Title: A holographic effective field theory for a strongly coupled metal with a Fermi surface

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Series: Quantum Fields and Strings

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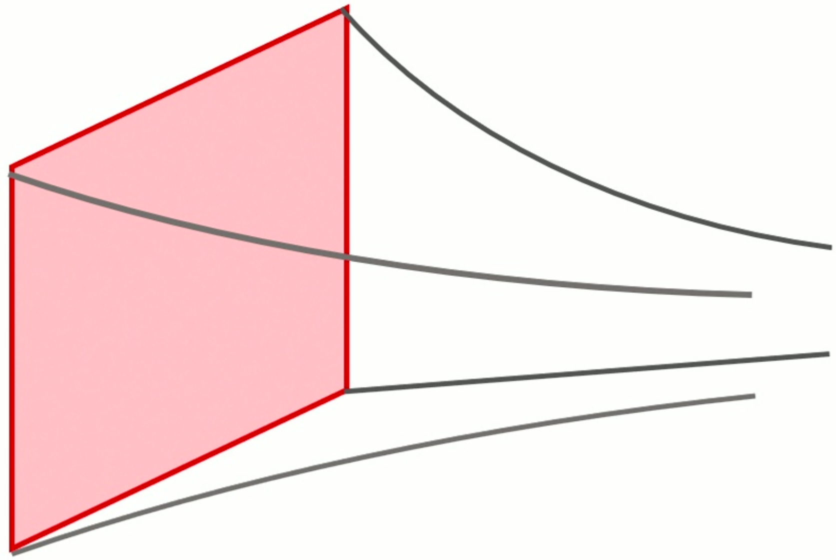
Abstract: The holographic duality between strongly coupled quantum field theories and weakly coupled gravitational theories in one higher dimension holds, in principle, the promise of understanding strongly coupled systems that occur in condensed matter physics, such as the "strange" metals that appear in materials such as high-Tc cuprates. Unfortunately, the holographic models of metals that have previously been studied have not been successful in capturing even the most basic physics that any realistic model of a metal should obey. In this talk, I will review the essential properties that any metal (strongly coupled or not) must satisfy, and propose a new holographic model that is consistent with these requirements. The new model is based on a radically different approach compared with previous holographic models of metals, and crucially relies on recent work that formulates in a precise way the conditions for an IR effective field theory to be "emergeable" from a UV theory at nonzero charge density. In particular, the holographic model I study is dual to a quantum field theory with a global symmetry group LU(1) -- the "loop group" whose elements are smooth functions from the circle into U(1). I present the results of a solution of the model and argue that its properties are qualitatively consistent with what one should expect to find in a strongly coupled metal.

Zoom Link: TBD

Holography



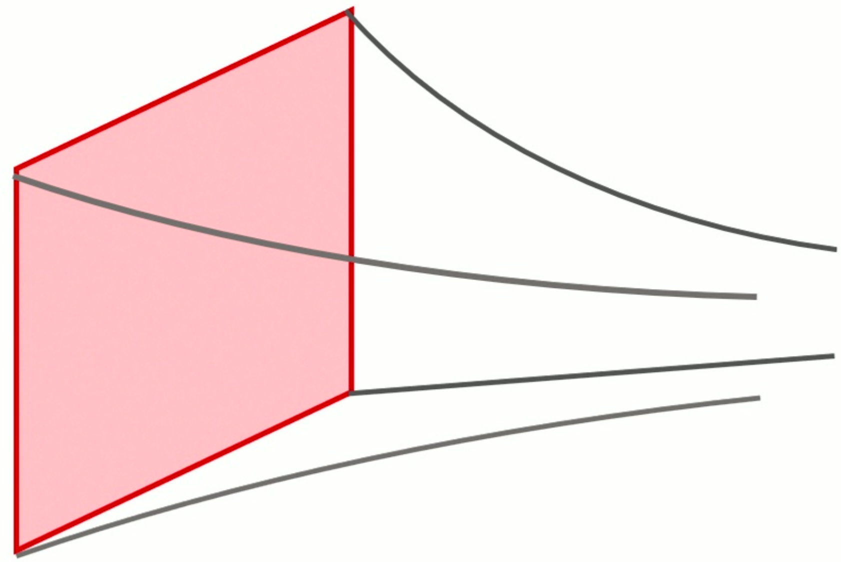
Quantum field theory
in D space-time dimensions

Duality

Quantum gravity
in D+1 space-time
dimensions

(weakly coupled, solves classical equations of motion)

Holography



Quantum field theory in D space-time dimensions

(strongly coupled)

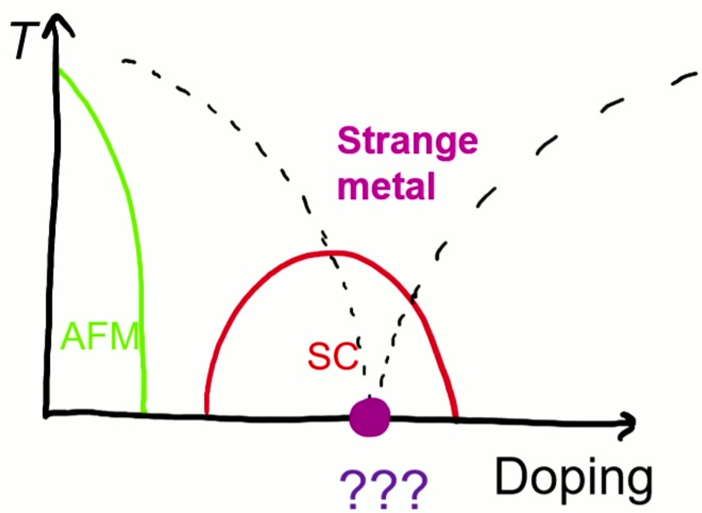
Duality ⟷

Quantum gravity in D+1 space-time dimensions

(weakly coupled, solves classical equations of motion)

Strange metals: example of non-Fermi liquid

Doped cuprates (e.g. YBCO = Yttrium Barium Copper Oxide)
High temperature superconductors (YBCO has T_c ~ 93 K)



Other examples in heavy fermion compounds, twisted bilayer graphene, …

Outline

- What are the fundamental properties of metals (beyond weak coupling)?
- Review: previous holographic models of metals, and why they are unsatisfactory
- Constructing a new holographic model
- Results from solution of the model

What is a metal?

UV

Quantum field theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

e.g. non-relativistic electron with chemical potential

$$H = \int d^d\mathbf{x} \left[-\frac{1}{2m} \Psi^\dagger \nabla^2 \Psi - \mu \Psi^\dagger \Psi + (\text{interactions}) \right]$$

↓

IR Effective field theory

What is a metal?

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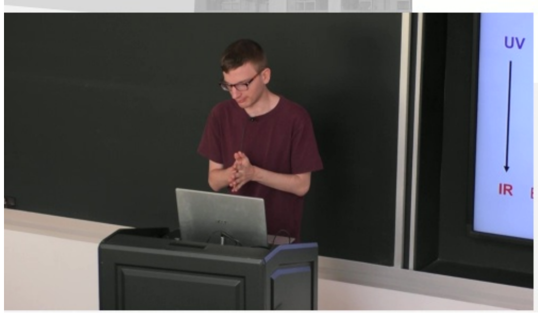
IR Effective field theory

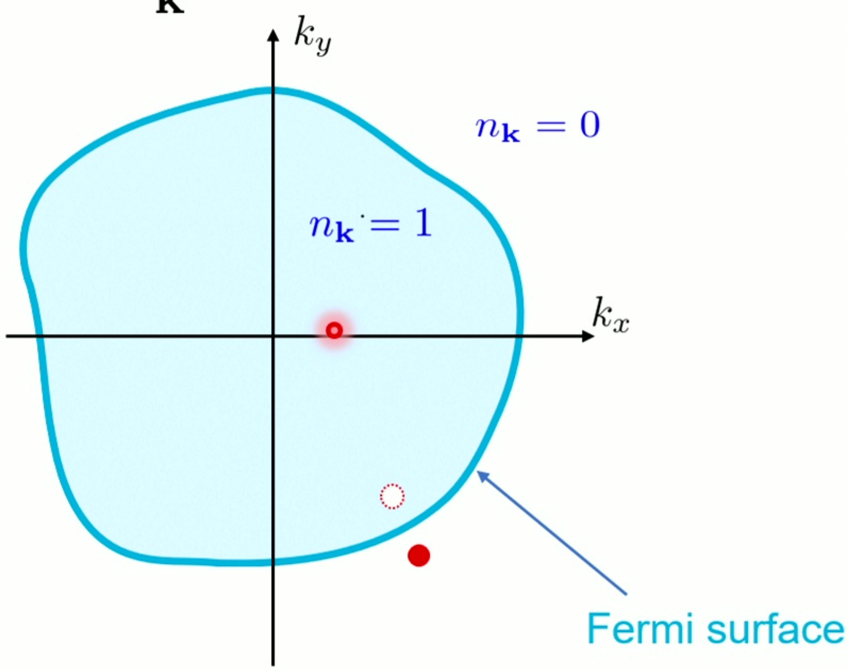
Metal or superfluid

Spontaneously breaks U(1)



IR theory of a metal: non-interacting electrons

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}}$$



UV-IR relation:

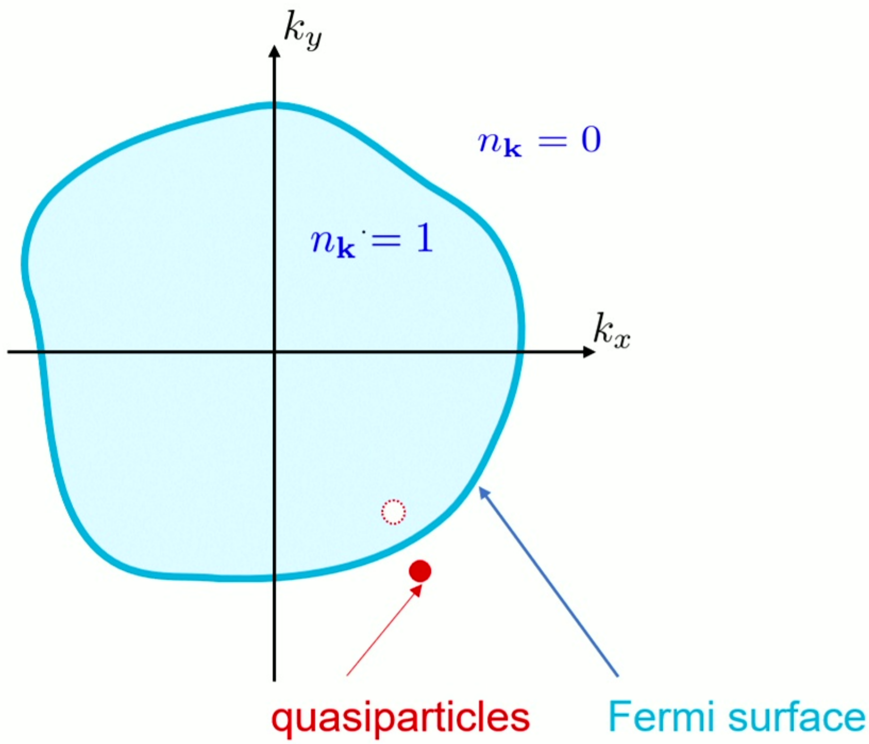
$$\boxed{\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}}$$

Volume enclosed by
Fermi surface

UV charge density

IR theory of a metal: Fermi liquid theory

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \hat{n}_{\mathbf{k}} \hat{n}_{\mathbf{k}'}$$



Volume enclosed by Fermi surface

UV

UV-IR relation (Luttinger's theorem)

$$\boxed{\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}}$$

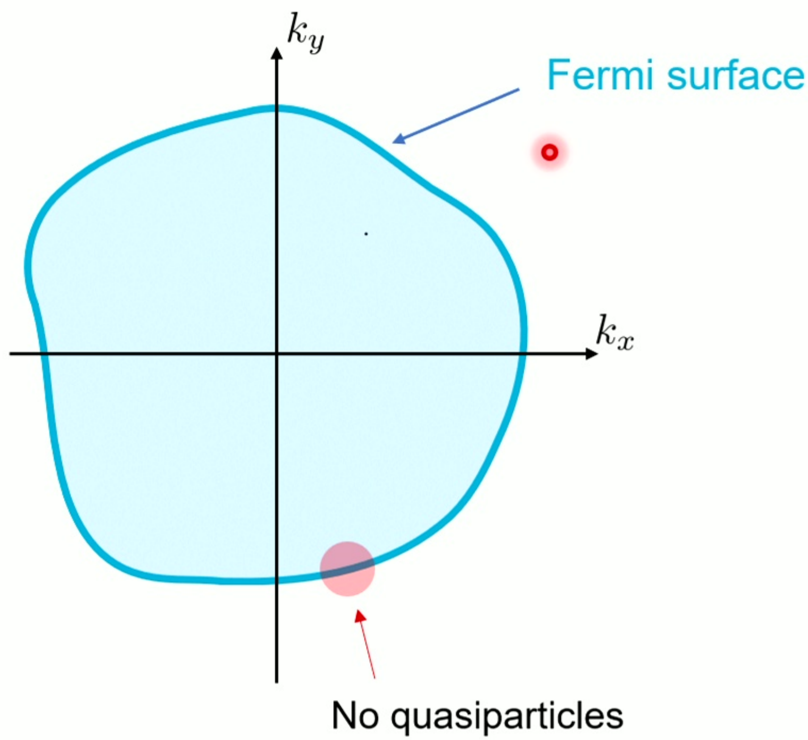
UV charge density

Fermi liquid theory represents a fixed-point under RG flow

IR

General features of metals (beyond Fermi liquid)

They still have a Fermi surface!



The Fermi surface still obeys Luttinger's theorem!

UV-IR relation
(Luttinger's theorem)

Volume enclosed by
Fermi surface

$$\boxed{\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}}$$

UV charge density

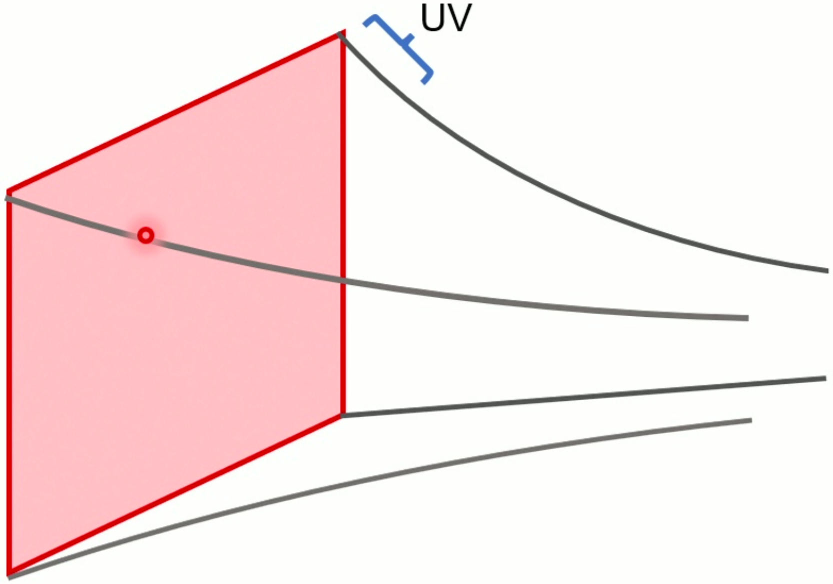
Holographic construction of metal

UV

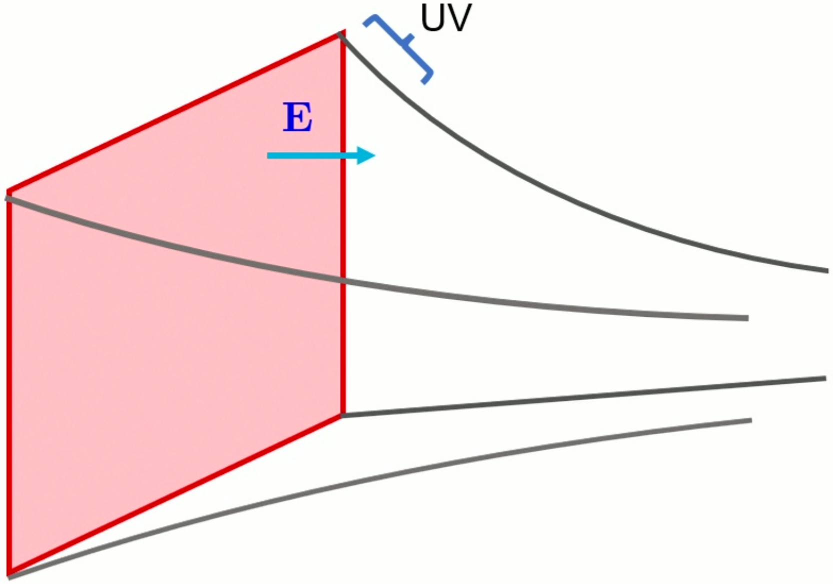
Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

For holography: take this to be a strongly coupled CFT perturbed by a chemical potential

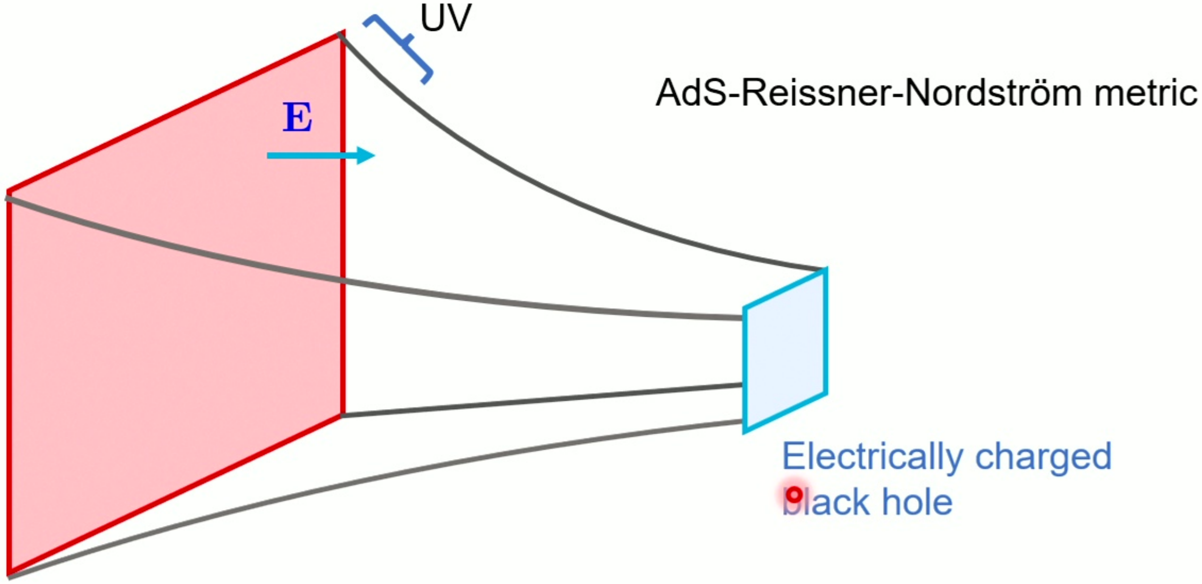
IR Effective field theory Metal



3D CFT ⟷ Asymptotically AdS_4 geometry in the UV region

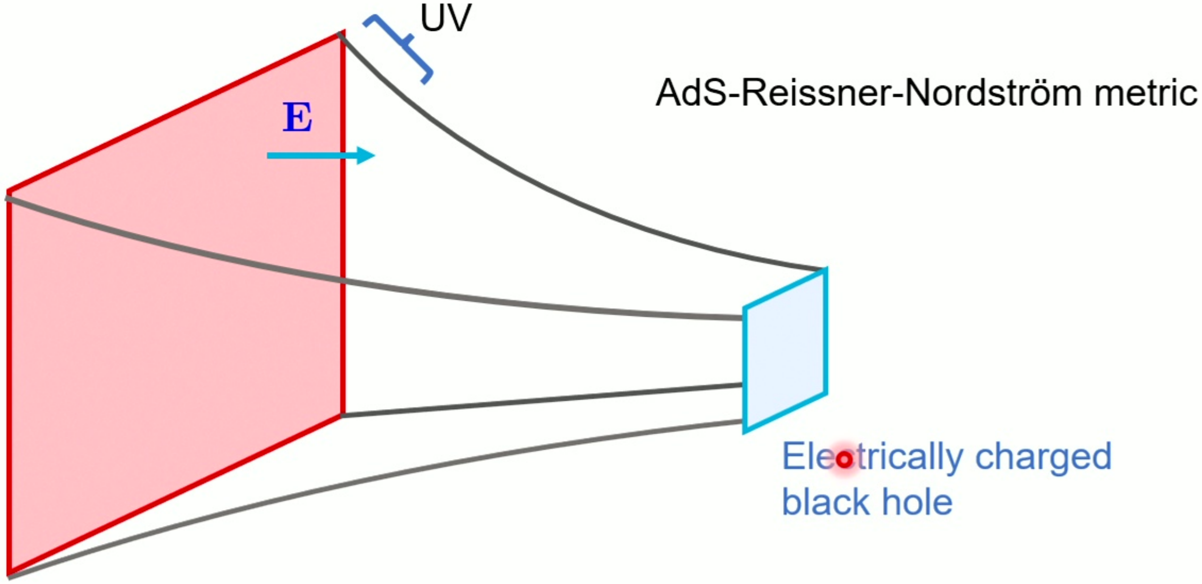


3D CFT	$\longleftrightarrow$	Asymptotically AdS_4 geometry in the UV region
Nonzero charge density	$\longleftrightarrow$	Electric field in the UV region in the direction normal to the boundary



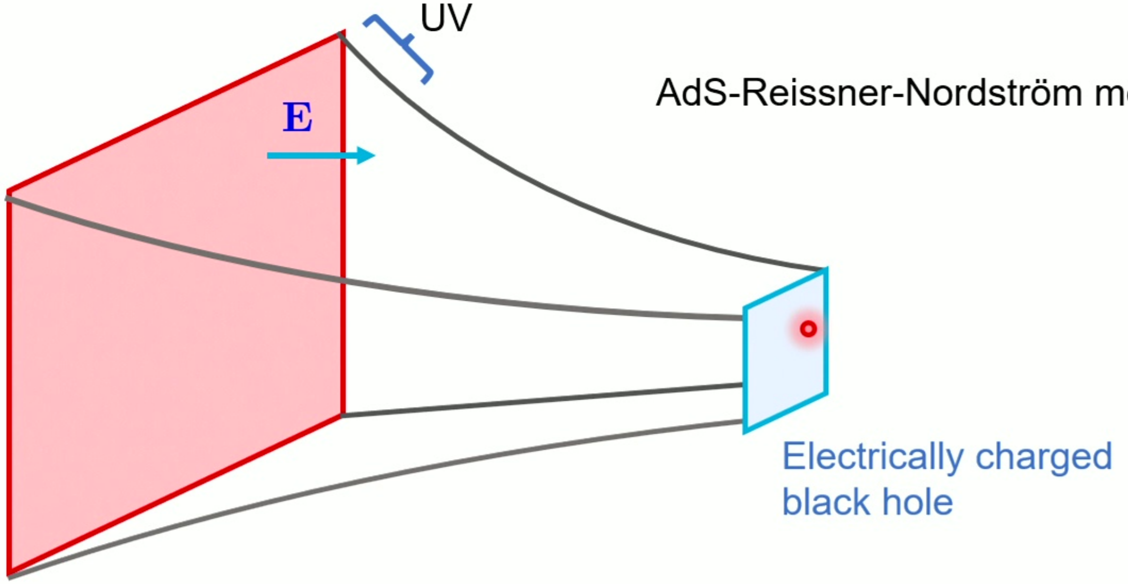
3D CFT $\longleftrightarrow$ Asymptotically AdS_4 geometry in the UV region

Nonzero charge density $\longleftrightarrow$ Electric field in the UV region in the direction normal to the boundary



The dual field theory has nonzero entropy density
even in the limit T=0 (!!)

3D CFT	⟷	Asymptotically AdS_4 geometry in the UV region
Nonzero charge density	⟷	Electric field in the UV region in the direction normal to the boundary



The dual field theory has nonzero entropy density
even in the limit T=0 (!!)

Variations of this model with dilaton fields, etc. can eliminate the entropy density but often at the cost of introducing other pathologies such as naked singularities in the bulk

3D CFT ⟷ Asymptotically AdS_4 geometry in the UV region

Nonzero charge density ⟷ Electric field in the UV region in the direction normal to the boundary

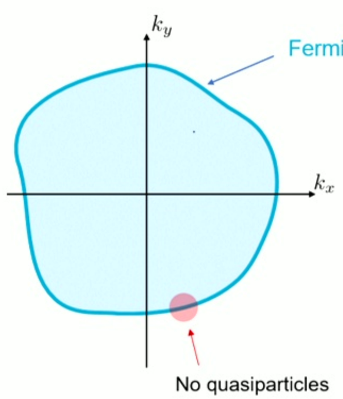
The missing Fermi surface

- These models have *no trace of a Fermi surface**

* at least not a Fermi surface satisfying Luttinger's theorem

Recall from before:

General features of metals (beyond Fermi liquid)



The Fermi surface still obeys Luttinger's theorem!

UV-IR relation (Luttinger's theorem)

Volume enclosed by Fermi surface

$$\boxed{\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}}$$

UV charge density

There are some hints that the Fermi surface may come back if we include quantum gravity corrections in the bulk

What are we actually trying to do?

The previous holographic models are attempting to find an exact description of this *entire* RG flow

UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

IR Effective field theory

Metal

What we usually do in condensed matter physics:

UV Some material, e.g. YBCO

???

IR We narrow down what the IR theory could be through *experimental* probes of the material

We want a holographic *effective* field theory

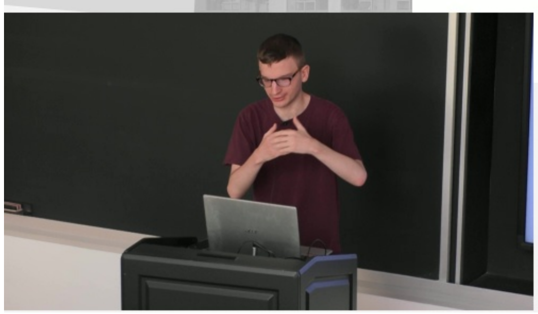
UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

Need to impose an "emergeability" condition

Holographic

IR Effective field theory

Metal



Emergeability condition

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

UV

Global U(1) symmetry
and continuous translation symmetry

Nonzero charge density
$\rho \neq 0$

IR

UV global symmetries have to act on the IR theory

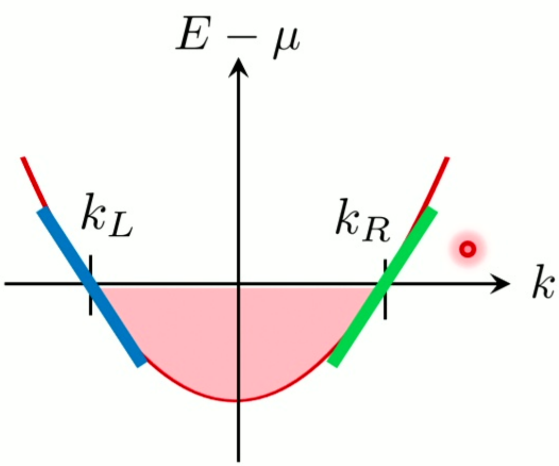
$\cancel{\rho_{\mathrm{IR}} = \rho_{\mathrm{UV}}}$

Charge density is *not* an RG invariant

Example: electron gas in 1 spatial dimension

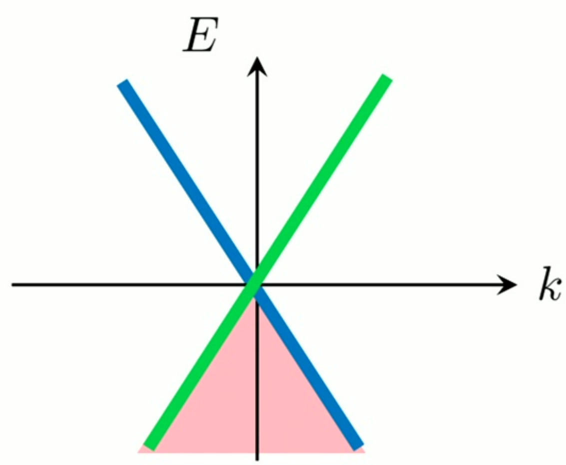
$$H = \int d^d\mathbf{x} \left[-\frac{1}{2m} \Psi^\dagger \nabla^2 \Psi - \mu \Psi^\dagger \Psi \right]$$

UV theory $(\rho \neq 0)$



IR theory: (1+1)-D Dirac fermion

$(\rho = 0)$



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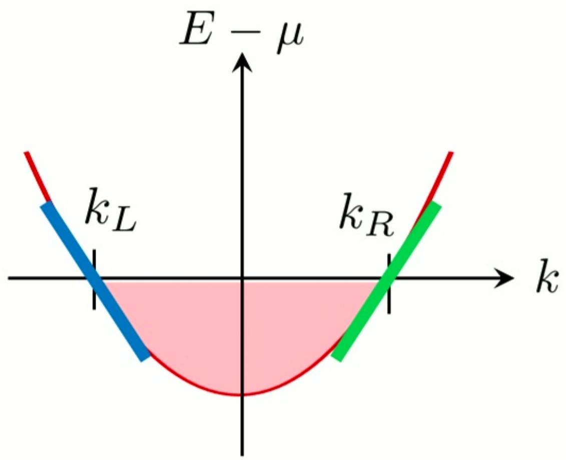

IR theory: (1+1)-D Dirac fermion

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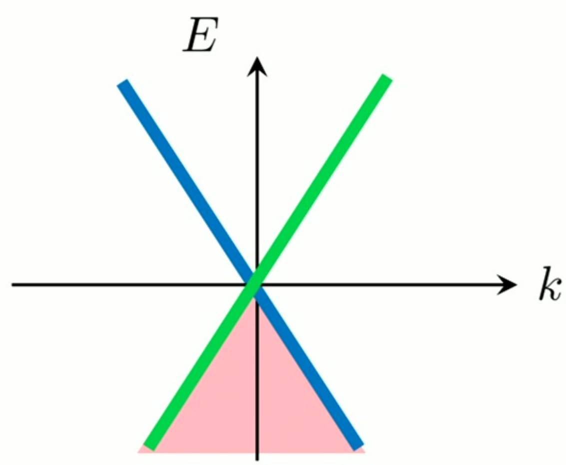


Emergent

$\mathrm{U}(1)_L \times \mathrm{U}(1)_R$

symmetry

Chiral anomaly

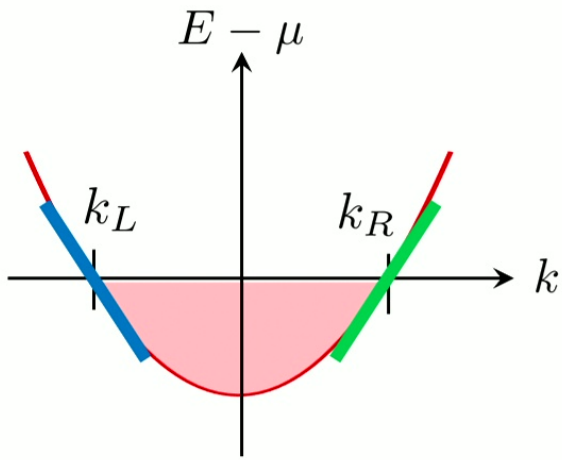
$$\partial_\mu (j^{(R)})^\mu = \frac{1}{2\pi} E$$

[Example of a ‘t Hooft anomaly]

Example: electron gas in 1 spatial dimension

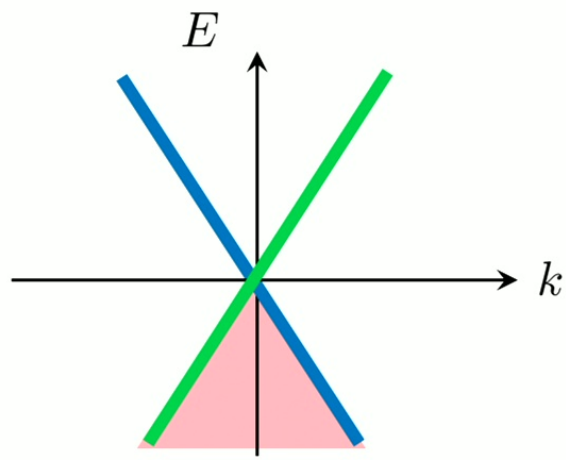
$$H = \int d^d\mathbf{x} \left[-\frac{1}{2m} \Psi^\dagger \nabla^2 \Psi - \mu \Psi^\dagger \Psi \right]$$

UV theory $(\rho \neq 0)$



IR theory: (1+1)-D Dirac fermion

$(\rho = 0)$



Emergent

$\mathrm{U}(1)_L \times \mathrm{U}(1)_R$

symmetry

Chiral anomaly

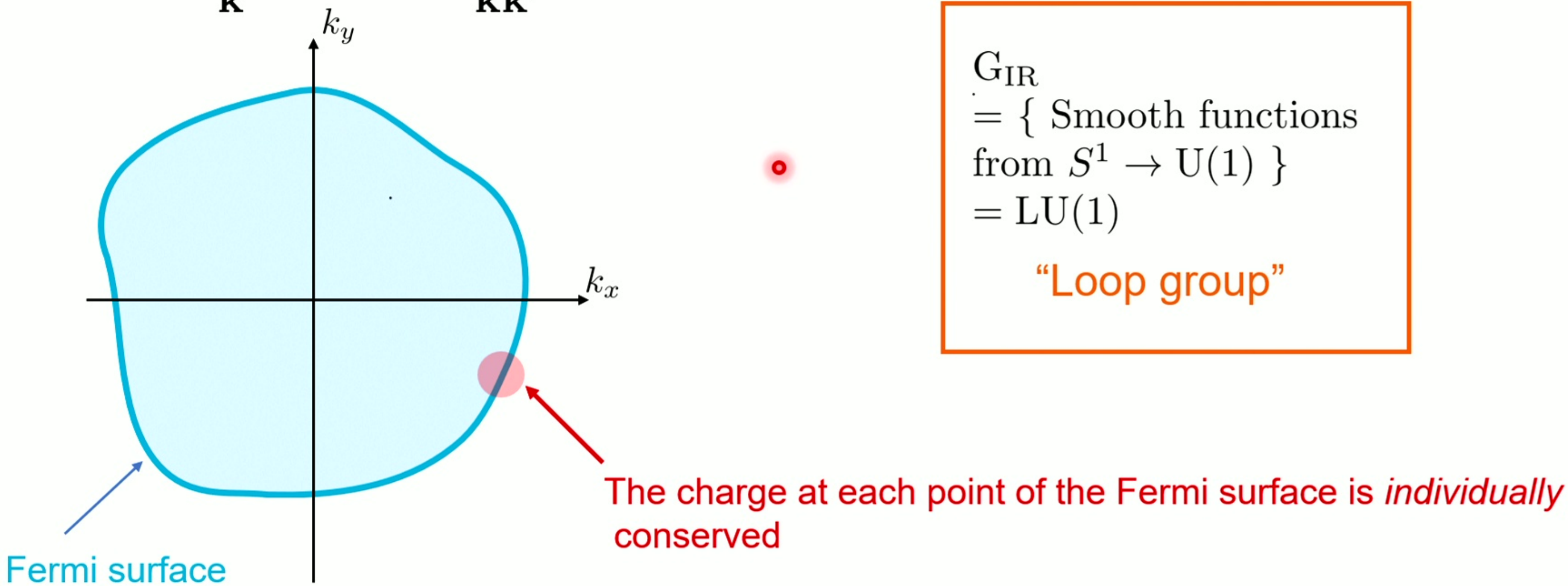
$$\partial_\mu (j^{(R)})^\mu = \frac{1}{2\pi} E$$

[Example of a ‘t Hooft anomaly]

Emergent symmetry of Fermi liquid theory in 2 spatial dimensions

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \hat{n}_{\mathbf{k}} \hat{n}_{\mathbf{k}'}$$

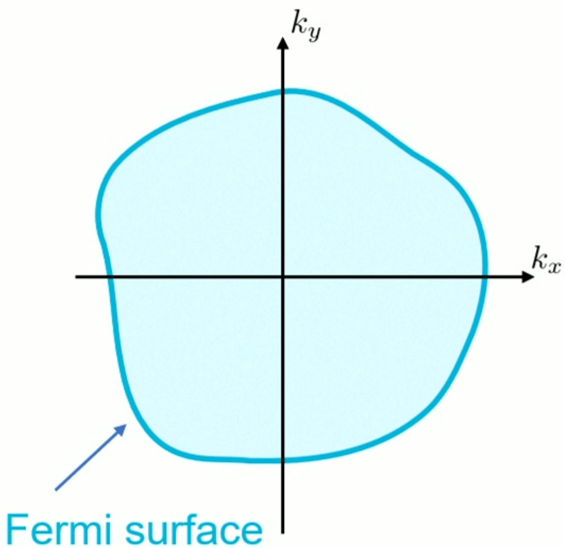


$$\begin{aligned} &\mathrm{G}_{\mathrm{IR}} \\ &= \{ \text{ Smooth functions} \\ &\text{from } S^1 \to \mathrm{U}(1) \ \} \\ &= \mathrm{LU}(1) \end{aligned}$$

"Loop group"

[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

- The emergent LU(1) symmetry (and its anomaly) turns out to be precisely the information needed to derive Luttinger's theorem



Volume enclosed by Fermi surface

UV-IR relation
(Luttinger's theorem)

$$\boxed{\rho = \frac{\mathcal{V}_{\mathcal{F}}}{(2\pi)^d}}$$

UV charge density

We want a holographic *effective* field theory

UV Continuum theory with global U(1) symmetry and continuous translation symmetry *at nonzero charge density* $\rho \neq 0$

Need to impose an "emergeability" condition

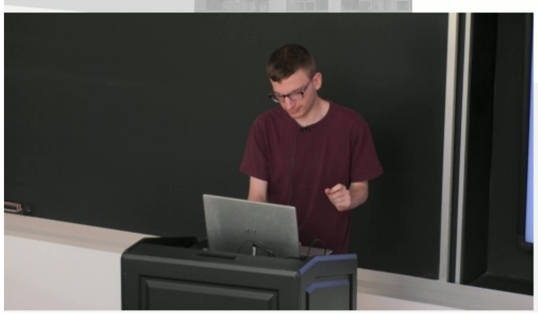
[DVE, R. Thorngren, T. Senthil, arXiv:2007.07896]

Just try to find a holographic effective field theory of a metal

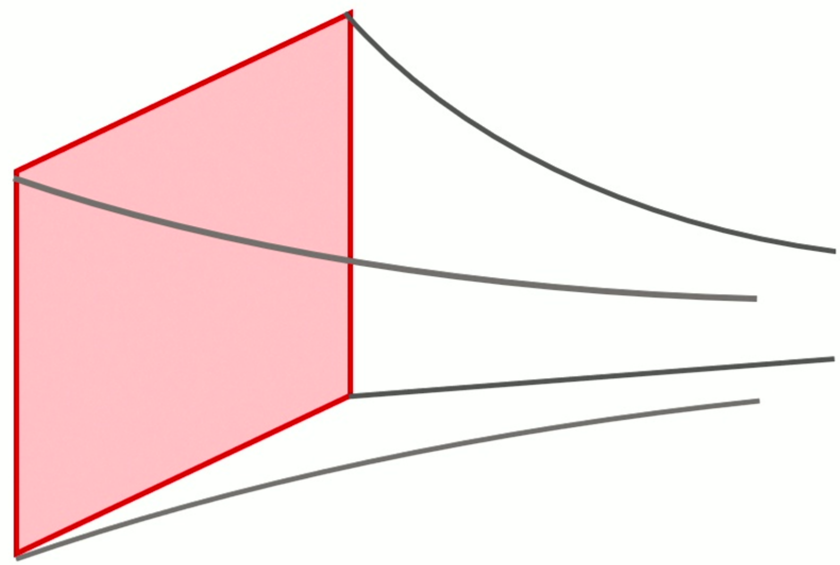
IR Effective field theory

Metal

Impose that the field theory has a global LU(1) symmetry (with anomaly)



Holography with a global LU(1) symmetry



Quantum field theory
in 3 space-time dimensions

Duality ⟷

Gravitational theory
in 4 space-time dimensions

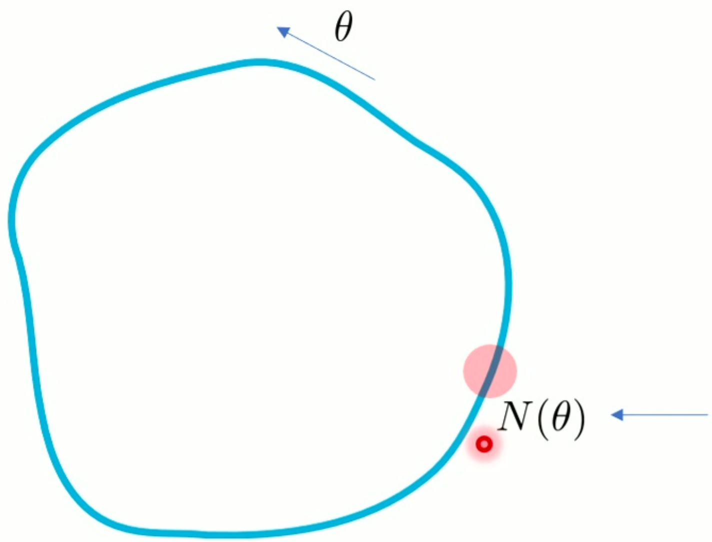
[with global LU(1) symmetry]

[with dynamical LU(1) gauge field]

[LU(1) has ‘t Hooft anomaly]

[Gauge field has a Chern-Simons term]

LU(1) gauge field



G_{IR}
$= \{$ Smooth functions from $S^1 \to \mathrm{U}(1)$ $\}$
$= \mathrm{LU}(1)$

"Loop group"

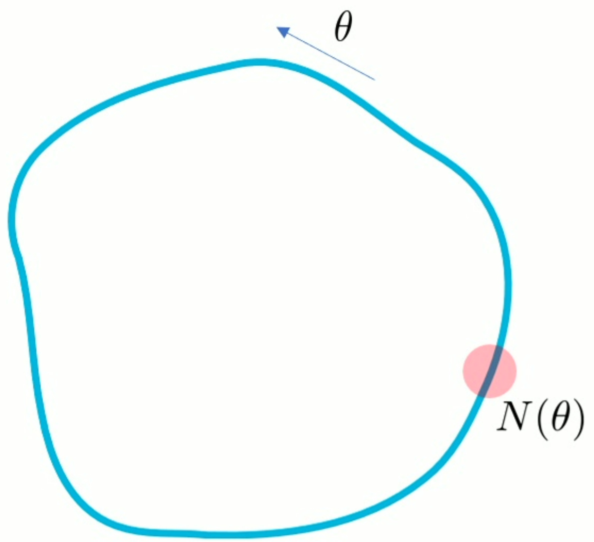
Generators are parameterized by θ

Total charge of the U(1) subgroup: $Q = \int N(\theta) d\theta$

LU(1) gauge field

G_{IR}
$= \{$ Smooth functions from $S^1 \to \mathrm{U}(1)$ $\}$
$= \mathrm{LU}(1)$

"Loop group"



Generators are parameterized by θ

Total charge of the U(1) subgroup: $Q = \int N(\theta) d\theta$

An $\mathrm{LU}(1)$ gauge field on a space-time M is a family of vector fields $a_\mu(\theta)$ parameterized by θ

Gauge transformation: $a_\mu(\theta) \to a_\mu(\theta) + \partial_\mu \lambda(\theta)$

This is basically equivalent to a $\mathrm{U}(1)$ gauge field on $M \times S^1$

The Chern-Simons term

The action for the LU(1) gauge field will include a Chern-Simons term

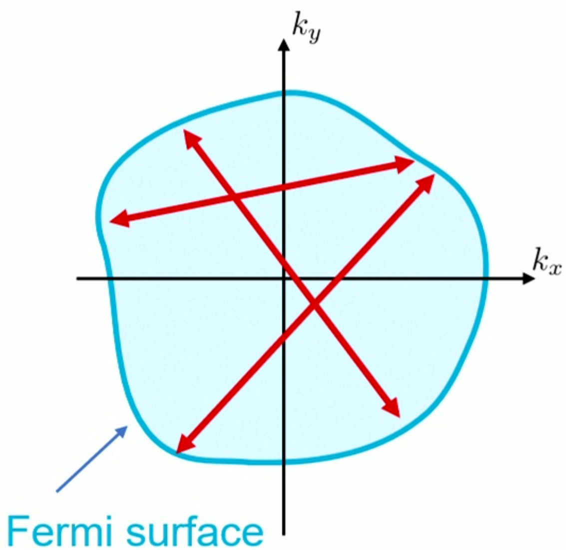
$$\frac{m}{24\pi^2}\int_{M_4\times S^1} A\wedge dA\wedge dA$$

$m\in\mathbb{Z}$

4D space-time

An important remark:

- The metric that satisfies the Einstein equations lives on M_4, *not* $M_4 \times S^1$
- In the dual QFT, there is no concept of "local" energy density on the Fermi surface.



The Hamiltonian couples different points of the the Fermi surface non-locally (even in Fermi liquid theory)

Another important remark

- LU(1) conservation law in 3-dimensional space-time M_3 is *not* the same as a U(1) conservation law in $M_3 \times S^1$

$\mathrm{LU}(1)$ conservation law in M_3 :

$$\partial_\mu j^\mu(\theta) = (\text{anomaly term})$$

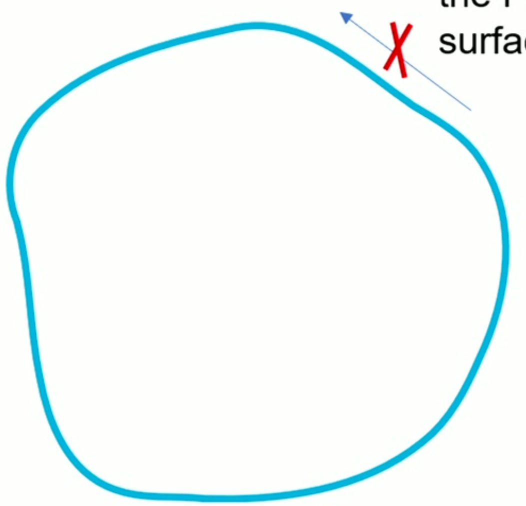
This index ranges over the 3 coordinates of M_3

$\mathrm{U}(1)$ conservation law in $M_3 \times S^1$

$$\partial_\mu j^\mu(\theta) + \partial_\theta j^\theta = (\text{anomaly term})$$

LU(1) conservation enforces that $j^\theta = 0$

No flow of charge *along* the Fermi surface



The Maxwell term in the bulk

Indices range over the 4 space-time coordinates, *not* including the Fermi surface coordinate θ

$$\int_{M_4} d^4x \sqrt{-g} \int d\theta \sqrt{g_{\theta\theta}}\, f_{\mu\nu}(\theta) f^{\mu\nu}(\theta)$$

$$f_{\mu\nu}(\theta) = \partial_\mu a_\nu(\theta) - \partial_\nu a_\mu(\theta)$$

Metric on the 4-D space-time

Metric on the Fermi surface (not dynamical)

Boundary conditions

- Asymptotic metric is AdS_4

$$ds^2 = \frac{L^2}{r^2}(-dt^2 + dx^2 + dy^2 + dz^2)$$

- Asymptotic solutions to the equations of motion for the LU(1) gauge field take the form

$$a_\alpha = a_\alpha^{(0)} + r a_\alpha^{(1)}$$

- The holographic dictionary tells us to identify

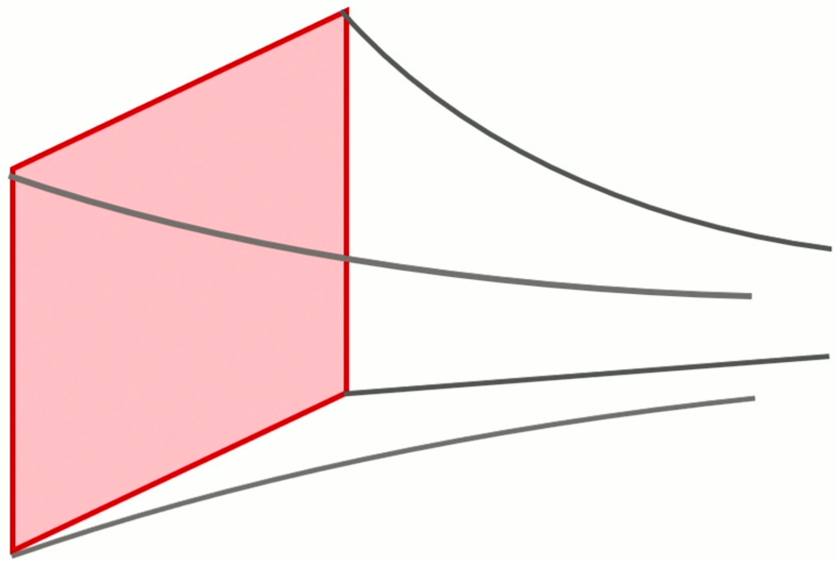
$$a_\alpha^{(0)} = A_\alpha$$

[background LU(1) gauge field in the dual QFT]

$$a_\alpha^{(1)} = \langle j_\alpha \rangle$$

[LU(1) current in the dual QFT]

The holographic model



Quantum field theory
in 3 space-time dimensions

Duality

$$S[g,a] = S_{\text{gravitational}} + S_{\text{Maxwell}} + S_{\text{Chern-Simons}}$$

[with global LU(1) symmetry]

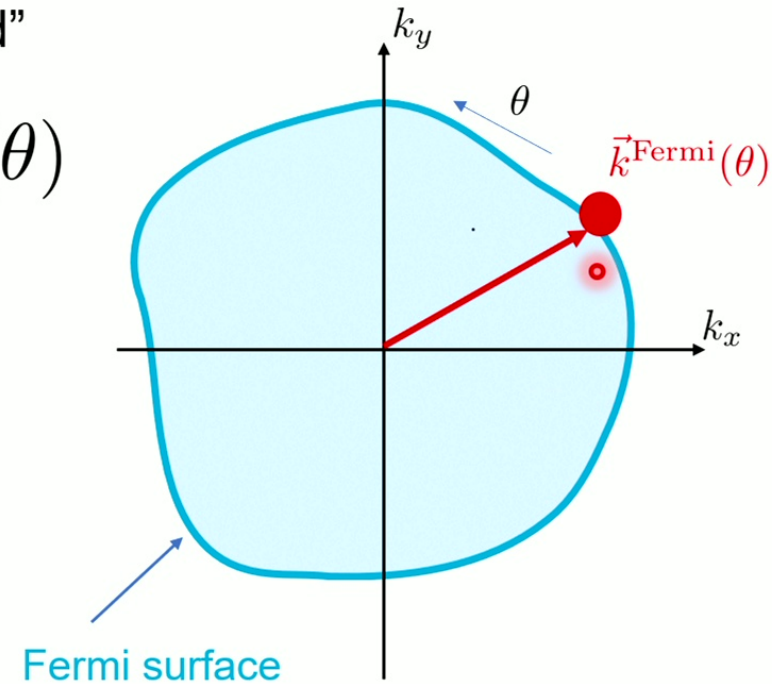
[LU(1) has anomaly]

“Bottom-up holography”

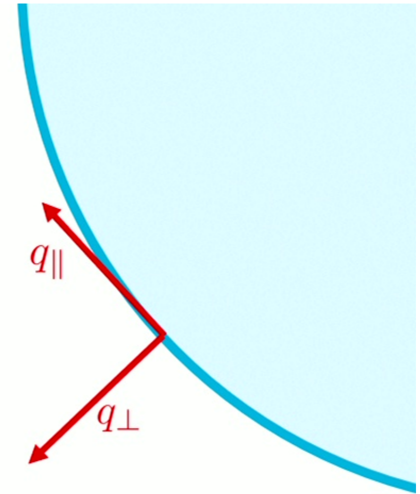
Properties to impose on the dual QFT

- The charge density of the LU(1) symmetry is *zero*
- We need to apply a “phase space magnetic field”

$$f_{\theta i} := \partial_\theta a_i - \partial_i a_\theta = \partial_\theta k_i^{\mathrm{Fermi}}(\theta)$$



Results from solution of the model

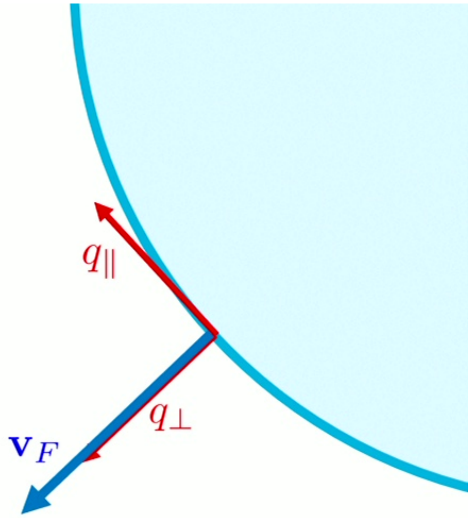


Chern-Simons level

Green's function of the LU(1) charge density [for $m > 0$]

$$G_{j^t(\theta) j^t(\theta')} = \left[-\frac{m|\partial_\theta \mathbf{k}_F(\theta)|}{(2\pi)^2} \frac{q_\perp}{\omega - q_\perp} + \frac{\alpha^{-1}\sqrt{g_{\theta\theta}} q_{\|}^2}{\sqrt{q_\perp^2 - \omega^2}} + O\left(\frac{1}{k_F}\right) \right] \delta(\theta - \theta')$$

Results from solution of the model



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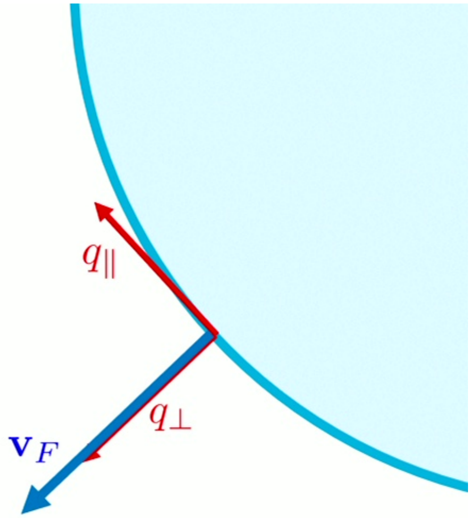
Fermi-liquid-like behavior
(undamped ballistic mode)

Results from solution of the model



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Fermi-liquid-like behavior (undamped ballistic mode)

Non-Fermi liquid corrections

Results from solution of the model

Optical conductivity: $j^i = \sigma^{ij}(\omega) E_j$ (at $\mathbf{q} = 0$)

$$\sigma^{ij}(\omega) = \frac{i}{\omega}\mathcal{D}^{ij} + \sigma^{ij}_{\text{inc}} + O\left(\frac{1}{k_F}\right)$$

"Drude" or "coherent" conductivity

"Incoherent" conductivity (absent in Fermi liquid theory)

A variant model

Instead of

$$g_{\theta\theta} \propto |\partial_\theta \mathbf{k}_F(\theta)|^2$$

We could set

$$g_{\theta\theta} \propto f_{\theta\mu} f_\theta{}^\mu$$

$$f_{\theta\mu} = \partial_\theta a_\mu - \partial_\mu a_\theta$$

A variant model

Instead of

$$g_{\theta\theta} \propto |\partial_\theta \mathbf{k}_F(\theta)|^2$$

We could set

$$g_{\theta\theta} \propto f_{\theta\mu} f_\theta{}^\mu$$

$$f_{\theta\mu} = \partial_\theta a_\mu - \partial_\mu a_\theta$$

Recall that we have to impose a "phase space magnetic field":

$$f_{\theta i} := \partial_\theta k_i^{\mathrm{Fermi}}(\theta)$$

$$\downarrow$$

$$g_{\theta\theta} \propto g_{xx} |\partial_\theta \mathbf{k}_F(\theta)|^2$$

Conclusions

- I have presented a new holographic model which incorporates the essential physics of strongly coupled metals, including the Fermi surface
- A jumping off point to build models of strongly coupled metals