Title: Gravitational Wave Theory

Speakers: Luis Lehner

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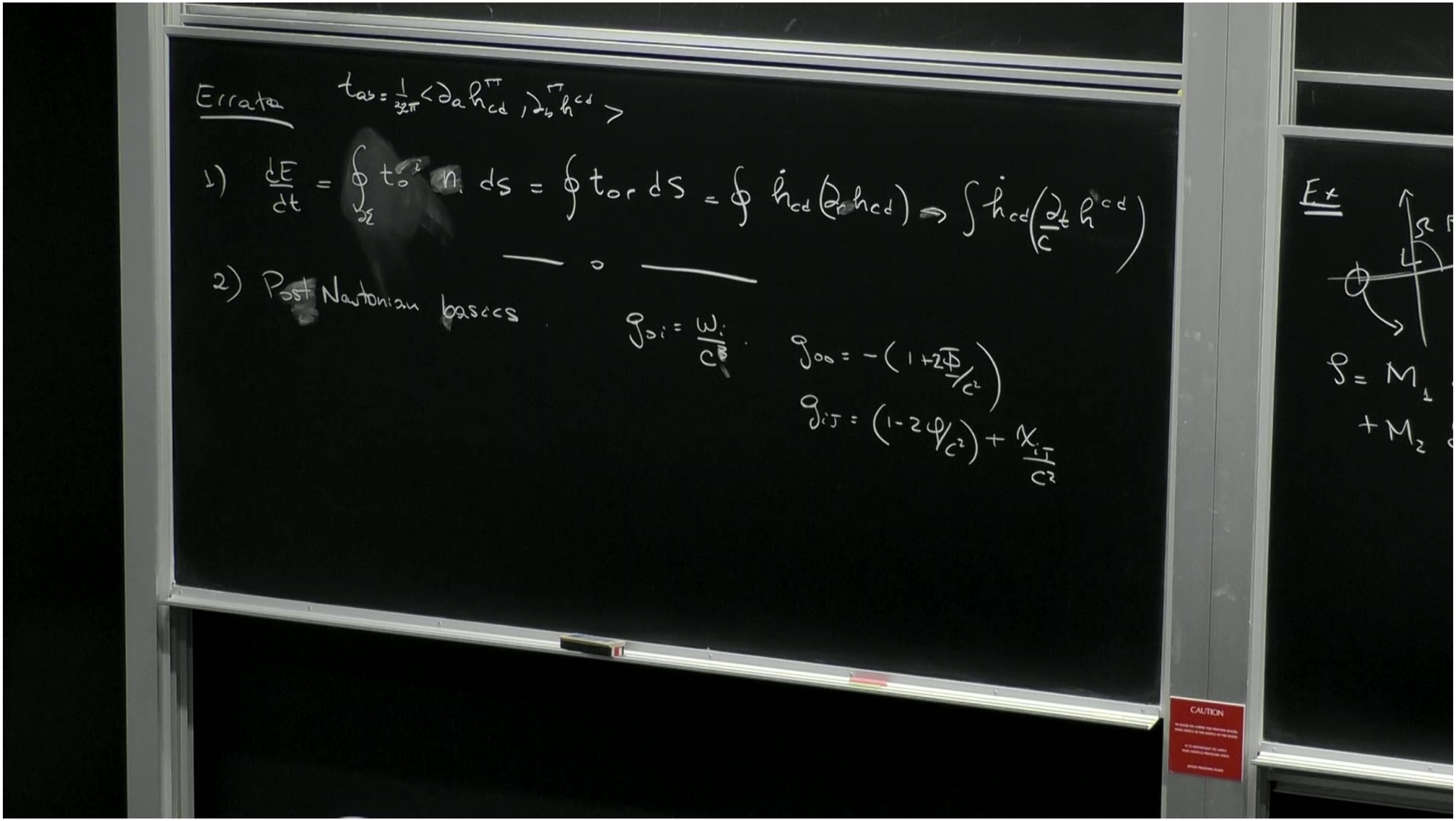
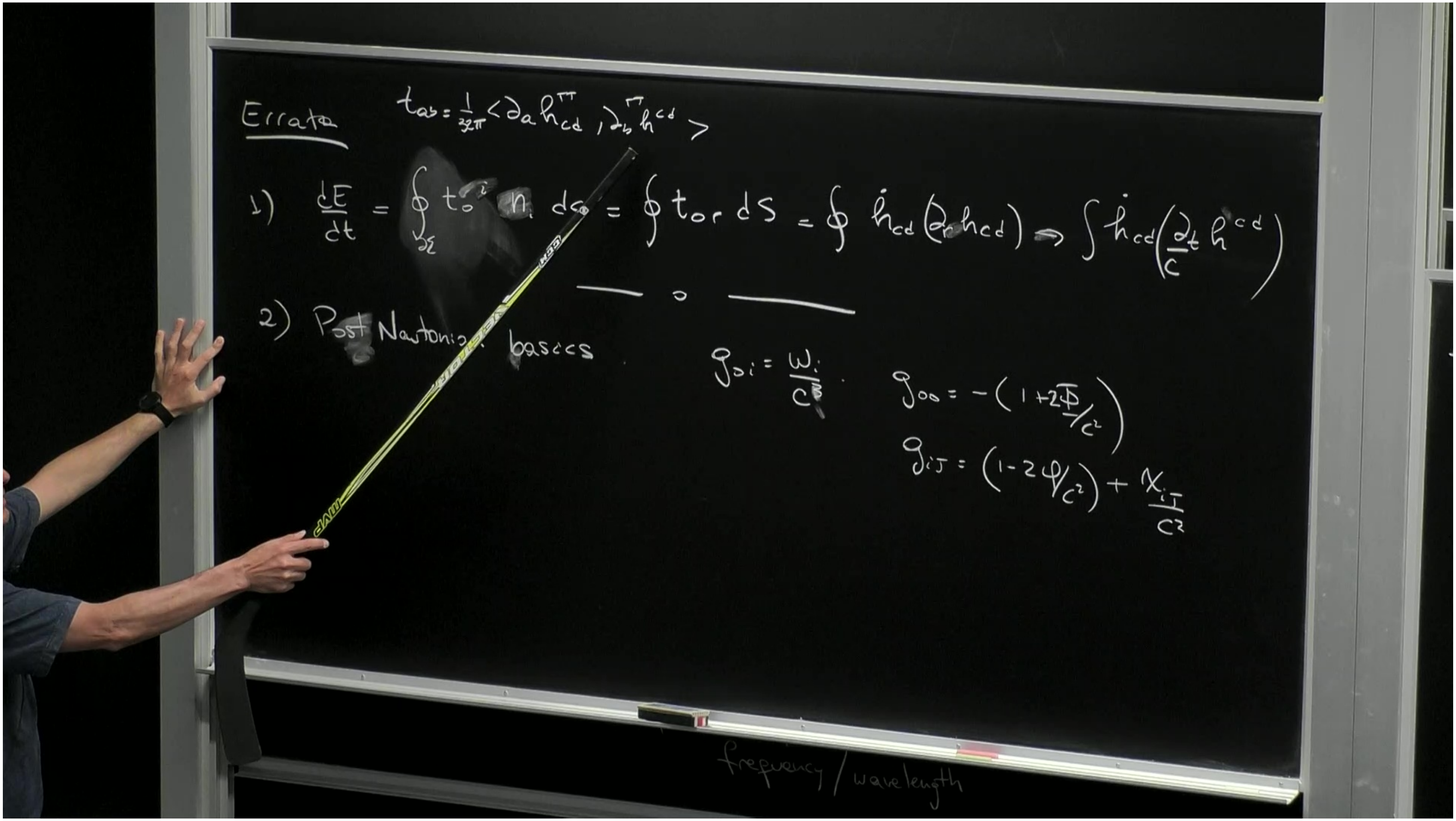
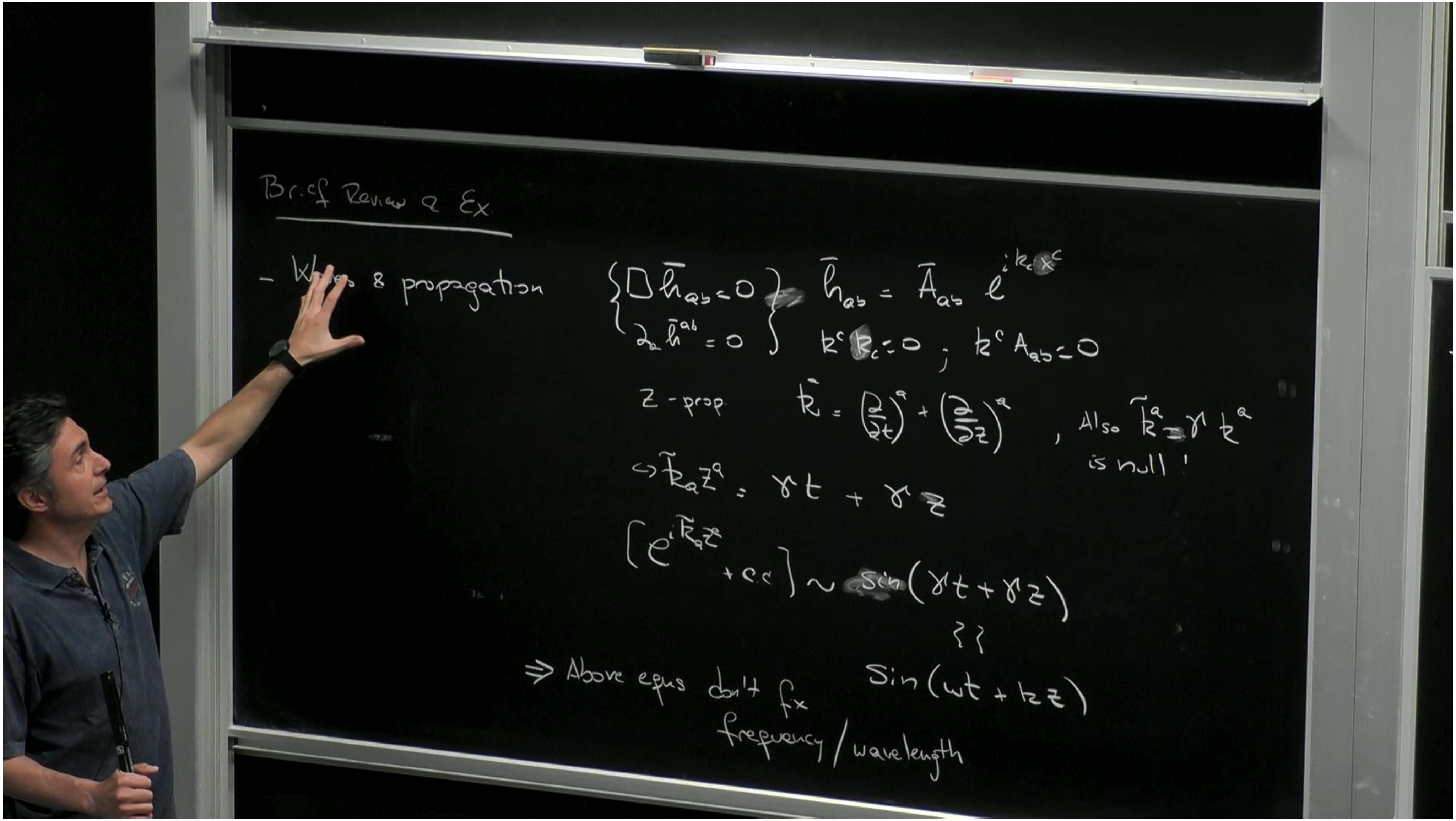
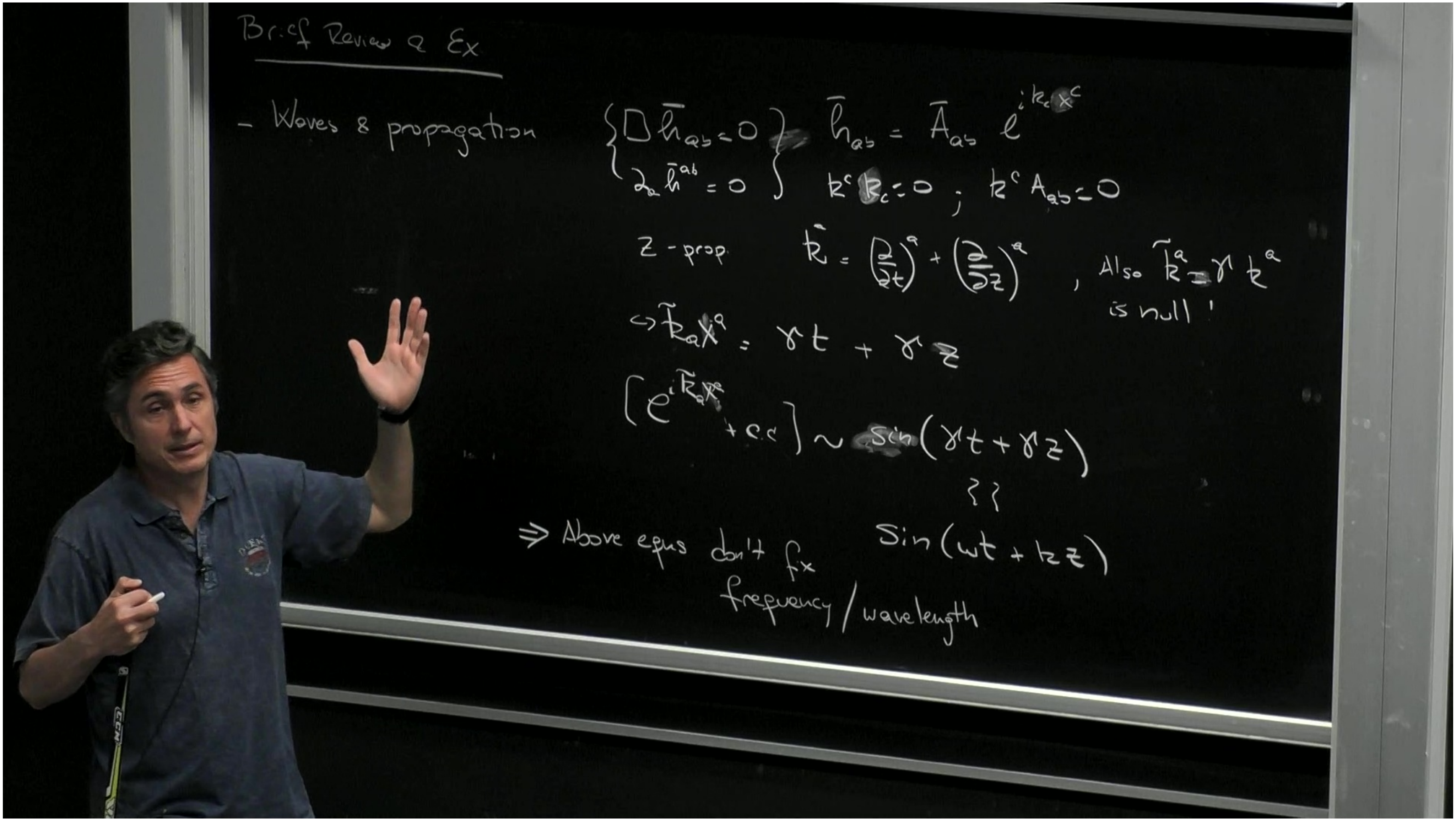
URL: https://pirsa.org/23060083


Errata
1) dE/dt = ∮ ds = ∮ ds
2) Post Newtonian basics
Ex
CAUTION


Errata
1)
2) Post Newtonian basics
frequency / wavelength


Brief Review & Ex
- Waves & propagation
Z - prop.
Also
is null !
⇒ Above eqns don't fix
frequency / wavelength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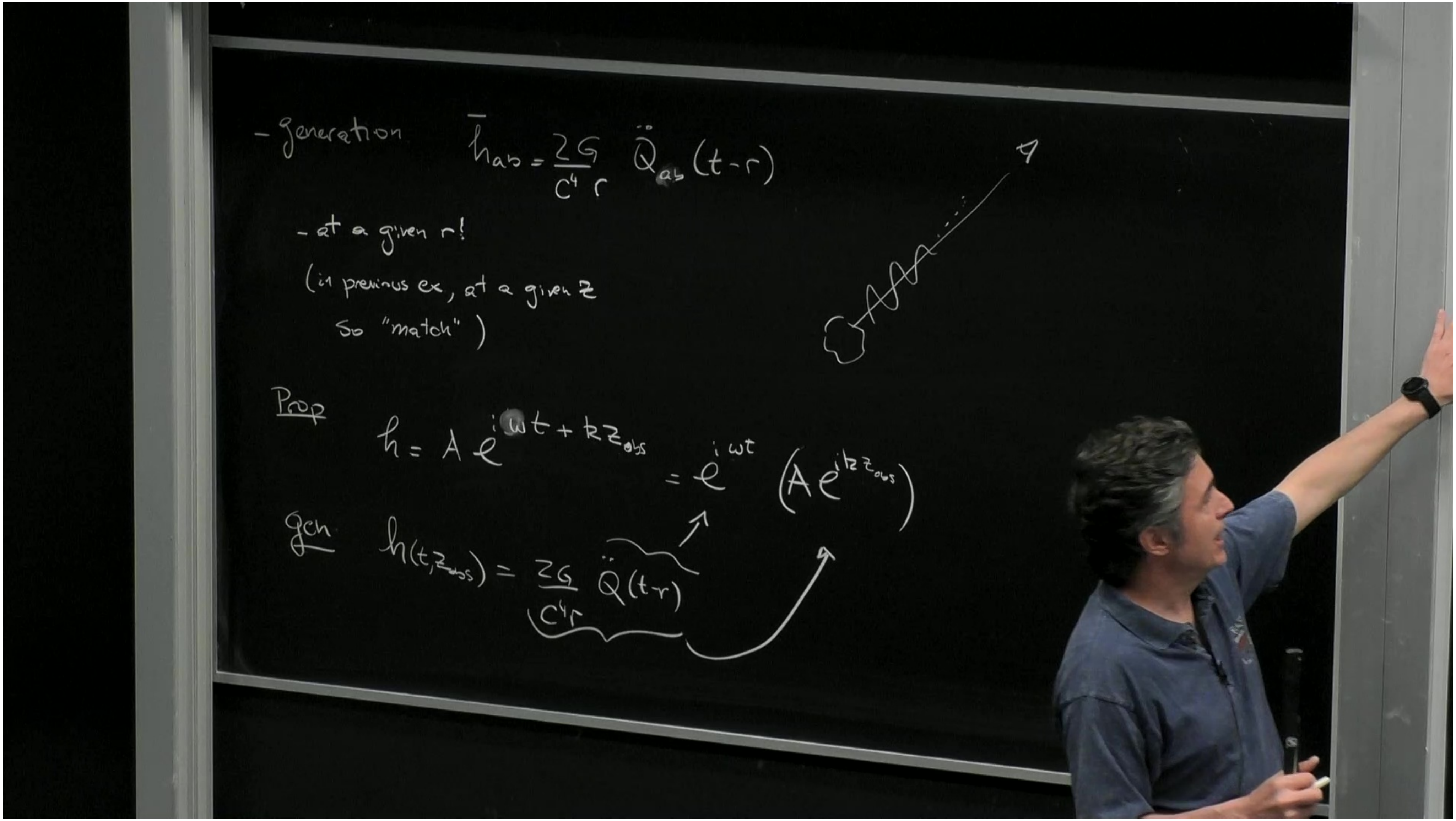
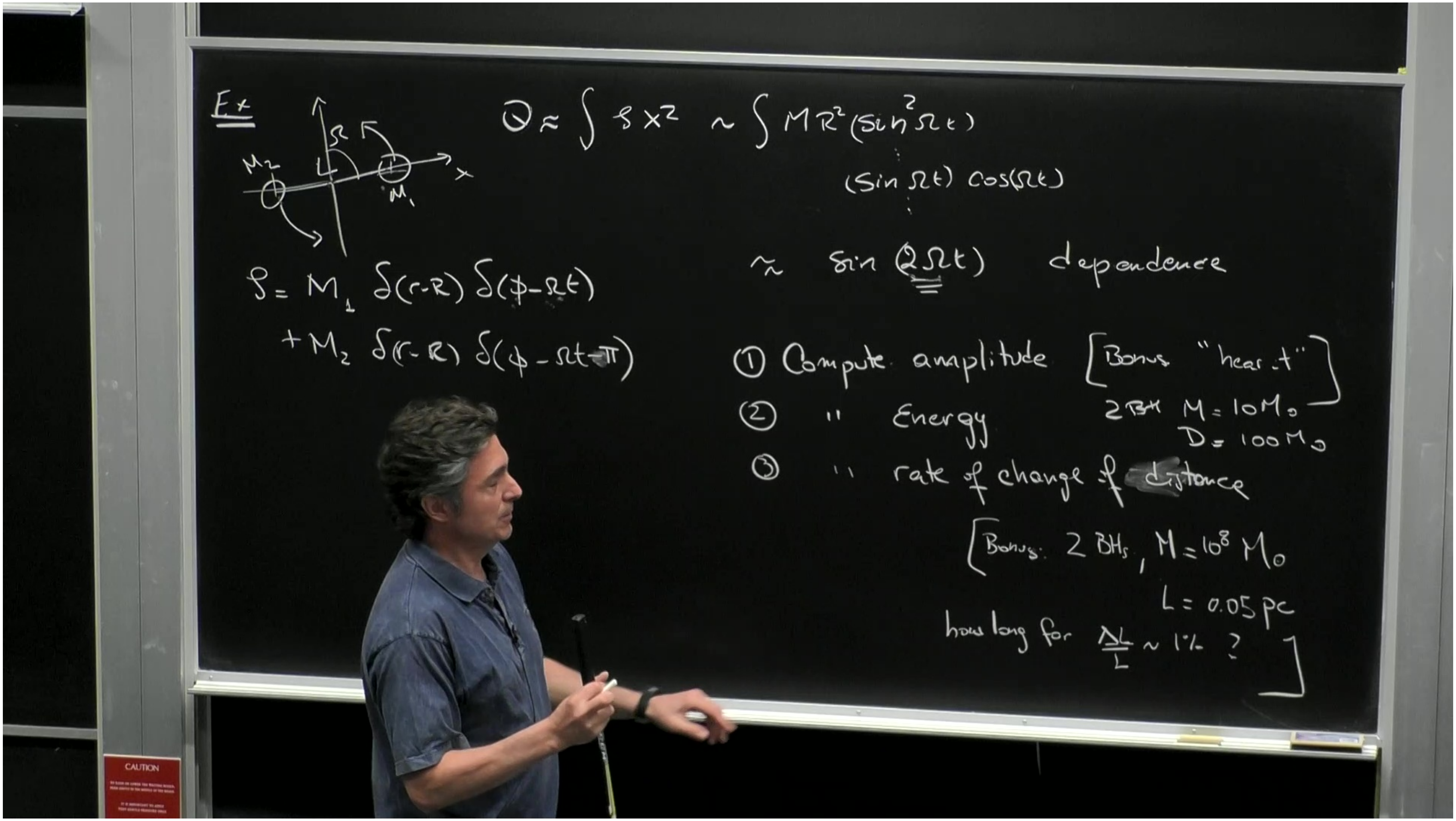
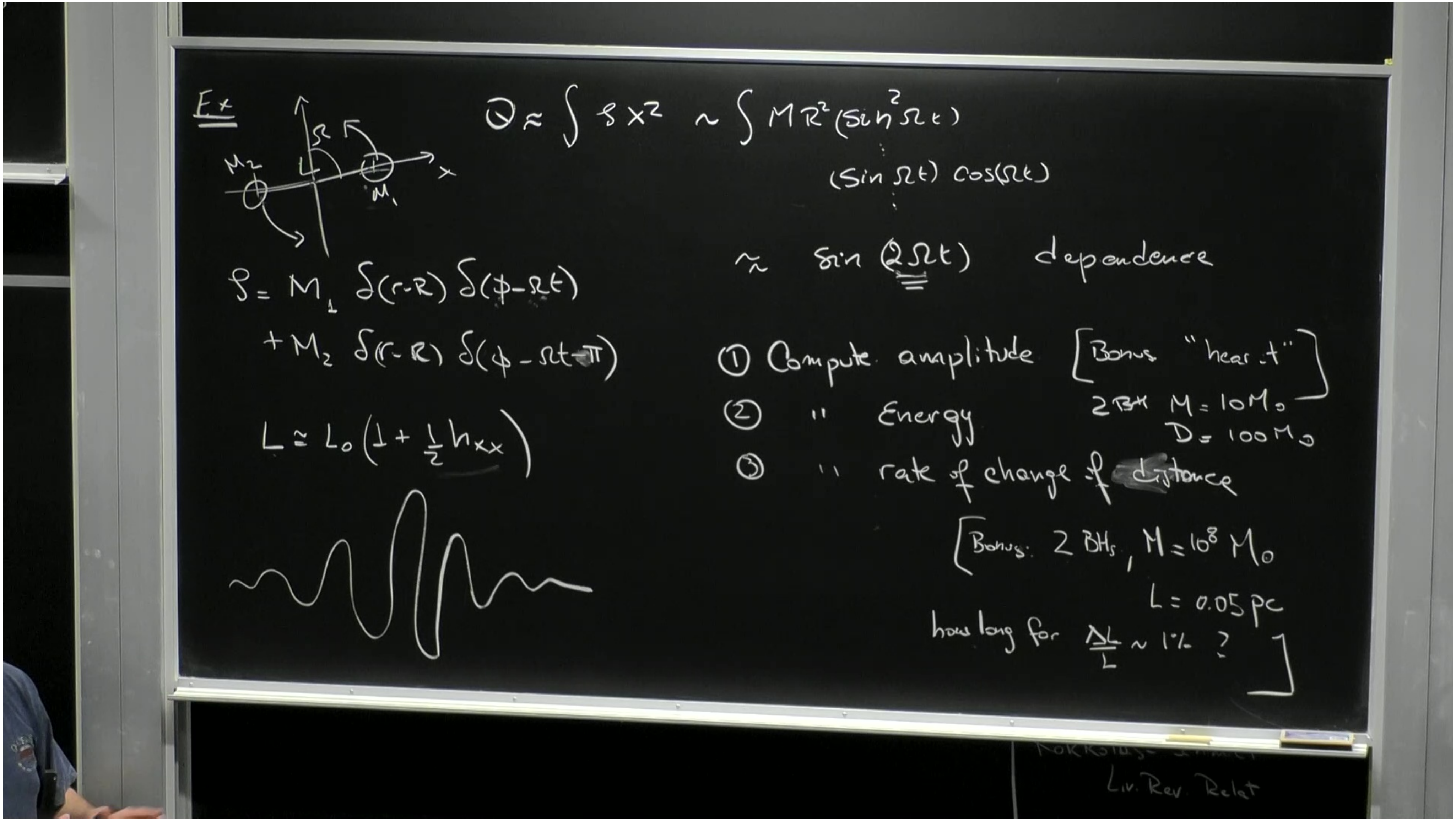

- generation
$\bar{h}_{ab} = \frac{2G}{c^4 r} \ddot{Q}_{ab}(t-r)$
- at a given r!
(in previous ex, at a given z
so "match")
Prop
$h = A e^{i\omega t + kz_{obs}} = e^{i\omega t}\left(A e^{ikz_{obs}}\right)$
gen
$h(t, z_{obs}) = \frac{2G}{c^4 r} \ddot{Q}(t-r)$


Ex
M2
M1
x
Q ≈ ∫ ρ x² ~ ∫ M R² (sin² Ωt)
(sin Ωt) cos(Ωt)
≈ sin (2Ωt) dependence
ρ = M1 δ(r-R) δ(φ-Ωt)
+ M2 δ(r-R) δ(φ-Ωt-π)
① Compute amplitude [Bonus "hear it"
2BH M = 10M⊙
D = 100M⊙]
② " Energy
③ " rate of change of distance
[Bonus: 2 BHs, M = 10⁸ M⊙
L = 0.05 pc
how long for ΔL/L ~ 1% ?]
CAUTION


Ex
M2
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Q ≈ ∫ ρ x² ~ ∫ M R² (sin² Ωt)
(sin Ωt) cos(Ωt)
~ sin (2Ωt) dependence
ρ = M1 δ(r−R) δ(φ−Ωt)
+ M2 δ(r−R) δ(φ−Ωt−π)
L ≈ L0 (1 + ½ hxx)
① Compute amplitude [Bonus "hear it"]
② " Energy
2 BH M = 10 M⊙
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③ " rate of change of distance
[Bonus: 2 BHs, M = 10⁸ M⊙
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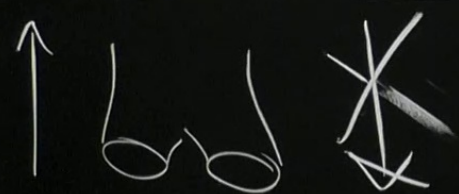
how long for ΔL/L ~ 1% ?]
Liv. Rev. Relat

Black Holes

- governed by 2 (+1?) parameters (M, a, q)
- Hide singularities, stable (ish!)
- Can Join but not bifurcate (why?)
- Ultimate state conjecture.

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \frac{1}{\left(1-\frac{2m}{r}\right)}dr^2 + r^2\, d\Omega^2.$$

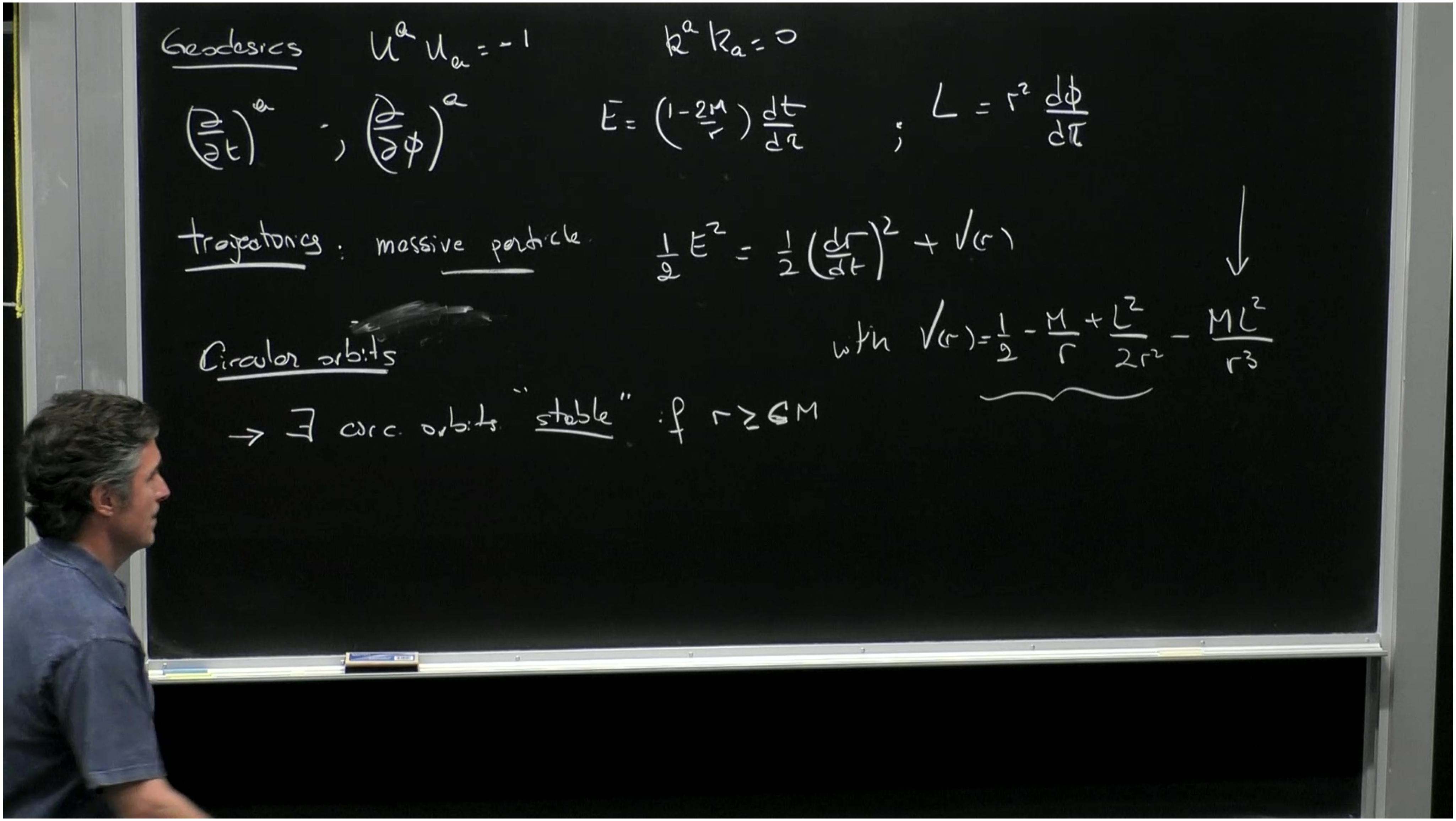


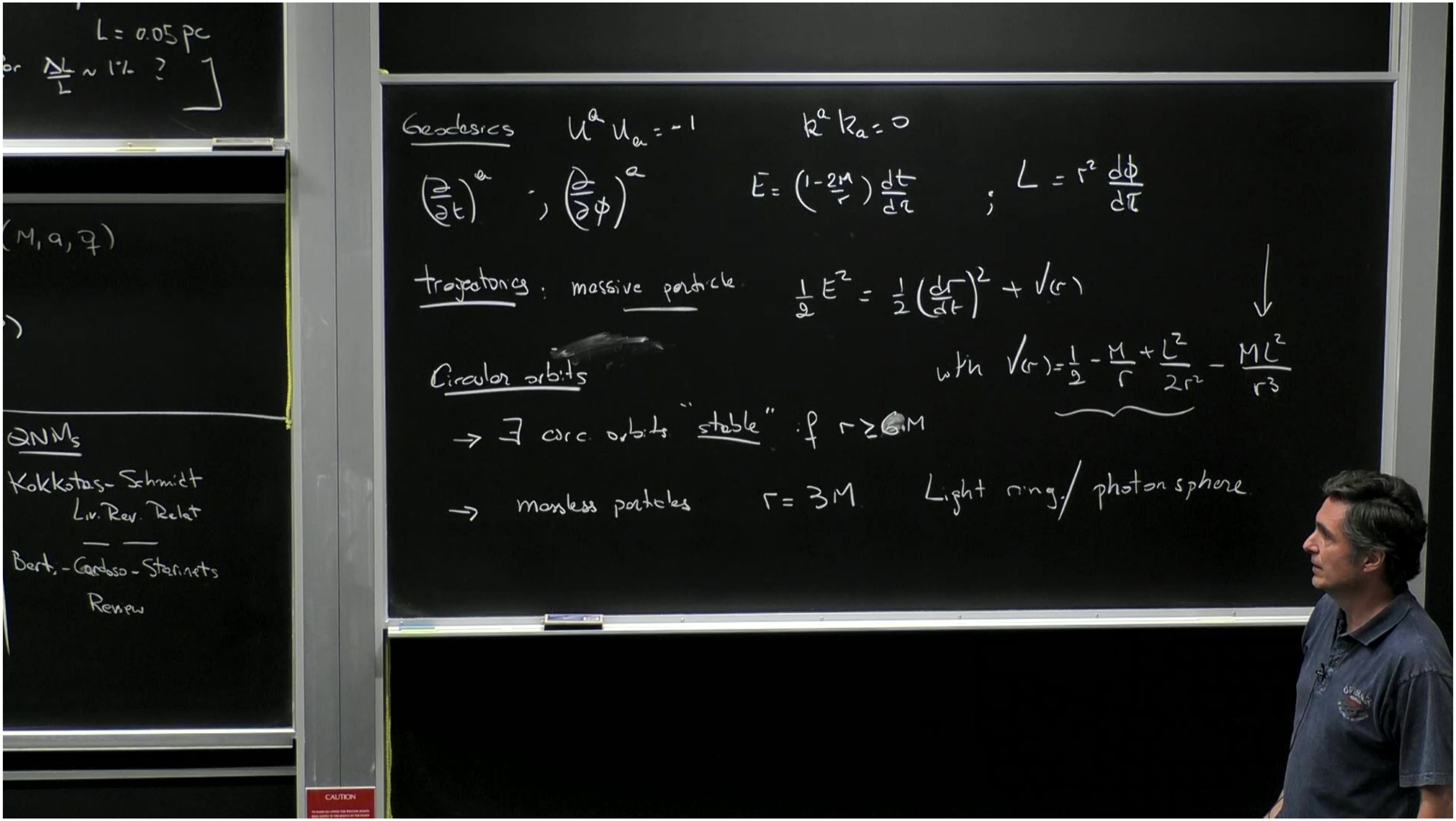
QNMs

Kokkotas - Schmidt
Liv. Rev. Relat

— —

Berti - Cardoso - Starinets
Review


Geodesics
$U^a U_a = -1$
$k^a k_a = 0$
$\left(\frac{\partial}{\partial t}\right)^a ; \left(\frac{\partial}{\partial \phi}\right)^a$
$E = \left(1 - \frac{2M}{r}\right)\frac{dt}{d\tau}$; $L = r^2 \frac{d\phi}{d\tau}$
trajectories : massive particle
$\frac{1}{2}E^2 = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V(r)$
with $V(r) = \frac{1}{2} - \frac{M}{r} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3}$
Circular orbits
→ ∃ circ. orbits "stable" if $r \geq 6M$


L = 0.05 pc
for ΔL/L ~ 1% ?
(M, a, q)
QNMs
Kokkotas - Schmidt
Liv. Rev. Relat
Berti - Cardoso - Starinets
Review
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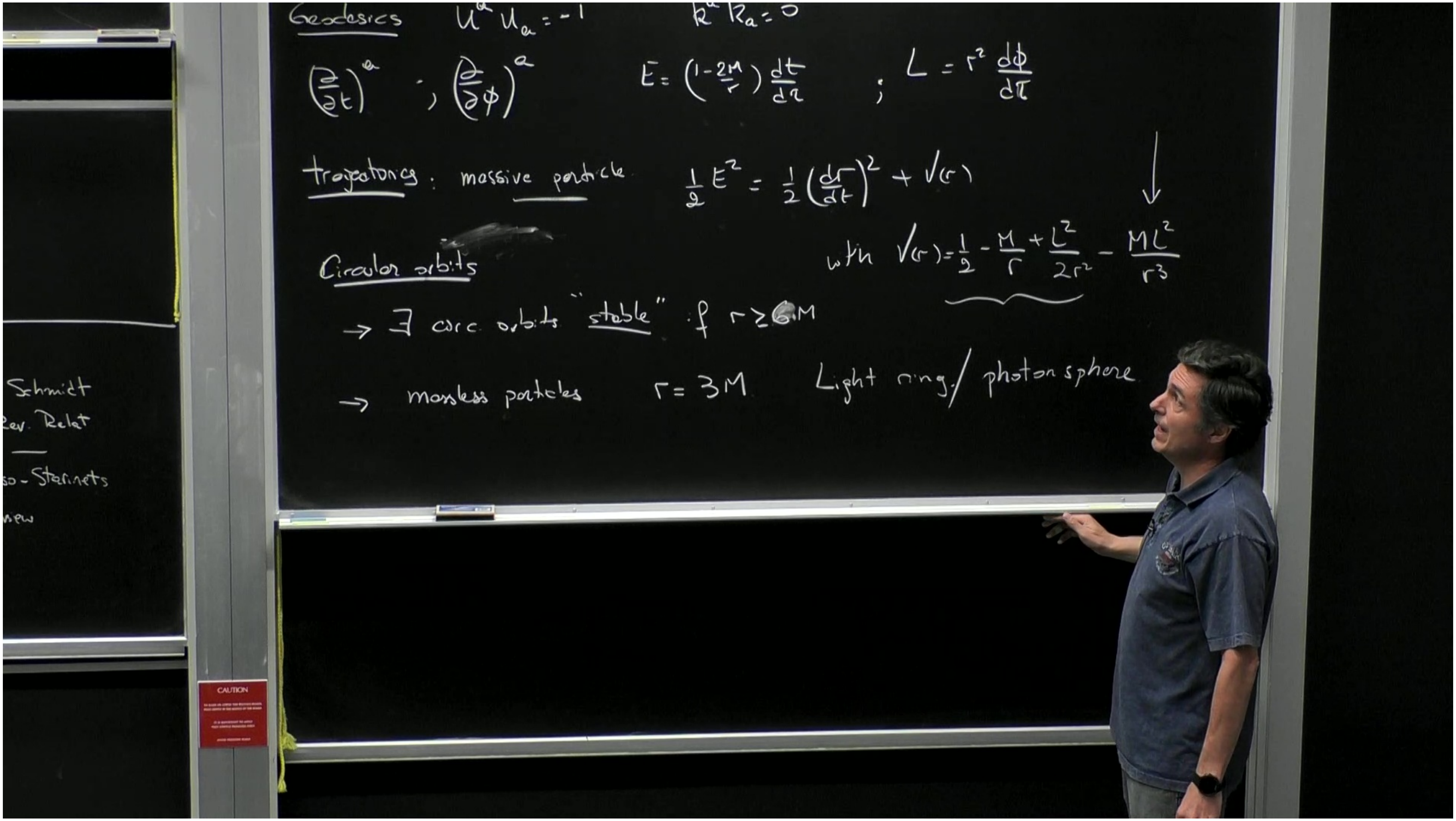
(∂/∂t)^a ; (∂/∂φ)^a   E = (1 - 2M/r) dt/dτ ; L = r^2 dφ/dτ
trajectories : massive particle   1/2 E^2 = 1/2 (dr/dt)^2 + V(r)
Circular orbits   with V(r) = 1/2 - M/r + L^2/2r^2 - ML^2/r^3
→ ∃ circ. orbits "stable" if r ≥ 6M
→ massless particles   r = 3M   Light ring / photon sphere
CAUTION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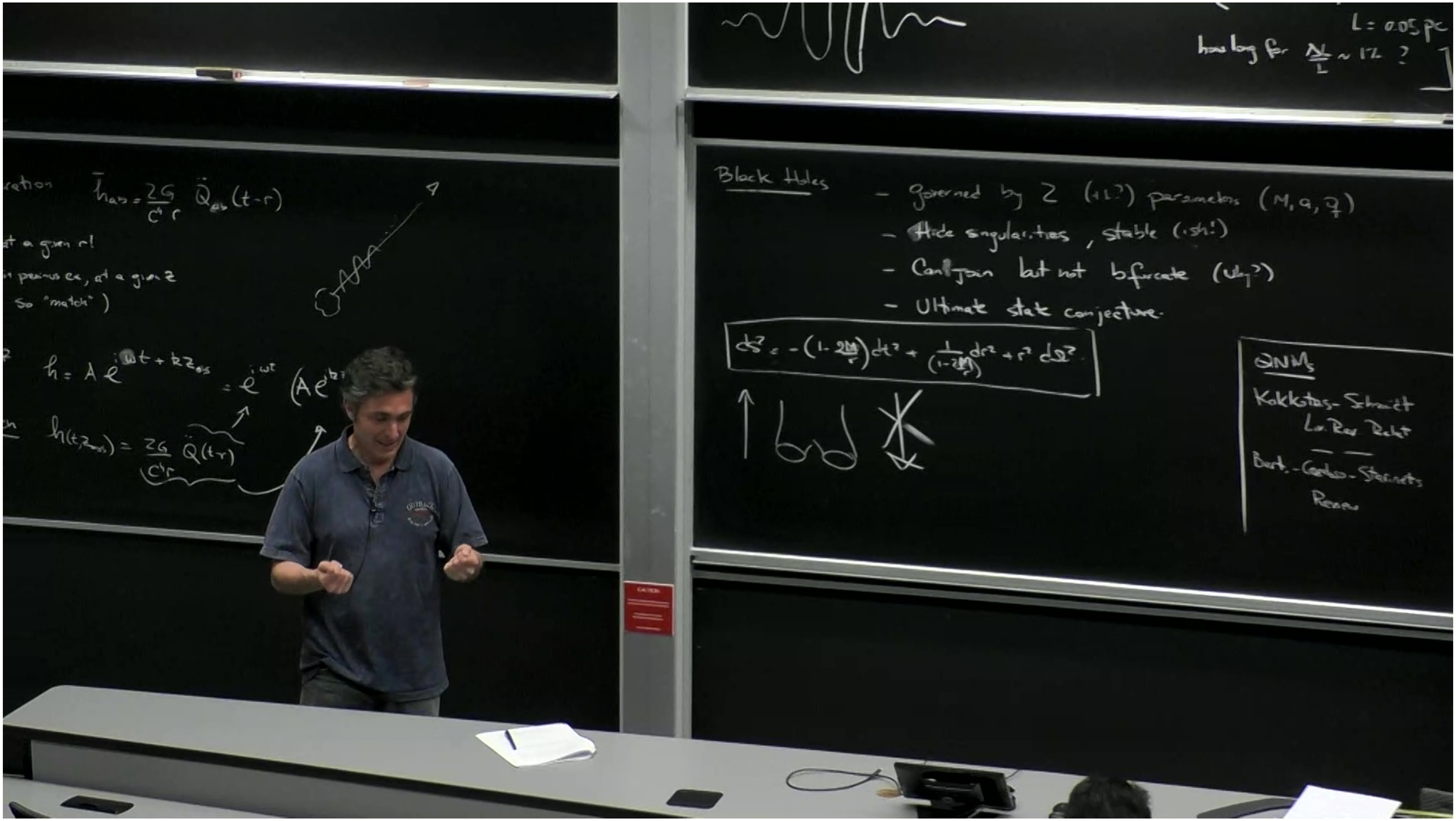
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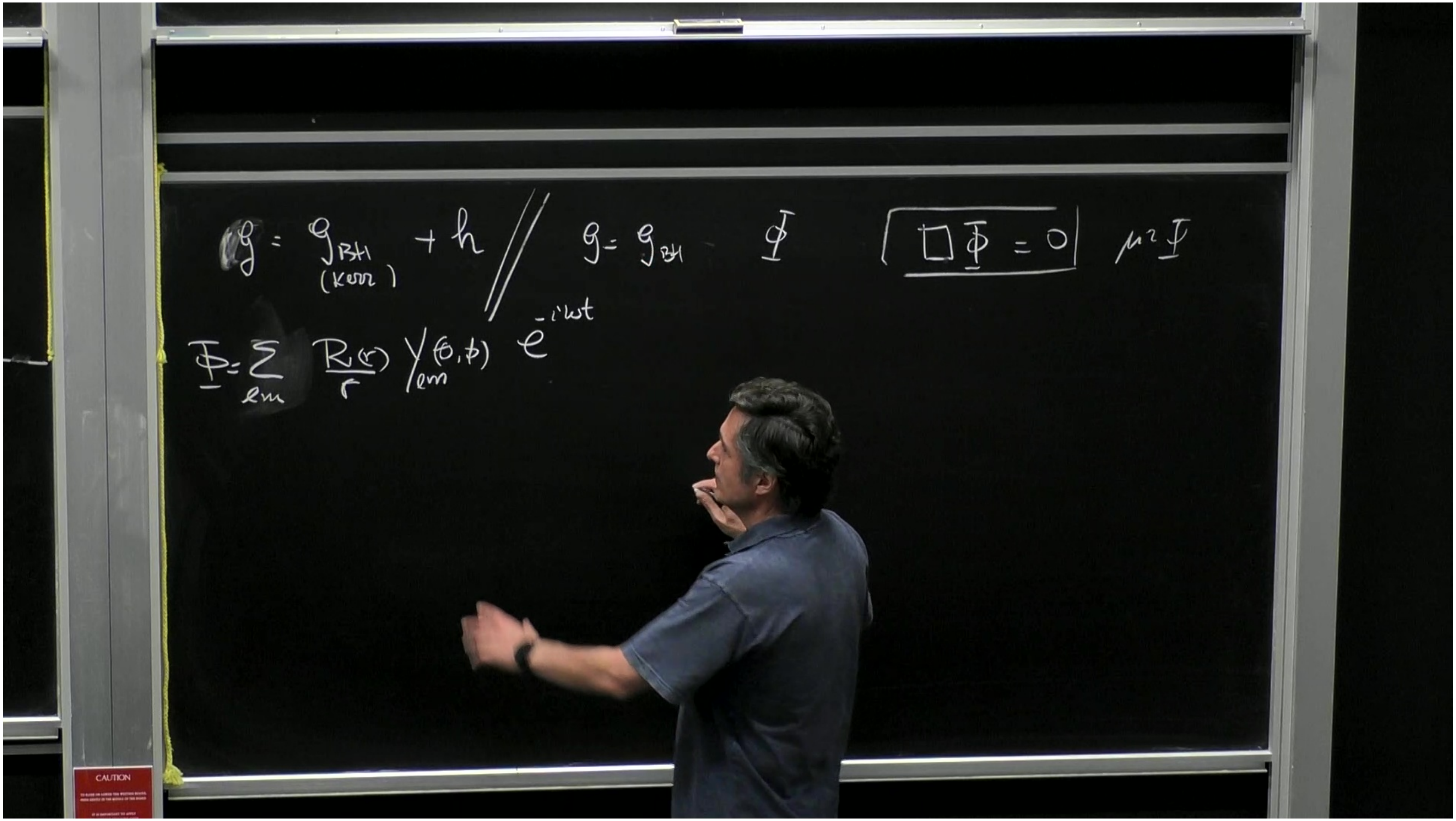
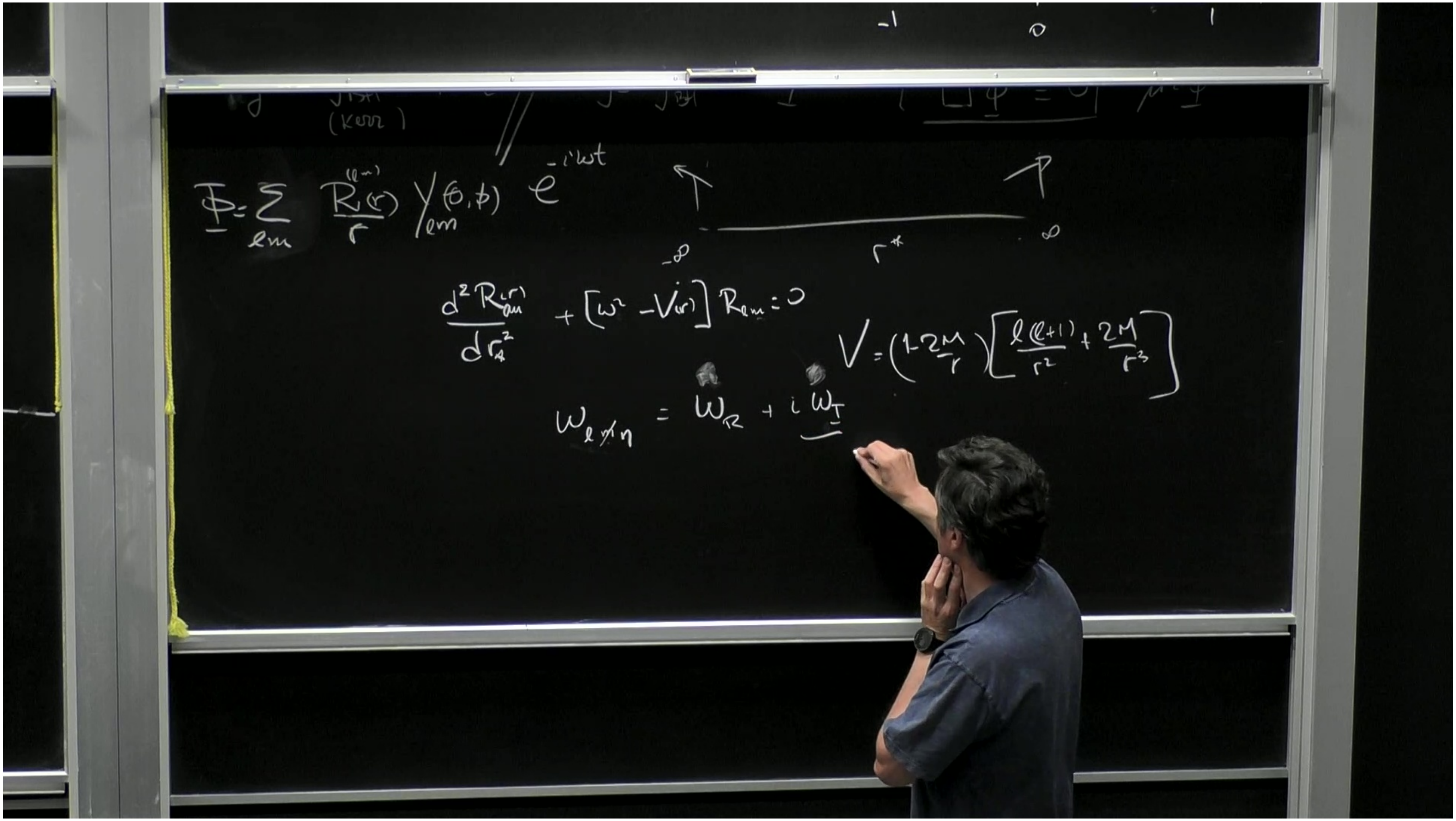
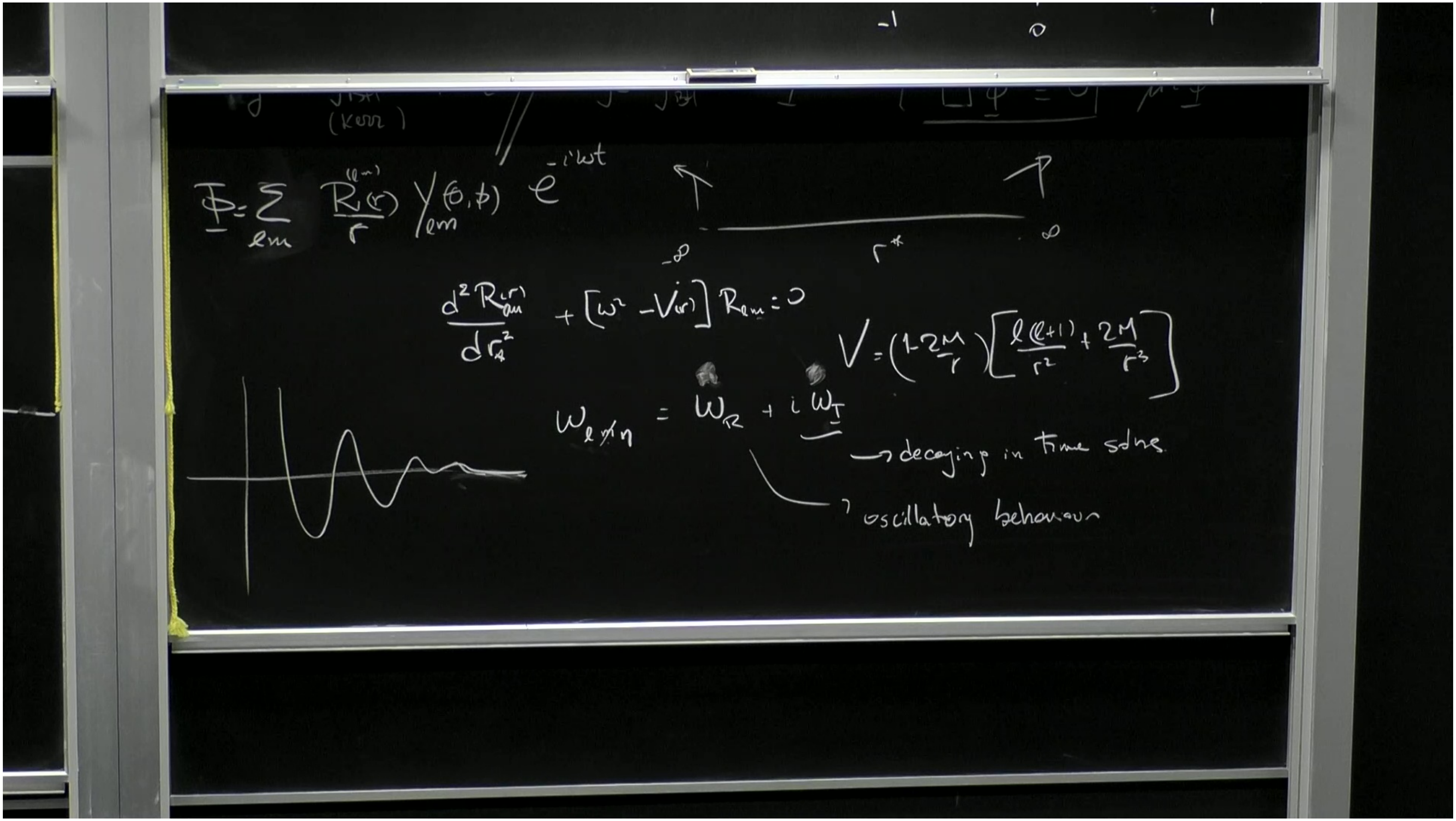
QNMs
Kokkotas-Schmidt
Bert.-Cardoso-Starinets
Review


$g = g_{BH} + h$ (Kerr)
$g = g_{BH}$ Φ
$\Box \Phi = 0$ $\mu^2 \Phi$
$\Phi = \sum_{lm} \frac{R(r)}{r} Y_{lm}(\theta,\phi) e^{-i\omega t}$


(Kerr)
$\Phi = \sum_{lm} \frac{R^{(lm)}(r)}{r} Y_{lm}(\theta,\phi)\, e^{-i\omega t}$
$-\infty$
r^*
∞
$\frac{d^2 R_{lm}(r)}{dr_*^2} + [\omega^2 - V(r)] R_{lm} = 0$
$V = \left(1 - \frac{2M}{r}\right)\left[\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right]$
$\omega_{lmn} = \omega_R + i\,\omega_I$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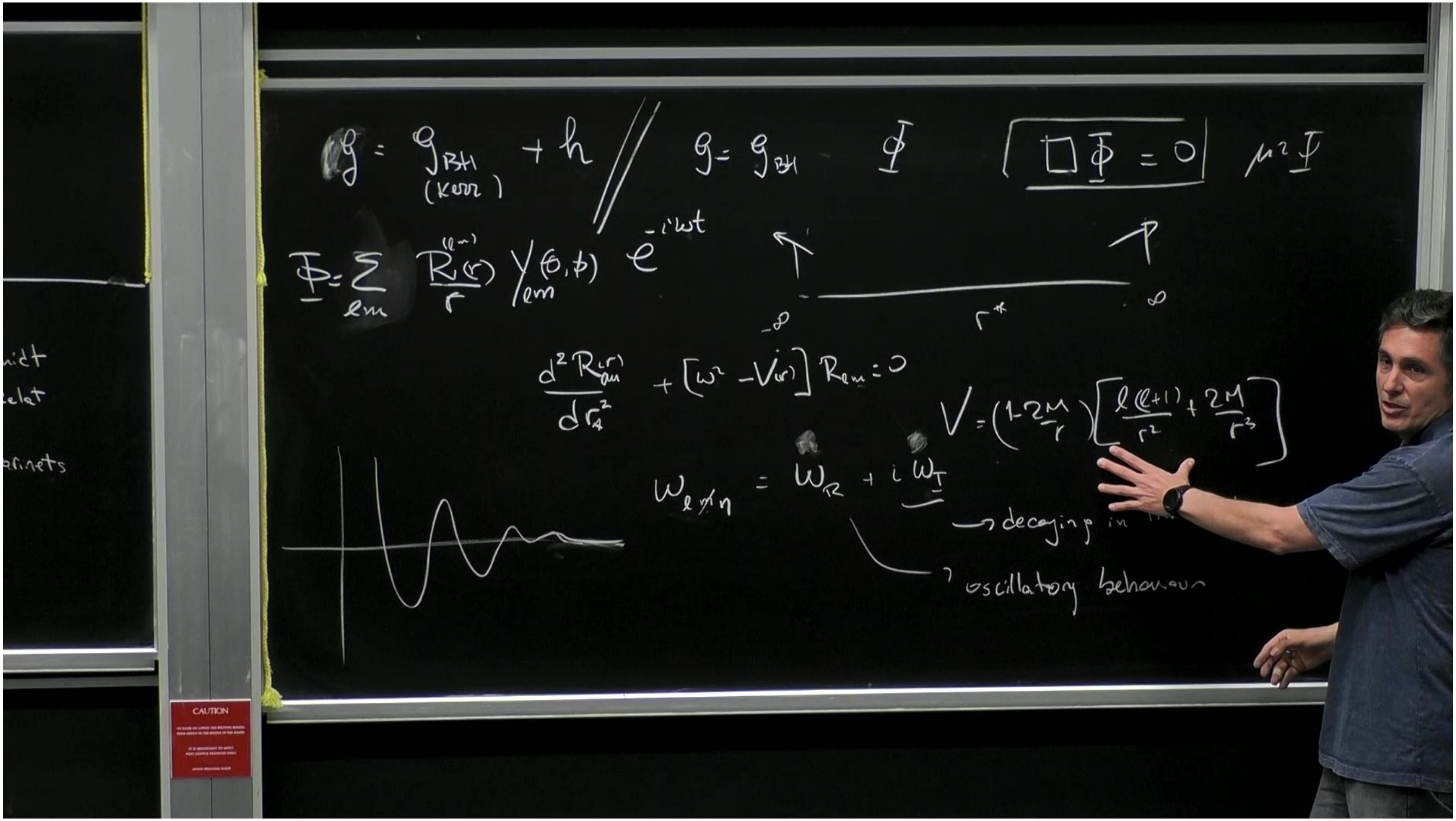
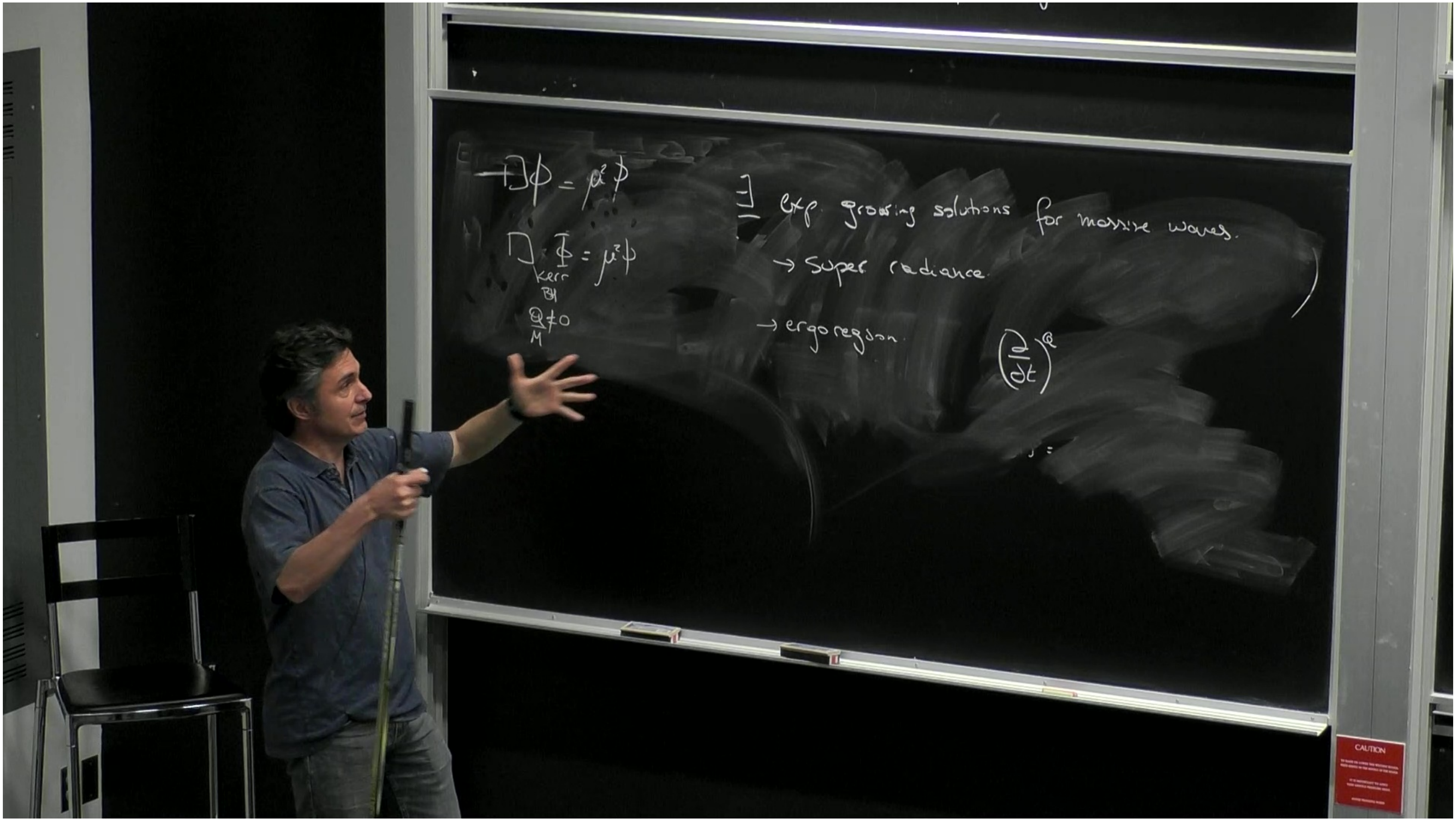
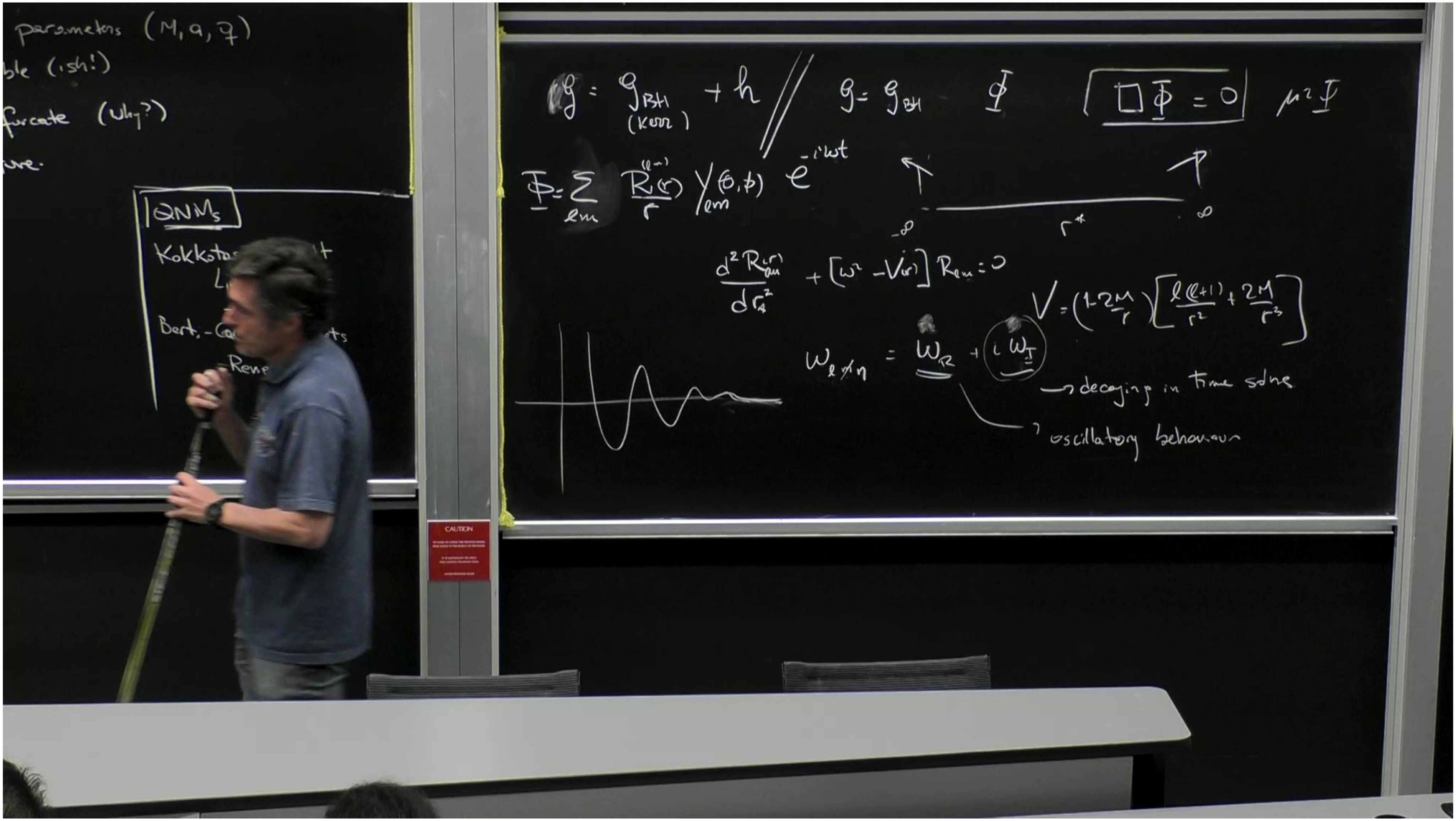
→ decaying in time solns.
→ oscillatory behaviour


g = g_BH + h
(Kerr)
g = g_BH
□Φ = 0
μ²Φ
Φ = Σ_ℓm R_ℓm(r)/r Y_ℓm(θ,φ) e^{-iωt}
-∞
r*
∞
d²R_ℓm(r)/dr*² + [ω² − V(r)] R_ℓm = 0
V = (1 − 2M/r) [ℓ(ℓ+1)/r² + 2M/r³]
ω_ℓmn = ω_R + i ω_I
→ decaying in
oscillatory behaviour
CAUTION


∃ exp. growing solutions for massive waves.
→ super radiance
→ ergoregion
CAUTION


parameters (M, a, q)
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Kokkotas
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$\mu^2 \Phi$
$\Phi = \sum_{\ell m} \frac{R^{(\ell m)}(r)}{r} Y_{\ell m}(\theta, \phi) e^{-i\omega t}$
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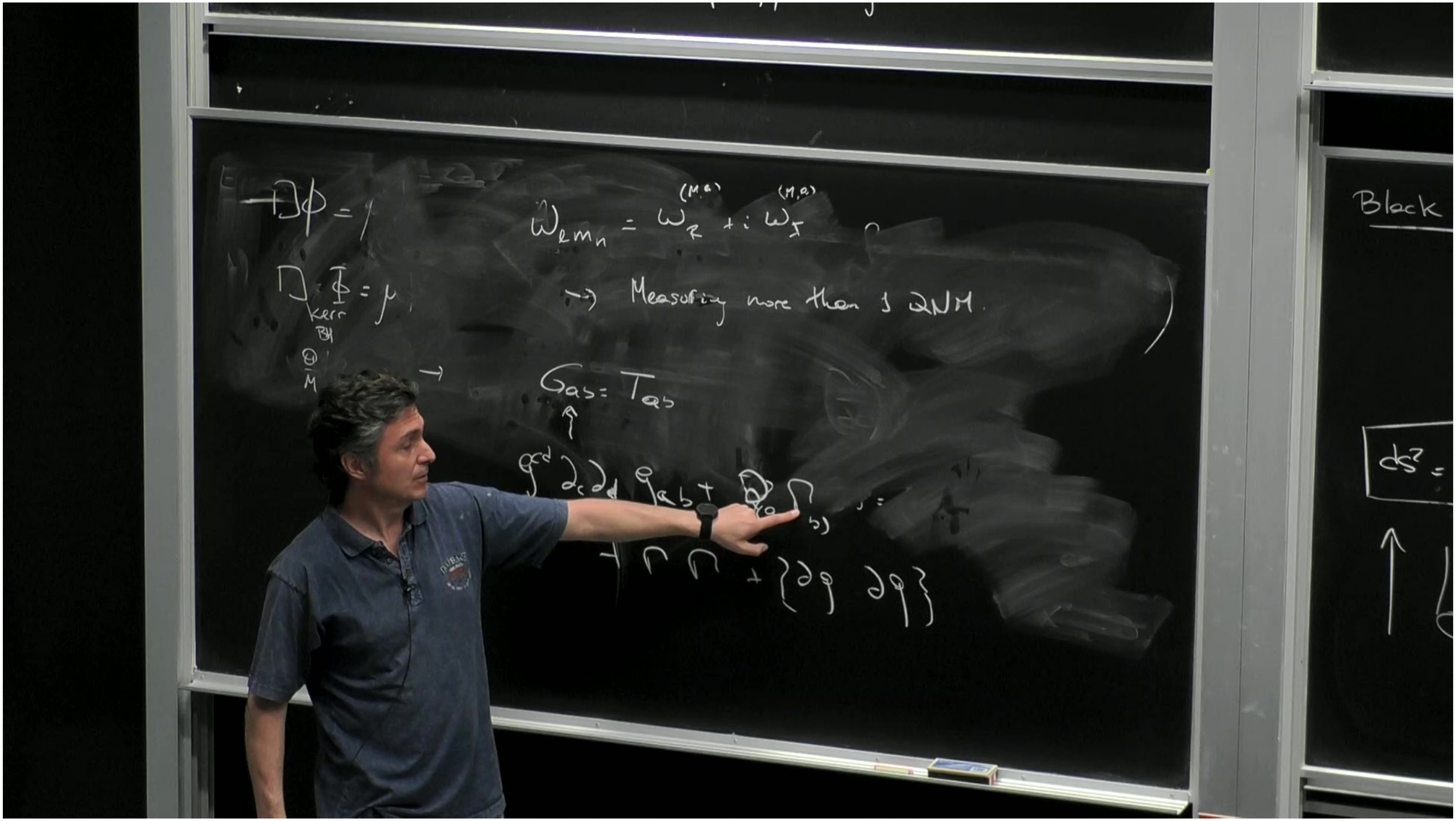
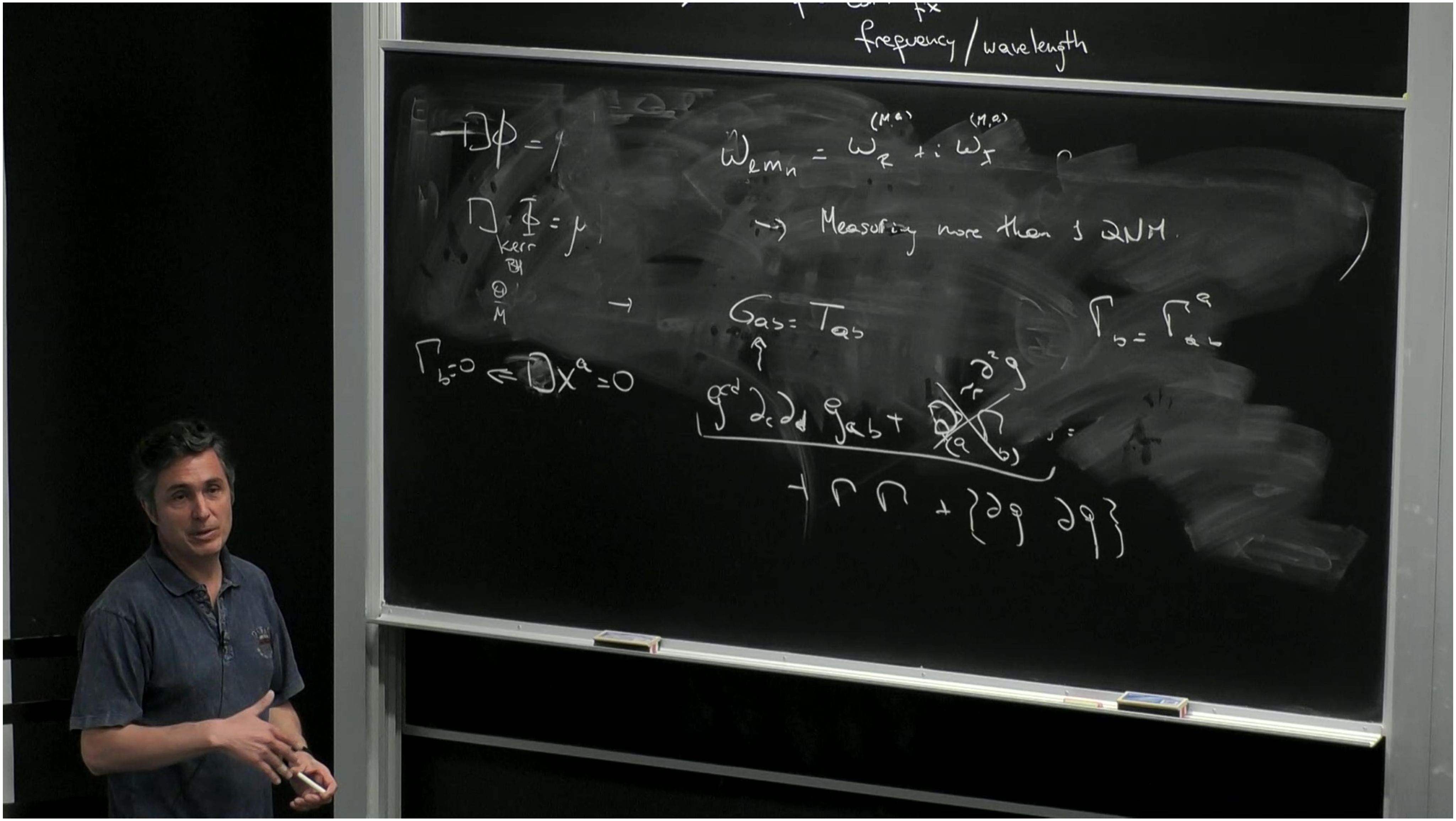
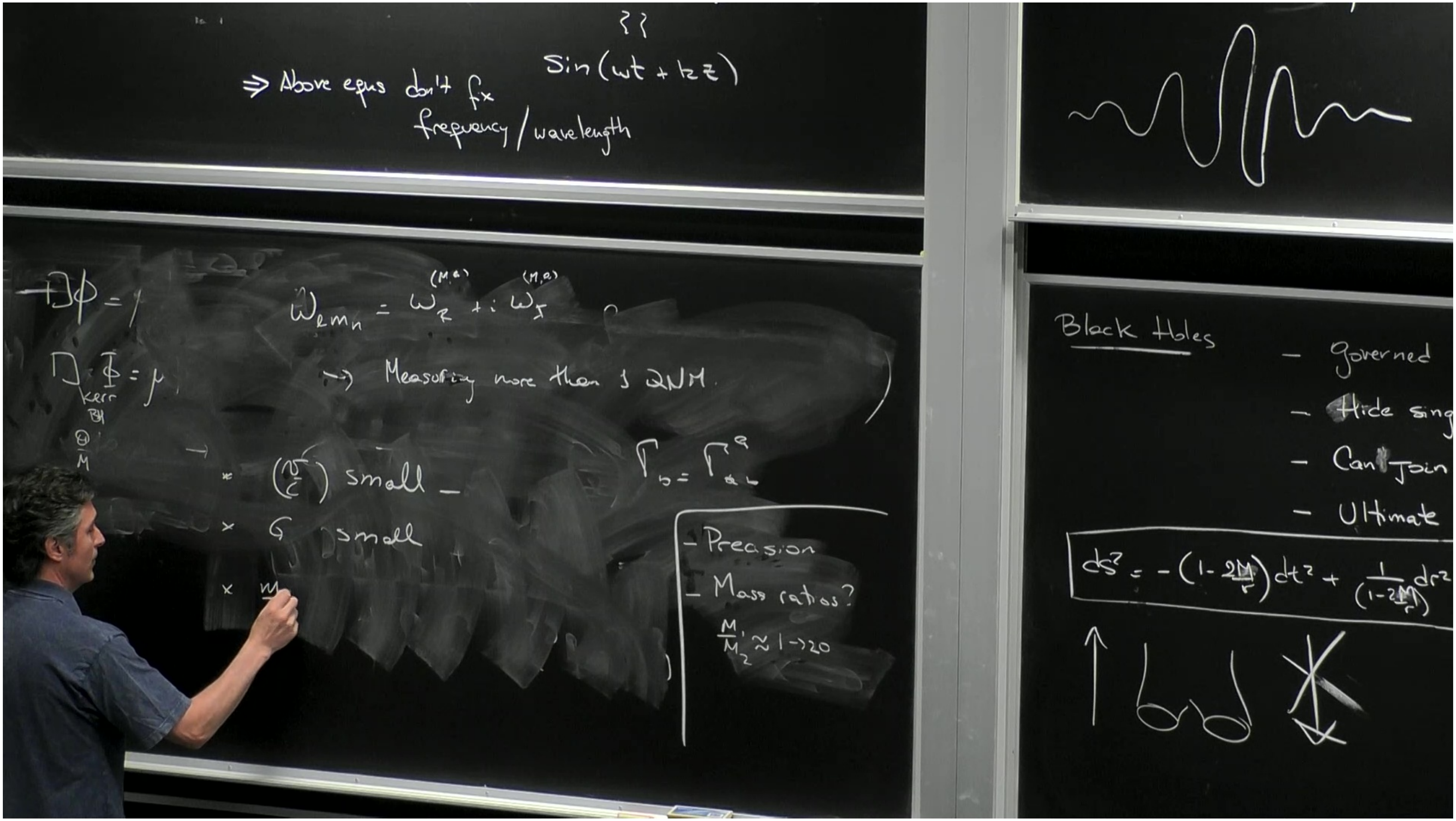
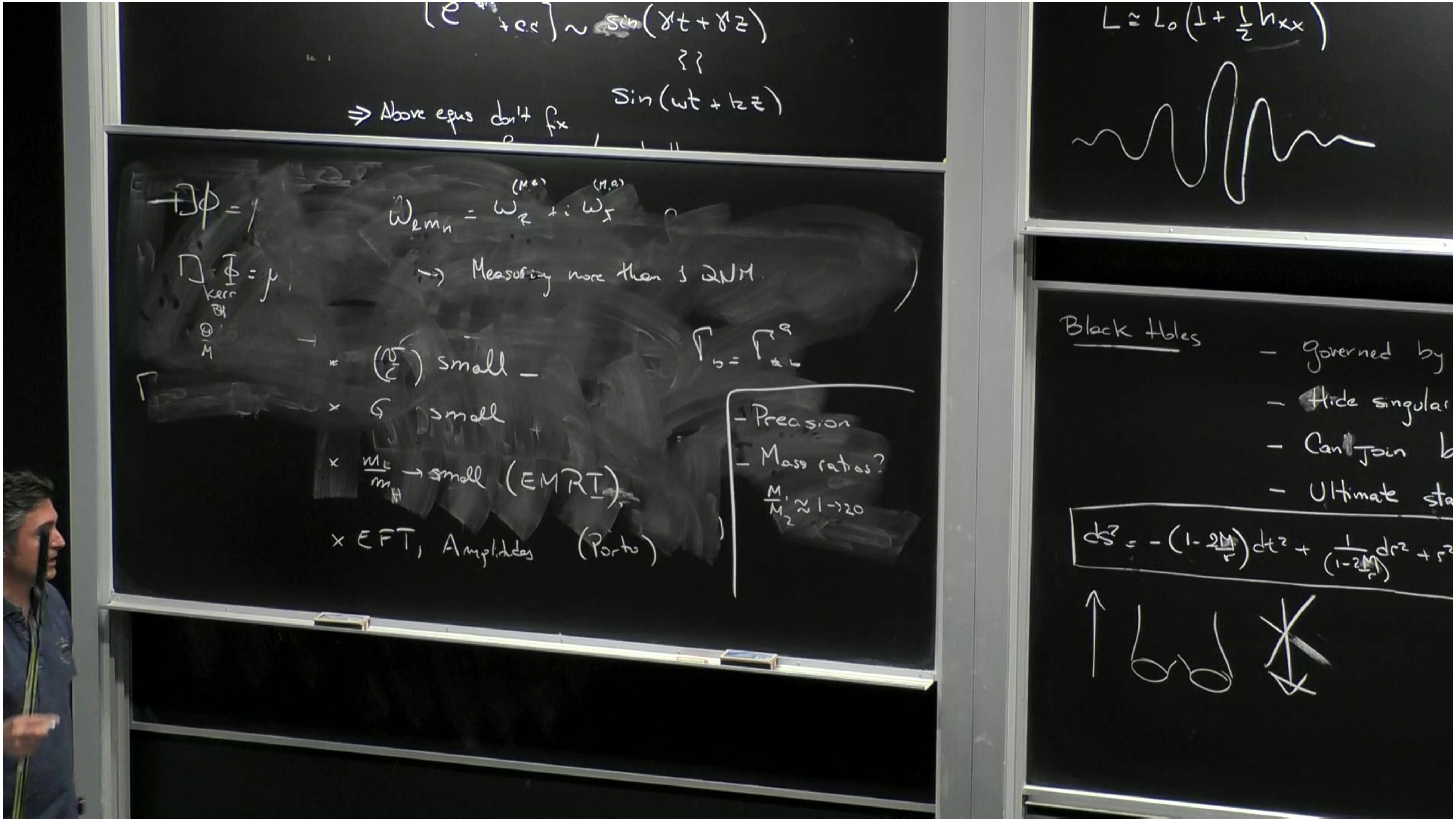

Measuring more than 1 QNM
Black


frequency / wavelength
Measuring more than 1 QNM


sin(wt + kz)
⇒ Above eqns don't fix frequency / wavelength
Measuring more than 1 QNM
small
small
- Precision
- Mass ratios?
Black Holes
- Governed
- Hide sing
- Can join
- Ultimate


⇒ Above eqns don't fix
sin(ωt + kz)
Measuring more than 1 QNM
small
small
small (EMRI)
× EFT, Amplitudes (Porto)
- Precision
- Mass ratios?
Black Holes
- Governed by
- Hide singula
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- Ultimate sta