Title: Gravitational Wave Theory

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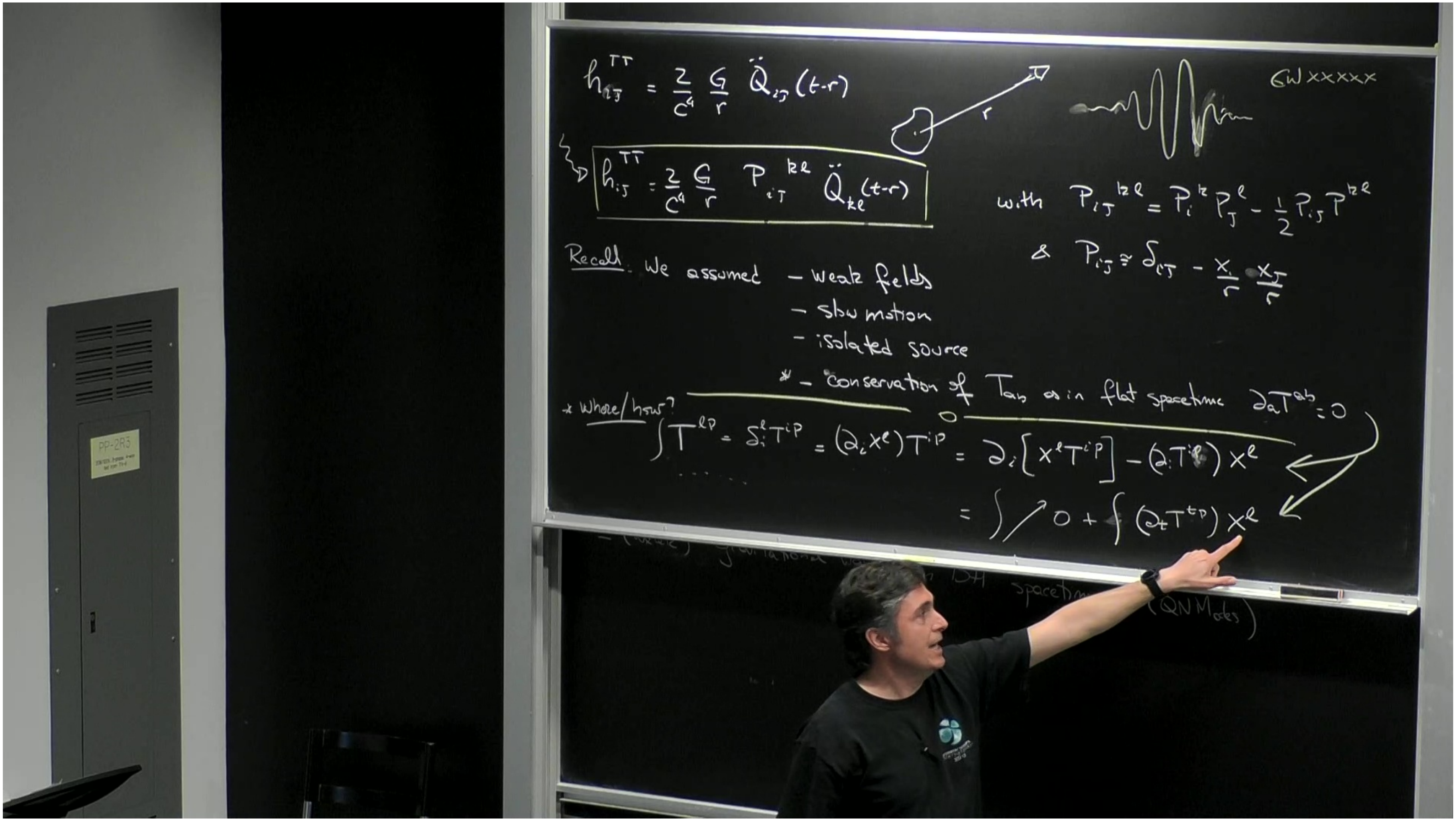
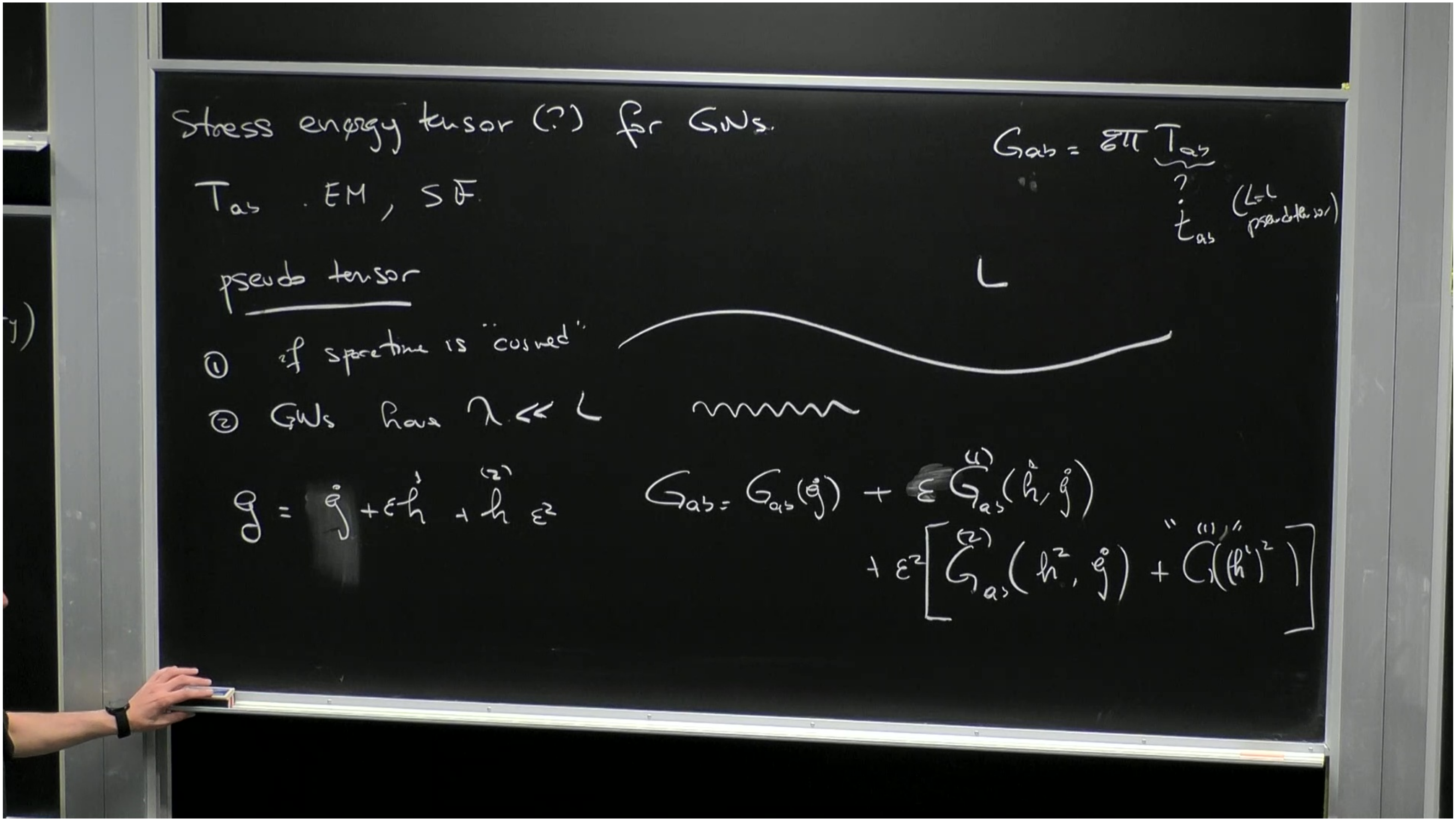
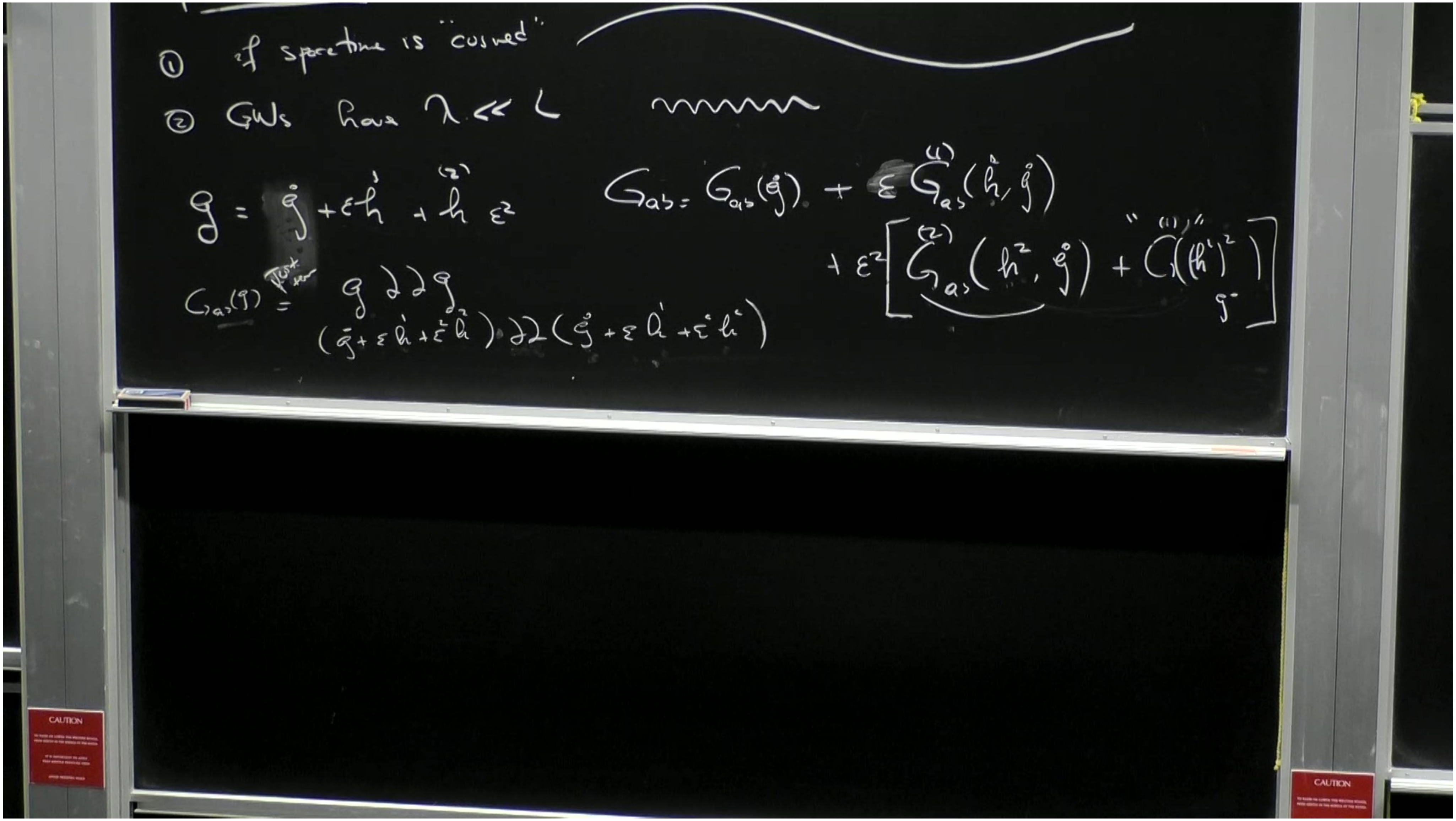
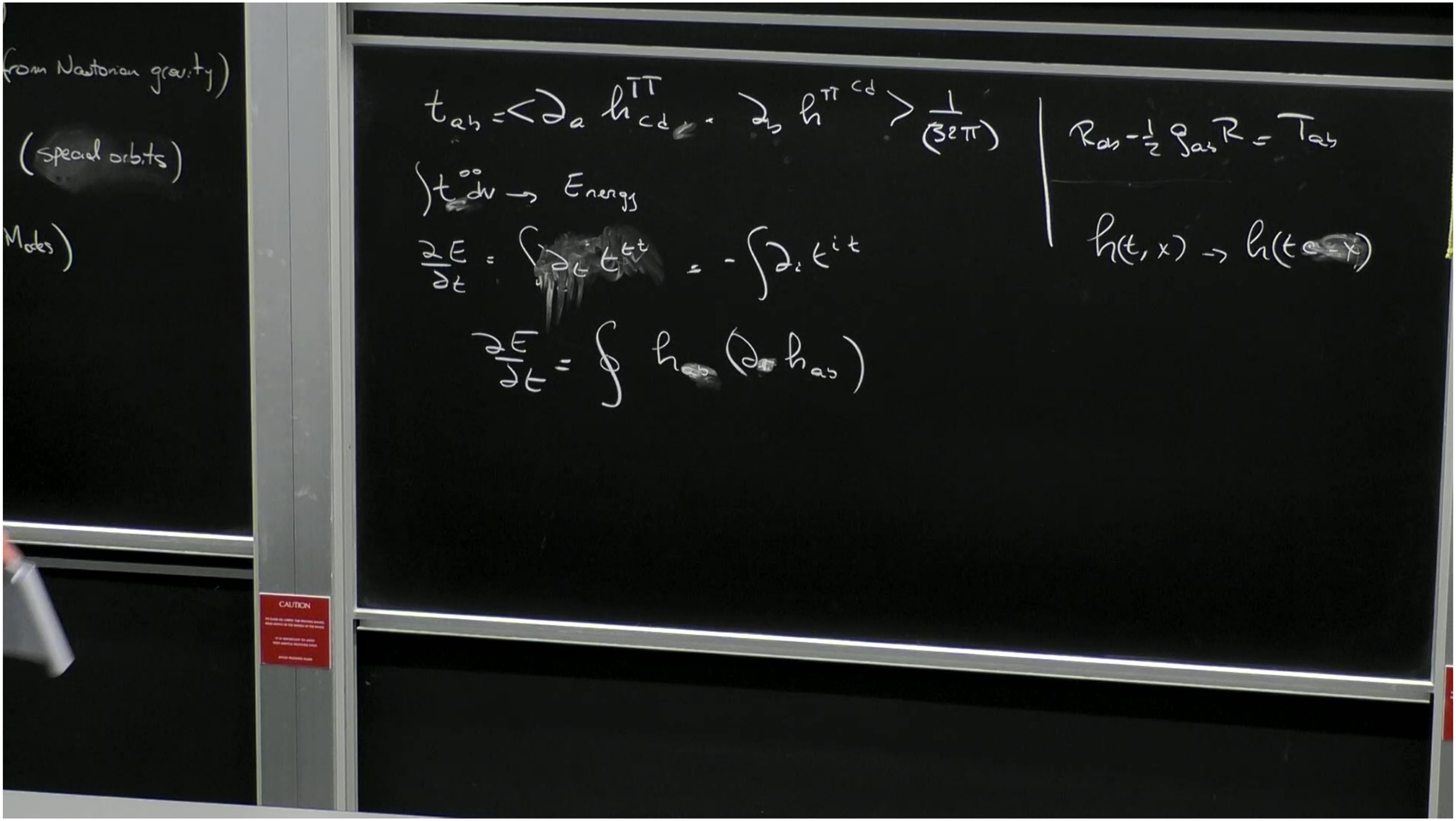

$h_{ij}^{TT} = \frac{2}{c^4}\frac{G}{r}\ddot{Q}_{ij}(t-r)$
r
GW xxxxx
$\boxed{h_{ij}^{TT} = \frac{2}{c^4}\frac{G}{r} P_{ij}{}^{kl}\ddot{Q}_{kl}(t-r)}$
with $P_{ij}{}^{kl} = P_i{}^k P_j{}^l - \frac{1}{2}P_{ij}P^{kl}$
& $P_{ij} \approx \delta_{ij} - \frac{x_i}{r}\frac{x_j}{r}$
Recall: We assumed - weak fields
- slow motion
- isolated source
* - conservation of T_{ab} as in flat spacetime $\partial_a T^{ab} = 0$
* where/how?
$\int T^{lp} = \delta^l_i T^{ip} = (\partial_i x^l) T^{ip} = \partial_i[x^l T^{ip}] - (\partial_i T^{ip}) x^l$
$= \int \nearrow 0 + \int (\partial_t T^{tp}) x^l$
(QNModes)

- Energy in grav. waves (how to define the undefinable)
- a PN "appetizer" (how to account for corrections from Newtonian gravity)
- (brief) BH review & key take home messages for GWs. (special orbits)
- (weak) gravitational waves in BH spacetimes (QNModes)


Stress energy tensor (?) for GWs.
$G_{ab} = 8\pi \underbrace{T_{ab}}_{?}$
t_{ab} (L-L pseudotensor)
T_{ab} . EM, SF
pseudo tensor
① if spacetime is "curved"
② GWs have $\lambda \ll L$
L
$g = \mathring{g} + \epsilon \overset{1}{h} + \overset{(2)}{h} \epsilon^2$
$G_{ab} = G_{ab}(\mathring{g}) + \epsilon \overset{(1)}{G}_{ab}(\overset{1}{h}, \mathring{g})$
$+ \epsilon^2 \left[\overset{(2)}{G}_{ab}(\overset{2}{h}, \mathring{g}) + \text{"}\overset{(1)}{G}\text{"}((\overset{1}{h})^2) \right]$


① if spacetime is "curved"
② GWs have $\lambda \ll L$
$g = \mathring{g} + \varepsilon \overset{1}{h} + \overset{(2)}{h} \varepsilon^2$
$G_{ab}(g) = g \partial\partial g$
$(\mathring{g} + \varepsilon \overset{1}{h} + \varepsilon^2 \overset{2}{h}) \partial\partial (\mathring{g} + \varepsilon \overset{1}{h} + \varepsilon^2 \overset{2}{h})$
$G_{ab} = G_{ab}(\mathring{g}) + \varepsilon \overset{(1)}{G}_{ab}(\overset{1}{h}, \mathring{g})$
$+ \varepsilon^2 \left[\overset{(2)}{G}_{ab}(\overset{2}{h}, \mathring{g}) + \text{"}\overset{(1)}{G}\text{"}((\overset{1}{h})^2, \mathring{g}) \right]$


rom Newtonian gravity)
(special orbits)
Modes)
$t_{ab} = \langle \partial_a h^{TT}_{cd} \, \partial_b h^{TT\,cd} \rangle \frac{1}{(32\pi)}$
$\int t^{00} dV \to$ Energy
$\frac{\partial E}{\partial t} = \int \partial_t t^{tt} = -\int \partial_i t^{it}$
$\frac{\partial E}{\partial t} = \oint h_{ab} (\partial_r h_{ab})$
$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}$
$h(t, x) \to h(t - x)$

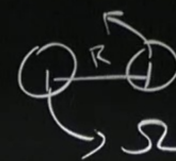
Newtonian gravity)

special orbits)

)

$$\frac{dE}{dt} = \frac{-G}{5c^5}\left(\dddot{Q}_{ab}\dddot{Q}^{ab}\right)$$

$$L \sim \frac{G}{c^5}\left(\frac{MR^2}{T^3}\right)^2$$



$$v^2 \sim \frac{GM}{r}$$

$$T \sim \frac{1}{\Omega} \sim \sqrt{\frac{R^3}{GM}}$$

$$L \sim \frac{G}{c^5}\frac{M^2R^4}{\left(\frac{R^3}{GM}\right)^3} \sim \frac{G^4}{c^5}\left(\frac{M}{R}\right)^2 \frac{v^6}{G^3}$$

$$\rightarrow \frac{G}{c^5}\left(\frac{M}{R}\right)^2 \frac{v^6}{c^6}\ \frac{G^2}{c^4}\ \frac{c^4}{G^2}\ c^6$$

$$\rightarrow \sim \frac{c^5}{G}\left(\frac{R_s}{R}\right)^2\left(\frac{v}{c}\right)^6$$

$\sim 10^{59}$ erg/s

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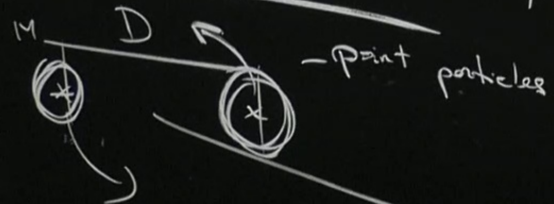
$\boxed{E_x}$



Compute: h

-

- Energy

$T \sim$

$$E = \frac{1}{2}\mu v^2 - \frac{M\mu}{r}$$

$$E^{\text{circular}} = -\frac{1}{2}\frac{M\mu}{r}$$

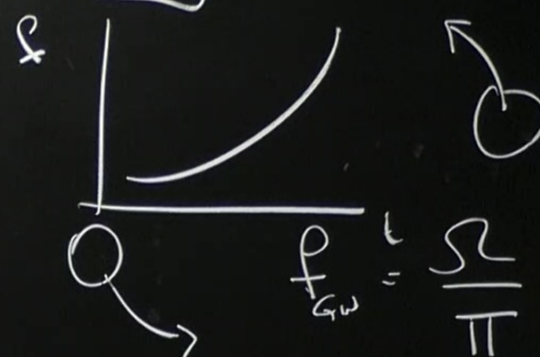
$$\frac{dE}{dt} \longrightarrow \frac{dr}{dt} < 0$$

obtain $\frac{d\Omega}{dt}$

$(D = r)$

$$\frac{dE}{dt} < 0$$

$\boxed{\text{Ex}}$



$$f_{GW} = \frac{\Omega}{\pi}$$

$$c = f\lambda$$

$$\hookrightarrow \underbrace{\dot{f}_{GW}} = \frac{96}{5}\frac{1}{c^5}\pi^{8/3}(GM_c)^{5/3}\,\underbrace{f_{GW}^{11/3}}$$

$M_c \equiv$ chirp mass $= M_{TOT}\left(\frac{\mu}{M}\right)^{3/5}$

$(D = r)$

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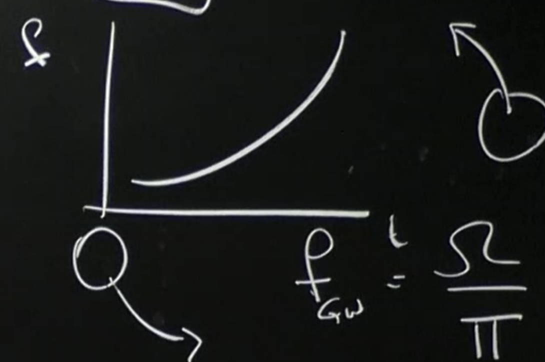

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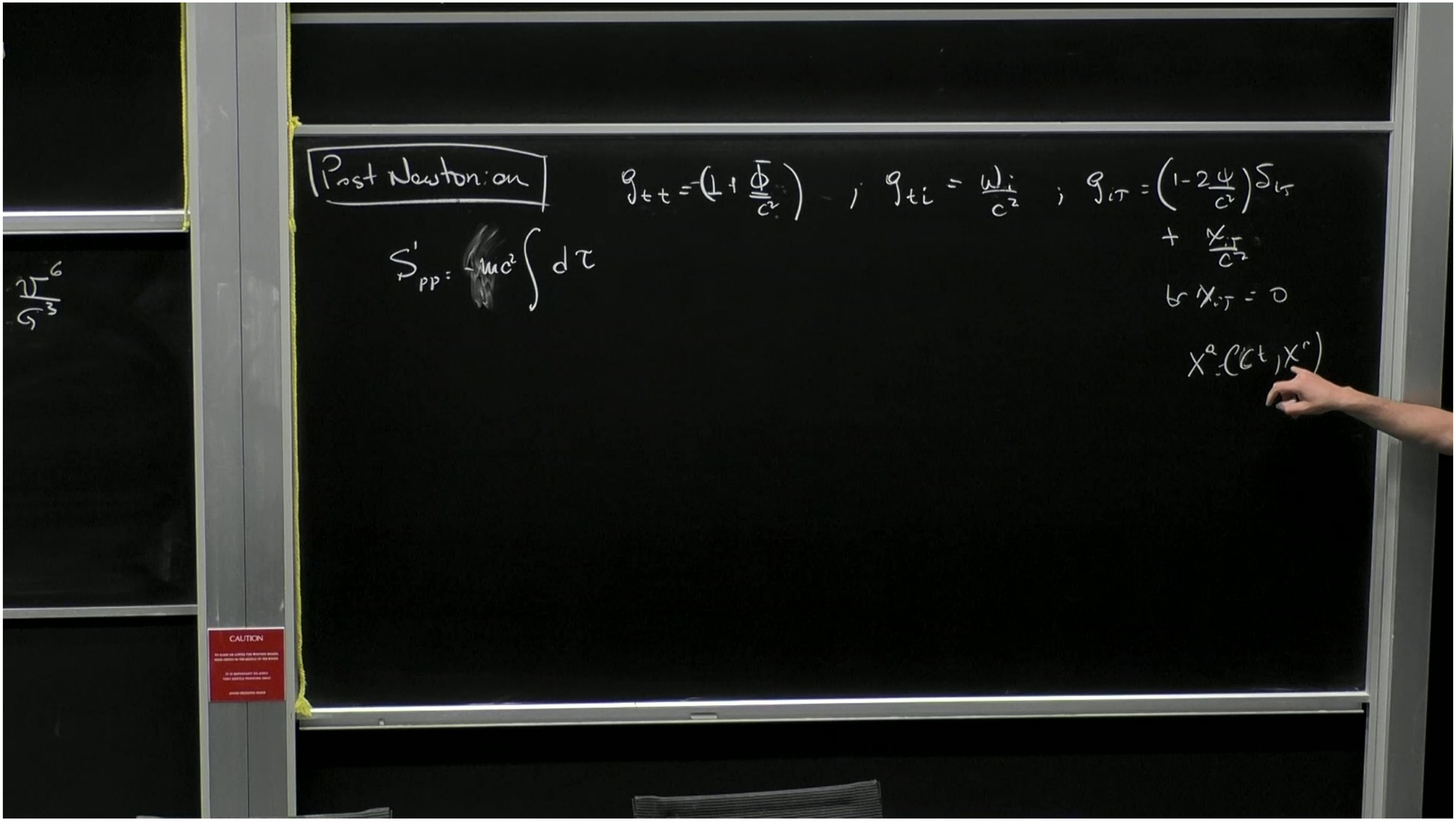
$$M_c \equiv \text{chirp mass} = M_{TOT}\left(\frac{\mu}{M_{TOT}}\right)^{3/5}$$


Post Newtonian
$g_{tt} = -\left(1 + \frac{\Phi}{c^2}\right)$; $g_{ti} = \frac{\omega_i}{c^2}$; $g_{ij} = \left(1 - 2\frac{\psi}{c^2}\right)\delta_{ij}$
$+ \frac{\chi_{ij}}{c^2}$
$\mathrm{tr}\,\chi_{ij} = 0$
$x^a = (ct, x^i)$
$S'_{pp} = -mc^2 \int d\tau$
CAUTION

$$M_c \equiv \text{chirp mass} = M_{TOT} \left(\frac{\mu}{M_{TOT}}\right)^{3/5}$$

$$= -mc \int \sqrt{c^2 + 2\Phi - 2\frac{\omega_i v^i}{c^2} - \left(1 - \frac{2\psi}{c^2}\right) + \frac{\phi v^2}{2c^4}}$$

$x^a = (ct, x^i)$

$v^i = \frac{dx^i}{dt}$

$$S_{pp} \approx -mc^2 + \underbrace{m\left(\frac{v^2}{2} - \phi\right) + m\left[\frac{\phi^2}{2c^2} - \frac{\phi v^2}{2c^2} - \frac{\psi v^2}{c^2} + \frac{v^4}{8c^4} + \frac{v_i \omega^i}{c^2} + \frac{\chi_{ij} v^i v^j}{2c^2}\right]}_{1PN}$$

- $c \to \infty$
- $G \to 0$

1PN $\left(\frac{M}{r}, \left(\frac{v}{c}\right)^2\right)$