Title: Petz recovery from subsystems in conformal field theory

Speakers: Shreya Vardhan

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Abstract: We probe the multipartite entanglement structure of the vacuum state of a CFT in 1+1 dimensions, using recovery operations that attempt to reconstruct the density matrix in some region from its reduced density matrices on smaller subregions. We use an explicit recovery channel known as the twirled Petz map, and study distance measures such as the fidelity, relative entropy, and trace distance between the original state and the recovered state. One setup we study in detail involves three contiguous intervals A, B and C on a spatial slice, where we can view these quantities as measuring correlations between A and C that are not mediated by the region B that lies between them. We show that each of the distance measures is both UV finite and independent of the operator content of the CFT, and hence depends only on the central charge and the cross-ratio of the intervals. We evaluate these universal quantities numerically using lattice simulations in critical spin chain models, and derive their analytic forms in the limit where A and C are close using the OPE expansion. We also compare the mutual information between various subsystems in the original and recovered states, which leads to a more qualitative understanding of the differences between them. Further, we introduce generalizations of the recovery operation to more than three adjacent intervals, for which the fidelity is again universal with respect to the operator content.

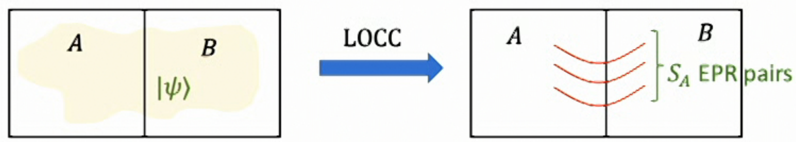
Background and Motivations

Bipartite entanglement in conformal field theory

- Consider a pure state $|\psi\rangle$ in a system AB.

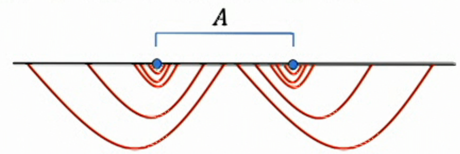
$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B \, |\psi\rangle \langle\psi|$$

- The entanglement entropy has a clear operational interpretation:



- For a single interval A in the vacuum state of a CFT in 1+1 D,

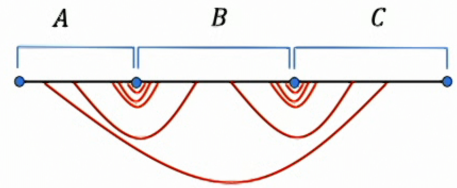
$$S_A = \frac{c}{3} \log \left(\frac{L_A}{\epsilon} \right)$$



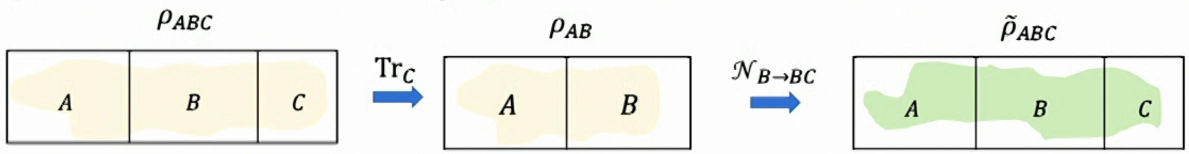
  - UV divergence reflects large amount of short-distance entanglement.
  - Grows with L_A, reflecting long-range correlations at criticality.
  - Does not depend on the operator content.
  - Useful for identifying c-function, RT formula, many other applications.

Questions about multipartite entanglement

- Now consider the reduced density matrix of the CFT vacuum state on three adjacent regions A, B, C. A naive model for correlations among these regions:



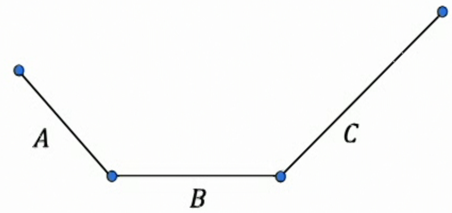
- Quantities like reflected entropy show that the state does not have this simple bipartite structure. Operational meaning not well-understood.
- In this talk, we will address the following question: To what extent are the correlations between A and C mediated by B?
- More precisely, consider the following operation:



- How close is the reconstructed state $\tilde{\rho}_{ABC}$ to the original state ρ_{ABC}?
- A first question: are there setups where $\tilde{\rho}_{ABC} = \rho_{ABC}$?

Review of a setup with perfect recovery

- In the CFT vacuum state, take B to be an interval on a spatial slice, and A and C to be null intervals on either side. Casini and Huerta



- In this setup, the conditional mutual information (CMI) vanishes:

$$I(A:C|B) \equiv S(AB) + S(BC) - S(B) - S(ABC) = 0\,.$$

- In any quantum mechanical system, vanishing of CMI is equivalent to perfect recovery using an explicit channel. Hayden, Jozsa, Petz, Winter
- Define the Petz map

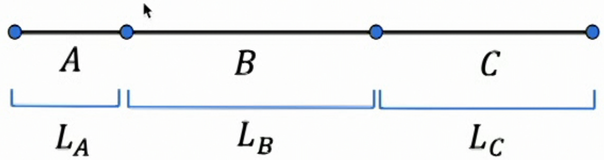
$$\mathcal{P}_{B\to BC}(\cdot) = \rho_{BC}^{\frac{1}{2}}\,\rho_B^{-\frac{1}{2}}\,(\cdot)\,\rho_B^{-\frac{1}{2}}\,\rho_{BC}^{\frac{1}{2}}$$

Then $I(A:C|B)_\rho = 0$ is equivalent to

$$\tilde{\rho}_{ABC} = \mathcal{P}_{B\to BC}(\rho_{AB}) = \rho_{ABC}\,.$$

CMI for intervals on a spatial slice

- Now consider three adjacent intervals on a spatial slice:



$$\eta = \frac{L_A L_C}{(L_A + L_B)(L_B + L_C)}$$

- The CMI is non-zero, and is given by
$$I(A:C|B) = -\frac{c}{3}\log(1-\eta)$$
The recovery operation cannot be carried out perfectly.
- Is the CMI a quantitative measure of how well the recovery works?
- In any quantum-mechanical system, we have the following inequality:
$$\max_\lambda F(\rho_{ABC}, \tilde{\rho}^{(\lambda)}_{ABC}) \geq e^{-I(A:C|B)/2}, \qquad \tilde{\rho}^{(\lambda)}_{ABC} = \rho_{BC}^{\frac{1}{2}-\frac{i\lambda}{2}} \rho_B^{-\frac{1}{2}+\frac{i\lambda}{2}} (\rho_{AB}) \rho_B^{-\frac{1}{2}-\frac{i\lambda}{2}} \rho_{BC}^{\frac{1}{2}+\frac{i\lambda}{2}}$$
where F is the fidelity. Junge, Renner, Sutter, Wilde, Winter
- When the intervals are far apart, $\eta \to 0$, and the fidelity approaches 1.
- For a direct quantitative answer to our operational question: evaluate $F(\rho_{ABC}, \tilde{\rho}^{(\lambda)}_{ABC})$.

Summary

- We consider a few different distance measures: fidelity, relative entropy, trace distance. Different operational interpretations, but show similar behaviors.
- In all cases, we use the explicit twirled Petz map.
- We use a replica trick to express each of these distance measures as a four-point function of twist operators in $\mathcal{S}_M$.
- Each of the distance measures turns out to be independent of the UV cutoff, and universal with respect to the operator content.
- We use the OPE to find behaviour in $\eta \to 1$ limit.
- We also evaluate these quantities numerically for all η using various critical spin chain models.
- In all regimes, the fidelity is better than the CMI lower bound.
- We also use both approaches to address a more qualitative question:
  What are the differences in correlations among different regions in the states ρ_{ABC} and $\tilde{\rho}_{ABC}$?

Plan

- Explain twist operator method for fidelity
- Summarize numerical results for fidelity and relative entropy
- Discuss OPE limits in twist operator formalism
- Discuss qualitative differences between ρ_{ABC} and $\tilde{\rho}_{ABC}$.

Replica trick and twist operators for fidelity

Replica trick

- The fidelity is defined as:

$$F(\rho,\sigma) = \mathrm{Tr}[(\sqrt{\rho}\sigma\sqrt{\rho})^{\frac{1}{2}}] = \mathrm{Tr}[(\rho\sigma)^{\frac{1}{2}}] = \lim_{k\to\frac{1}{2}} \mathrm{Tr}[(\rho\sigma)^k]$$

- Combining this with the definition of the twirled Petz map:

$$F(\tilde{\rho}^{(\lambda)}_{ABC}, \rho_{ABC}) = \lim_{k\to\frac{1}{2}} \lim_{\substack{n_1\to-\frac{1}{2}+\frac{i\lambda}{2},\\ n_2\to-\frac{1}{2}-\frac{i\lambda}{2}\\ m_1\to\frac{1}{2}-\frac{i\lambda}{2},\\ m_2\to\frac{1}{2}+\frac{i\lambda}{2}}} F_{k,n_1,n_2,m_1,m_2}$$

where

$$F_{k,n_1,n_2,m_1,m_2} = \mathrm{Tr}[(\rho_{BC}^{m_1}\rho_B^{n_1}\rho_{AB}\rho_B^{n_2}\rho_{BC}^{m_2}\rho_{ABC})^k]$$

- Try to evaluate this for integer values of all parameters, assume that the resulting expression is analytic in the parameters, and continue to non-integer values.
- We will refer to the above limit as the "replica limit."

Replica trick

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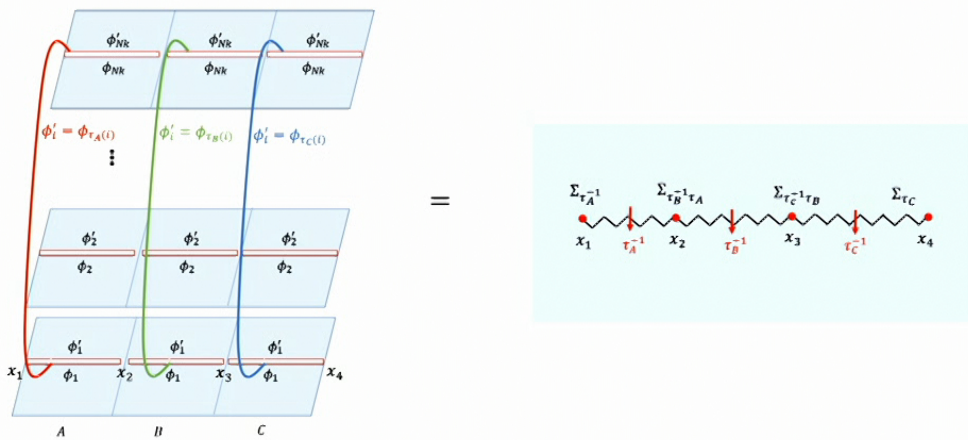
Representation in terms of twist operators

- We can write
$$F_{k,n_1,n_2,m_1,m_2} = \mathrm{Tr}[(\rho_{BC}^{m_1}\rho_B^{n_1}\rho_{AB}\rho_B^{n_2}\rho_{BC}^{m_2}\rho_{ABC})^k]$$
as a path integral on Nk copies, $N = n_1 + m_1 + n_2 + m_2 + 2$, glued together according to some permutations $\tau_{A,B,C}$ in regions A, B, C.
- This path integral can be rewritten as a four-point function of twist operators in $\mathcal{S}_{Nk}$ placed at the endpoints of the intervals.



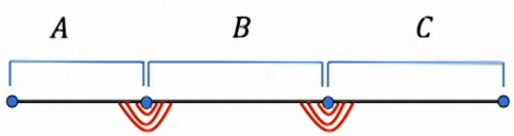
- $\Sigma_\tau(p)$: $\psi_I(x^\mu) \mapsto \psi_{\tau(I)}(x^\mu)$ on going anticlockwise around p

Twist operator dimensions and UV finiteness

- The twist operators are primary, with dimensions determined by cycle structure.
- In general, if τ has m cycles of length a_1, ..., a_m:

$$\Delta_\tau = \sum_{i=1}^{m} \frac{c}{24}\left(a_i - \frac{1}{a_i}\right)$$

- Plugging in the cycle structures of the twist operators appearing in the fidelity, all dimensions go to zero in the replica limit.
- For general values of m_i, n_i, k, we need to include an overall factor to cancel dependence on UV cutoff. This factor goes to 1 when dimensions go to zero, indicating that the fidelity is independent of the UV cutoff.
- Simpler combinations of the reduced density matrices are bad approximations to ρ_{ABC}: $F(\rho_{ABC}, \rho_A \otimes \rho_{BC})$ goes to zero as $\epsilon \to 0$.



- In contrast, $\tilde{\rho}_{ABC}$ is able to capture the short-distance entanglement structure of ρ_{ABC}.

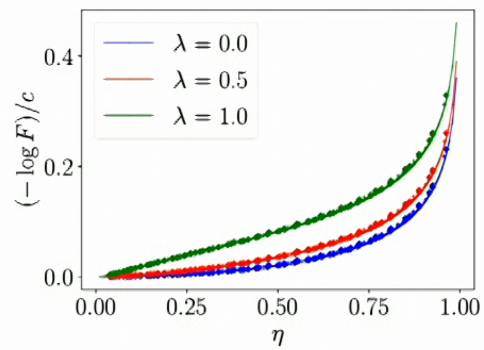
Universality of the fidelity

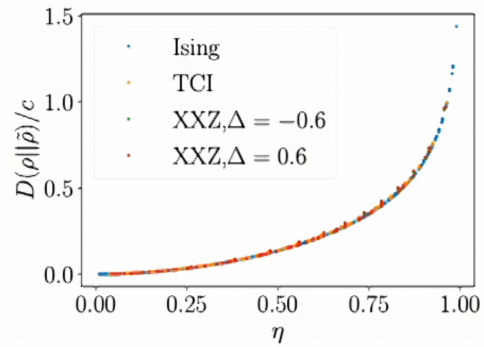
- To evaluate a four-point function of twist operators such as
$$\langle \Sigma_{\tau_A^{-1}}(x_1)\Sigma_{\tau_B^{-1}\tau_A}(x_2)\Sigma_{\tau_C^{-1}\tau_B}(x_3)\Sigma_{\tau_C}(x_4)\rangle$$
we can find a map to a *covering space* on which fields are single-valued.
- Path integral on the base space with twist operator insertions = path integral on covering space without operator insertions.
- Genus of covering space is determined by cycle structure of twist operators through the Riemann-Hurwitz formula.
- For twist operators appearing in F_{k,n_1,n_2,m_1,n_2}, the genus is zero.
- For zero genus,
$$F_{k,n_1,n_2,m_1,m_2} = \langle \Sigma_{\tau_A^{-1}}(x_1)\Sigma_{\tau_B^{-1}\tau_A}(x_2)\Sigma_{\tau_C^{-1}\tau_B}(x_3)\Sigma_{\tau_C}(x_4)\rangle = e^{S_L} Z_1^{1-Nk}$$
where S_L is the Louiville action, determined by the covering map.
$$S_L = -c\, f_{k,n_1,n_2,m_1,m_2}(\eta)$$
- In the replica limit, we get:
$$-\log F = c\, f(\eta)\,.$$
for some universal function f.
- Not possible to find an explicit expression for f_{k,n_1,n_2,m_1,m_2} using this method.

Numerical results for fidelity and relative entropy

Verifying universality in critical spin chains

- We consider lattice realizations of:
  - Ising CFT, $c = 0.5$ (solid line).
  - Tricritical Ising model, $c = 0.7$ (diamond markers).
  - Free boson CFT, $c = 1$ ("+" markers).
- For each λ, the curves for all models collapse together.
- We can also consider
$$D_\alpha(\rho||\sigma) = \frac{1}{\alpha - 1} \log \mathrm{Tr}[\rho^\alpha \sigma^{1-\alpha}], \quad \alpha \in [0,1]$$
- $\alpha \to 1$ limit is relative entropy,
$$D(\rho||\sigma) = \mathrm{Tr}[\rho \log \rho] - \mathrm{Tr}[\rho \log \sigma].$$
- $D_\alpha(\rho_{ABC}||\tilde{\rho}_{ABC})/c$ is the same for all 2D CFTs.
- The trace distance also depends only on c and η.





Comparison to conditional mutual information

- Consider the maximum and average values of $\log F$ over λ,

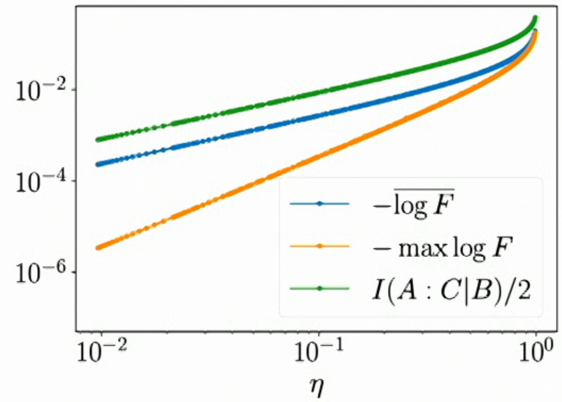
$$-\log \max_{\lambda} F = -\log F^{(\lambda=0)}$$

$$-\overline{\log F} = -\int_{-\infty}^{\infty} d\lambda\, \beta(\lambda)\, \log F\left(\rho_{ABC}, \tilde{\rho}_{ABC}^{(\lambda)}\right)$$

- General information-theoretic bounds tells us that Junge, Renner, Sutter, Wilde, Winter

$$-\log\max F \leq I(A:C|B)/2, \quad -\overline{\log F} \leq I(A:C|B)/2\,.$$

- The bounds are not saturated:



Behaviour at small and large η

- In the $\eta \to 1$ limit,

$$
\begin{aligned}
I(A:C|B)/2 &= -\frac{c}{6}\log(1-\eta) \\
-\log F(\rho_{ABC}, \tilde{\rho}^{(\lambda)}_{ABC}) &= -\frac{c}{9}\log(1-\eta) + \mathcal{O}(1) \\
D(\rho_{ABC}||\tilde{\rho}^{\lambda}_{ABC}) &= -\frac{c}{3}\log(1-\eta) + \mathcal{O}(1),
\end{aligned}
$$

- We will explain these behaviors using the OPE expansion, and relate the $\mathcal{O}(1)$ contributions to other entanglement quantities.
- In the $\eta \to 0$ limit:

$$
\begin{aligned}
I(A:C|B)/2 &\approx \frac{c}{6}\eta \\
-\log\max F &\approx a_1\, c\, \eta^2 \\
-\log \overline{F} &\approx a_2\, c\, \eta \\
D^{(\lambda=0)} &\approx a_3\, c\, \eta^2.
\end{aligned}
$$

- The replica trick will lead to answers inconsistent with numerical results in this limit.

OPE limits of fidelity

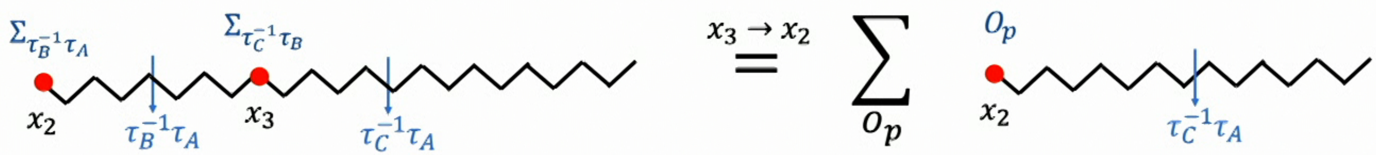
$\eta \to 1$ limit

- When $\eta = 1$, the region B disappears, $\rho_{ABC} = \rho_{AC}$, and $\tilde{\rho}_{ABC} = \rho_A \otimes \rho_C$.
- We know that $F(\rho_{AC}, \rho_A \otimes \rho_C)$ vanishes in the continuum limit, so $F(\rho_{ABC}, \tilde{\rho}_{ABC})$ should approach 0 as $\eta \to 1$.
- Using OPE for operators at the endpoints of B, we get the expansion

$$\langle \Sigma_{\tau_A^{-1}}(x_1) \Sigma_{\tau_B^{-1}\tau_A}(x_2) \Sigma_{\tau_C^{-1}\tau_B}(x_3) \Sigma_{\tau_C}(x_4) \rangle$$

$$= f_1(x_{ij}, \Delta_i) \sum_p \frac{f_{23\bar{O}_p} f_{1O_p4}}{c_{O_p}} (1-\eta)^{\Delta_p} (1-\bar{\eta})^{\bar{\Delta}_p} g_{\Delta_i;\Delta_p,\bar{\Delta}_p}(1-\eta, 1-\bar{\eta})$$

- Key input: which operators appear in the OPE?

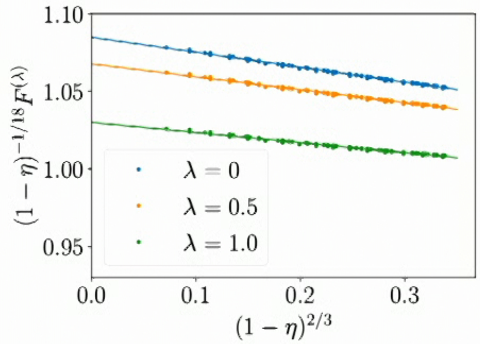


- The lowest dimension such operator is $\Sigma_{\tau_C^{-1}\tau_A}$. We can use its cycle structure to see its dimension, which is $c/9$ in the replica limit.
- Next-to-leading contribution from a fractional mode of the stress tensor acting on $\Sigma_{\tau_C^{-1}\tau_A}$, which has dimension $c/9 + 2/3$ in the replica limit.

- This gives the prediction

$$F(\rho_{ABC}, \tilde{\rho}_{ABC}) = b_0(1-\eta)^{c/9} + b_1 c(1-\eta)^{c/9+2/3}$$

for some constants b_0, b_1.



- Recall that

$$b_0 = \frac{f_{23\bar{O}_a} f_{1O_a4}}{c_{O_a}}$$

We can show that for any two adjacent intervals R and S,

$$f_{1O_a4} \propto \mathrm{Tr}[((\rho_R \otimes \rho_S)\, \rho_{RS})^{\frac{1}{2}}]$$

and

$$\frac{f_{23\bar{O}_a}}{c_{O_a}} \propto \frac{\left(\mathrm{Tr}_S\left[\mathrm{Tr}_R\left(\rho_{RS}^{\frac{3}{2}-\frac{i\lambda}{2}}\rho_R^{-\frac{1}{2}+\frac{i\lambda}{2}}\right)\mathrm{Tr}_R\left(\rho_{RS}^{\frac{3}{2}+\frac{i\lambda}{2}}\rho_R^{-\frac{1}{2}-\frac{i\lambda}{2}}\right)\right]\right)^{\frac{1}{2}}}{(\mathrm{Tr}[\rho_S^3])^{\frac{1}{2}}}$$

- How should the second quantity be interpreted?

$\eta \to 0$ limit

- Recall that in this limit, the CMI vanishes. So from the information-theoretic lower bound, the fidelity should approach 1.
- We can see that leading contribution to OPE comes from an operator whose dimension goes to zero in replica limit.
- So the leading contribution is a constant. We can also see that the constant is 1.
- However, we can argue in this case that all other OPE coefficients and conformal block coefficients vanish in the replica limit.
- To see this, we can note that the same OPE coefficients will also contribute to the quantity

$$\langle \Sigma_{\tau_A^{-1}}(x_1)\Sigma_{\tau_B^{-1}\tau_A}(x_2)\Sigma_{\tau_A^{-1}\tau_B}(x_3)\Sigma_{\tau_A}(x_4)\rangle$$

which becomes equal to $\mathrm{Tr}[\rho_{ABC}] = 1$ in the replica limit.

- Seems to imply that F approaches 1 faster than any power of η, which is inconsistent with numerics.
- It seems that the replica limit does not commute with the OPE limit. Same issue in all $\alpha-$ relative entropies.

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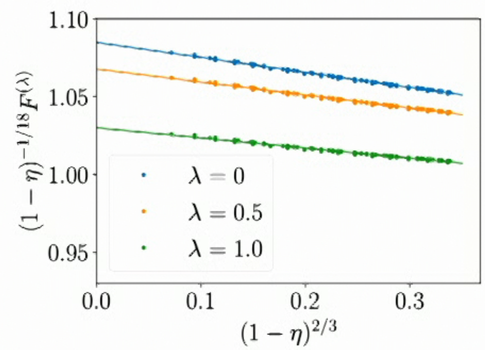

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Structure of correlations in ρ_{ABC} and $\tilde{\rho}_{ABC}$

Difference in $I(A:B)$ in $\eta \to 1$ limit

- The reduced density matrices on A and BC are unchanged between ρ_{ABC} and $\tilde{\rho}_{ABC}$, but reduced density matrices on AB and AC are changed.
- Consider the difference in $I(A:B)$ between ρ_{ABC} and $\tilde{\rho}_{ABC}$,

$$\delta I(A:B) = I(A:B)_{\rho} - I(A:B)_{\tilde{\rho}} > 0$$

- Can be evaluated using similar techniques to the distance measures. UV finite and universal.
- In the $\eta \to 1$ limit, $\delta I(A:B)$ approaches a non-zero constant.
  - Recall that $\eta = \frac{L_A L_C}{(L_A+L_B)(L_B+L_C)}$, so the region B vanishes in this limit.
  - At the same time, the distance measures between ρ_{ABC} and $\tilde{\rho}_{ABC}$ diverge.

The non-zero constant value seems to result from a competition between these two effects. Can be seen both analytically and numerically.

Difference in $I(A:C)$ in the $\eta \to 1$ limit

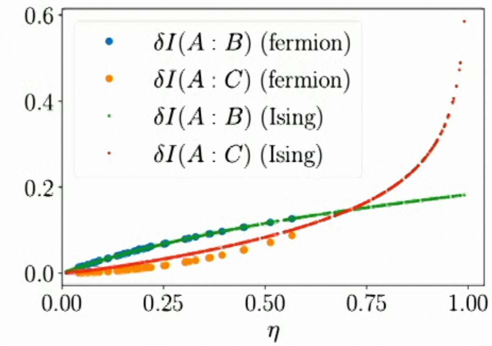
- The difference
$$\delta I(A:C) = I(A:C)_\rho - I(A:C)_{\tilde{\rho}}$$
is also UV finite, but not universal.

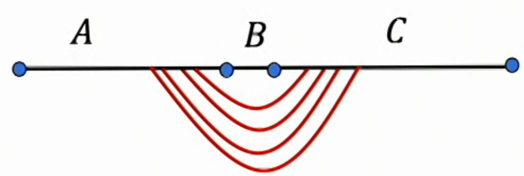
- $\delta I(A:C)$ can be seen as a more direct measure of "correlations between A and C that are not mediated by B" than $I(A:C|B)$ or $F(\rho_{ABC}, \tilde{\rho}_{ABC})$.

- In the $\eta \to 1$ limit, it has a universal divergence:
$$\delta I(A:C) = -\frac{c}{3}\log(1-\eta), \quad \eta \to 1$$

- This shows that a diverging amount of correlations between A and C are not present in $\tilde{\rho}_{ABC}$, and are therefore likely to be direct correlations.

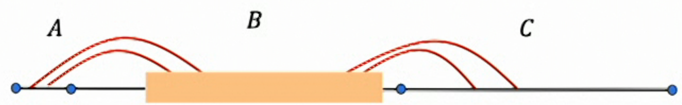
- Divergence is same as that of the CMI.





Mutual information differences in $\eta \to 0$ limit

- For small η, we find
$$\delta I(A:C) \propto \eta^{1+2\Delta}\,,$$
where Δ is the dimension of the second-lowest-dimension primary operator.
- In contrast, recall leading behaviour of $I(A:C)$: Calabrese, Cardy, Tonni
$$I(A:C)_\rho \propto \eta^{2\Delta}, \quad \eta \to 0\,.$$
- The leading behaviour of the CMI thus precisely agrees between ρ_{ABC} and $\tilde{\rho}_{ABC}$. Shows in a precise way that these leading correlations are mediated by B, and are not of the Bell pair type.



- There is a qualitative as well as quantitative change in entanglement between A and C on going from the near-interval limit to the far-interval limit.
- In the $\eta \to 0$ limit, $\delta I(A:B) \propto \eta$, and in particular much larger than $\delta I(A:C)$.

Difference in $I(A:C)$ in the $\eta \to 1$ limit

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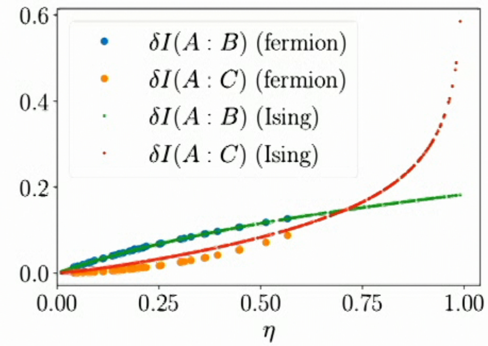
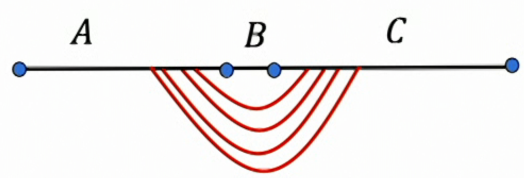
is also UV finite, but not universal.
- $\delta I(A:C)$ can be seen as a more direct measure of "correlations between A and C that are not mediated by B" than $I(A:C|B)$ or $F(\rho_{ABC}, \tilde{\rho}_{ABC})$.
- In the $\eta \to 1$ limit, it has a universal divergence:
$$\delta I(A:C) = -\frac{c}{3}\log(1-\eta), \quad \eta \to 1$$
- This shows that a diverging amount of correlations between A and C are not present in $\tilde{\rho}_{ABC}$, and are therefore likely to be direct correlations.
- Divergence is same as that of the CMI.





Comparison to c-theorem

- Recall that the entropic c-theorem in (1+1) dimensions tells us that in the vacuum state of any relativistic QFT,

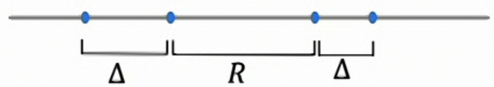
$$\frac{d}{dR}(RS'(R)) \leq 0\,.$$

  The inequality is saturated in CFTs.
- We can rearrange the above equation to:

$$-S''(R)\Delta^2 \geq \frac{S'(R)}{R}\Delta^2$$

$$\Rightarrow \quad 2S(R+\Delta) - S(R) - S(R+2\Delta) \geq \frac{S'(R)}{R}\Delta^2\,.$$

- The LHS can be seen as $I(A:C|B)$ for the following configuration:



- While the c-theorem inequality is saturated in CFT, we found that the general inequality $I(A:C|B) \geq -\log F$ is not.
- Is there a stronger version of the QI inequality, which does coincide with the c-theorem, and in particular is saturated in CFTs?

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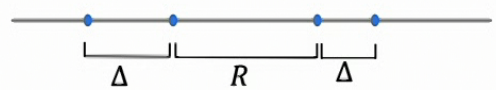

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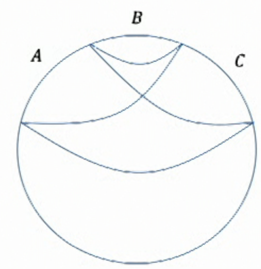
Further questions

- Is there some holographic dual of $-\log F$ or relative entropy between ρ_{ABC} and $\tilde{\rho}_{ABC}$?



- Can differences between ρ_{ABC} and $\tilde{\rho}_{ABC}$ provide some new insight into topological phase transitions or non-equilibrium dynamics?
- Are there classes of states where the QI inequality is saturated, and if so what is their structure?

Thank you!