

Title: Gravitational Wave Theory

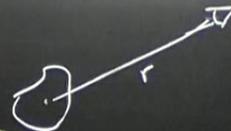
Speakers: Luis Lehner

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Date: June 29, 2023 - 9:00 AM

URL: <https://pirsa.org/23060082>

$$h_{ij}^{TT} = \frac{2}{c^4} \frac{G}{r} \ddot{Q}_{ij}(t-r)$$



GWxxxxx

$$h_{ij}^{TT} = \frac{2}{c^4} \frac{G}{r} P_{ij}^{kl} \ddot{Q}_{kl}(t-r)$$

with $P_{ij}^{kl} = P_i^k P_j^l - \frac{1}{2} P_{ij} P^{kl}$

& $P_{ij} \approx \delta_{ij} - \frac{x_i x_j}{r^2}$

- Recall: We assumed
- weak fields
 - slow motion
 - isolated source

* - conservation of T_{ab} as in flat spacetime $\partial_a T^{ab} = 0$

* Where/how?

$$\int T^{lp} = \int \partial_i T^{ip} = (\partial_i x^e) T^{ip} = \partial_i [x^e T^{ip}] - (\partial_i T^{ie}) x^e$$

$$= \int \cancel{0} + \int (\partial_t T^{tp}) x^e$$



- Energy in grav. waves (how to define the undefinable)
- a PN "appetizer" (how to account for corrections from Newtonian gravity)
- (brief) BH review & key take home messages for GWs. (special orbits)
- (what) gravitational waves in BH spacetimes (QN Modes)

Stress energy tensor (?) for GWs.

T_{ab} EM, SF

pseudo tensor

① if spacetime is "curved"

② GWs have $\lambda \ll L$

$$g = \overset{\circ}{g} + \epsilon h^{\circ} + h^{\circ\circ} \epsilon^2$$

$$G_{ab} = G_{ab}(\overset{\circ}{g}) + \epsilon G_{ab}^{(1)}(h^{\circ}, \overset{\circ}{g}) + \epsilon^2 \left[G_{ab}^{(2)}(h^{\circ\circ}, \overset{\circ}{g}) + G_{ab}^{(1)}(h^{\circ})^2 \right]$$

$$G_{ab} = 8\pi \underbrace{T_{ab}}_{?} \quad (L=L \text{ pseudo tensor})$$

L



① if spacetime is "curved":

② GWs have $\lambda \ll L$ 

$$g = \bar{g} + \epsilon h^{(1)} + h^{(2)} \epsilon^2$$

$$G_{ab} = G_{ab}(\bar{g}) + \epsilon G_{ab}^{(1)}(h^{(1)}, \bar{g})$$

$$+ \epsilon^2 \left[G_{ab}^{(2)}(h^{(2)}, \bar{g}) + \overset{\text{"(1)"}{}}{C} \left((h^{(1)})^2 \right) \right]$$

$$G_{ab}(\bar{g}) \stackrel{\text{Post-Mink}}{=} \bar{g} \partial \partial \bar{g} \\ (\bar{g} + \epsilon h^{(1)} + \epsilon^2 h^{(2)}) \partial \partial (\bar{g} + \epsilon h^{(1)} + \epsilon^2 h^{(2)})$$

CAUTION

BE CAREFUL IN USING THIS EQUIPMENT
WITH CHILDREN OR THE ELDERLY OR THE SICK
IF IN DOUBT CONSULT AN ADULT
FOR SAFETY INSTRUCTIONS

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(from Newtonian gravity)

(special orbits)

Modes)

$$t_{ab} = \left\langle \partial_a h^{\pi\pi}_{cd} - \partial_b h^{\pi\pi cd} \right\rangle \frac{1}{(32\pi)}$$

$t_{ab} \rightarrow$ Energy

$$\frac{\partial E}{\partial t} = \int \partial_t t^{tt} = - \int \partial_i t^{it}$$

$$\frac{\partial E}{\partial t} = \int h_{ab} (\partial_a h_{ab})$$

$$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}$$

$$h(t, x) \rightarrow h(t, x)$$

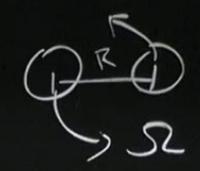
CAUTION
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Newtonian gravity)

special orbits)

$$\frac{dE}{dt} = \frac{1}{5} \frac{G}{c^5} (\ddot{Q}_{ij} \ddot{Q}^{ij})$$

$$L \sim \frac{G}{c^5} \left(\frac{MR^2}{T^3} \right)^2$$



$$v^2 \sim \frac{GM}{r}$$

$$T \sim \frac{1}{\Omega} \sim \sqrt{\frac{R^3}{GM}}$$

$$L \sim \frac{G}{c^5} \frac{M^2 R^4}{\left(\frac{GM}{R} \right)^3} \sim \frac{G^4}{c^5} \left(\frac{M}{R} \right)^2$$

$$\sim \frac{G}{c^5} \left(\frac{R^2}{T^3} \right)^2$$

$$\sim \frac{G}{c^5} \left(\frac{R^2}{\left(\frac{R^3}{GM} \right)^3} \right)^2$$

$$\sim \frac{G}{c^5} \left(\frac{GM^3}{R^3} \right)^2$$

$$\sim \frac{G^3 M^6}{c^{10} R^3}$$

$\sim 10^{59} \text{ erg/s}$

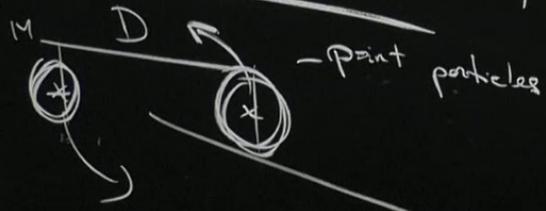


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E_x



Compute: h
- Energy

$$E = \frac{1}{2} \mu v^2 - \frac{M\mu}{r}$$

$$E_{\text{circular}} = -\frac{1}{2} \frac{M\mu}{r}$$

$$\frac{dE}{dt} \rightarrow \frac{dr}{dt} < 0$$

obtain $\frac{d\Omega}{dt}$

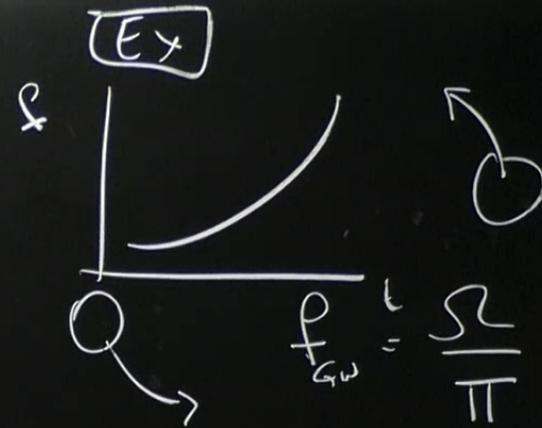
(D=r)

$$\frac{dE}{dt} < 0$$

$$c = f \lambda$$

$$\hookrightarrow \dot{P}_{\text{GW}} = \frac{96}{5} \frac{1}{c^5} \pi^{5/3} (GM_c)^{5/3} \left[\frac{P}{P_{\text{GW}}} \right]^{11/3}$$

$$M_c \equiv \text{chirp mass} = M_{\text{TOT}} \left(\frac{\mu}{M} \right)^{3/5}$$



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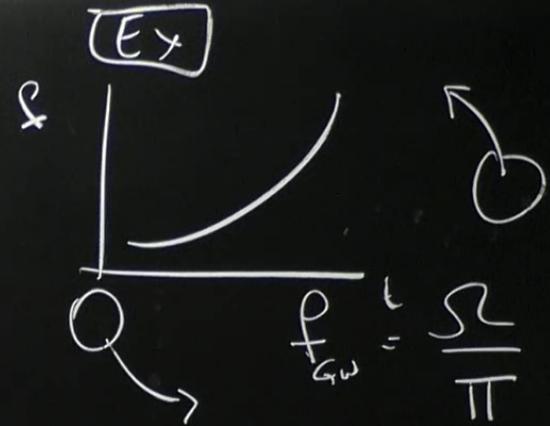
(D=r)

$$\frac{dE}{dt} < 0$$

$$c = f \lambda$$

$$\rho_{\text{GW}} = \frac{96}{5} \frac{1}{c^5} \pi^{90/5} (GM_c)^{5/3} \left[\frac{\rho^{11/3}}{f_{\text{GW}}} \right]$$

$$M_c \equiv \text{chirp mass} = M_{\text{TOT}} \left(\frac{\mu}{M_{\text{TOT}}} \right)^{3/5}$$



$\frac{dP}{dt}$

Post Newtonian

$$g_{tt} = -\left(1 + \frac{\Phi}{c^2}\right) ; g_{ti} = \frac{w_i}{c^2} ; g_{ij} = \left(1 - 2\frac{\Phi}{c^2}\right)\delta_{ij}$$

$$S'_{pp} = mc^2 \int d\tau$$

$$+ \frac{X_{ij}}{c^2}$$

$$\text{or } X_{ij} = 0$$

$$X^a(t, X^i)$$

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$$ct \rightarrow +_{GW} = \frac{1}{5} c^3$$

$$M_c \equiv \text{chirp mass} = M_{TOT} \left(\frac{\mu}{M_{TOT}} \right)^{3/5}$$

$\downarrow -g_{00} \frac{dx^0}{dt} \frac{dx^0}{dt}$
 $\downarrow X_{,T} = 0$

$$S_{pp} \approx -mc \int \sqrt{c^2 + 2\Phi - 2\frac{\omega_i v^i}{c^2} - \left(1 - \frac{2\psi}{c^2}\right) + \frac{\phi v^2}{2c^4}}$$

$$S_{pp} \approx -mc^2 + m \left(\frac{v^2}{2} - \phi \right) + m \left[\frac{\phi^2}{2c^2} - \frac{\phi v^2}{2c^2} - \frac{\psi v^2}{c^2} + \frac{v^4}{8c^4} + \frac{v_i \omega^i}{c^2} + \frac{X_{,T} v^i v^j}{2c^2} \right]$$

$\downarrow X^a = (ct, X^i)$
 $v^i = \frac{dx^i}{dt}$

- $C \rightarrow \infty$
 - $G \rightarrow 0$

1PN $\left(\frac{M}{r}, \left(\frac{v}{c} \right)^2 \right)$