Title: Talk 41 - Mutual Information of Holographic Generalized Free Fields

Speakers: Pedro Jorge MartÃ-nez

Collection: It from Qubit 2023

Date: July 31, 2023 - 4:30 PM

URL: https://pirsa.org/23070010

Abstract: We study Generalized Free Fields (GFF) from the point of view of information measures. We begin by reviewing conformal GFF, their holographic representation, and the multiple possible assignations of algebras to a single spacetime region that arise in these theories. We will focus on manifestations of these features present in the Mutual Information (MI) of holographic GFF. First, we show that the MI can be expected to be finite even if the AdS dual space is of infinite volume. Then, we present the long-distance limit of the MI for regions with arbitrary boundaries in the light cone for the causal and entanglement wedge algebras. The pinching limit of these surfaces shows the GFF behaves as an interacting model from the MI point of view. The entanglement wedge algebra choice allows these models to ``fake" causality, giving results consistent with their role in the description of large N models. Finally, we explore the short distance limit of the MI. Interestingly, we find that the GFF has a leading volume term rather than an area term and a logarithmic term in any dimension rather than only for even dimensions as in ordinary CFTs. We also find the dependence of some subleading terms on the conformal dimension of the GFF.

Mutual Information of Generalized Free Fields

Authors: Horacio Casini, Valentín Benedetti, **PJM**

Based on: arXiv:2210.00013 [hep-th]

IfQ2023 - Perimeter Institute

Index

- GFFs & Algebras
- Finiteness of Mutual Information
- Short Distance MI: Volume Law
- Long Distance MI: Algebra-Region
- Conclusions

GFFs & Algebras

We **define** GFFs by Kallén Lehmann + Wick Theorem [*]

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \int_0^\infty ds\, \rho(s)\, W_0(x-y,s)$$

e.g. $\rho(s) = s^{\Delta - d/2}$ for conformal GFF of weight Δ .

GFFs meet Wightman's axioms **but not the time slice axiom** [**], i.e. the algebra of a finite time strip does not generate the full operator algebra of the theory.

Enlarging the strip adds new operators to the algebra.

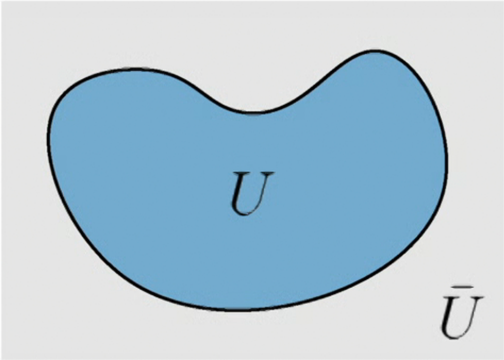
[*] See e.g. Yngvason, '94.

GFFs & Algebras: QFT point of view

Ordinary relativistic QFTs assign unique algebras to topologically trivial causally complete regions e.g. U. These trivially meets **Haag Duality:** $\mathcal{A}(U) = \mathcal{A}'(\bar{U})$

GFFs have more than one algebra, except for spheres. Min & Max algebras can be seen to be different [*].

Conformal GFFs have a holographic description that put these algebra choices in a more familiar set-up. **We define Mutual Information for these theories.**



[*] e.g. Liu & Leutheusser '22; Faulkner & Li '22, Nebabu & Qi '23, many others.

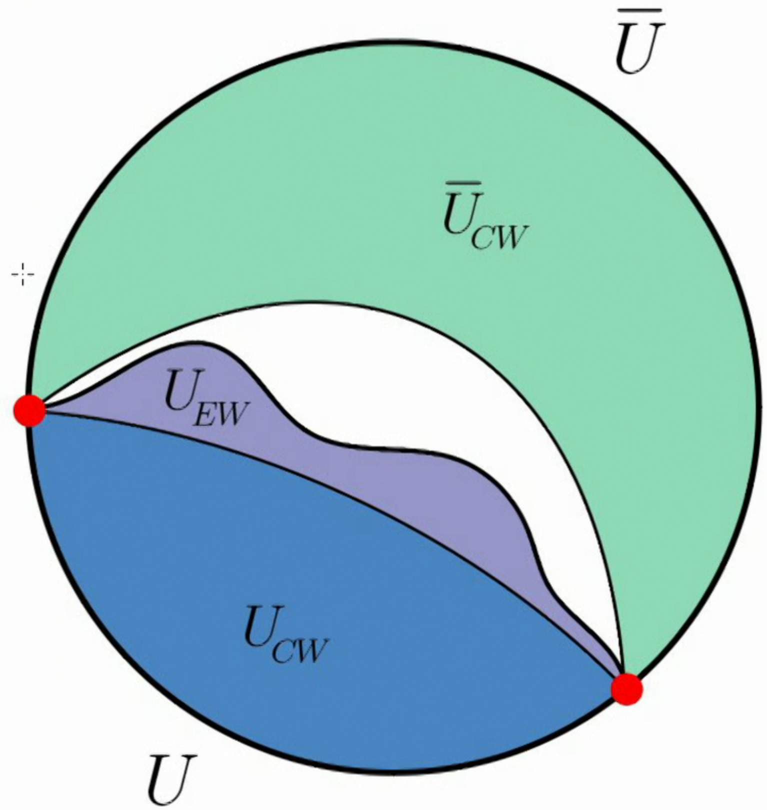
GFFs & Algebras: Holographic point of view

We map the problem to **free QFTs** in (a)AdS.

The choice of boundary GFF algebra maps to the choice of bulk region:

- Causal Wedge
- Entanglement Wedge
- etc.

Many recent papers use a similar set-up [*].



[*] This definition extends to other wedges. || [**] FLM '13. $I(A,B) \equiv S(A) + S(B) - S(A \cup B)$

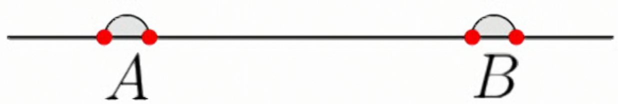
Finiteness of MI

The Holographic GFF **Mutual Information (MI)** [*]

$$I_{GFF;EW}(A,B) \equiv I_{\varphi}(A_{EW}, B_{EW})$$

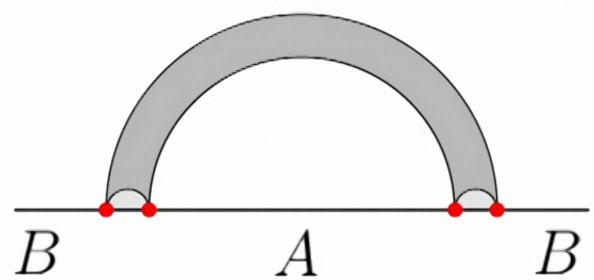
Long Distance [**]:

$$I_{GFF;EW}(A,B) = I_{CFT}(A,B)$$



Short Distance:

$$I_{GFF;EW}(A,B) \neq I_{CFT}(A,B)$$



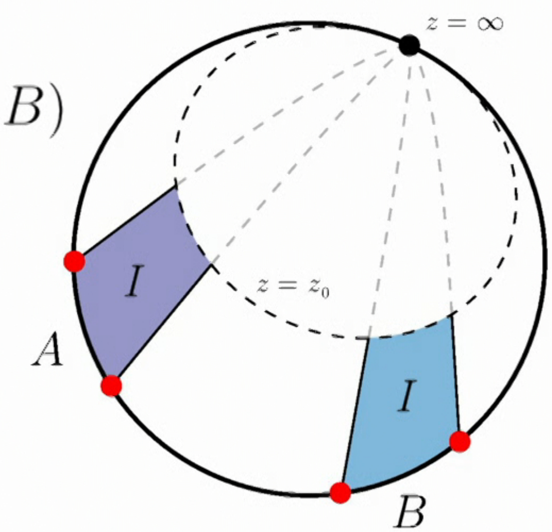
[*] See Casini & Huerta '05. $I_0(\kappa_n, A, B)$ flat space MI of scalar of mass κ_n . Sum converges.

Finiteness of MI

Does the infinite AdS Volume lead to non-finite MI?
Near the boundary, area grows but correlators decay.
A Poincaré-patch motivated algebra tests convergence and yields a convergent KK like decomposition, i.e. [*]

$$I_\varphi(A \times I, B \times I) = \sum_n I_0(\kappa_n, A, B)$$

Convergence ends at U.Bound $\Delta \to (d-2)/2$ as expected.



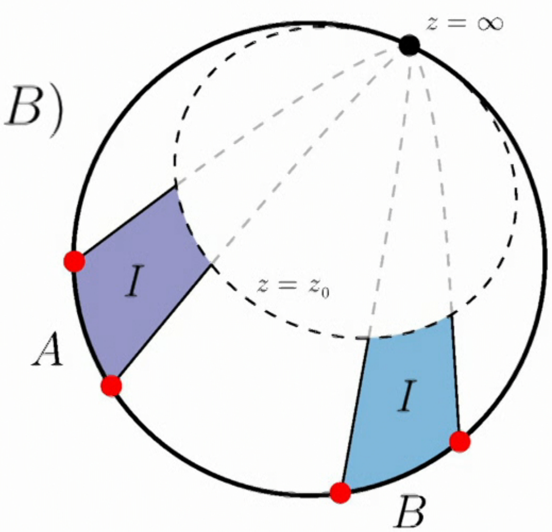
[*] See Casini & Huerta '05. $I_0(\kappa_n, A, B)$ flat space MI of scalar of mass κ_n . Sum converges.

Finiteness of MI

Does the infinite AdS Volume lead to non-finite MI?
Near the boundary, area grows but correlators decay.
A Poincaré-patch motivated algebra tests convergence and yields a convergent KK like decomposition, i.e. [*]

$$I_\varphi(A \times I, B \times I) = \sum_n I_0(\kappa_n, A, B)$$

We will always work with finite MI.

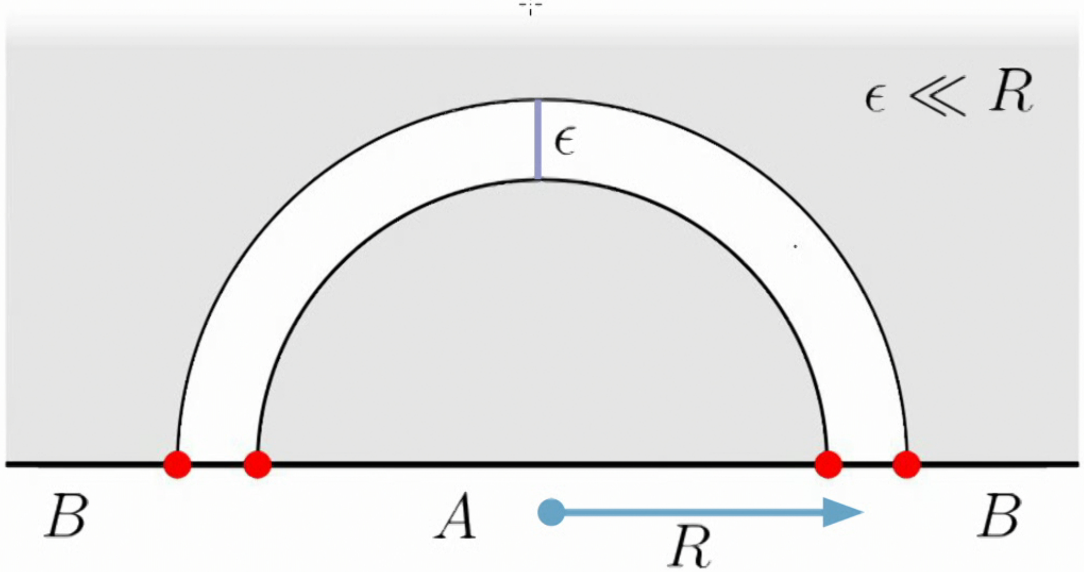


[*] CW, EW and Max Alg all coincide for spheres.

Short Distance MI: Volume Law

We compute a short distance MI expansion between two nearby spheres [*] for a conformal GFF via holography.

$$I_{GFF;EW}(A, B) \equiv I_{\varphi}(A_{EW}, B_{EW})$$



[*] El-Showk & K. Papadodimas '12. [**] k_d leading MI coeff. for planar boundaries and massless scalars.

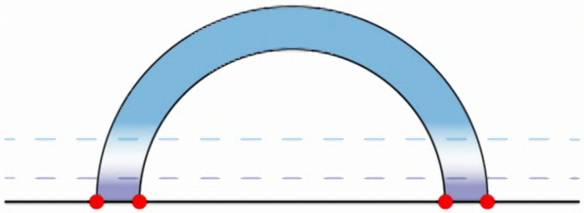
Short Distance MI: Volume Law

The short distance MI expansion for a conformal GFF is [*][**]

$$I_{GFF} = \boxed{k_{d+1}\frac{\pi^{d/2}}{\Gamma(d/2)}\frac{R^{d-1}}{\epsilon^{d-1}}} + \boxed{k_d\frac{(\Delta-\frac{d}{2})\pi^{d-1}}{\Gamma(d-1)}\frac{R^{d-2}}{\epsilon^{d-2}}} + \cdots + \boxed{c_0\log(R/\epsilon)}$$

$\boxed{c_{d-3},\ldots}$ look like $\int dA\,\frac{\mathcal{R}^a\,(K^2)^b\,(m^2)^c}{l(x)^{d-1-2\,(a+b+c)}}$

$\boxed{c_{d-2},\ldots}$ depend on Δ and K_∂^2

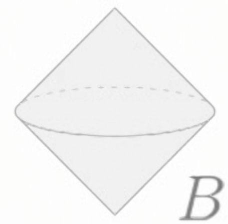


For reference, the ordinary CFT MI is,

$$I = \boxed{k_d\frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)}\frac{R^{d-2}}{\epsilon^{d-2}}} + \boxed{\tilde{c}_{d-4}\frac{R^{d-4}}{\epsilon^{d-4}}} + \cdots + \begin{cases}(-)^d\,2A\log(R/\epsilon) & d \text{ even.}\\ (-)^d\,F & d \text{ odd.}\end{cases}$$

[*] Casini, Testé & Torroba '21 || [**] Wall '11 & refs within. || [***] 0-measure region removal.

Long Distance MI: Algebra-Region



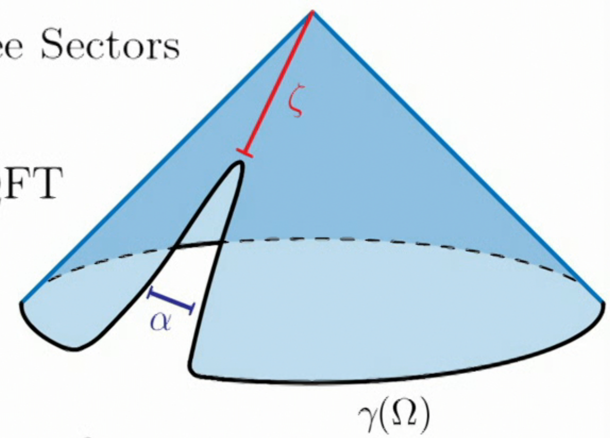
Pinching [*]: We deform the sphere $A \to \gamma(\Omega)$ and take the the causal development of blue region.

- Interacting [**]:

$$\lim_{\alpha > \zeta \to 0} I(\gamma(\Omega), B) \begin{cases} = 0 & \text{No Free Sectors} \\ < I(A,B) & \text{Other} \\ = I(A,B) & \text{Free QFT} \end{cases}$$

- Causal [***]:

$$\lim_{\zeta > \alpha \to 0} I(\gamma(\Omega), B) \begin{cases} = I(A,B) & \text{Causal} \\ < I(A,B) & \text{Non Causal} \end{cases}$$



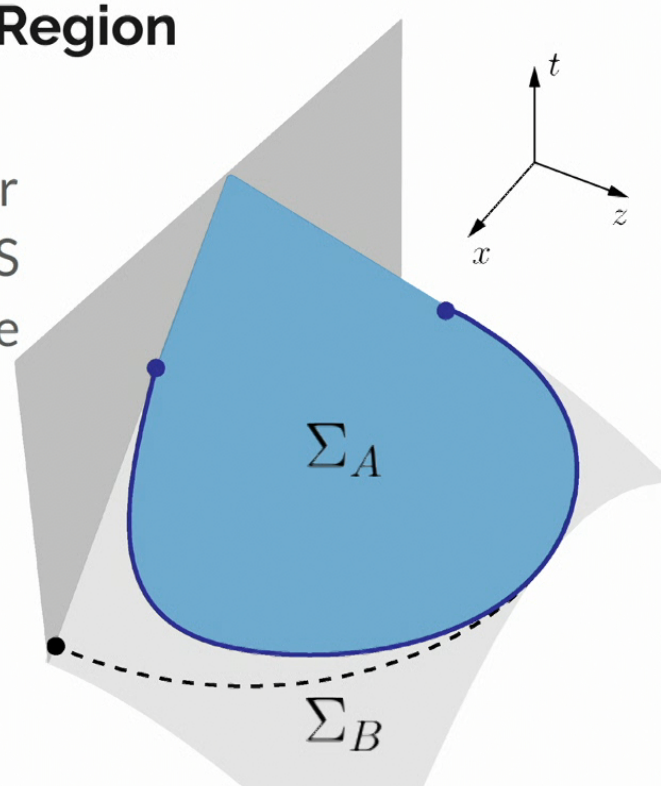
[*] Cardy '13; Agón & Faulkner '16; Casini, Testé & Torroba '21, among others.

Long Distance MI: Algebra-Region

The holographic computation for conformally coupled fields in AdS [*] localizes on the past null cone and **depends on algebra choice**:

$$I(A,B) \sim \left(\frac{C_A C_B}{L^4}\right)^{\Delta}$$

$$\boxed{C_A = \frac{1}{2\pi}\int_{\Sigma_A} dA}$$



Long Distance MI: Algebra-Region

Causal Wedge Results

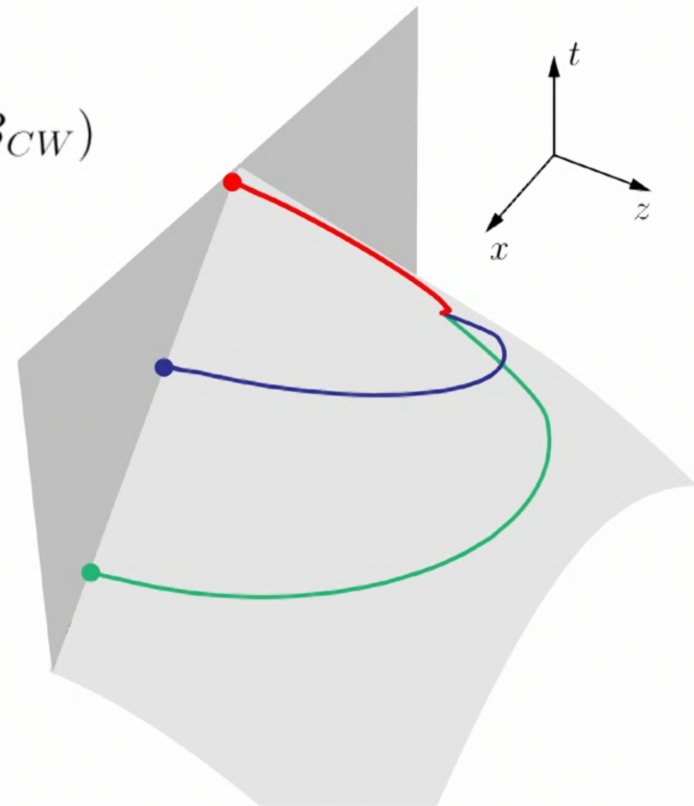
$$I_{GFF;CW}(A, B) \equiv I_{\varphi}(A_{CW}, B_{CW})$$

- Interacting: **YES**

$$\lim_{\alpha>\zeta\to 0} I(\gamma(\Omega), B) \sim \left(\frac{\zeta}{\alpha}\right)^{\frac{1}{2}}$$

- Causal: **NO**

$$\lim_{\zeta>\alpha\to 0} I(\gamma(\Omega), B) = 0$$



Long Distance MI: Algebra-Region

Entanglement Wedge Results

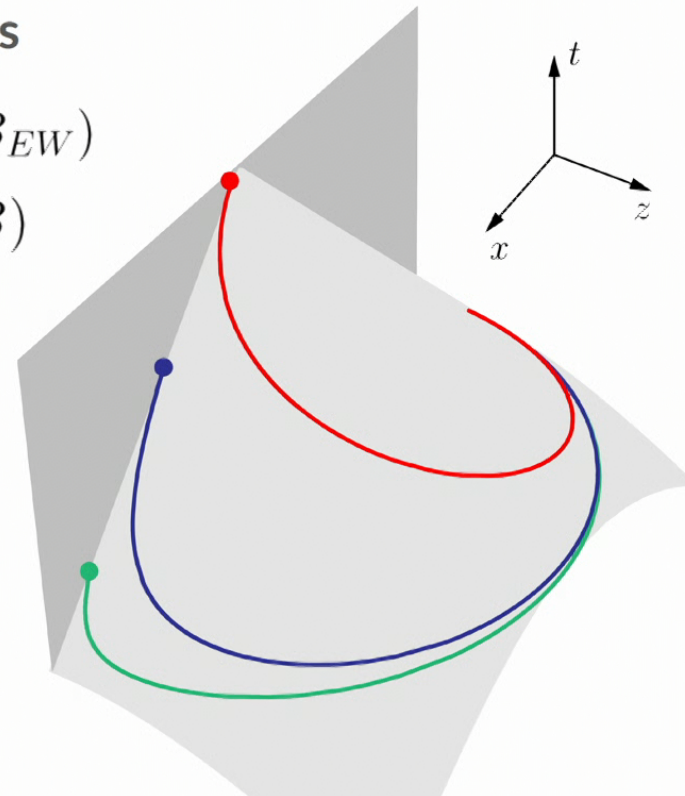
$$
\begin{aligned}
I_{GFF;EW}(A,B) &\equiv I_{\varphi}(A_{EW}, B_{EW}) \\
&= I_{CFT}(A,B)
\end{aligned}
$$

- Interacting: **YES**

$$\lim_{\alpha > \zeta \to 0} I(\gamma(\Omega), B) \sim \left(\frac{\zeta}{\alpha}\right)^{\frac{1}{3}}$$

- Causal: **YES** (“fake”)

$$\lim_{\zeta > \alpha \to 0} I(\gamma(\Omega), B) = I(A,B)$$



Conclusions

- GFFs are QFTs that admit more than one possible algebra for topologically trivial causally complete regions.
- Holographic GFFs map the possible GFFs algebras to bulk free field algebras in asymptotically AdS spacetimes.
- We defined holographic GFFs Mutual Information:
  - Short Distance: Volume Law
    - No connection to large N theory limit.
    - CFT origin of Volume Law?
  - Long Distance: Algebra-Region
    - Explicit dependence on algebra choice.
    - GFFs are interacting and non causal, but EW MI is blind to non causality.

Thanks!