Title: Gravitational Wave Theory

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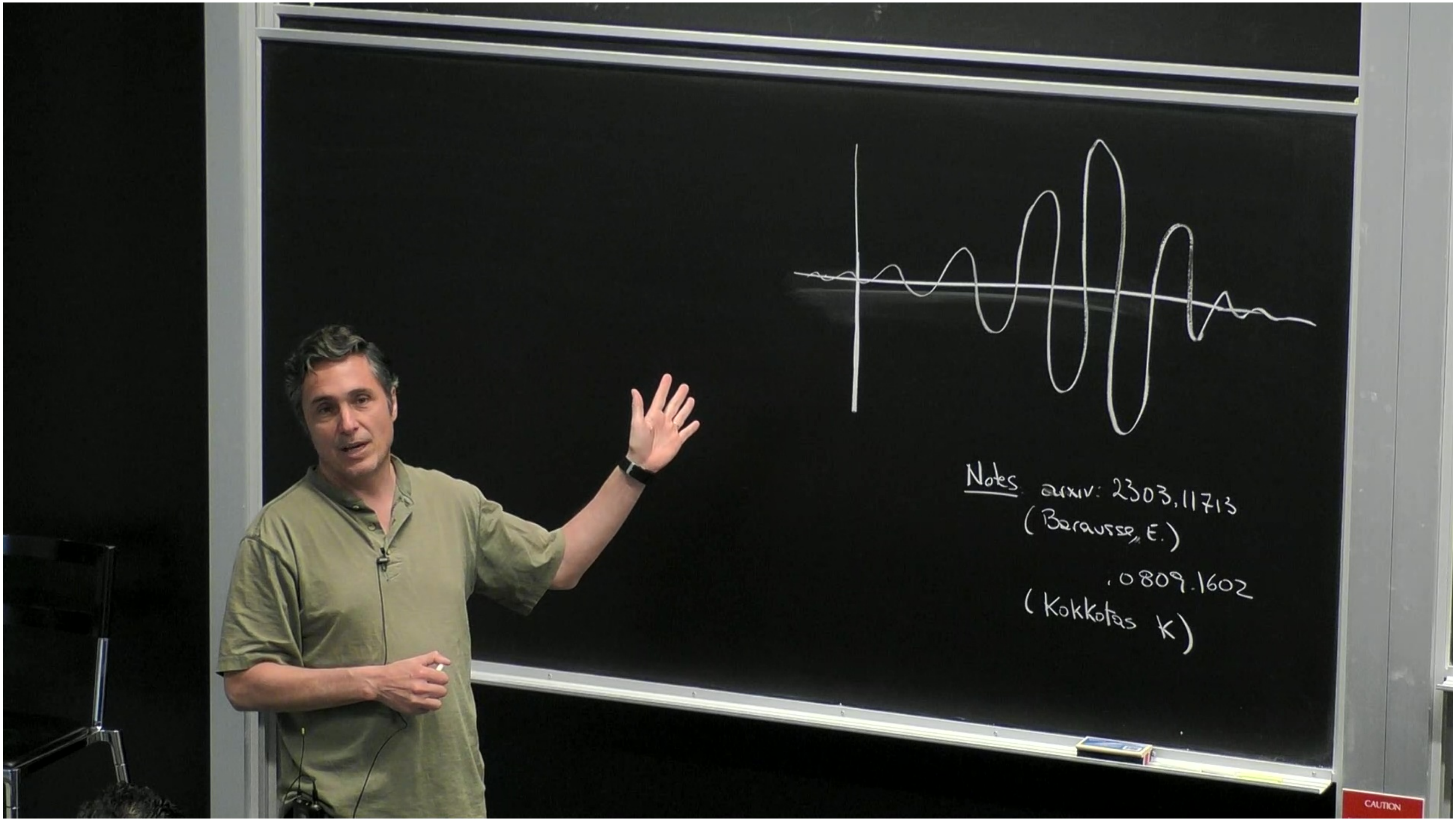
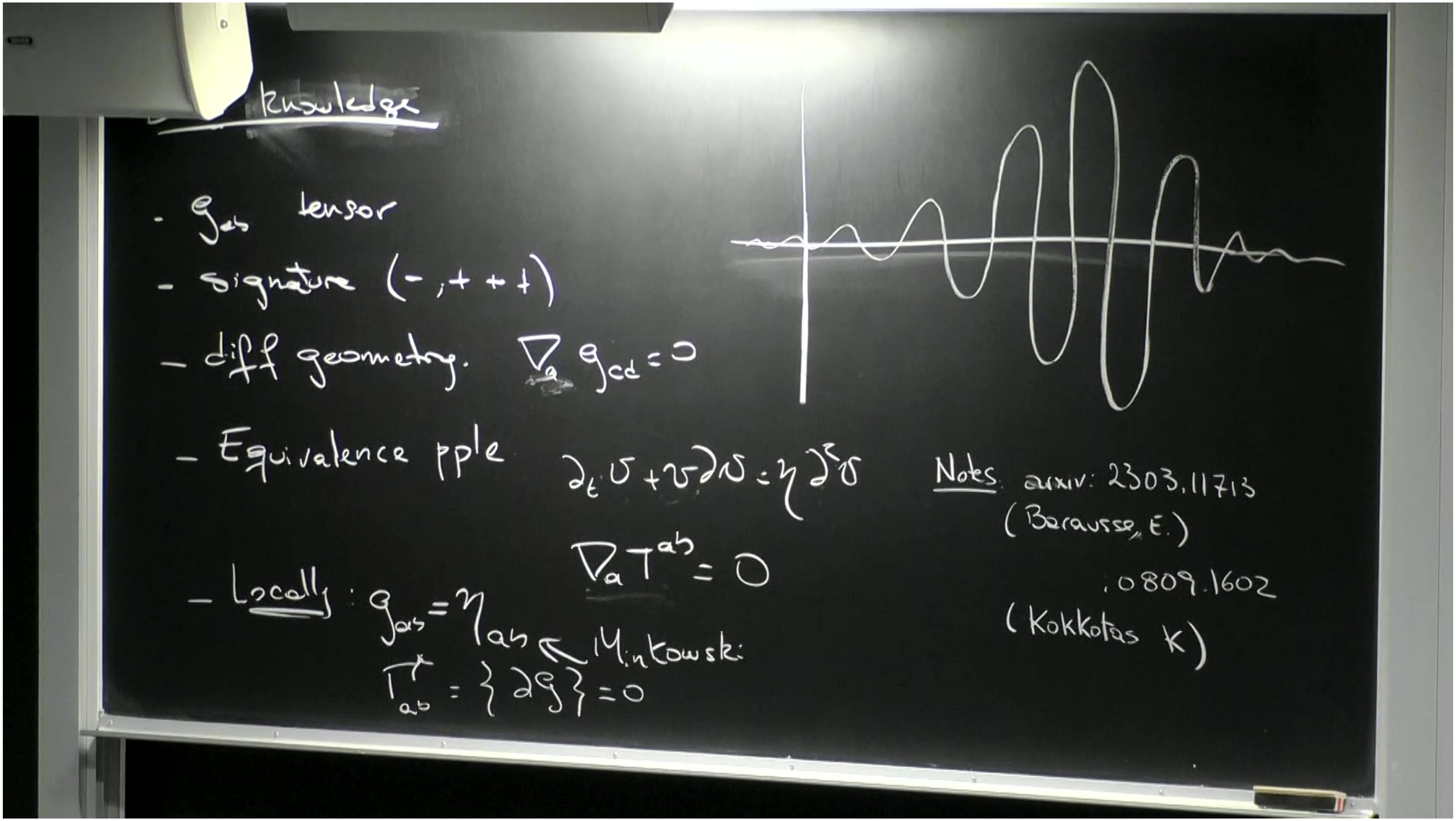
URL: https://pirsa.org/23060081


Notes. arxiv: 2303.11713
(Barausse, E.)
.0809.1602
(Kokkotas K)
CAUTION


knowledge
- g_{ab} tensor
- signature (-, + + +)
- diff geometry. $\nabla_a g_{cd} = 0$
- Equivalence pp'le
$\partial_t v + v \partial v = \eta \partial^2 v$
$\nabla_a T^{ab} = 0$
- Locally: $g_{ab} = \eta_{ab}$ ← Minkowski
$\Gamma^c_{ab} = \{\partial g\} = 0$
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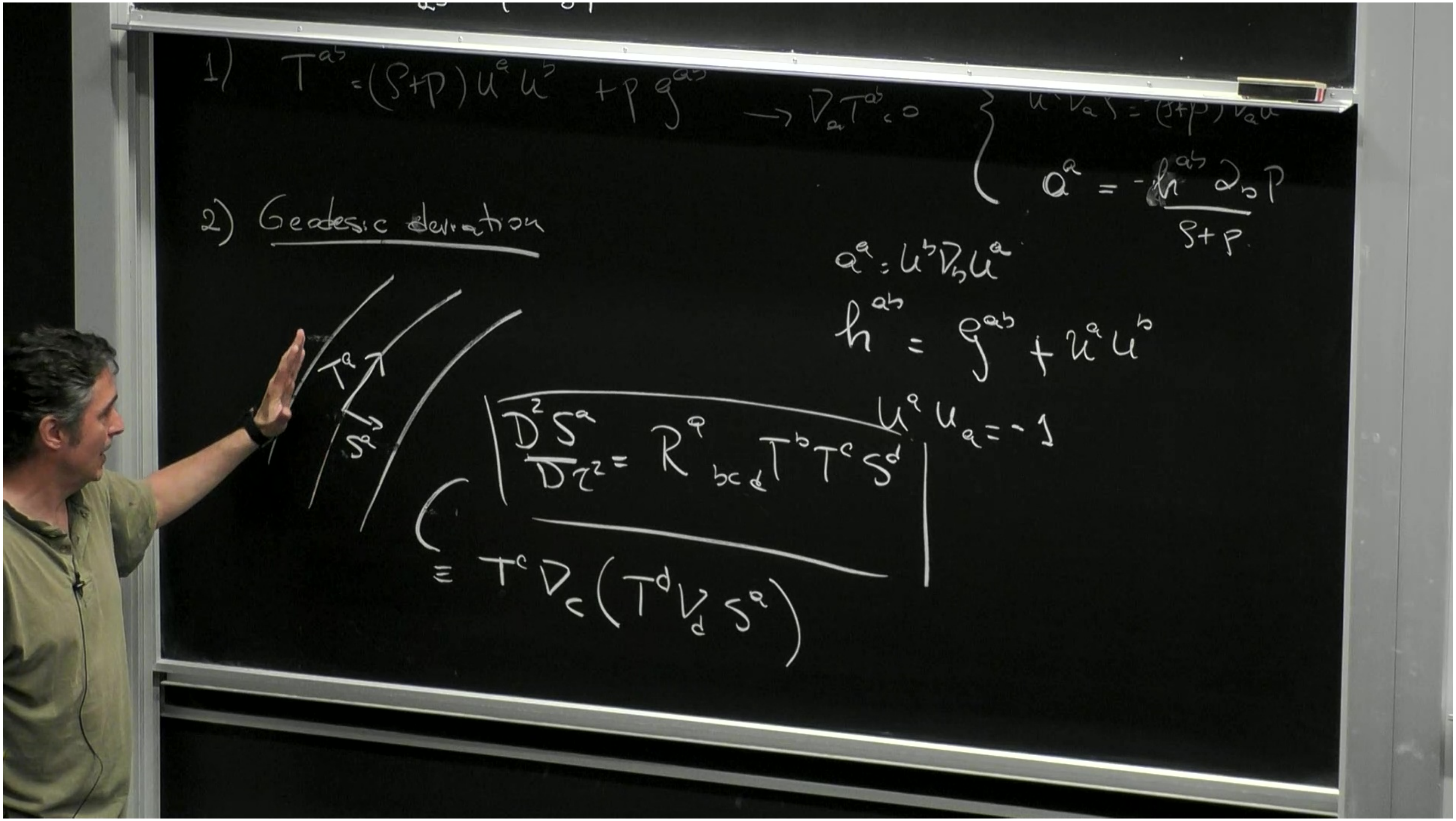
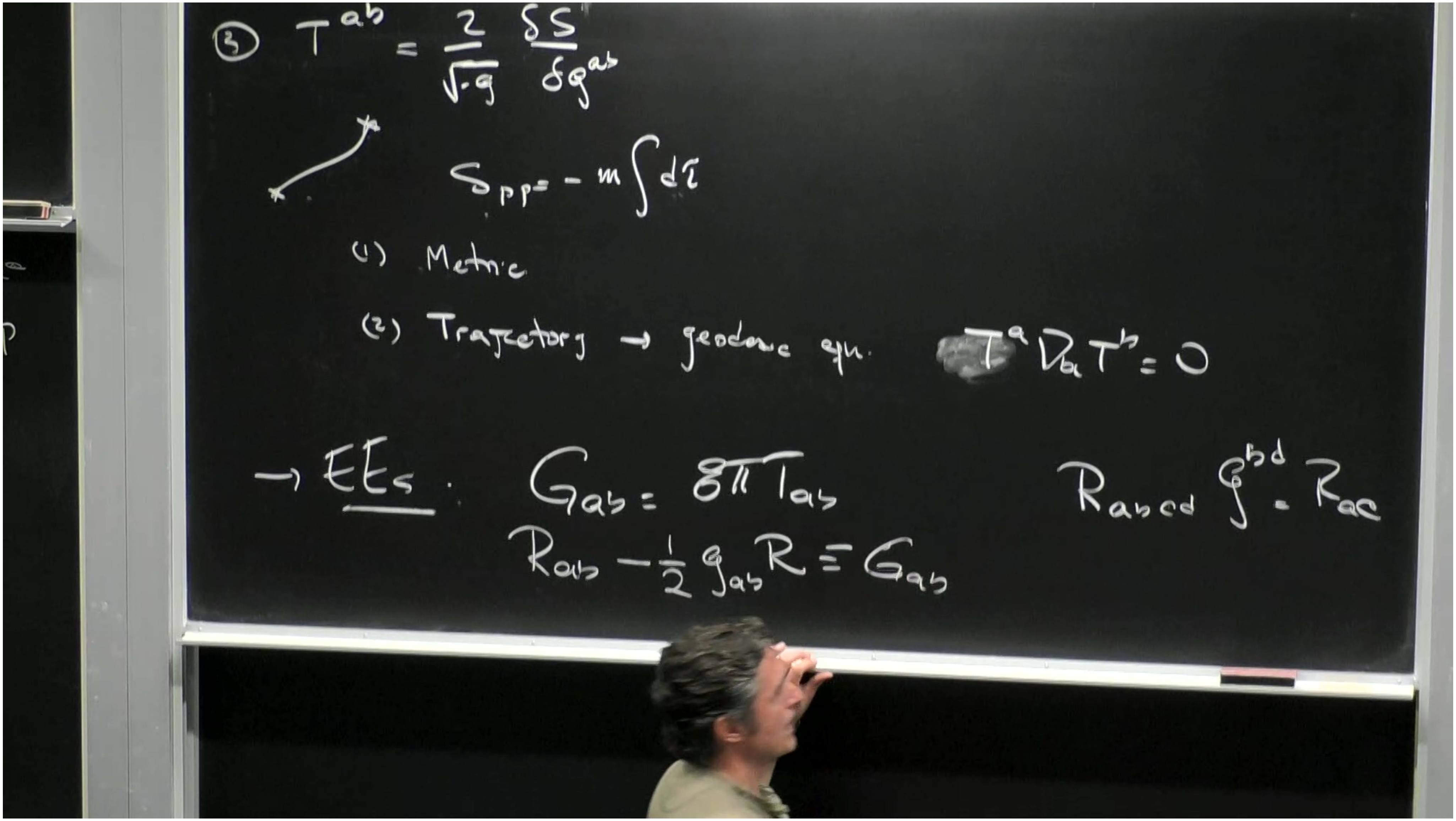

1) $T^{ab} = (\rho+p)u^a u^b + p g^{ab} \longrightarrow \nabla_a T^{ab} = 0$
$u^a \nabla_a \rho = -(\rho+p)\nabla_a u^a$
$a^a = -\frac{h^{ab}\partial_b p}{\rho+p}$
2) Geodesic deviation
$a^a = u^b \nabla_b u^a$
$h^{ab} = g^{ab} + u^a u^b$
$u^a u_a = -1$
T^a
S^a
$\frac{D^2 S^a}{D\tau^2} = R^a{}_{bcd} T^b T^c S^d$
$= T^c \nabla_c (T^d \nabla_d S^a)$

③ $T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$

$S_{pp} = -m \int d\tau$

(1) Metric

(2) Trajectory → geodesic eqn. $T^a \nabla_a T^b = 0$


③ $T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$
$S_{pp} = -m \int d\tau$
(1) Metric
(2) Trajectory → geodesic eqn. $T^a D_a T^b = 0$
→ EEs. $G_{ab} = 8\pi T_{ab}$
$R_{ab} - \frac{1}{2} g_{ab} R \equiv G_{ab}$
$R_{abcd}\, g^{bd} = R_{ac}$

$$- \; g_{ab} = \eta_{ab} + h_{ab} \quad ; \quad g^{ab} = \eta^{ab} - h^{ab}$$

$$h^{ab} = \eta^{ac}\eta^{bd} h_{cd}$$

$$h = \eta^{ab} h_{ab}$$

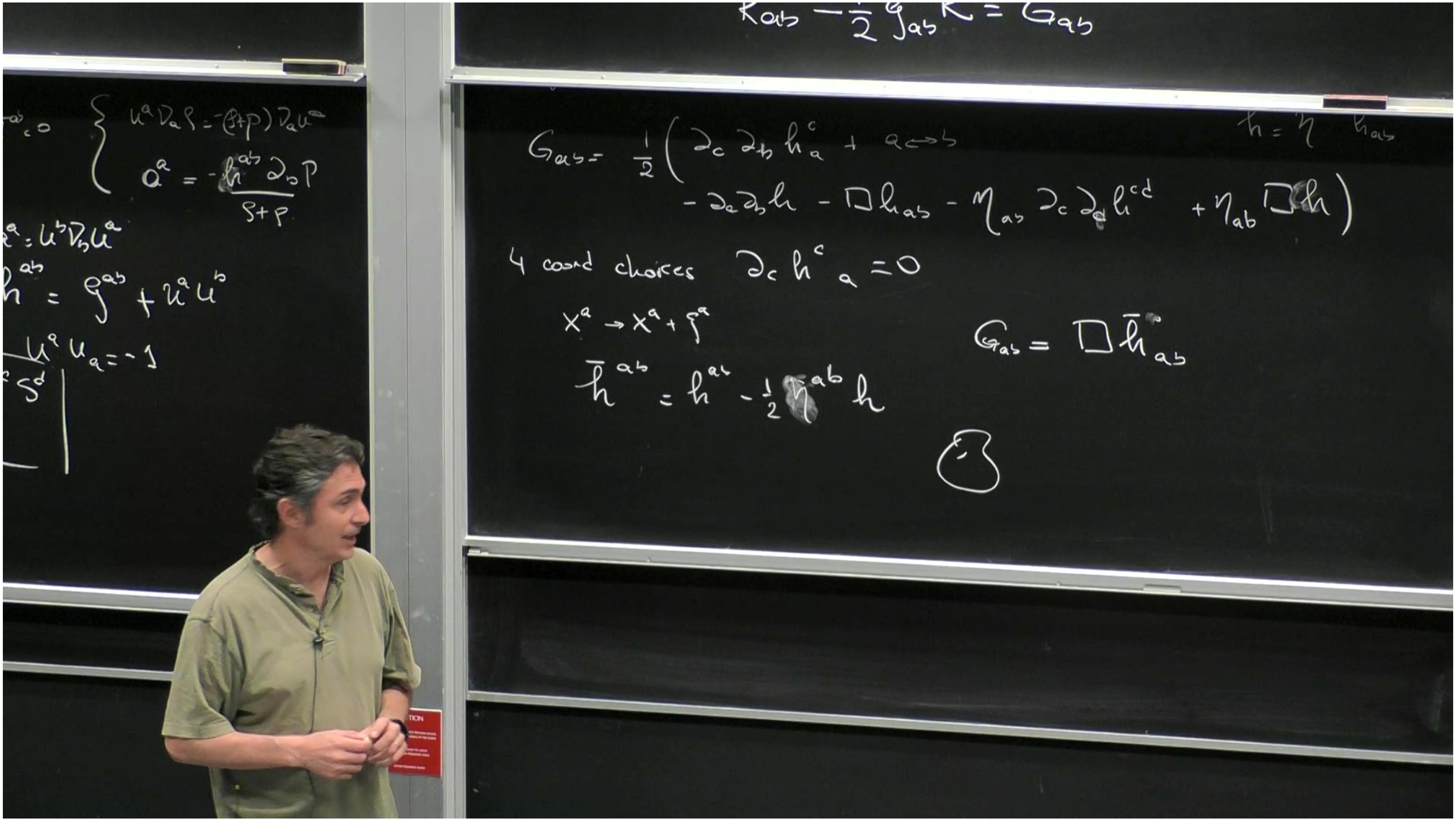
$$G_{ab} = \frac{1}{2}\Big(\partial_c \partial_b h^c{}_a + a \leftrightarrow b - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \Big)$$

4 coord choices $\partial_c h^c{}_a = 0$

$$x^a \to x^a + \xi^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2}\eta^{ab} h$$

$$G_{ab} = \Box \bar{h}_{ab}$$



$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$
$u^a \nabla_a \rho = -(\rho+p) \nabla_a u^a$
$a^a = -\frac{h^{ab} \partial_b p}{\rho + p}$
$a^a = u^b \nabla_b u^a$
$h^{ab} = g^{ab} + u^a u^b$
$u^a u_a = -1$
$G_{ab} = \frac{1}{2}\Big(\partial_c \partial_b h^c{}_a + a \leftrightarrow b - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \Big)$
$h = \eta\ h_{ab}$
4 coord choices $\partial_c h^c{}_a = 0$
$x^a \to x^a + \xi^a$
$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$
$G_{ab} = \Box \bar{h}_{ab}$

$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$h = \eta^{ab} h_{ab}$$

$$G_{ab} = \frac{1}{2}\Big(\partial_c \partial_b h^c{}_a + a \leftrightarrow b$$
$$- \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \Big)$$

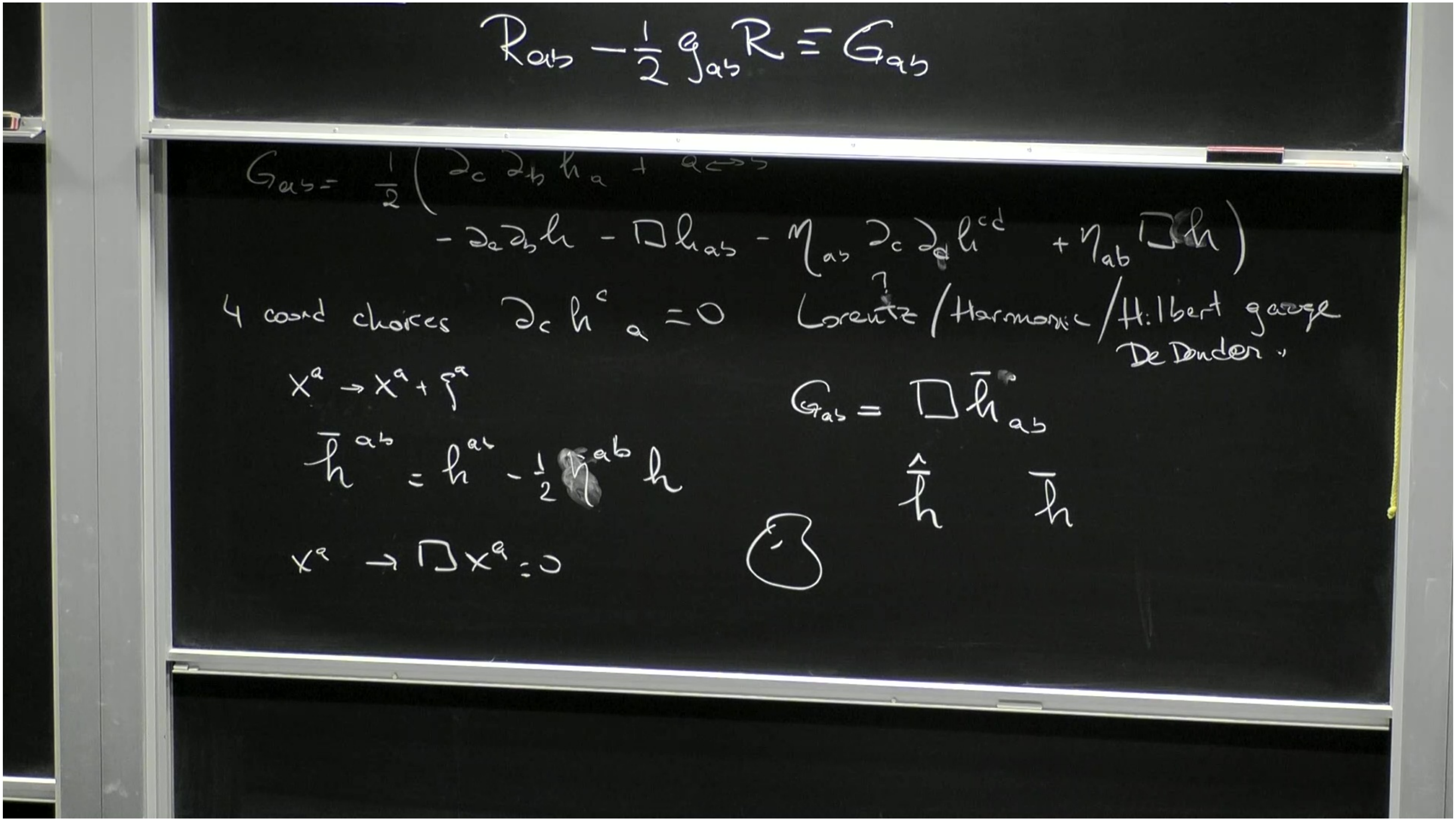
4 coord choices $\partial_c h^c{}_a = 0$ Lorentz / Harmonic / Hilbert gauge

$$x^a \to x^a + \xi^a$$

$$G_{ab} = \Box \bar{h}_{ab}$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$\hat{h} \qquad \bar{h}$$


$R_{ab} - \frac{1}{2} g_{ab} R \equiv G_{ab}$
$G_{ab} = \frac{1}{2} \big(\partial_c \partial_b h_a + a \leftrightarrow b$
$- \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \big)$
4 coord choices $\partial_c h^c{}_a = 0$
Lorentz / Harmonic / Hilbert gauge
De Donder
$x^a \to x^a + \xi^a$
$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$
$x^a \to \Box x^a = 0$
$G_{ab} = \Box \bar{h}_{ab}$
$\hat{h}$ $\bar{h}$

$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2}\Big(\partial_c \partial_b h^c{}_a + a \leftrightarrow b$$

$$- \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \Big)$$

4 coord choices $\partial_c h^c{}_a = 0$ Lorentz / Harmonic / Hilbert gauge

De Donder

$x^a \to x^a + \xi^a$

$G_{ab} = \Box \bar{h}_{ab}$

$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$

$\hat{\bar{h}}$ $\bar{h}$

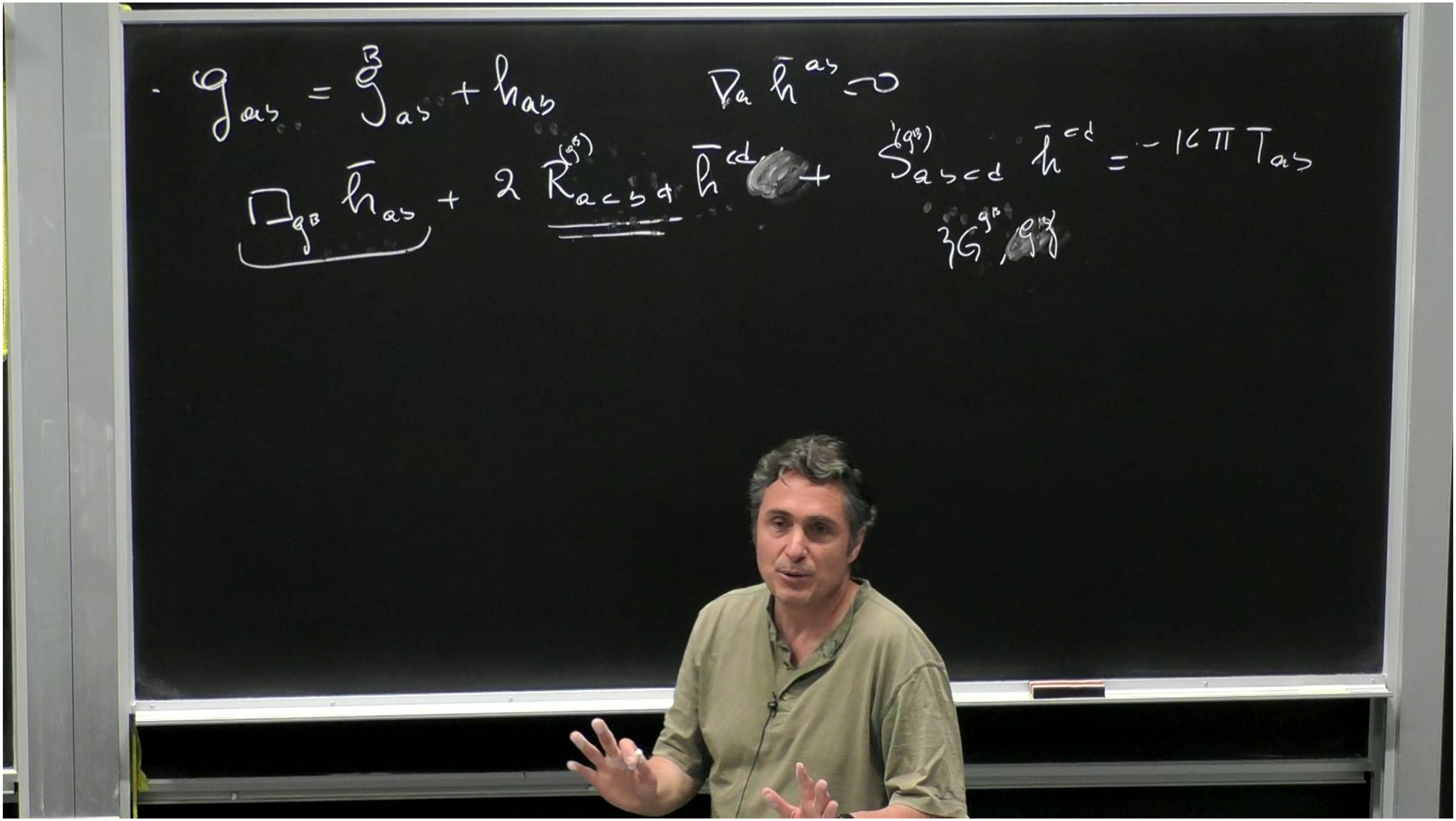
$x^a \to \Box x^a = 0$

$\Box (x^a + y^a) = 0$

$\Box \xi^a$

$$g_{ab} = \overset{B}{g}_{ab} + h_{ab} \qquad \nabla_a \bar{h}^{ab} = 0$$

$$\underline{\Box_{g^B} \bar{h}_{ab}} + 2\, \underline{\underline{\overset{(g^B)}{R}_{acbd}}}\, \bar{h}^{cd} + \overset{(g^B)}{S}_{abcd}\, \bar{h}^{cd} = -16\pi T_{ab}$$


$g_{ab} = \overset{B}{g}_{ab} + h_{ab}$
$\nabla_a \bar{h}^{ab} = 0$
$\Box_{g^B} \bar{h}_{ab} + 2 R^{(g^B)}_{acbd} \bar{h}^{cd} + S^{(g^B)}_{abcd} \bar{h}^{cd} = -16\pi T_{ab}$

$$g_{ab} = \overset{B}{g}_{ab} + h_{ab} \qquad \nabla_a \bar{h}^{ab} = 0$$

$$\underbrace{\Box_{g^B} \bar{h}_{ab}} + 2 \underline{\underline{R^{(g^B)}_{acbd}}}\, \bar{h}^{cd} + S^{(g^B)}_{abcd} \bar{h}^{cd} = -16\pi T_{ab}$$

$$\exists G^{g^B}, g^B$$

$$z$$

$$\underline{\partial_a \bar{h}^{ab}} = 0$$

$$k^a = \left(\frac{\partial}{\partial t}\right)^a + \left(\frac{\partial}{\partial z}\right)^a$$

$$k_a = -dt_a + dz_a$$

$$\Box \bar{h}_{ab} \to \bar{h}_{ab} = A_{ab}\, e^{i k_c x^c}$$

$$\hookrightarrow k^a A_{ab} = 0 \qquad k_a A^{ab} = 0$$

$$k^a k_a = 0 \qquad A^{ta} = A^{za}$$

$$g_{ab} = \mathring{g}_{ab} + h_{ab}$$

$$\underbrace{\Box_{g_B} \bar{h}_{ab}} + 2 \underline{R^{(g_B)}_{acbd}} \, \bar{h}^{cd} + S^{(g_B)}_{abcd} \, \bar{h}^{c}$$

$$3 G^{g_B}$$

$$\partial_a \bar{h}^{ab} = 0$$

z

$$k^a = \left(\frac{\partial}{\partial t}\right)^a + \left(\frac{\partial}{\partial z}\right)^a$$

$$k_a = -dt_a + dz_a$$

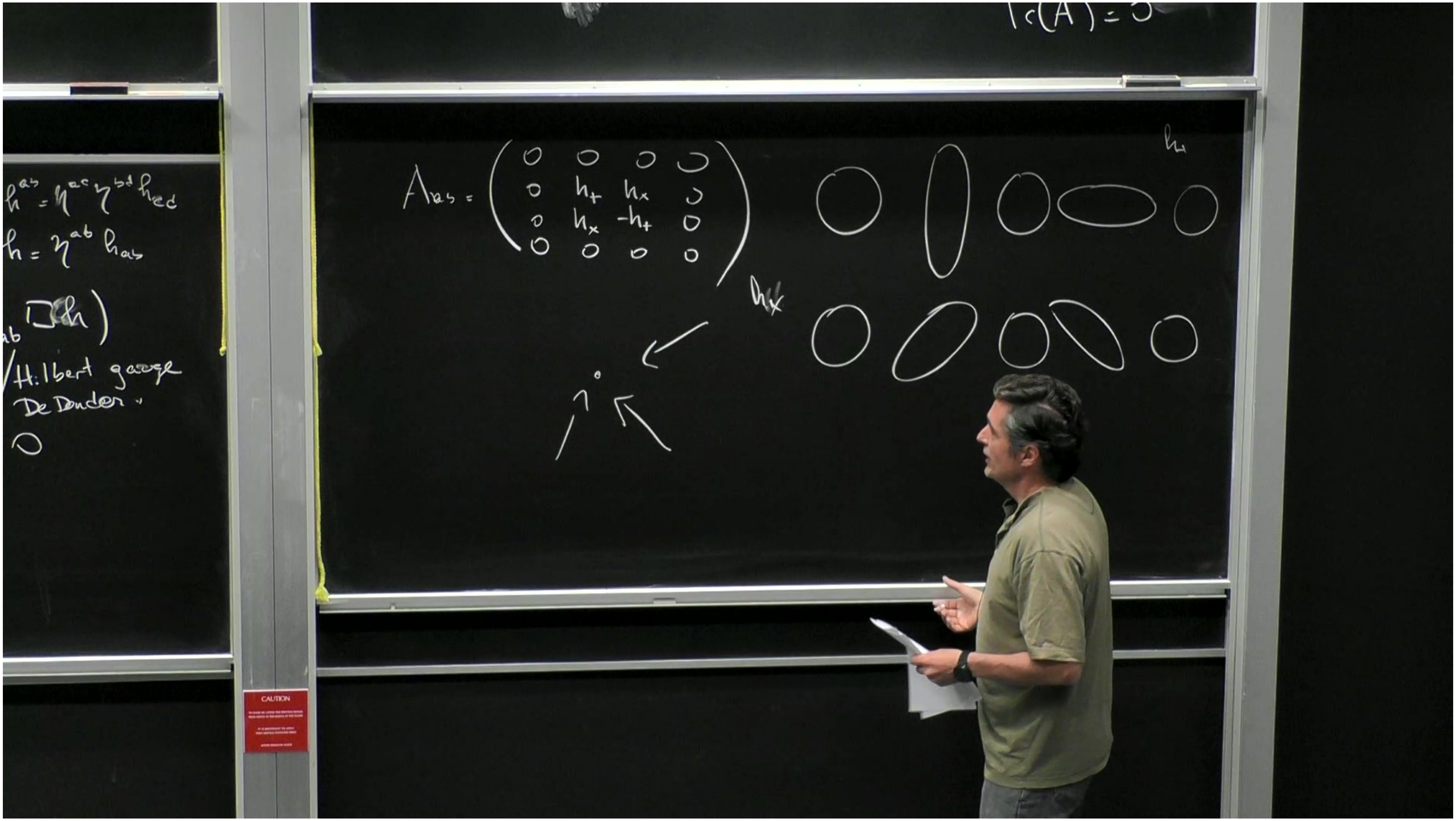
$$\Box \bar{h}_{ab} \to \bar{h}_{ab} = A_{ab} \, e^{i k_c x^c}$$

$$\hookrightarrow k^a A_{ab} = 0 \qquad k_a A^{ab} = 0$$

$$k^a k_a = 0 \qquad A^{ta} = A^{za}$$

$$A^{ti} = 0$$

$$\mathrm{Tr}(A) = 0$$


$h^{ab} = \eta^{ac}\eta^{bd} h_{cd}$
$h = \eta^{ab} h_{ab}$
Hilbert gauge
De Donder
$A_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
h_+
$h_\times$

$$g_{ab} = \eta_{ab} + h^{TT}_{ab}$$

$$\frac{d^2 x^a}{d\tau^2} + \Gamma^a_{\ bc}\frac{dx^b}{d\tau}\frac{dx^c}{d\tau} = 0$$

$$\frac{dx}{d\tau} \to \frac{dx}{dt}$$

$$x^a = (t, x^i)$$

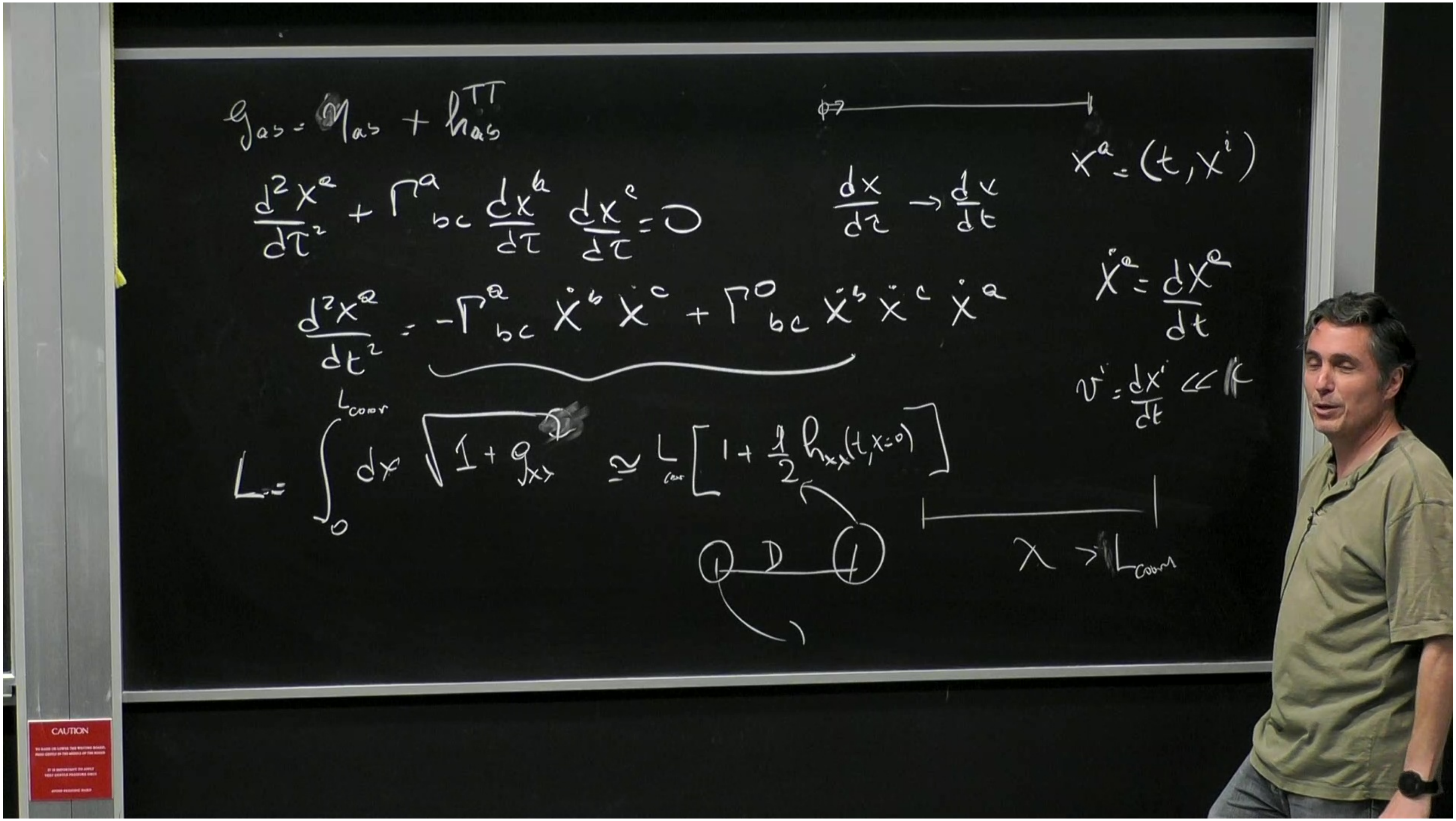
$$\frac{d^2 x^a}{dt^2} = \underbrace{-\Gamma^a_{\ bc}\dot{x}^b \dot{x}^c + \Gamma^0_{\ bc}\dot{x}^b \dot{x}^c \dot{x}^a}$$

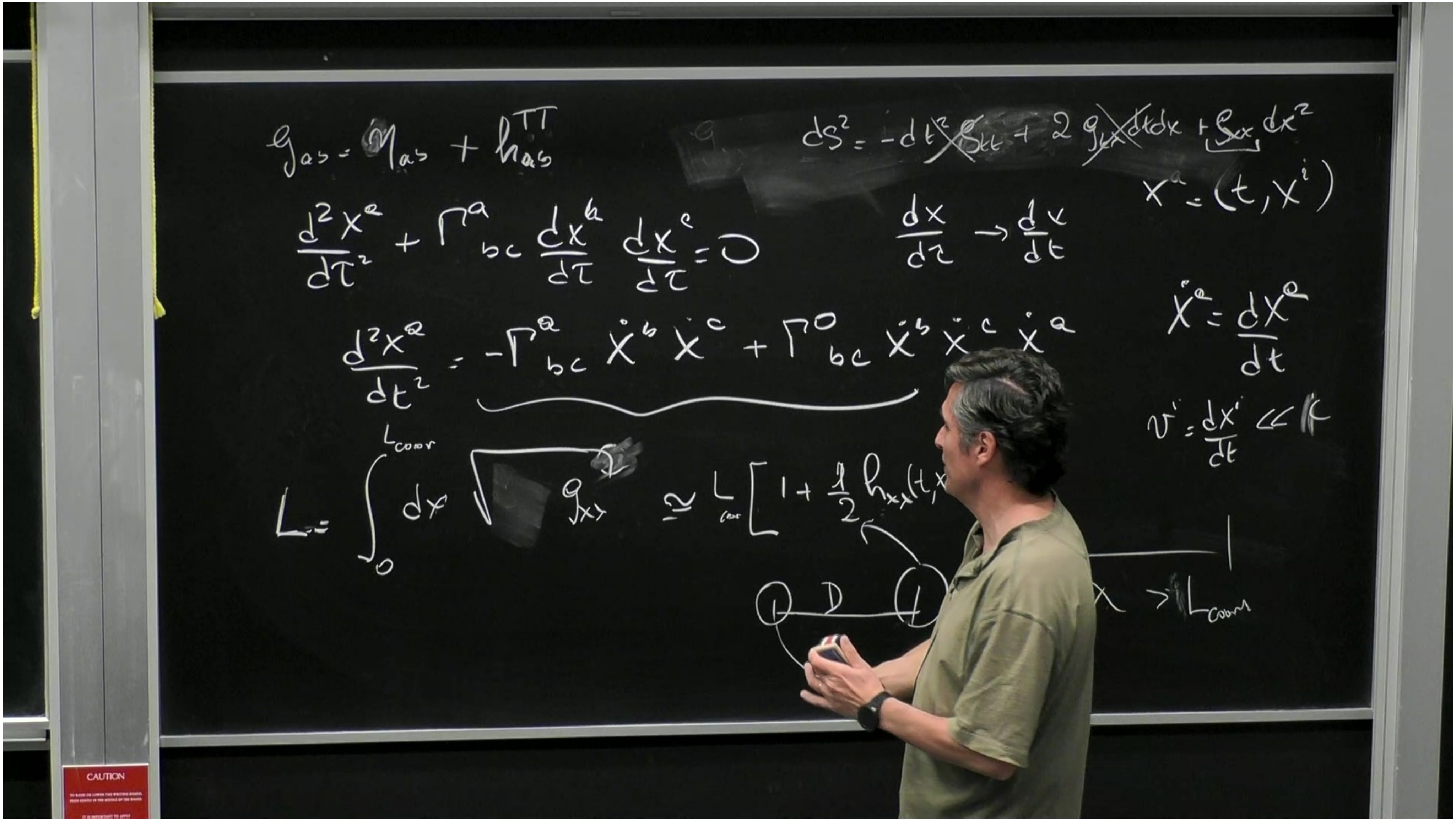
$$\dot{x}^a = \frac{dx^a}{dt}$$

$$v^i = \frac{dx^i}{dt} \ll c$$

$$L = \int_0^{L_{coor}} dx \sqrt{1 + g_{xx}} \simeq L_{cor}\left[1 + \frac{1}{2} h_{xx}(t, x=0)\right]$$

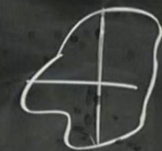
$$x \to L_{coor}$$


$g_{ab} = \eta_{ab} + h^{TT}_{ab}$
$\frac{d^2 x^a}{d\tau^2} + \Gamma^a_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$
$\frac{dx}{d\tau} \to \frac{dx}{dt}$
$x^a = (t, x^i)$
$\frac{d^2 x^a}{dt^2} = -\Gamma^a_{bc} \dot{x}^b \dot{x}^c + \Gamma^0_{bc} \dot{x}^b \dot{x}^c \dot{x}^a$
$\dot{x}^a = \frac{dx^a}{dt}$
$v^i = \frac{dx^i}{dt} \ll c$
$L = \int_0^{L_{coor}} dx \sqrt{1 + g_{xx}} \simeq L_{coor}\left[1 + \frac{1}{2} h_{xx}(t, x=0)\right]$
$x \to L_{coom}$
CAUTION


$g_{ab} = \eta_{ab} + h^{TT}_{ab}$
$ds^2 = -dt^2 g_{tt} + 2 g_{tx} dt dx + g_{xx} dx^2$
$x^a = (t, x^i)$
$\frac{d^2 x^a}{d\tau^2} + \Gamma^a{}_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$
$\frac{dx}{d\tau} \rightarrow \frac{dx}{dt}$
$\dot{x}^a = \frac{dx^a}{dt}$
$\frac{d^2 x^a}{dt^2} = -\Gamma^a{}_{bc} \dot{x}^b \dot{x}^c + \Gamma^0{}_{bc} \dot{x}^b \dot{x}^c \dot{x}^a$
$v^i = \frac{dx^i}{dt} \ll c$
$L = \int_0^{L_{coor}} dx \sqrt{g_{xx}} \simeq L_{coor}\left[1 + \frac{1}{2} h_{xx}(t, x\right.$
$x \rightarrow L_{coor}$

$$\Box \bar{h}_{ab} = -16\pi T_{ab}$$

$$\Box G = \delta(t)\,\delta^3(\vec{x}) \to h_{ab} = 4\int \frac{T_{ab}(t - |\vec{x}-\vec{x}'|, \vec{x}')}{|\vec{x}-\vec{x}'|}\, d^3x'$$



$(t, \vec{x})$

$$h_{ab} = \frac{4}{r}\int T_{ab}(t-r, x')$$

(1) Assume $|\vec{x}| \gg |x'|$

(2) Slow dynamics. $\left(\frac{v}{c}\right) \ll 1$

$$x^i x^j \partial_t^2 T_{ij} = (\partial_t T)$$

$$\partial_a T^{ab} = 0 \to \partial_t T^{tb} = -\partial_i T^{ib}$$

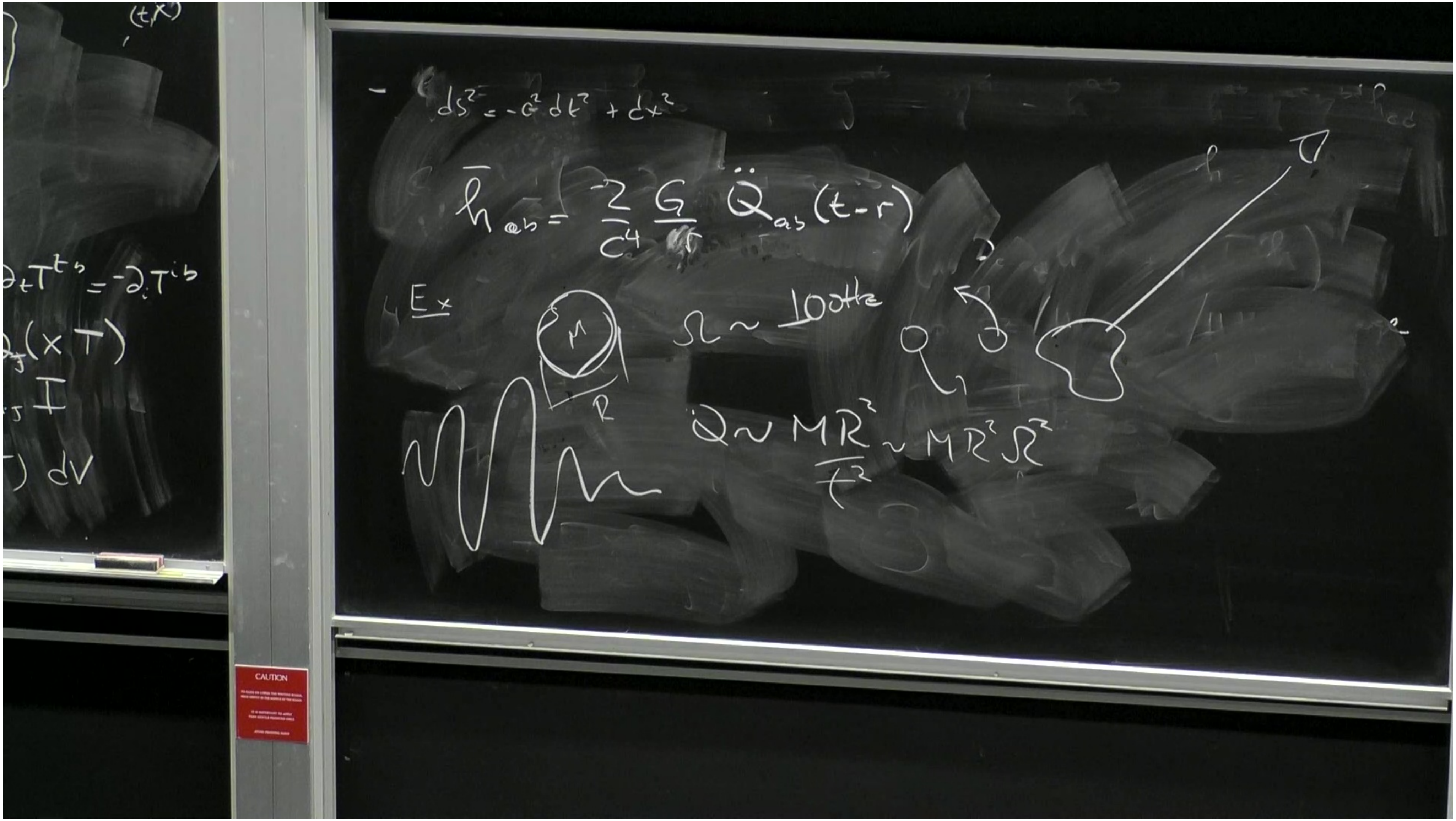
$$\int d^3x\, T^{ab} = \frac{1}{2}\int \left(\partial_t^2 T^{00} x^a x^b\right)$$

$$\partial_j (x T)$$

$$\sim \ddot{Q}$$

$$Q_{ij} \equiv I_{ij} - \frac{1}{3}\delta_{ij} I$$

$$I^{ij} = \int (\rho x^i x^j)\, dV$$


$\partial_t T^{tb} = -\partial_i T^{ib}$
$ds^2 = -c^2 dt^2 + dx^2$
$\bar{h}_{ab} = \frac{2}{c^4}\frac{G}{r}\ddot{Q}_{ab}(t-r)$
Ex
M
R
$\Omega \sim 100\,\mathrm{Hz}$
$\ddot{Q} \sim \frac{MR^2}{T^2} \sim MR^2\Omega^2$