Title: Gravitational Wave Theory

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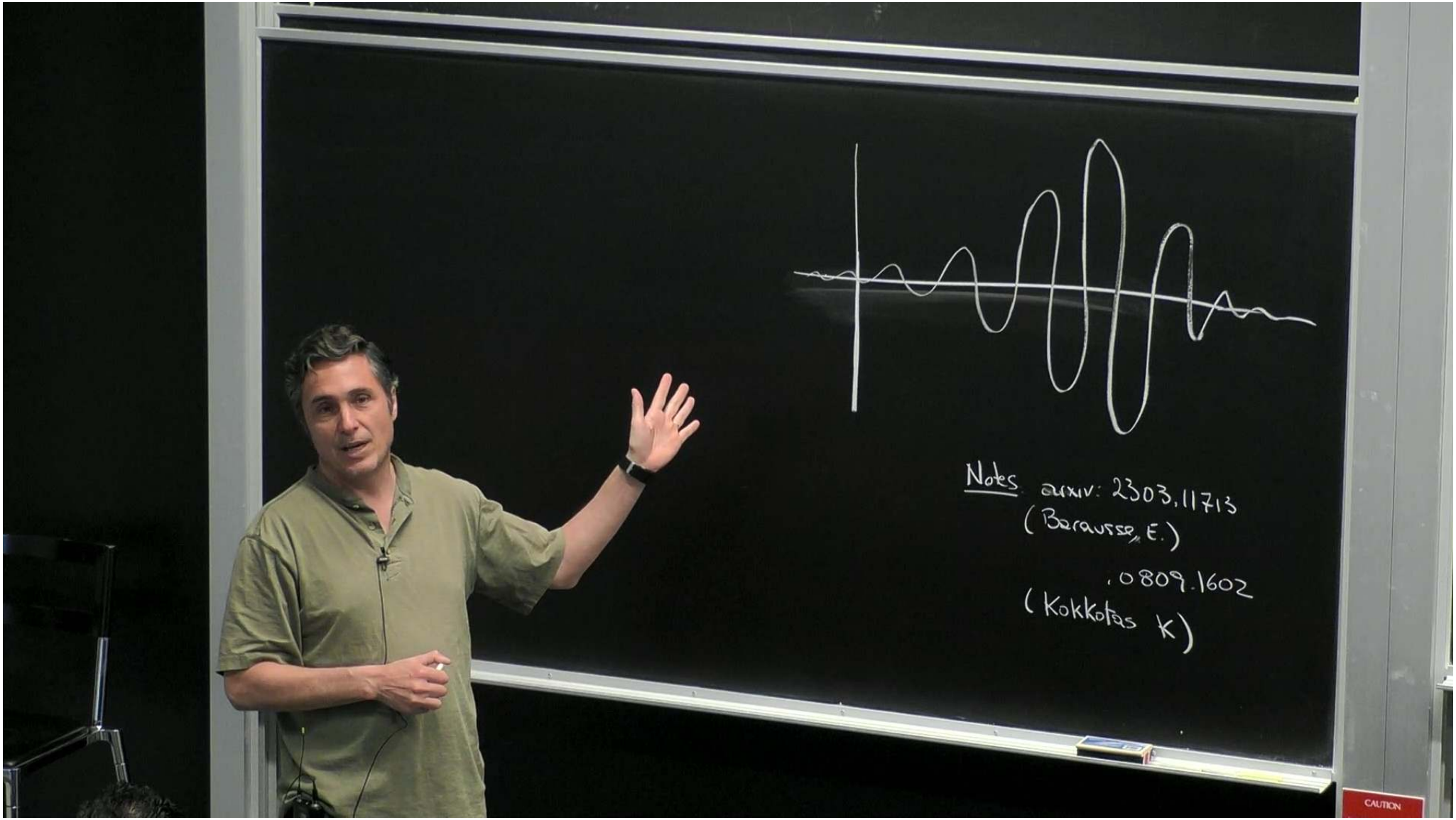
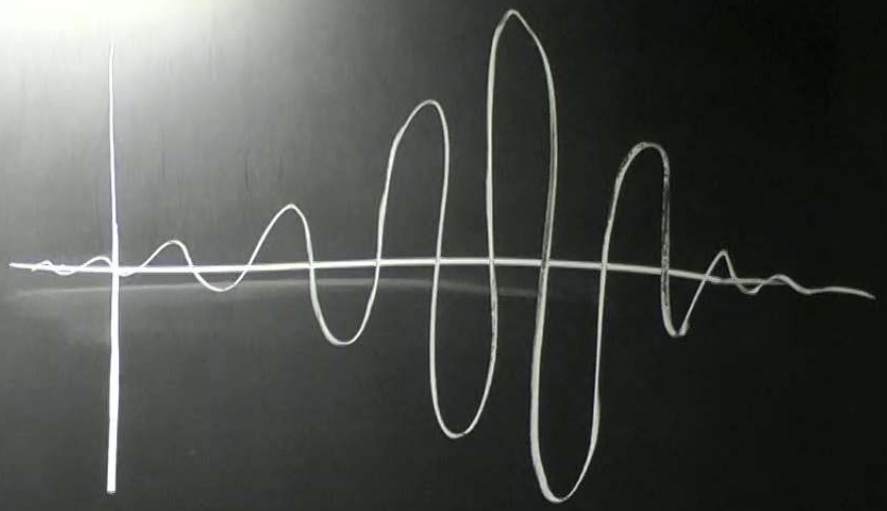
URL: https://pirsa.org/23060081


Notes arxiv: 2303.11713
(Barausse, E.)
0809.1602
(Kokkotas K)
CAUTION

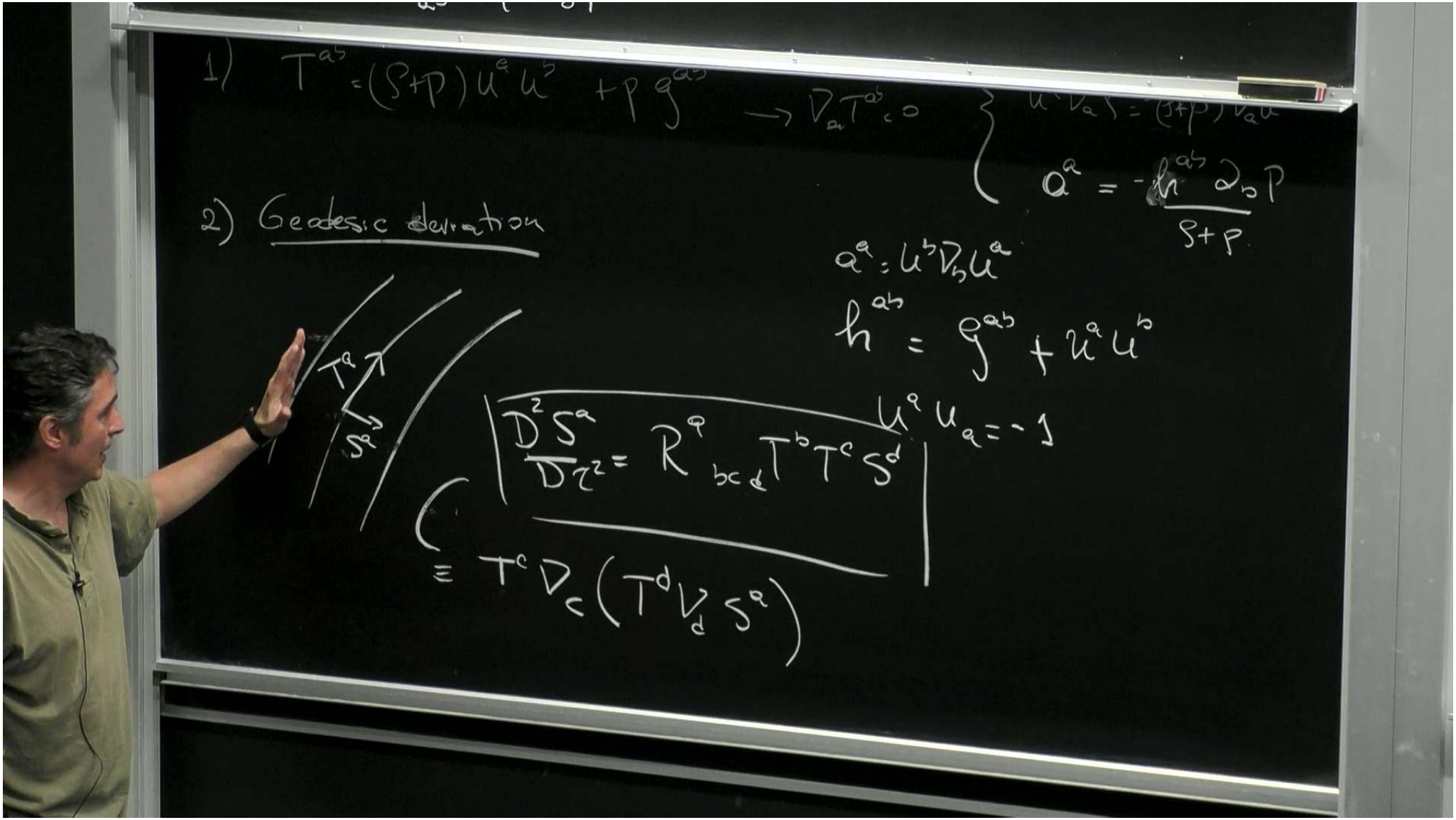
Knowledge

- g_{ab} tensor
- signature $(-,+++)$
- diff geometry. $\nabla_a g_{cd} = 0$
- Equivalence ppl $\partial_t v + v\partial v = \eta \partial^2 v$
- Locally: $g_{ab} = \eta_{ab}$ ⇐ Minkowski: $\Gamma^c_{ab} = \{ag\} = 0$



$\nabla_a T^{ab} = 0$

Notes: arxiv: 2303.11713 (Berausse, E.)
; 0809.1602 (Kokkotas K)

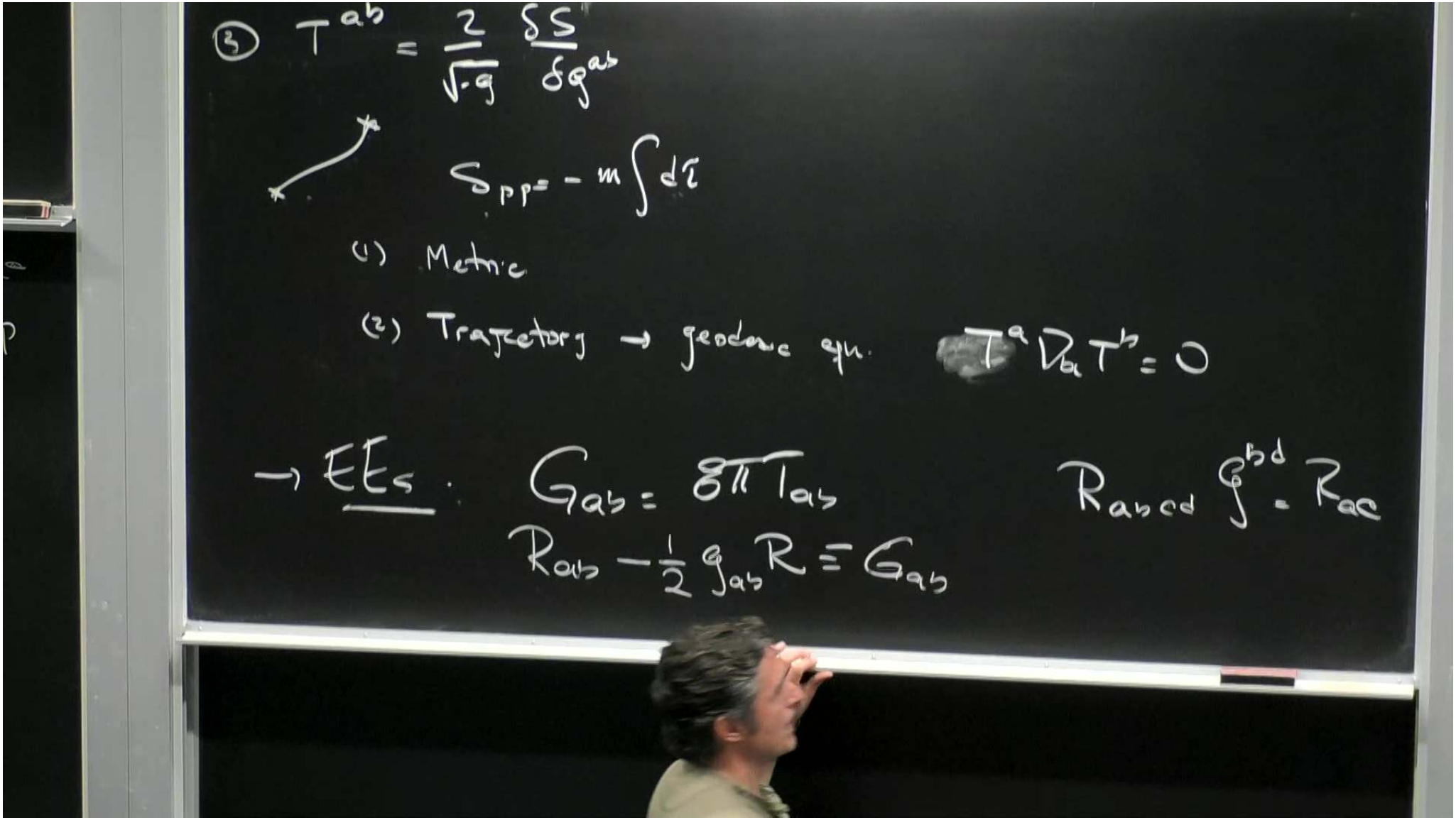

1) $T^{ab} = (\rho + p) u^a u^b + p g^{ab} \rightarrow \nabla_a T^{ab} = 0$
$u^a \nabla_a \rho = -(\rho + p) \nabla_a u^a$
$a^a = -\frac{h^{ab} \nabla_b p}{\rho + p}$
2) Geodesic deviation
$a^a = u^b \nabla_b u^a$
$h^{ab} = g^{ab} + u^a u^b$
$u^a u_a = -1$
T^a
S^a
$\frac{D^2 S^a}{D\tau^2} = R^a{}_{bcd} T^b T^c S^d$
$\equiv T^c \nabla_c (T^d \nabla_d S^a)$

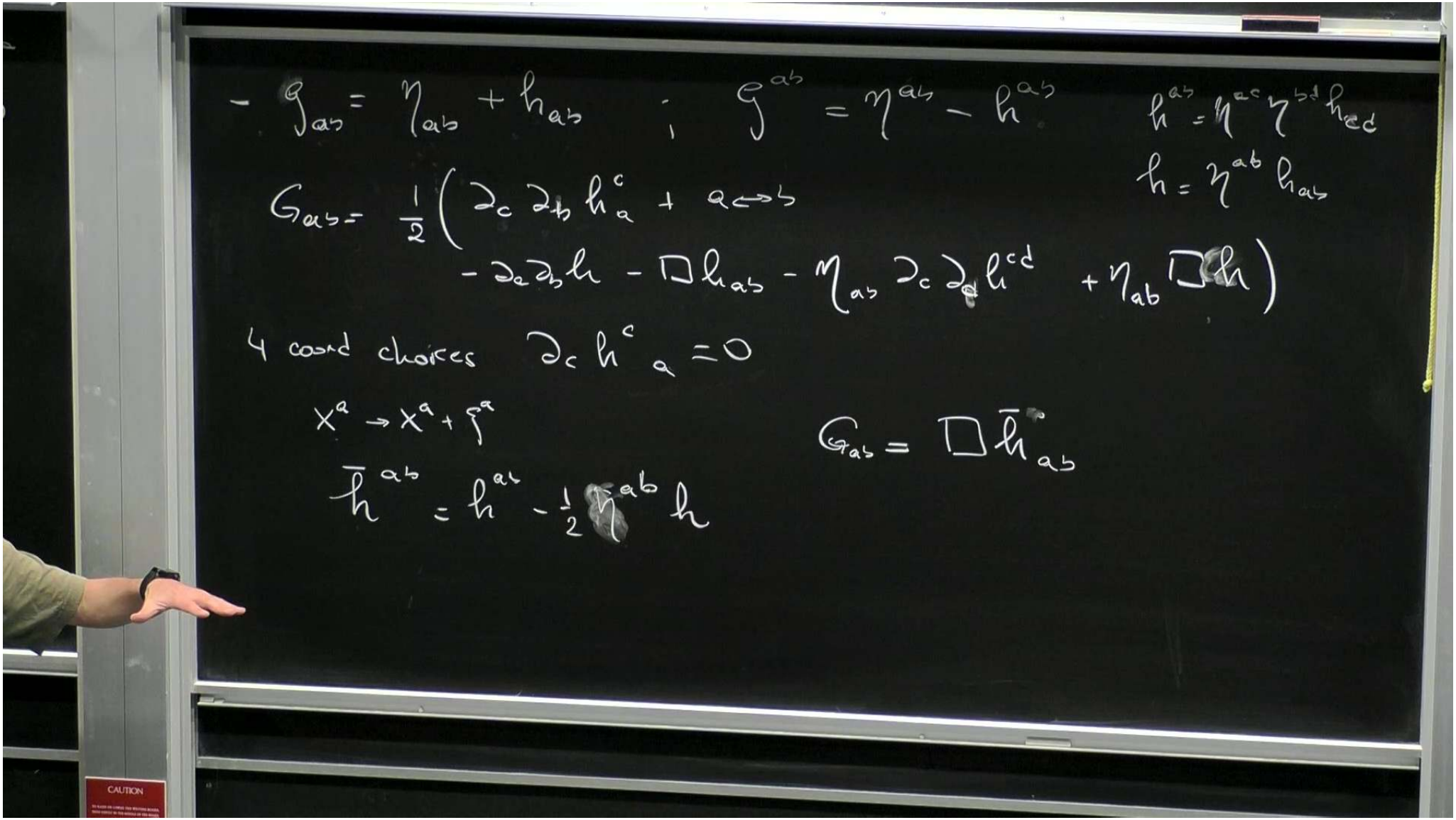
③ $T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$

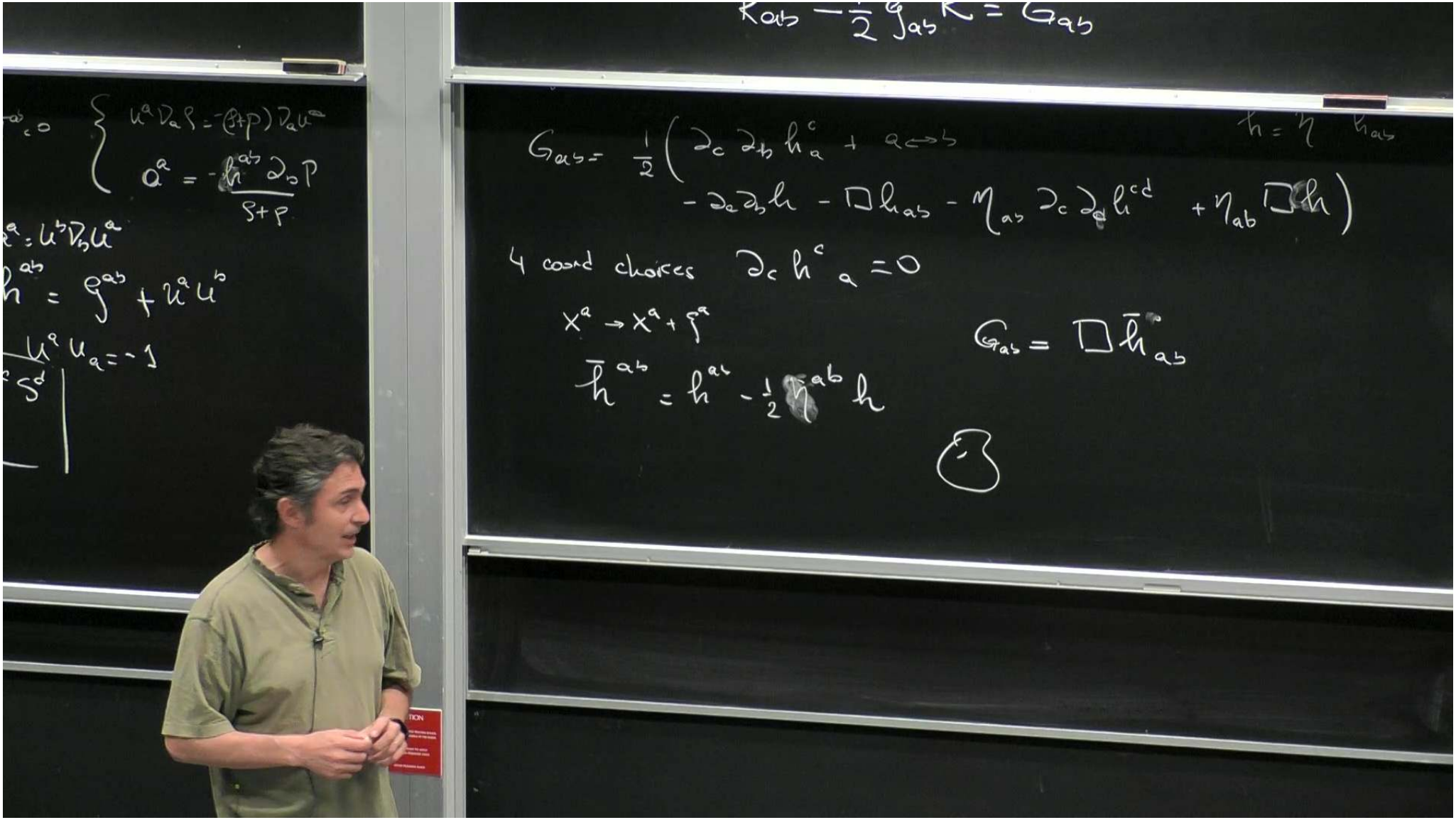
$S_{pp} = -m \int d\tau$

(1) Metric

(2) Trajectory → geodesic eq. $T^a D_a T^b = 0$


③ $T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ab}}$
$S_{pp} = -m \int d\tau$
(1) Metric
(2) Trajectory → geodesic eqn. $T^a \nabla_a T^b = 0$
→ EEs. $G_{ab} = 8\pi T_{ab}$
$R_{abcd} g^{bd} = R_{ac}$
$R_{ab} - \frac{1}{2} g_{ab} R \equiv G_{ab}$


$g_{ab} = \eta_{ab} + h_{ab}$; $g^{ab} = \eta^{ab} - h^{ab}$
$h^{ab} = \eta^{ac}\eta^{bd} h_{cd}$
$h = \eta^{ab} h_{ab}$
$G_{ab} = \frac{1}{2}\left(\partial_c \partial_b h^c_a + a \leftrightarrow b \right.$
$\left. - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \right)$
4 coord choices $\partial_c h^c{}_a = 0$
$x^a \to x^a + \xi^a$
$G_{ab} = \Box \bar{h}_{ab}$
$\bar{h}^{ab} = h^{ab} - \frac{1}{2}\eta^{ab} h$
CAUTION


$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$
$\begin{cases} u^a \nabla_a \rho = -(\rho + p) \nabla_a u^a \\ a^a = -\frac{h^{ab} \partial_b p}{\rho + p} \end{cases}$
$h = \eta \quad h_{ab}$
$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h^c_a + a \leftrightarrow b \right.$
$\left. - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \right)$
4 coord choices $\partial_c h^c_{\ a} = 0$
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$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$
$G_{ab} = \Box \bar{h}_{ab}$

$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2}\left(\partial_c \partial_b h^c_a + a \leftrightarrow b \right.$$

$$\left. - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \right)$$

$$h = \eta^{ab} h_{ab}$$

4 coord choices $\partial_c h^c{}_a = 0$ Lorentz / Harmonic / Hilbert gauge

$x^a \to x^a + \xi^a$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$G_{ab} = \Box \bar{h}_{ab}$$

$\hat{\bar{h}}$ $\bar{h}$

$$R_{ab} - \frac{1}{2} g_{ab} R \equiv G_{ab}$$

$$G_{ab} = \frac{1}{2}\Big(\partial_c \partial_b h_a + a \leftrightarrow b$$

$$- \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \Big)$$

4 coord choices $\partial_c h^c{}_a = 0$ Lorentz / Harmonic / Hilbert gauge

De Donder ?

$$x^a \to x^a + \xi^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$x^a \to \Box x^a = 0$$

$$G_{ab} = \Box \bar{h}_{ab}$$

$\hat{\bar{h}}$ $\bar{h}$

$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2}\left(\partial_c \partial_b h_a + \text{a} \leftrightarrow \text{b} \right.$$

$$\left. - \partial_a \partial_b h - \Box h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \Box h \right)$$

4 coord choices $\partial_c h^c{}_a = 0$ Lorentz / Harmonic / Hilbert gauge
De Donder

$x^a \to x^a + \xi^a$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$x^a \to \Box x^a = 0$$

$$\Box(x^a + \xi^a) = 0$$

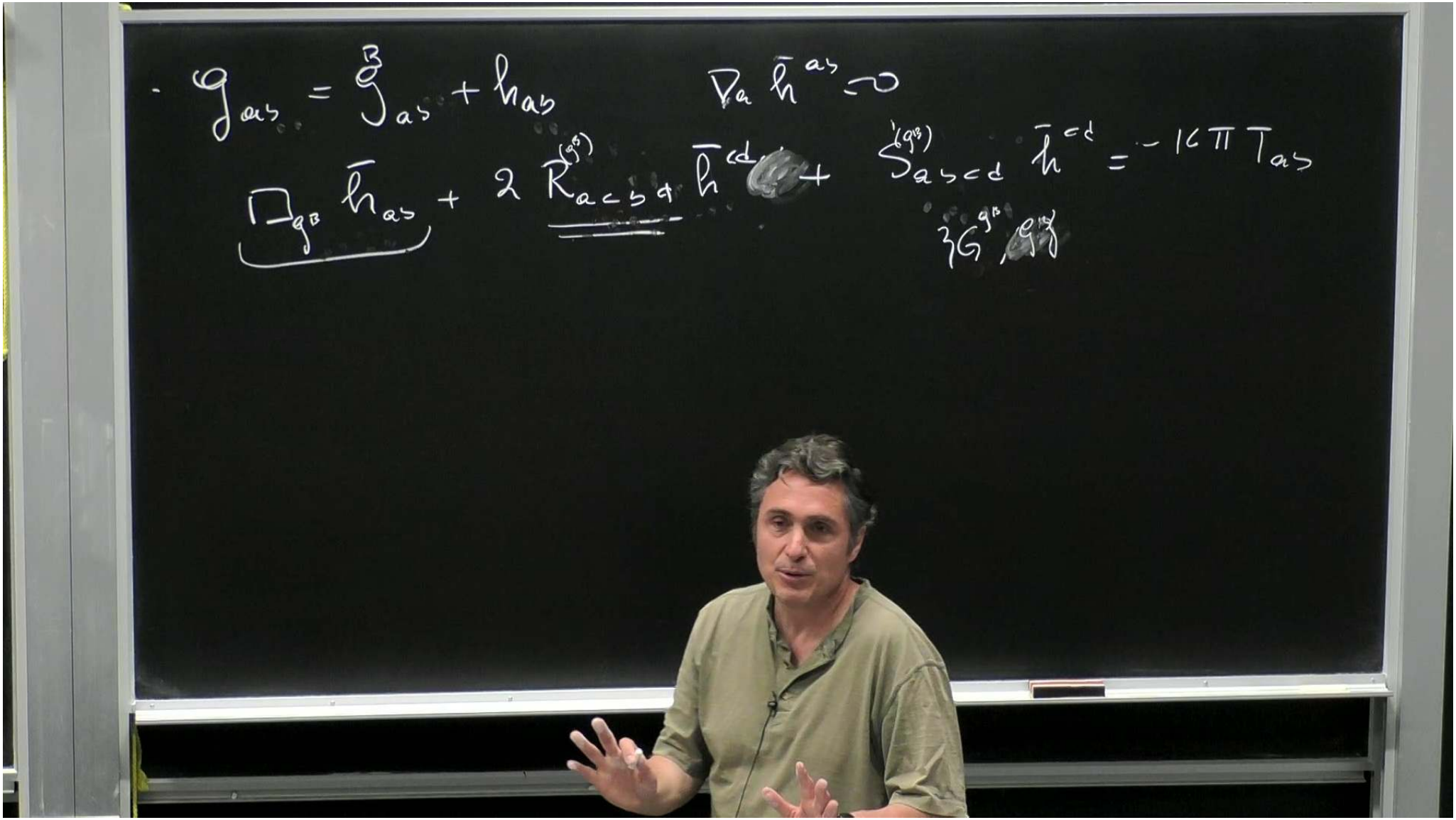
$$G_{ab} = \Box \bar{h}_{ab}$$

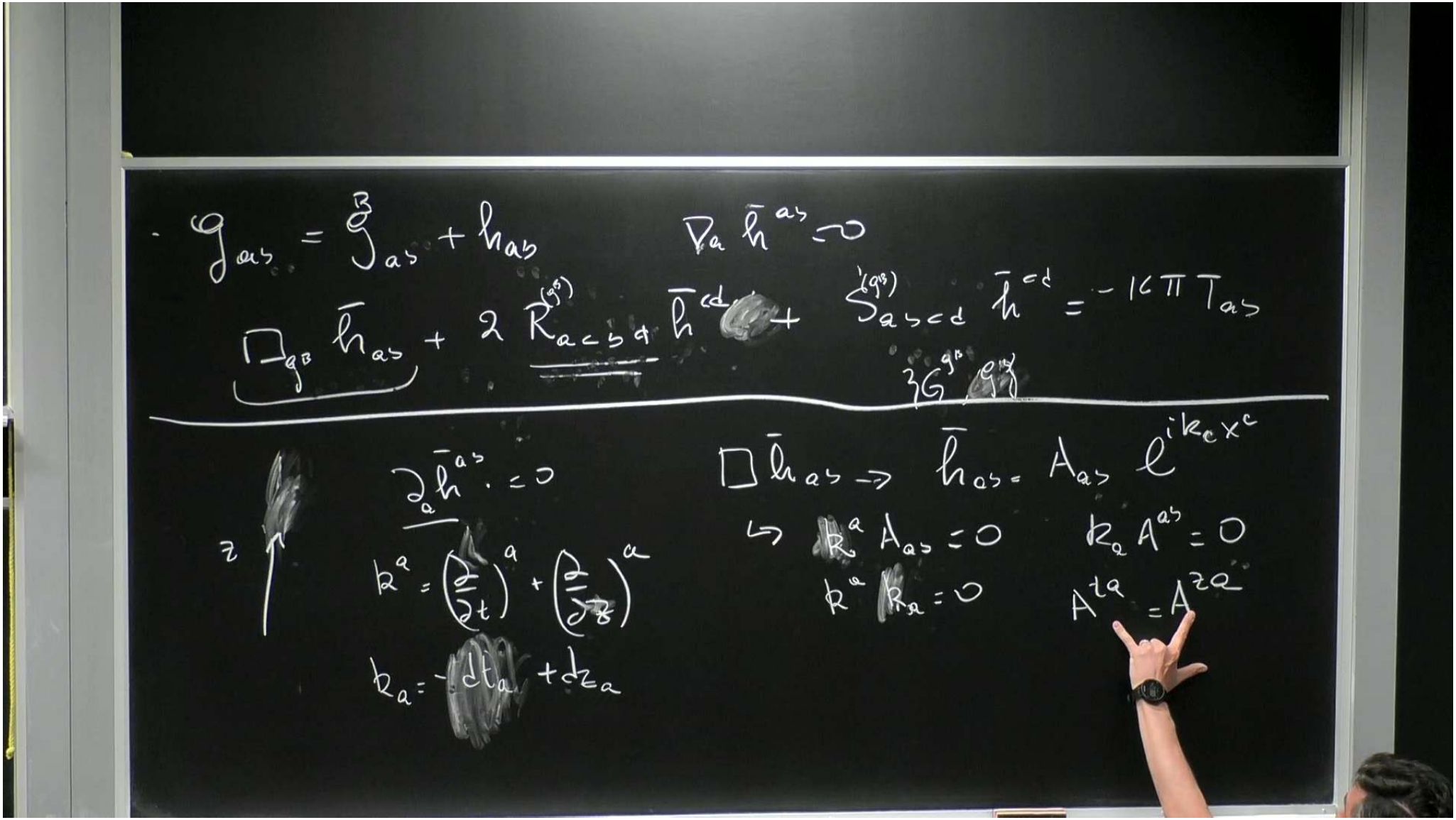
$\hat{\bar{h}}$ $\bar{h}$

$\Box \xi^a$

$$g_{ab} = \bar{g}_{ab} + h_{ab} \qquad \nabla_a \bar{h}^{ab} = 0$$

$$\underline{\Box_{\bar{g}} \bar{h}_{ab}} + 2 \bar{R}^{(\bar{g})}_{acbd} \bar{h}^{cd} + S^{(\bar{g})}_{abcd} \bar{h}^{cd} = -16\pi T_{ab}$$


$g_{ab} = \bar{g}_{ab} + h_{ab}$
$\nabla_a \bar{h}^{ab} = 0$
$\Box_{\bar{g}} \bar{h}_{ab} + 2 \bar{R}^{(\bar{g})}_{acbd} \bar{h}^{cd} + \bar{S}^{(\bar{g})}_{ab cd} \bar{h}^{cd} = -16\pi T_{ab}$
$\frac{1}{2} G^{\bar{g}} \bar{g}$


$g_{ab} = \bar{g}_{ab} + h_{ab}$
$\nabla_a \bar{h}^{ab} = 0$
$\Box_{\bar{g}} \bar{h}_{ab} + 2 R^{(\bar{g})}_{acbd} \bar{h}^{cd} + S^{(\bar{g})}_{ab cd} \bar{h}^{cd} = -16\pi T_{ab}$
$\partial_a \bar{h}^{ab} = 0$
$\Box \bar{h}_{ab} \Rightarrow \bar{h}_{ab} = A_{ab} e^{ik_c x^c}$
$k^a A_{ab} = 0$
$k_a A^{ab} = 0$
$k^a k_a = 0$
$k_a = -dt_a + dz_a$

$g_{ab} = \eta_{ab} + h_{ab}$

$\Box_{\eta} \bar{h}_{ab} + 2 R^{(\eta)}_{acbd} \bar{h}^{cd} + S^{(\eta\eta)}_{abcd} \bar{h}^{cd}$

$\partial_a \bar{h}^{ab} = 0$

$\Box \bar{h}_{ab} \to \bar{h}_{ab} = A_{ab} e^{i k_c x^c}$

$\hookrightarrow k^a A_{ab} = 0$

$k^a k_a = 0$

$k_a A^{ab} = 0$

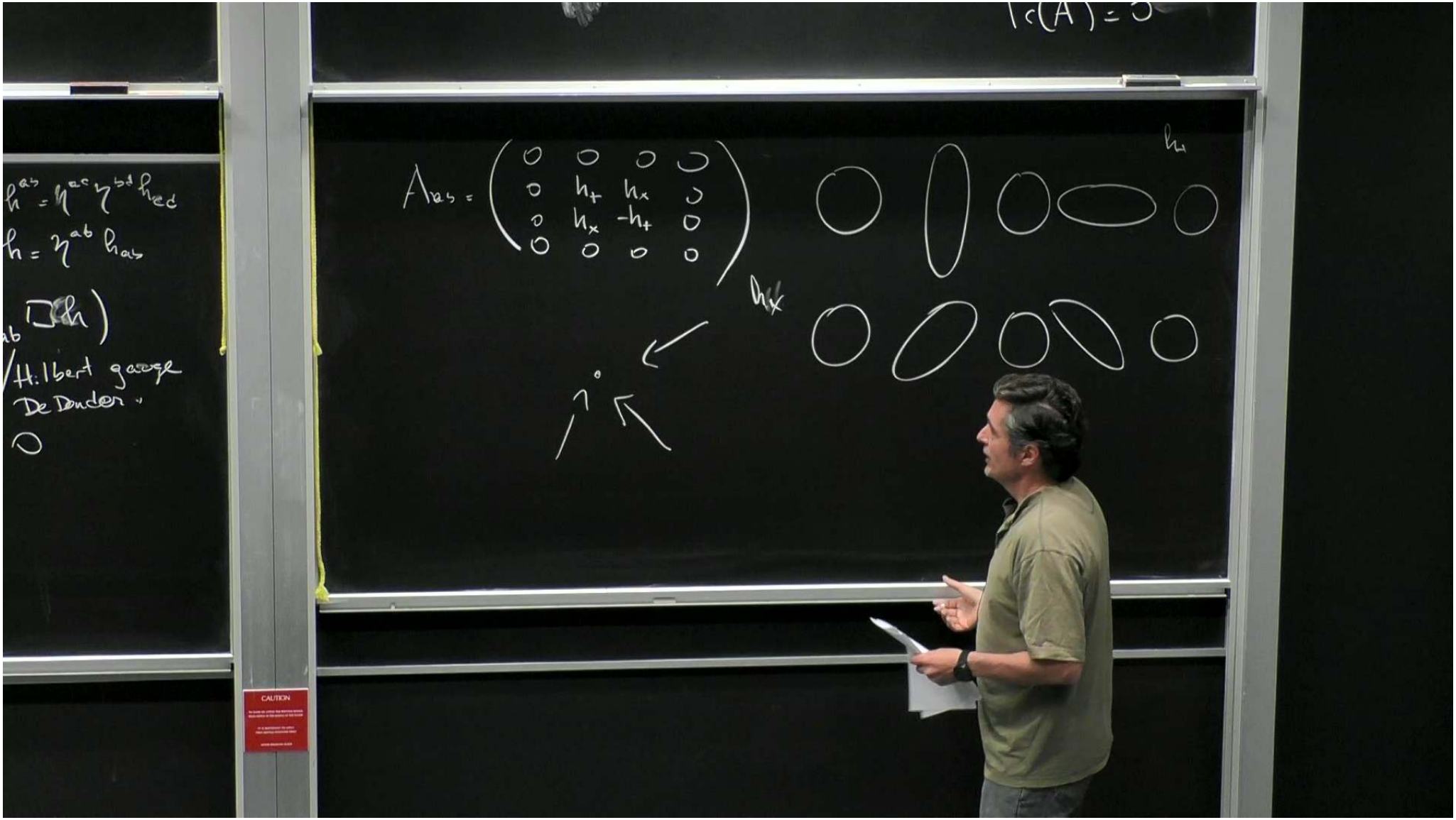
$A^{tb} = A^{zb}$

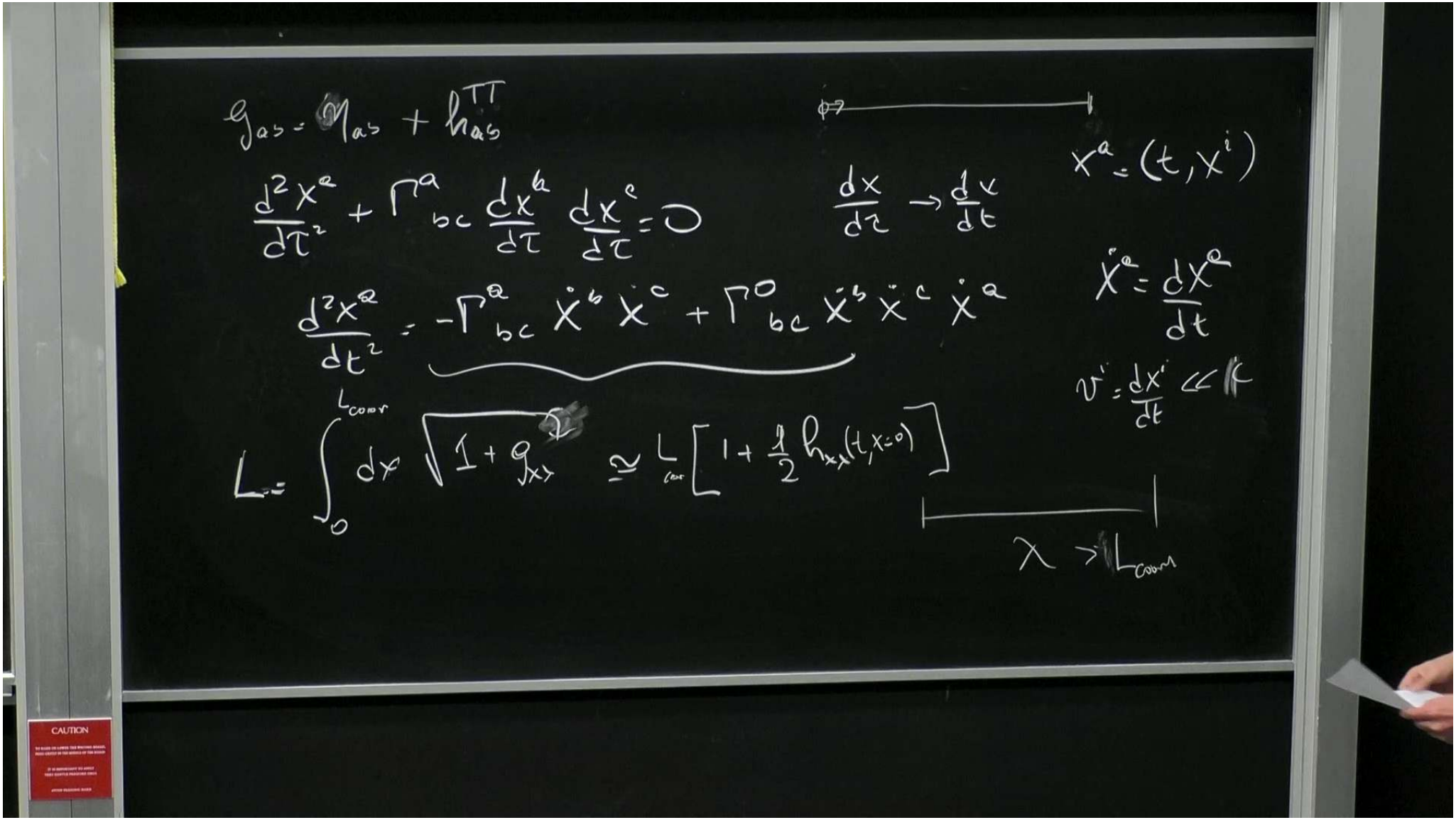
$A^{ti} = 0$

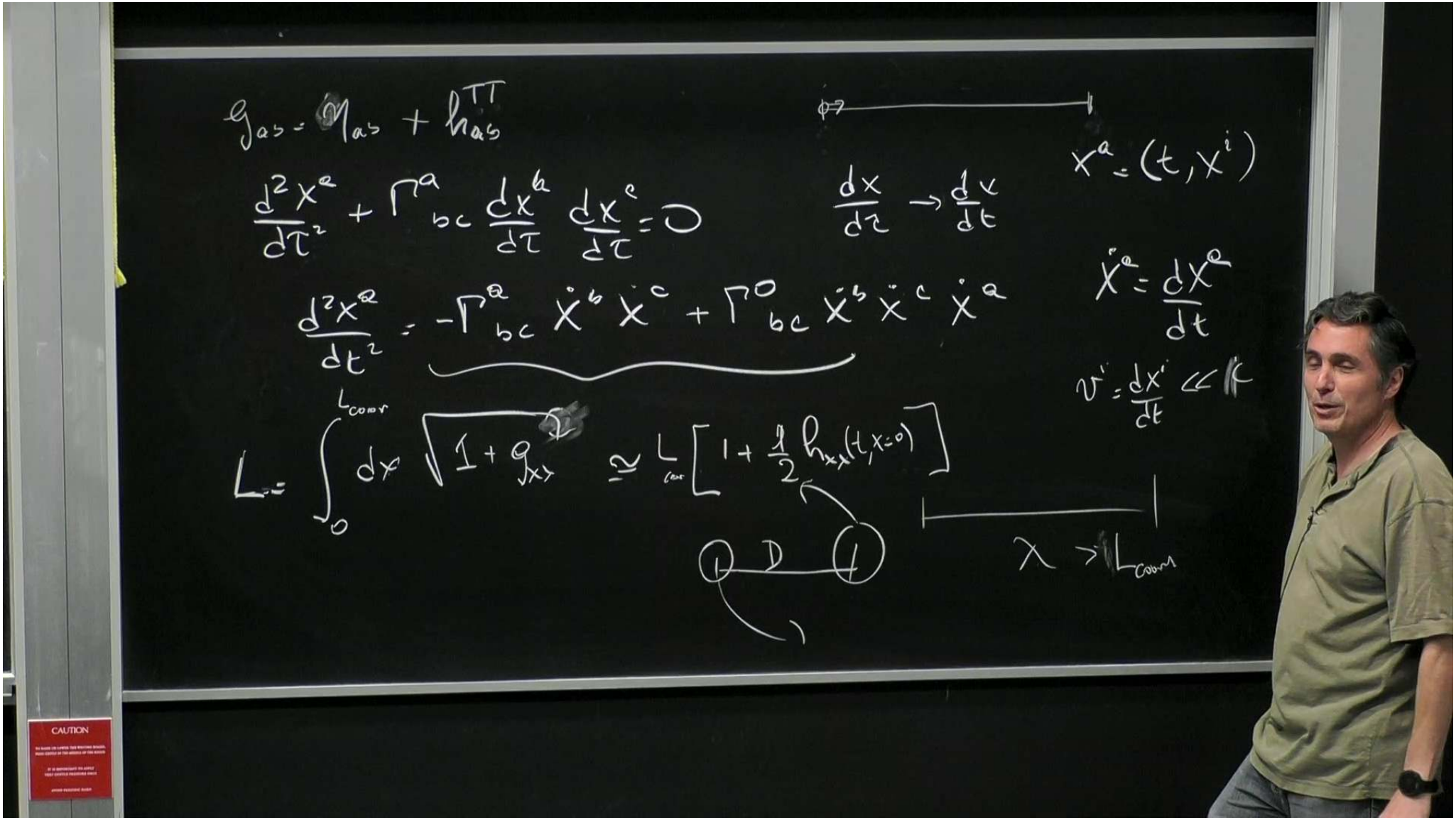
$Tr(A) = 0$

$k^a = \left(\frac{\partial}{\partial t}\right)^a + \left(\frac{\partial}{\partial z}\right)^a$

$k_a = -dt_a + dz_a$


Hilbert gauge
De Donder
CAUTION


$g_{ab} = \eta_{ab} + h^{TT}_{ab}$
$\frac{d^2x^a}{d\tau^2} + \Gamma^a_{bc}\frac{dx^b}{d\tau}\frac{dx^c}{d\tau} = 0$
$\frac{dx}{d\tau} \to \frac{dx}{dt}$
$x^a = (t, x^i)$
$\frac{d^2x^a}{dt^2} = -\Gamma^a_{bc}\dot{x}^b\dot{x}^c + \Gamma^0_{bc}\dot{x}^b\dot{x}^c\dot{x}^a$
$\dot{x}^a = \frac{dx^a}{dt}$
$v^i = \frac{dx^i}{dt} \ll 1$
$L = \int_0^{L_{coord}} dx \sqrt{1 + g_{xx}} \simeq L_{coord}\left[1 + \frac{1}{2}h_{xx}(t, x=0)\right]$
$\lambda > L_{coord}$
CAUTION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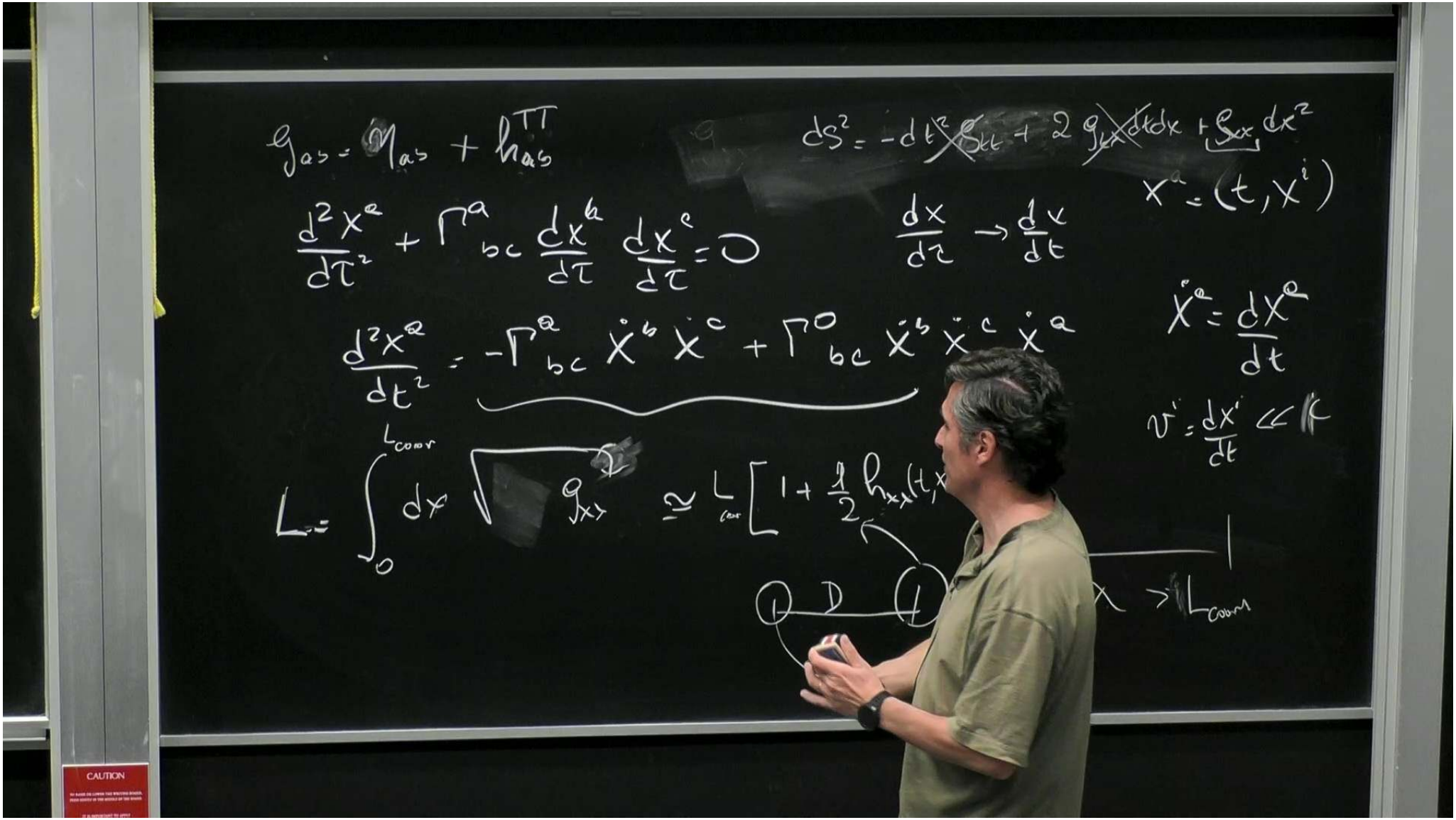
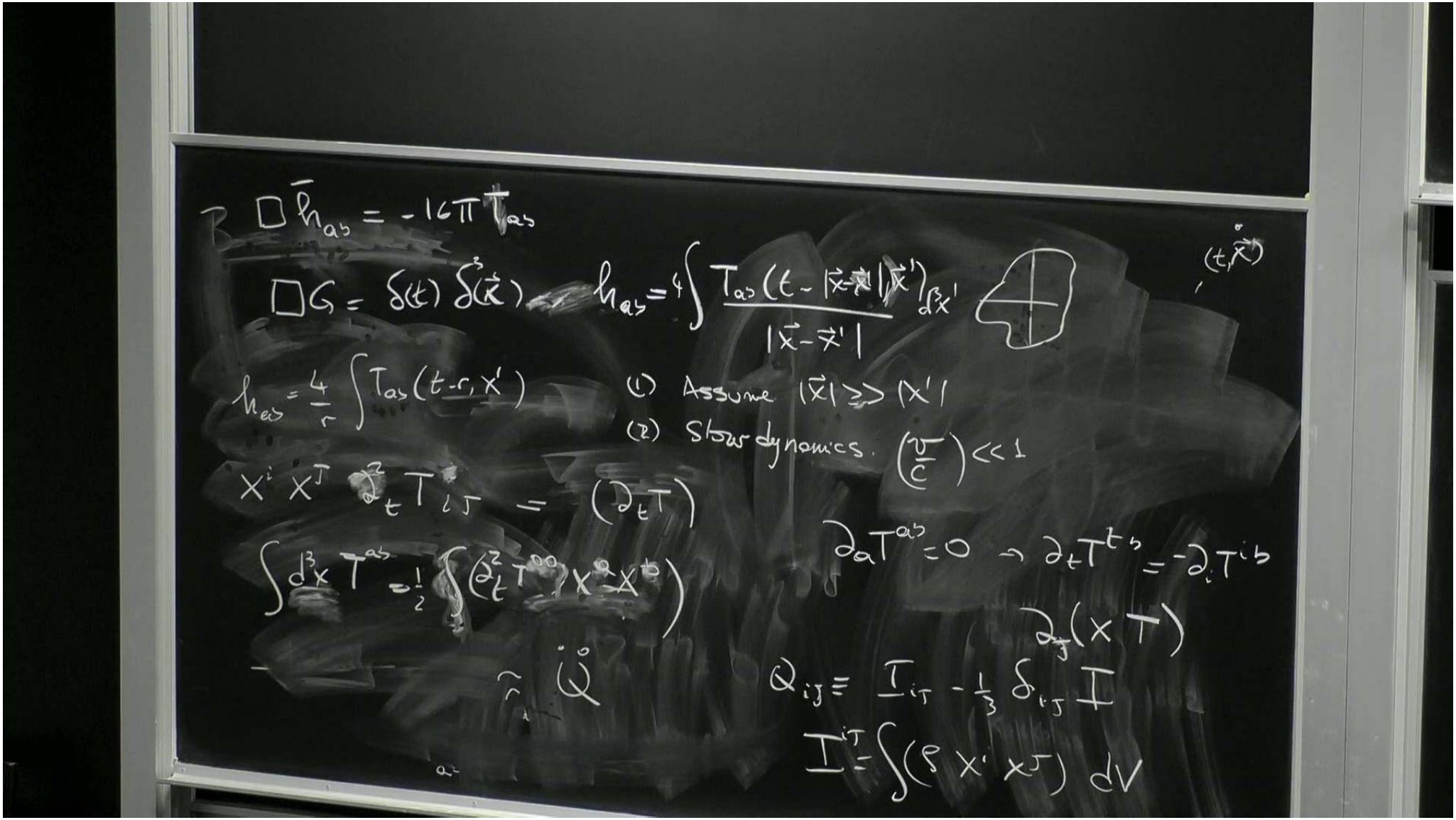
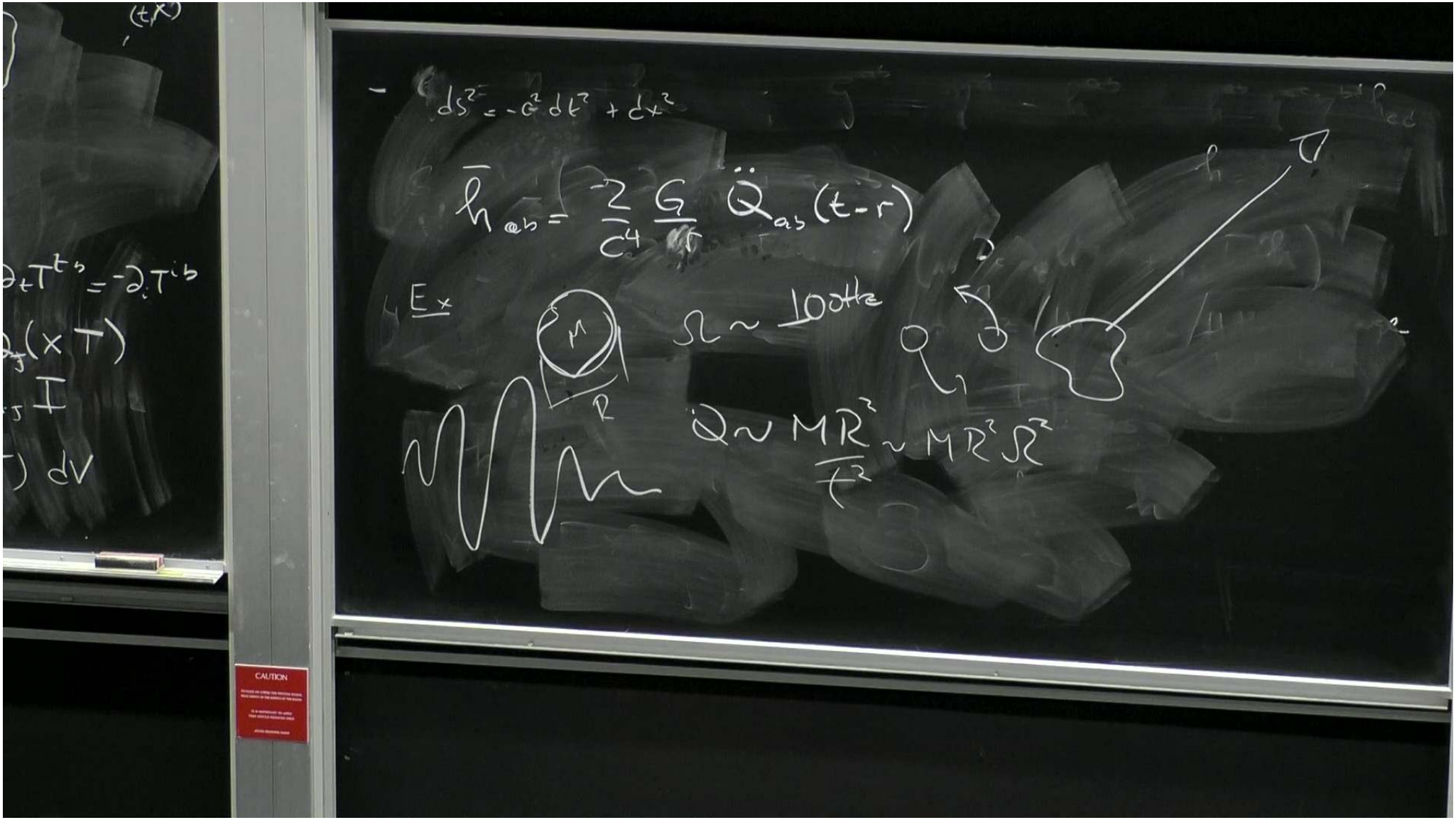
$\lambda > L_{coord}$
CAUTION


$g_{\mu\nu} = \eta_{\mu\nu} + h^{TT}_{\mu\nu}$
$ds^2 = -dt^2 g_{tt} + 2 g_{tx} dt dx + g_{xx} dx^2$
$x^a = (t, x^i)$
$\frac{d^2x^a}{d\tau^2} + \Gamma^a_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$
$\frac{dx}{d\tau} \to \frac{dx}{dt}$
$\dot{x}^a = \frac{dx^a}{dt}$
$\frac{d^2x^a}{dt^2} = -\Gamma^a_{bc} \dot{x}^b \dot{x}^c + \Gamma^0_{bc} \dot{x}^b \dot{x}^c \dot{x}^a$
$v^i = \frac{dx^i}{dt} \ll 1$
$L = \int_0^{L_{coor}} dx \sqrt{g_{xx}} \simeq L_{coor} \left[1 + \frac{1}{2} h_{xx}(t, x \right.$
$\lambda > L_{coor}$


$\Box \bar{h}_{ab} = -16\pi T_{ab}$
$\Box G = \delta(t)\,\delta^3(\vec{x})$
$h_{ab} = 4\int \frac{T_{ab}(t-|\vec{x}-\vec{x}'|,\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'$
$(t,\vec{x})$
$h_{ab} = \frac{4}{r}\int T_{ab}(t-r,x')$
(1) Assume $|\vec{x}| \gg |x'|$
(2) Slow dynamics. $\left(\frac{v}{c}\right) \ll 1$
$x^i x^j \partial_t^2 T_{ij} = (\partial_t T)$
$\int d^3x\, T^{ab} = \frac{1}{2}\int (\partial_t^2 T^{00}) x^a x^b$
$\partial_a T^{ab} = 0 \rightarrow \partial_t T^{tb} = -\partial_i T^{ib}$
$\partial_j(xT)$
$\sim \ddot{Q}$
$Q_{ij} \equiv I_{ij} - \frac{1}{3}\delta_{ij} I$
$I^{ij} = \int (\rho\, x^i x^j)\, dV$


$ds^2 = -c^2 dt^2 + dx^2$
$\bar{h}_{ab} = \frac{2}{c^4} \frac{G}{r} \ddot{Q}_{ab}(t-r)$
Ex
M
R
$\Omega \sim 10\,\text{kHz}$
$Q \sim \frac{MR^2}{T^2} \sim MR^2\Omega^2$
CAUTION