} =$$



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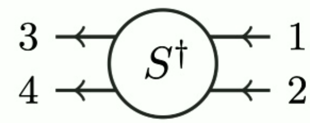

$$\langle 0|a_4 a_3 a_2^\dagger a_1^\dagger|0\rangle = {}_{\text{in}}\langle 43|21\rangle_{\text{in}} = 0$$

$$\langle 0|b_4 b_3 b_2^\dagger b_1^\dagger|0\rangle = {}_{\text{in}}\langle 43|21\rangle_{\text{in}} = 0$$

4 PT ASYMPTOTIC MEASUREMENTS

$$b = S^\dagger a S, \quad b^\dagger = S^\dagger a^\dagger S$$

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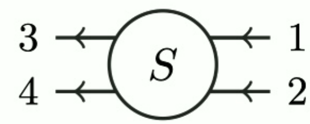

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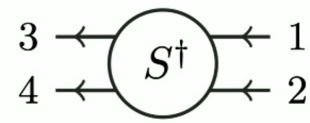

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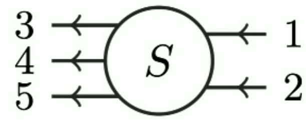
$$\langle 0|b_4 b_3 b_2^\dagger b_1^\dagger|0\rangle = {}_{\text{in}}\langle 43|21\rangle_{\text{in}} = 0$$

Time flows to the left in all diagrams

5 PT ASYMPTOTIC MEASUREMENTS

$$\langle 0|b_5 b_4 b_3 a_2^\dagger a_1^\dagger|0\rangle = {}_{\text{in}}\langle 543|S|21\rangle_{\text{in}} =$$



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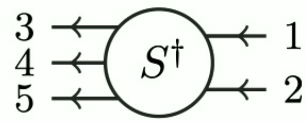
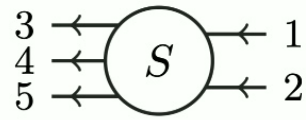


(plus forward terms and Hermitian conjugates)

5 PT ASYMPTOTIC MEASUREMENTS

$$\langle 0|b_5 b_4 b_3 a_2^\dagger a_1^\dagger|0\rangle \;=\; {}_{\rm in}\langle 543|S|21\rangle_{\rm in} \;=$$



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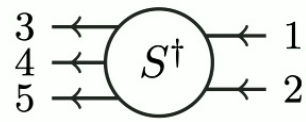
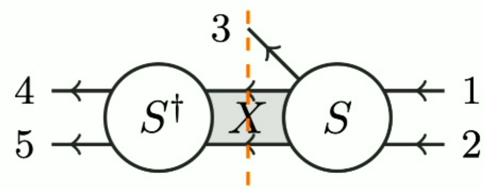


$$\langle 0|a_5 a_4 b_3 a_2^\dagger a_1^\dagger|0\rangle = {}_{\rm in}\langle 54|b_3|21\rangle_{\rm in} \;=$$



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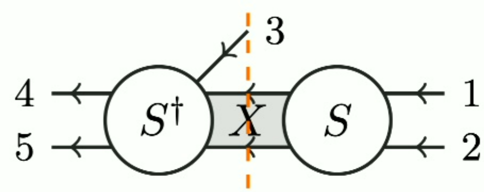

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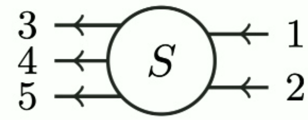

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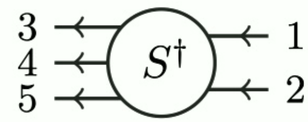
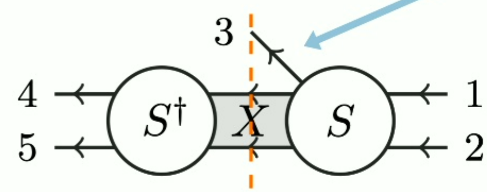


Insert a complete set of states X, integrate inclusively over phase space

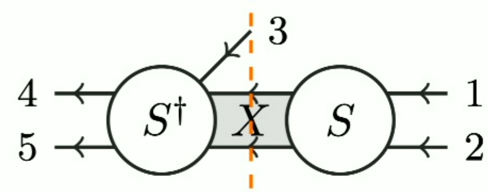
$b = S^\dagger a S$

$$\langle 0|a_5 a_4 b_3 a_2^\dagger a_1^\dagger|0\rangle = {}_{\text{in}}\langle 54|b_3|21\rangle_{\text{in}} =$$



$b^\dagger = S^\dagger a^\dagger S$

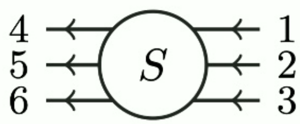
$$\langle 0|a_5 a_4 b_3^\dagger a_2^\dagger a_1^\dagger|0\rangle = {}_{\text{in}}\langle 54|b_3^\dagger|21\rangle_{\text{in}} =$$



(plus forward terms and Hermitian conjugates)

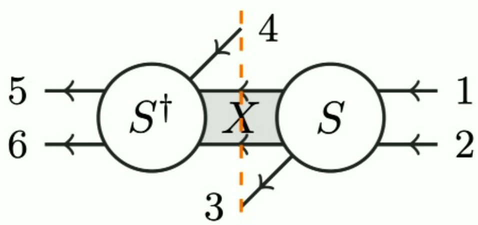
Example 6 pt asymptotic measurements

$\langle 0|b_6 b_5 b_4 a_3^\dagger a_2^\dagger a_1^\dagger|0\rangle$



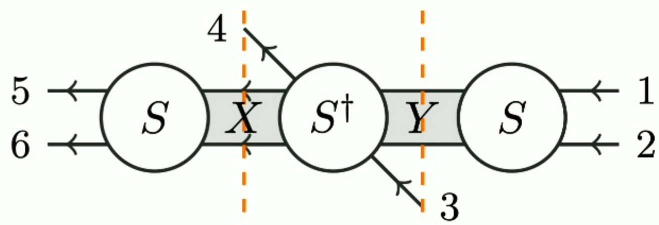
Scattering amplitudes

$\langle 0|a_6 a_5 b_4^\dagger b_3 a_2^\dagger a_1^\dagger|0\rangle$



Inclusive amplitudes

$\langle 0|b_6 b_5 a_4 b_3^\dagger a_2^\dagger a_1^\dagger|0\rangle$



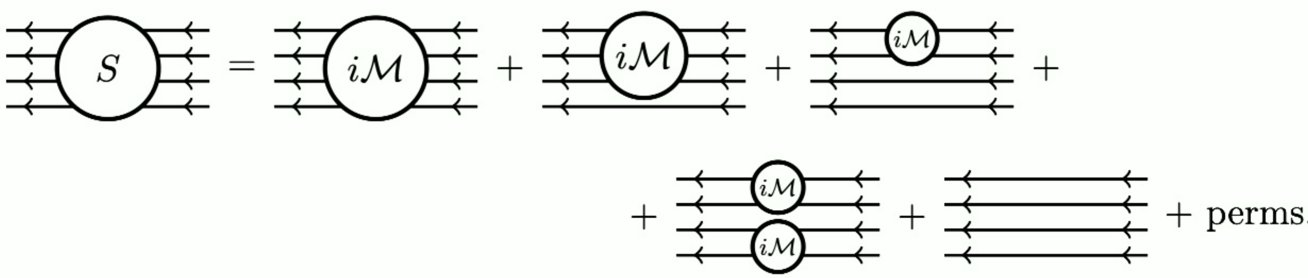
Out-of-time-ordered correlators

Example n-pt asymptotic measurements

More generally:

$$\langle 0|\underbrace{a\cdots a}_{k_{2s}}S\underbrace{a^\dagger\cdots a^\dagger}_{k_{2s-1}}\underbrace{a\cdots a}_{k_{2s-2}}S^\dagger\cdots S^\dagger\underbrace{a^\dagger\cdots a^\dagger}_{k_3}\underbrace{a\cdots a}_{k_2}S\underbrace{a^\dagger\cdots a^\dagger}_{k_1}|0\rangle$$

We expand in terms of connected components:



Physical interpretation of asymptotic observables

$${}_{\text{in}}\langle 0|b_n \cdots b_{j+1} a_j^\dagger \cdots a_1^\dagger|0\rangle_{\text{in}}$$: Scattering amplitude

$${}_{\text{in}}\langle 54|b_3|21\rangle_{\text{in}}$$: Expectation value of electromagnetic field in a scattering experiment / Gravitational waveform detected by LIGO-Virgo-KAGRA

$$\lim_{p_3 \to p_4} {}_{\text{in}}\langle 65|b_4 b_3^\dagger|21\rangle_{\text{in}}$$: Inclusive cross section / inclusive particle number

$${}_{\text{in}}\langle 6|b_5^\dagger a_4 b_3^\dagger a_2^\dagger|1\rangle_{\text{in}}$$: Out-of-time-ordered correlator

[See e.g. Shenker, Stanford 2013; Maldacena, Shenker, Stanford 2015; Kosower, Maybee, O'Connell 2018; Caron-Huot 2022]

Takeaway points:

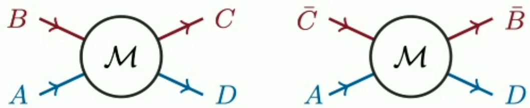
- S-matrix only one of exponentially many asymptotic observables
- Asymptotic observables are physical; already being measured and computed

In this talk:

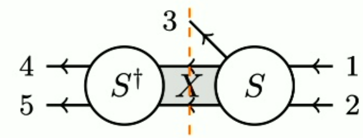
- Relate asymptotic observables to one another via analytic continuations

Outline

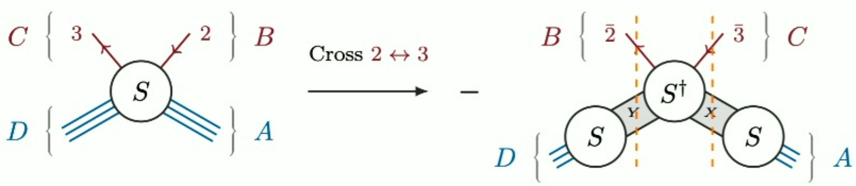
1. Introduction



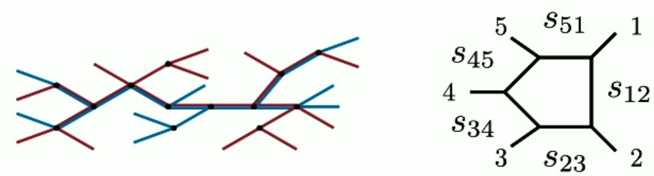
2. What can be measured asymptotically?



3. Crossing equation



4. Examples

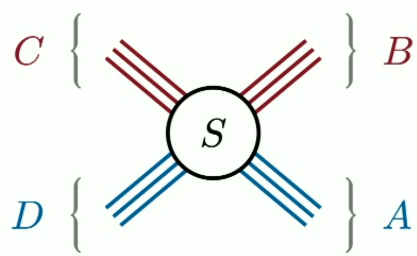


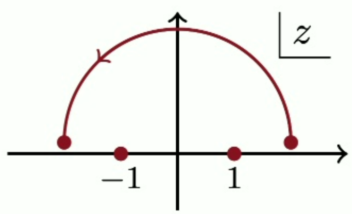
Contour of continuation

We propose an **on-shell** contour of analytic continuation which exchanges **incoming and outgoing** states with a parameter z

$$p_b^\mu(z) = \left(z p_b^+,\, \tfrac{1}{z} p_b^-,\, p_b^\perp\right), \qquad p_c^\mu(z) = \left(z p_c^+,\, \tfrac{1}{z} p_c^-,\, p_c^\perp\right),$$

for all $b \in B$ for all $c \in C$

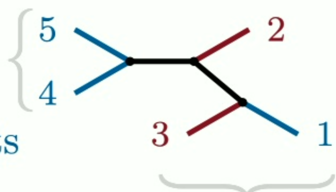




CROSSING PATH IN PRACTICE

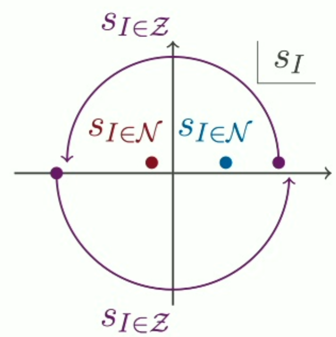
s_{45} *stays fixed*

Uniform invariants stay fixed

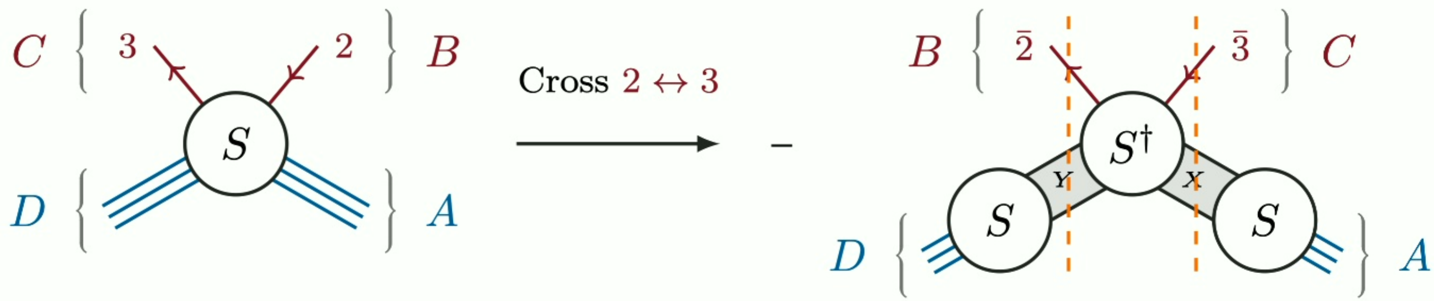


s_{13} *rotates*

Mixed invariants rotate



Crossing Equation for 2-particle crossing:



[See also Bros 1986]

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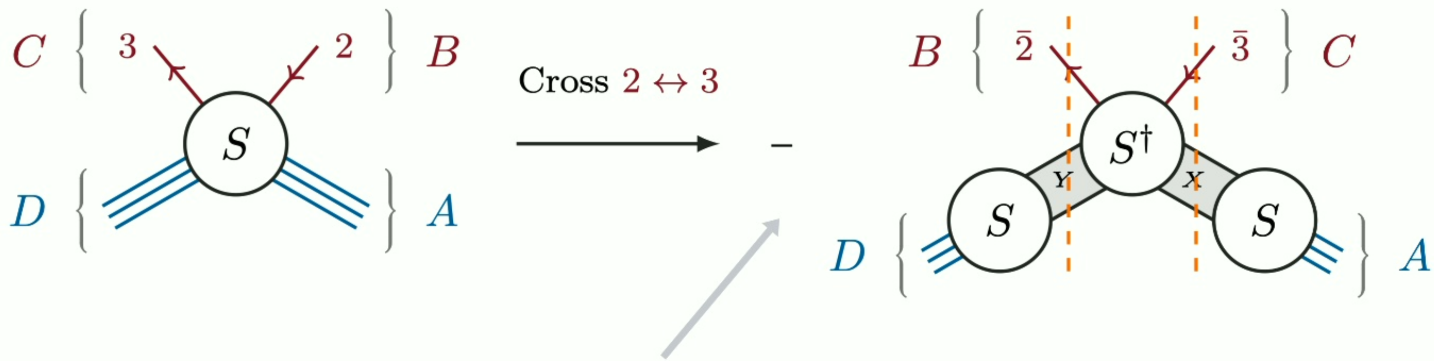
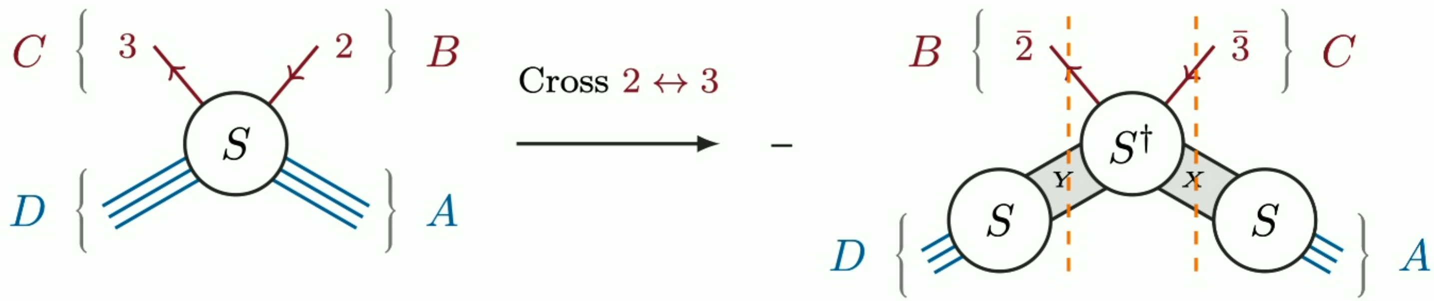
Crossing Equation for 2-particle crossing:



Minus sign from $S = \mathbb{1} + i(2\pi)^{\mathrm{D}}\delta^{\mathrm{D}}(\Sigma p_i)\mathcal{M}$

[See also Bros 1986]

Crossing Equation for 2-particle crossing:



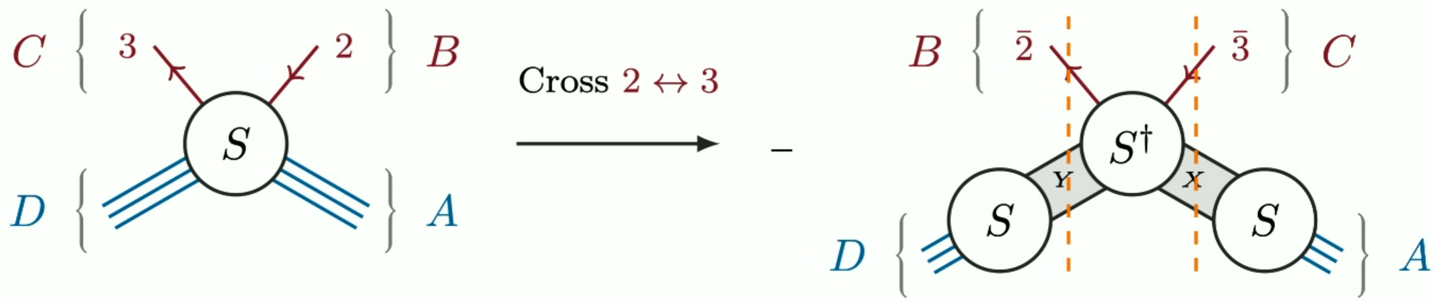
Evidence:

- Loop-level examples and tree-level proof (*part 4*)
- Axiomatic quantum field theory, assuming analyticity,

$$[b, a^\dagger] \quad \xleftrightarrow{\text{Cross } B\leftrightarrow C} \quad [b^\dagger, a]$$

[See also Bros 1986]

Crossing Equation for 2-particle crossing:



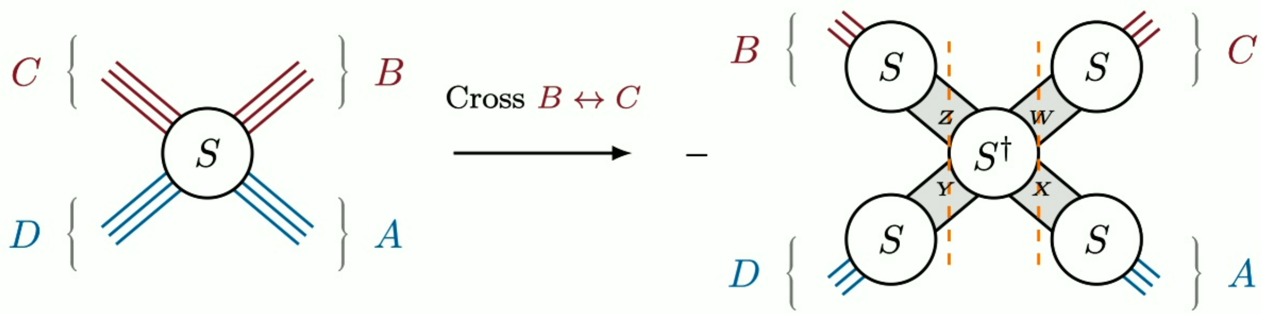
$$\mathcal{G}_{AB\to CD} - \mathcal{G}_{AC\to BD} = \int \mathrm{d}^{\mathrm{D}}x\, \mathrm{e}^{i(p_c - p_b)\cdot x} \langle D | [j(x/2), j(-x/2)] | A \rangle$$

Use causality, unitarity, mass gap to show $\mathcal{G}_{AB\to CD} - \mathcal{G}_{AC\to BD} = 0$:

$$[b, a^\dagger] \quad \overset{\longleftrightarrow}{\text{Cross } B\leftrightarrow C} \quad [b^\dagger, a]$$

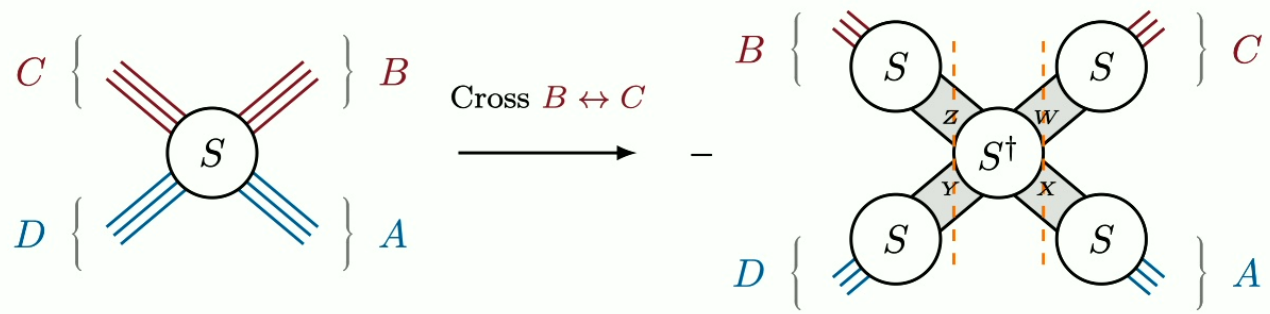
[Bros, Epstein, Glaser 1964, 1965; Itzykson, Zuber 1987]

Crossing proposal for multi-particle crossing:



$$[S_{DC\leftarrow BA}]_{\circlearrowright s_I, \circlearrowleft s_J} = \sum\!\!\!\!\!\!\int_{X,Y,Z,W} S_{D\leftarrow Z} S_{B\leftarrow Y} S^{\dagger}_{YZ\leftarrow XW} S_{W\leftarrow C} S_{X\leftarrow A} \,.$$

Crossing proposal for multi-particle crossing:



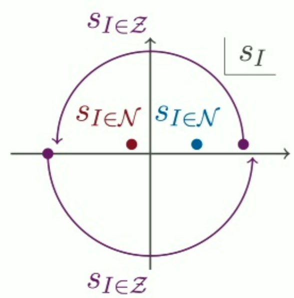
$$[S_{DC\leftarrow BA}]_{\circlearrowright s_I, \curvearrowleft s_J} = \sum\!\!\!\!\!\!\!\int_{X,Y,Z,W} S_{D\leftarrow Z} S_{B\leftarrow Y} S^\dagger_{YZ\leftarrow XW} S_{W\leftarrow C} S_{X\leftarrow A} \,.$$

Evidence:

- Loop-level examples and tree-level proof (*part 4*)
- Symmetry in AD & BC

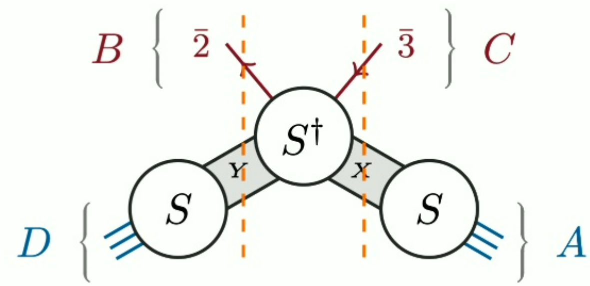
Proving the crossing equation involves comparing:

(I)



The analytic continuation of S via the prescribed path

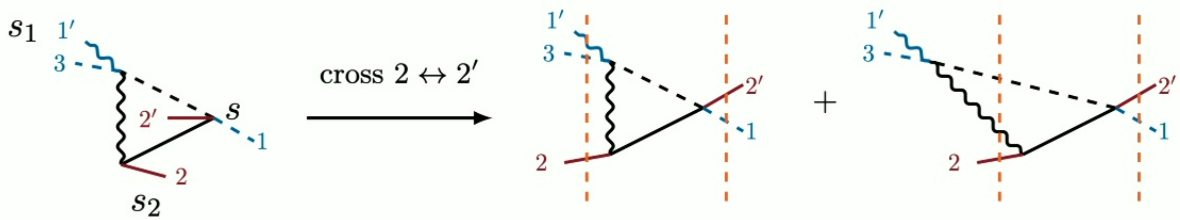
(II)



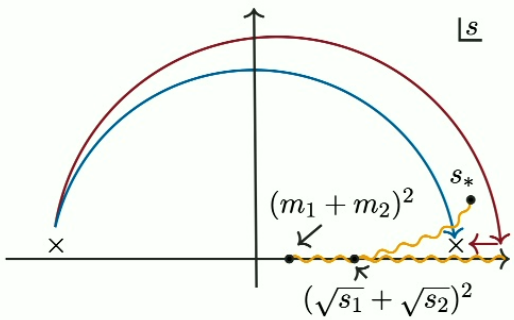
Computing the corresponding observables explicitly

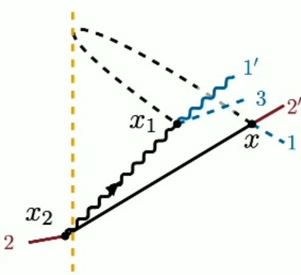
Continuing around singularities

Local analyticity can be subtle: might need to continue past **anomalous thresholds**



Expected from axiomatic field theory





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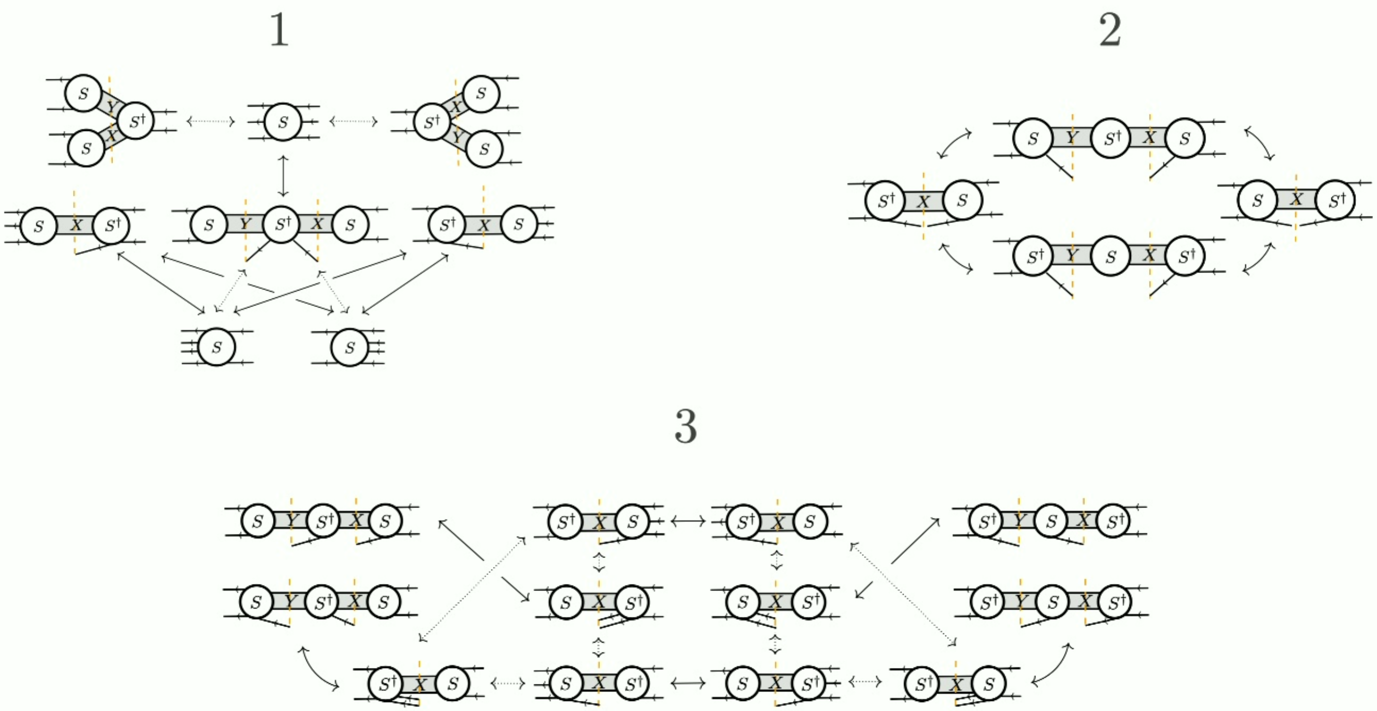
Families of observables



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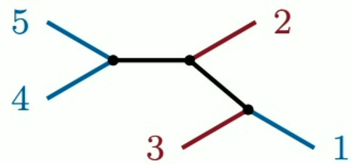
Tree-Level example revisited

$$\mathcal{M}_{543\leftarrow 21} =$$



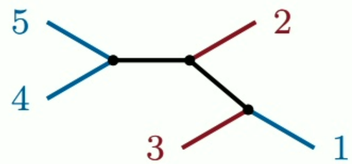
$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)}$$

(I) Analytic continuation path: s_{13} rotates, s_{45} stays fixed,

$$[\mathcal{M}_{543\leftarrow 21}]_{s_{13}\circlearrowright} = \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}$$

Tree-Level example revisited

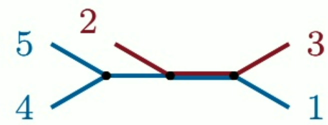
$$\mathcal{M}_{543\leftarrow 21} =$$

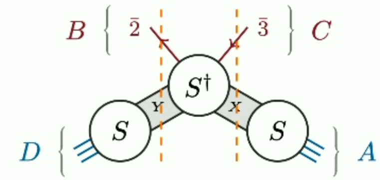


$$= \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2)}$$

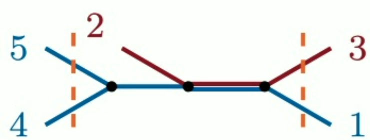
(I) Analytic continuation path: s_{13} rotates, s_{45} stays fixed,

$$[\mathcal{M}_{543\leftarrow 21}]_{s_{13}\curvearrowright} = \frac{g^3}{(s_{45} - m_{45}^2 + i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}$$

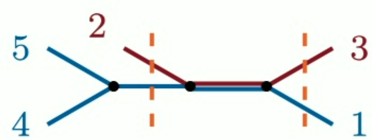
(II) Crossing prediction: all ways of fitting 

into 

Allowed patterns:

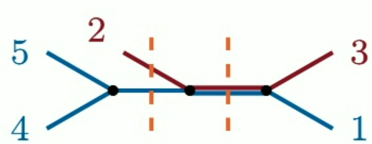


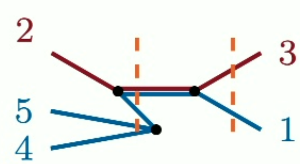
$$= \frac{g^3}{(s_{45} - m_{45}^2 - i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}$$

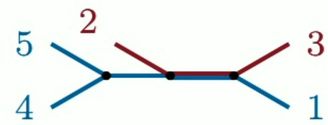


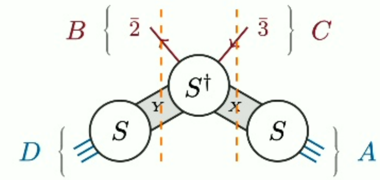
$$= -2\pi i \delta(s_{45} - m_{45}^2) \frac{g^3}{(s_{13} - m_{13}^2 - i\varepsilon)}$$

Example disallowed patterns:

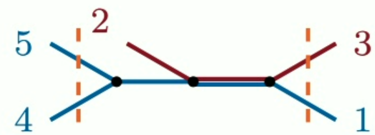




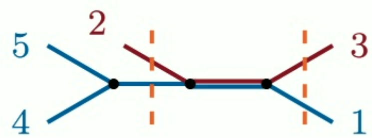
(II) Crossing prediction: all ways of fitting 

into 

Allowed patterns:



$$= \frac{g^3}{(s_{45} - m_{45}^2 - i\varepsilon)(s_{13} - m_{13}^2 - i\varepsilon)}$$

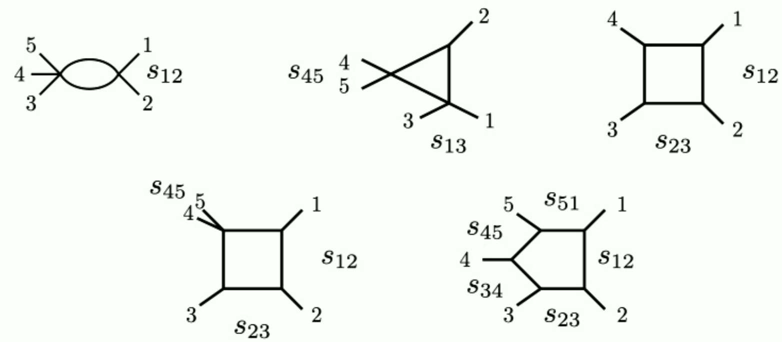


$$= -2\pi i \delta(s_{45} - m_{45}^2) \frac{g^3}{(s_{13} - m_{13}^2 - i\varepsilon)}$$

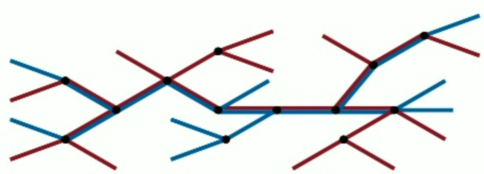
Comparing **(I)** and **(II)** verifies the crossing equation.

Perturbation theory checks

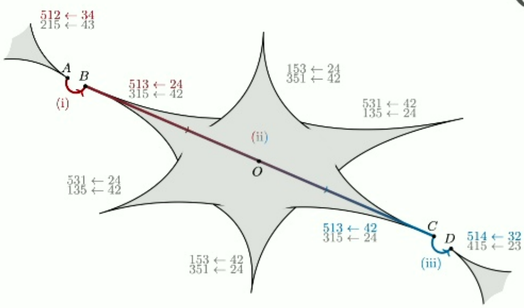
- Checked all D-dim massless basis integrals for an expansion around D=4

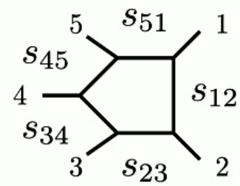


- Proof at any multiplicity at tree level (highly nontrivial)



Crossing check for pentagon





$$\left[\boldsymbol{I}_0^{(34\to215)}\right]_{2\leftrightarrow3} - \left[\boldsymbol{I}_0^{(24\to315)}\right]^* + \mathrm{Cut}_{s_{51}}\boldsymbol{I}_0^{(34\to215)} \stackrel{?}{=} 0$$

$$\left[\boldsymbol{I}_0^{(34\to215)}\right]_{2\leftrightarrow3} = \mathcal{P}\exp\left(\epsilon\int_{\gamma_{2\leftrightarrow3}} \mathrm{d}\boldsymbol{\Omega}\right)\cdot\boldsymbol{I}_0^{(34\to215)}$$

$$\left[\boldsymbol{I}_0^{(34\to215)}\right]_{2\leftrightarrow3} = \begin{pmatrix} -1 & -i\pi & \frac{7\pi^2}{12} & \frac{7\zeta_3}{3}+\frac{i\pi^3}{4} & -\frac{73\pi^4}{1440}+\frac{7i\pi\zeta_3}{3} \\ -\mathbf{1}_4 & \mathbf{0}_4 & \frac{\pi^2}{12}\mathbf{1}_4 & \frac{7\zeta_3}{3}\mathbf{1}_4 & \frac{47\pi^4}{1440}\mathbf{1}_4 \\ 2 & 2i\pi & -\frac{5\pi^2}{6} & -\frac{14\zeta_3}{3}-\frac{i\pi^3}{6} & -\frac{13\pi^4}{240}-\frac{14i\pi\zeta_3}{3} \\ 2 & 2i\pi & -\frac{7\pi^2}{6} & -\frac{20\zeta_3}{3}-\frac{i\pi^3}{6} & -\frac{7\pi^4}{144}-\frac{14i\pi\zeta_3}{3} \\ 2 & 0 & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3} & -\frac{43\pi^4}{720} \\ 2 & -2i\pi & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3}+\frac{i\pi^3}{6} & -\frac{43\pi^4}{720}+\frac{14i\pi\zeta_3}{3} \\ 2 & 0 & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3} & -\frac{43\pi^4}{720} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix}1\\ \epsilon\\ \epsilon^2\\ \epsilon^3\\ \epsilon^4\end{pmatrix}$$

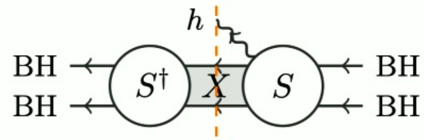
$$\left[\boldsymbol{I}_0^{(24\to315)}\right]^* = \begin{pmatrix} -1 & i\pi & \frac{7\pi^2}{12} & \frac{7\zeta_3}{3}-\frac{i\pi^3}{4} & -\frac{73\pi^4}{1440}-\frac{7i\pi\zeta_3}{3} \\ -\mathbf{1}_4 & \mathbf{0}_4 & \frac{\pi^2}{12}\mathbf{1}_4 & \frac{7\zeta_3}{3}\mathbf{1}_4 & \frac{47\pi^4}{1440}\mathbf{1}_4 \\ 2 & -2i\pi & -\frac{5\pi^2}{6} & -\frac{14\zeta_3}{3}+\frac{i\pi^3}{6} & -\frac{13\pi^4}{240}+\frac{14i\pi\zeta_3}{3} \\ 2 & -2i\pi & -\frac{7\pi^2}{6} & -\frac{20\zeta_3}{3}+\frac{i\pi^3}{6} & -\frac{7\pi^4}{144}+\frac{14i\pi\zeta_3}{3} \\ 2 & 0 & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3} & -\frac{43\pi^4}{720} \\ 2 & 2i\pi & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3}-\frac{i\pi^3}{6} & -\frac{43\pi^4}{720}-\frac{14i\pi\zeta_3}{3} \\ 2 & 0 & -\frac{\pi^2}{2} & -\frac{20\zeta_3}{3} & -\frac{43\pi^4}{720} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix}1\\ \epsilon\\ \epsilon^2\\ \epsilon^3\\ \epsilon^4\end{pmatrix}$$

$$\mathrm{Cut}_{s_{51}}\boldsymbol{I}_0^{(34\to215)} = \begin{pmatrix} 0 & 2i\pi & 0 & -\frac{i\pi^3}{2} & -\frac{14}{3}i\pi\zeta_3 \\ \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\ 0 & -4i\pi & 0 & \frac{i\pi^3}{3} & \frac{28i\pi\zeta_3}{3} \\ 0 & -4i\pi & 0 & \frac{i\pi^3}{3} & \frac{28i\pi\zeta_3}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4i\pi & 0 & -\frac{i\pi^3}{3} & -\frac{28}{3}i\pi\zeta_3 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix}\cdot\begin{pmatrix}1\\ \epsilon\\ \epsilon^2\\ \epsilon^3\\ \epsilon^4\end{pmatrix}$$

Emission in black-hole scattering

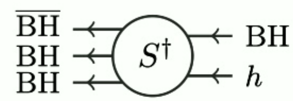
Waveform in LIGO-Virgo-KAGRA obtained as an in-in expectation value

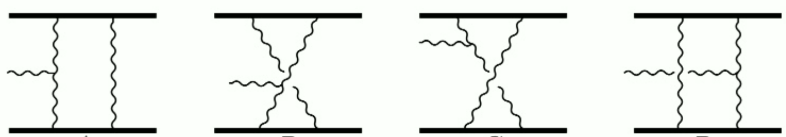
$$ {}_{\text{in}}\langle 54|b_3|12\rangle_{\text{in}} \quad = $$



[Kosower, Maybee, O'Connell 2018]

Here, **analytically continue** the 5-pt amplitude in one-loop computations





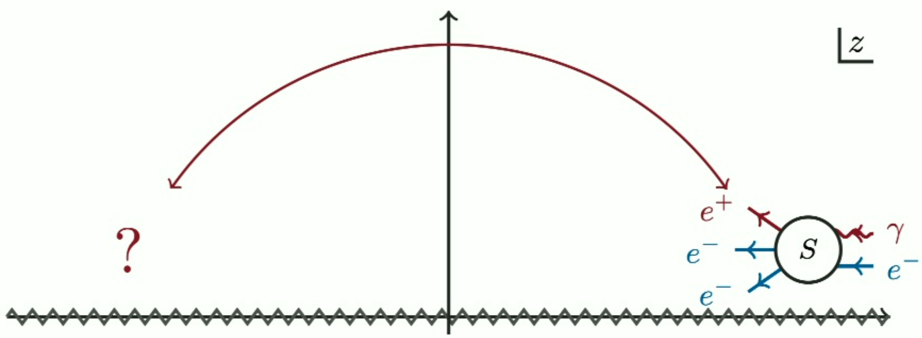
[See also Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini 2023; Herderschee, Roiban, Teng 2023; Elkhidir, O'Connell, Sergola, Vazquez-Holm 2023]

Conclusions

· Exponentially many **asymptotic observables**, e.g. gravitational waveforms, out-of-time-ordered correlators and in-in expectation values

· **New version of crossing symmetry:**

S-matrix contains a host of asymptotic observables which are related by analytic continuations between different channels

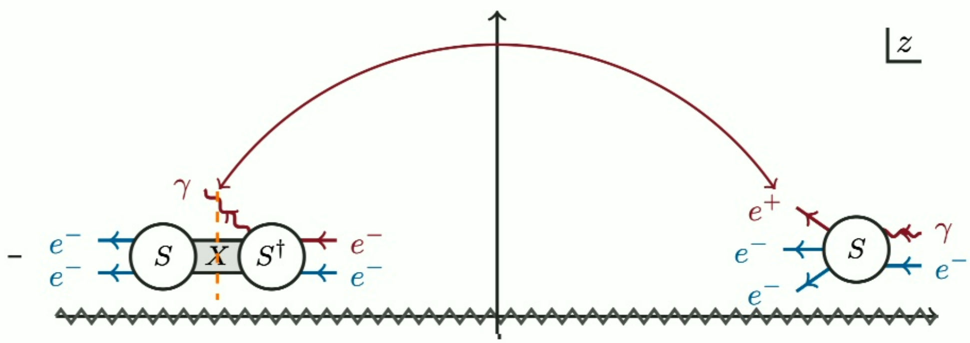


Conclusions

- Exponentially many **asymptotic observables**, e.g. gravitational waveforms, out-of-time-ordered correlators and in-in expectation values

- **New version of crossing symmetry:**

S-matrix contains a host of asymptotic observables which are related by analytic continuations between different channels



THANKS!