Title: Talk 124 - von Neumann algebras in JT gravity with matter

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Abstract: We quantize JT gravity with matter on the spatial interval with two asymptotically AdS boundaries. We consider the von Neumann algebra generated by the right Hamiltonian and the gravitationally dressed matter operators on the right boundary. We prove that the commutant of this algebra is the analogously defined left boundary algebra and that both algebras are type II infinity factors. These algebras provide a precise notion of the entanglement wedge away from the semiclassical limit.

von Neumann algebras in JT gravity with matter

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Based on 2303.04701

See also 2301.07257 by Penington and Witten

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Introduction: why von Neumann algebras?

- A *von Neumann (vN) algebra* is a $*$-algebra of bounded operators that equals its double-commutant: $\mathcal{A} = \mathcal{A}''$.
- Physically, a vN algebra models a subsystem of a quantum system. Example: an entanglement wedge (EW).
- **[Harlow '16]** studied finite-dimensional QECC as a toy model for AdS/CFT, where the EW was treated as a *Type I vN algebra*

$$\mathcal{H}_{bulk} = \bigoplus_{\alpha} \mathcal{H}_{L,\alpha} \otimes \mathcal{H}_{R,\alpha}$$

- I will discuss a different toy model, JT gravity with matter, and show that the EWs are instead modeled by *Type II_∞ factors*. These algebras give us a natural language for understanding bulk emergence.
- Furthermore, I will demonstrate how vN algebras put the replica trick on completely firm conceptual ground.

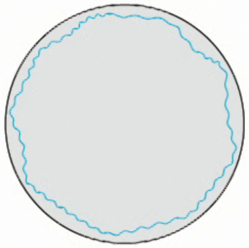
JT gravity with matter

- The Euclidean action is

$$I[g,\phi,\varphi] = \chi S_0 + I_{JT}[g,\phi] + I_{matter}[g,\varphi]$$

$$I_{JT}[g,\phi] = -\left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2) + \int_{\partial\mathcal{M}}\sqrt{h}\phi(K-1)\right]$$

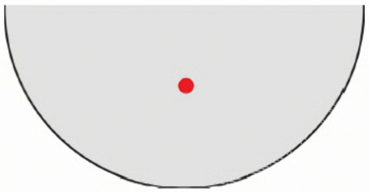
- On an asymptotically AdS boundary, we take both the boundary length and dilaton to infinity while fixing their ratio, which determines β.



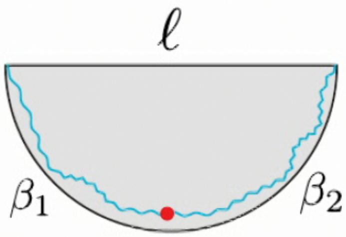
- The disk diagram computes the partition function of the putative boundary dual. We may also insert matter operators on the AdS boundary. These insertions are dual to insertions of boundary operators $\mathcal{O}_i$.

The Hilbert space

- States in the Hilbert space $\mathcal{H}_0$ may be prepared using half of the Euclidean hyperbolic disk.

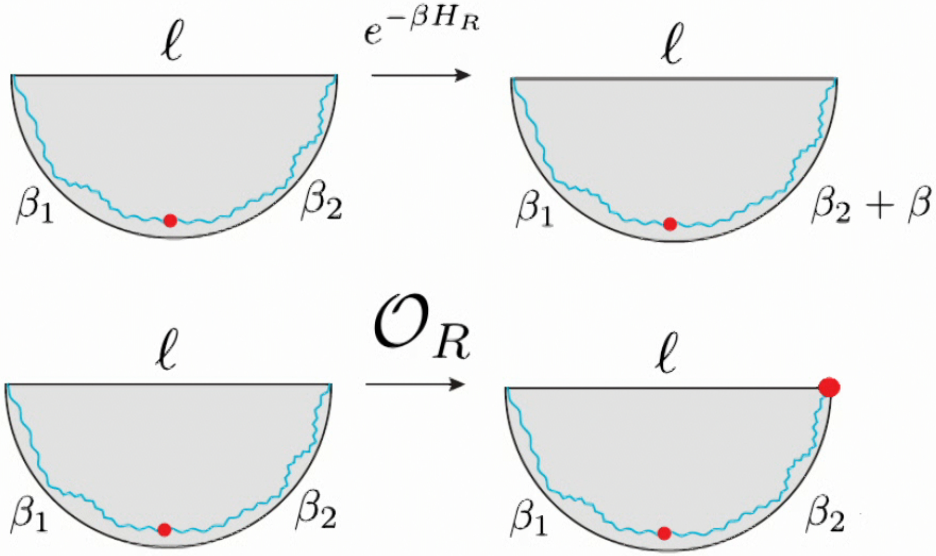


- In JT gravity with matter, states may also be prepared by cutting the disk path integral.



Boundary operators

- We define $\mathcal{O}_R$ and H_R as follows:



- $\mathcal{O}_L$ and H_L are defined similarly. There exist simple expressions for these operators acting on $\mathcal{H}_0 \otimes L^2(\mathbb{R})$.

Boundary von Neumann algebras

- We can define a vN algebra by first defining a $*$-algebra and then taking its double-commutant. Let $\mathcal{A}_{R,0}$ be the $*$-algebra of all words made from the letters $\mathcal{O}_{i,R}$ and H_R (with Boltzmann factors inserted to keep the operators bounded). For example,

$$H_R e^{-\beta H_R}, \quad e^{-\beta_1 H_R} \mathcal{O}_{i,R} e^{-\beta_2 H_R} H_R^2 \mathcal{O}_{j,R}, \quad \text{etc}$$

Results

- Let $a \in \mathcal{A}_L'$. Then

$$a = \sum_{i=1}^{\infty} a_i, \quad a_i \in \mathcal{A}_{R,0},$$

where the sum converges when inserted in between any bra and ket. Hence, $\mathcal{A}_L' \subset \mathcal{A}_R$. Also, $\mathcal{A}_R \subset \mathcal{A}_L'$. Hence,

$$\mathcal{A}_L' = \mathcal{A}_R, \quad \mathcal{A}_R' = \mathcal{A}_L.$$

- $\mathcal{A}_L$ and $\mathcal{A}_R$ are *factors*

$$\mathcal{A}_L \cap \mathcal{A}_R = \mathbb{C} \cdot 1.$$

- For any vN algebra $\mathcal{A}$, we have that

$$(\mathcal{A} \cap \mathcal{A}')' = (\mathcal{A} \cup \mathcal{A}')''.$$

Setting $\mathcal{A} = \mathcal{A}_R$, we obtain

$$(\mathcal{A}_R \cup \mathcal{A}_L)'' = (\mathbb{C} \cdot 1)' = \mathcal{B}(\mathcal{H}).$$

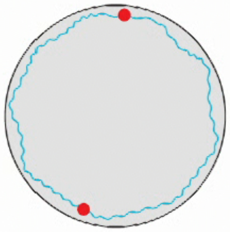
Results: physical interpretation

- These results are valid for any value of G_N. Because spacetime itself may be subject to huge quantum fluctuations, the only way we can make sense of the entanglement wedge is via the algebras $\mathcal{A}_R$ and $\mathcal{A}_L$.
- We interpret
$$(\mathcal{A}_R \cup \mathcal{A}_L)'' = \mathcal{B}(\mathcal{H})$$
to mean that any bulk operator can be reconstructed from $\text{CFT}_L \cup \text{CFT}_R$, as one might expect from semiclassical considerations.
- The right entanglement wedge is defined to be $\mathcal{A}_R$. These operators can be reconstructed from H_R and the right single-trace matter operators.
- Resonates with the idea that modular-flowed single-trace operators are sufficient to reconstruct the entanglement wedge. [JLMS '15] [Faulkner-Lewkowycz '17] [Leutheusser-Liu '22]

Traces

- The Euclidean path integral furnishes a natural example of a trace.

$$\text{tr } e^{-\beta_1 H} \mathcal{O} e^{-\beta_2 H} \mathcal{O} =$$

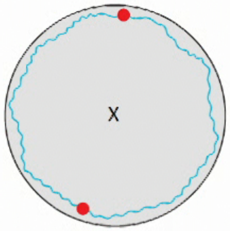


- The existence of the trace implies that $\mathcal{A}_R$ is not type III. The lack of Hilbert space factorization implies that $\mathcal{A}_R$ is not type I. We are left to conclude that $\mathcal{A}_R$ is a Type II factor. This trace is unique up to normalization. It is a Type II_∞ factor because $\text{tr } 1 = \infty$.
- The entropy is completely well-defined up to a state-independent additive constant, rendering the replica trick obsolete.

Traces

- One could consider the disk with a defect:

$$\widetilde{\mathrm{tr}}\; e^{-\beta_1 H}\mathcal{O}e^{-\beta_2 H}\mathcal{O} =$$

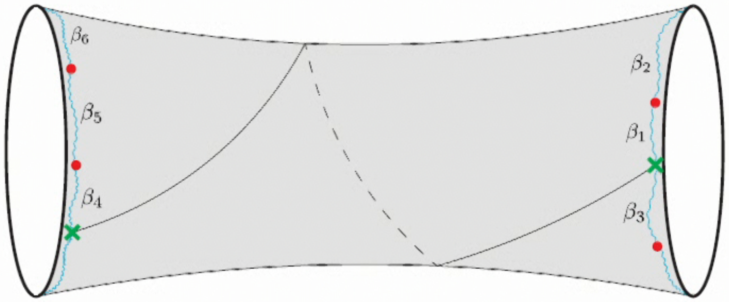


  One can choose the defect so that the trace is positive, obeying $\widetilde{\mathrm{tr}}\; a^\dagger a > 0$.

- This is an alternate trace defined on $\mathcal{A}_{R,0}$, which is a *dense subalgebra* of $\mathcal{A}_R$. It cannot be extended to a continuous trace on all of $\mathcal{A}_R$, because a factor admits a unique (faithful-semifinite-normal) trace up to normalization.

Traces

- Another trace to consider is the canonical trace Tr on $\mathcal{H}$.
- However, for $a_R \in \mathcal{A}_R$ and $a_L \in \mathcal{A}_L$, Tr $a_L a_R = \infty$, because it is given by the gravitational path integral on a double-trumpet geometry.
- For example, Tr $e^{-\beta_6 H_L}\mathcal{O}_L e^{-\beta_5 H_L}\mathcal{O}_L e^{-\beta_4 H_L} e^{-\beta_3 H_R}\mathcal{O}_R e^{-\beta_2 H_R}\mathcal{O}_R e^{-\beta_1 H_R}$ is



- This is divergent due to the sum over the marked geodesic and the matter divergence as the geodesic neck of the double-trumpet becomes small.

The code subspace

- So far, I have only discussed the bulk JT+matter theory. I have not commented on how this theory is embedded in the boundary dual.
- [Lin '22] has discussed the bulk-to-boundary map, where the boundary theory consists of SYK_L and SYK_R in an appropriate limit.
- The state $|\beta = 0\rangle$ is dual to a maximally entangled state of SYK_L and SYK_R, and H_R and $\mathcal{O}_R$ are respectively reconstructed by H_{SYK_R} and M_{SYK_R}. In the appropriate limits, the bulk-to-boundary map is isometric.
- Because JT+matter is realized on a code subspace, we expect that

$$\mathrm{Tr}\ e^{-\beta_L H_L} e^{-\beta_R H_R} < \mathrm{Tr}_{SYK_L} e^{-\beta_L H_L} \mathrm{Tr}_{SYK_R} e^{-\beta_R H_R}$$

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$$\overbrace{\mathrm{Tr}\; e^{-\beta_L H_L} e^{-\beta_R H_R}}^{\infty} < \overbrace{\mathrm{Tr}_{SYK_L} e^{-\beta_L H_L} \mathrm{Tr}_{SYK_R} e^{-\beta_R H_R}}^{e^{2S_0}}$$

Discussion

- We defined vN algebra factors $\mathcal{A}_R$, $\mathcal{A}_L$ that algebraically define the entanglement wedges, even in the non-semiclassical regime.
- We showed that $\mathcal{A}'_R = \mathcal{A}_L$, and $\mathcal{A}_R$ and $\mathcal{A}_L$ are factors.
- Analogous results have been proven in the math literature **[Ricard '03]**, but in a system physicists would call "q-deformed JT gravity with matter."
- Would be interesting to arrive at the usual definition of the EW (using the QES formula) by taking semiclassical limits of $\mathcal{A}_L$ and $\mathcal{A}_R$.
- Consider JT gravity with one AdS boundary and one end-of-the-world brane boundary. Is the algebra of boundary observables type I?
- By studying more carefully the embedding of the bulk into the boundary, it would be interesting to derive corrections to bulk physics at finite S_0.