

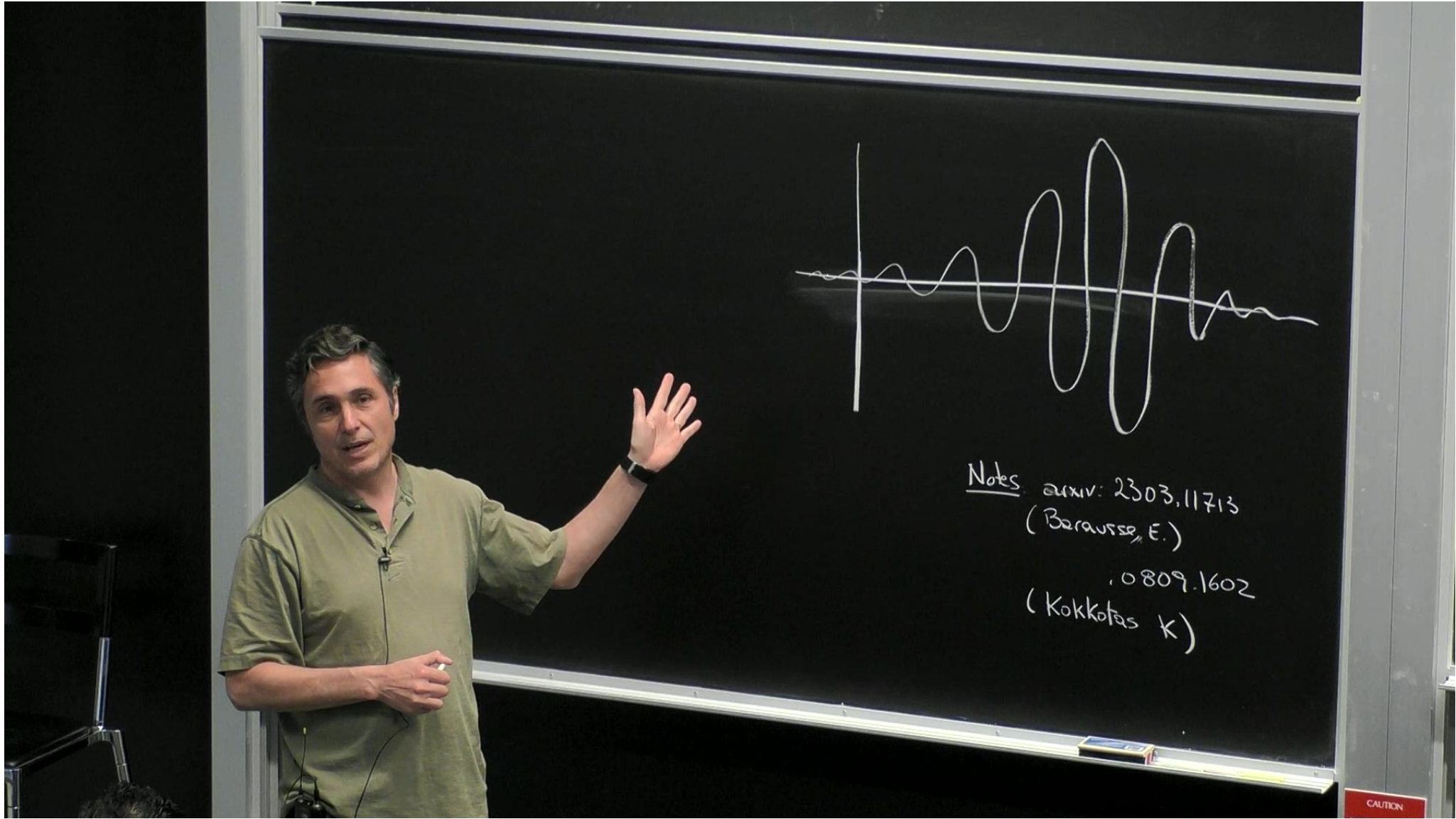
Title: Gravitational Wave Theory

Speakers: Luis Lehner

Collection: TRISEP 2023

Date: June 28, 2023 - 9:00 AM

URL: <https://pirsa.org/23060081>

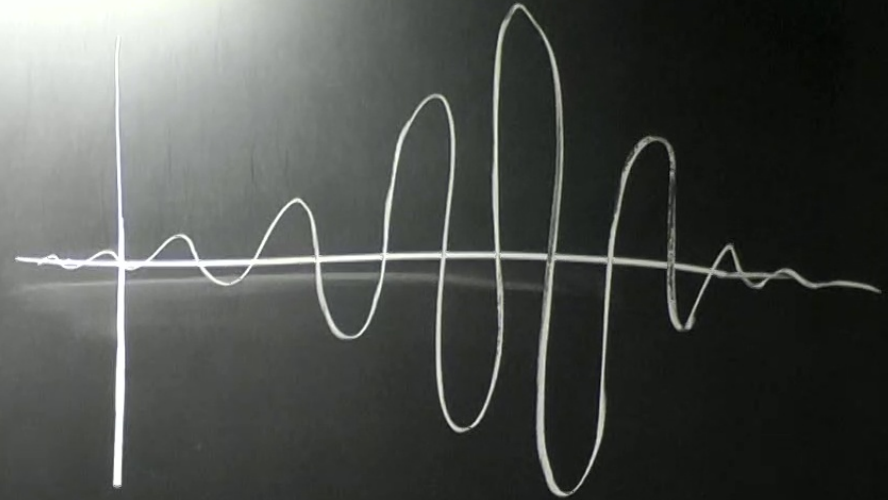


Knowledge

- g_{ab} tensor
- signature $(-, + + +)$
- diff geometry. $\nabla_a g_{cd} = 0$

- Equivalence principle $\partial_\epsilon \psi + \psi \partial_\epsilon \psi = \eta \partial_\epsilon \psi$

- Locally: $g_{ab} = \eta_{ab} \leftarrow$ Minkowski:
 $\Gamma_{ab}^\alpha = \{ \partial g \} = 0$



Notes: arxiv: 2303.11713
(Barausse, E.)

0809.1602

(Kokkotas K)

$$1) T^{ab} = (S+P)u^a u^b + P g^{ab} \rightarrow \nabla_a T^{ab} = 0$$

$$u^a u_a = -(S+P)u^a u_a$$

$$a^a = -\frac{h^{ab} \partial_b P}{S+P}$$

2) Geodesic deviation

$$a^a = u^b \nabla_b u^a$$


$$h^{ab} = g^{ab} + u^a u^b$$



$$\left| \frac{D^2 S^a}{D\tau^2} = R^a{}_{bcd} T^b T^c S^d \right| \quad u^a u_a = -1$$

$$\equiv T^c \nabla_c (T^d \nabla_d S^a)$$

$$\textcircled{3} \quad T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$$


$$S_{pp} = -m \int d\tilde{t}$$

(1) Metric

(2) Trajectory \rightarrow geodesic eqn. $T^a \nabla_a T^b = 0$

$$\textcircled{3} T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}}$$



$$S_{pp} = -m \int d\tau$$

(1) Metric

(2) Trajectory \rightarrow geodesic eqn.

$$T^a \nabla_b T^b = 0$$

\rightarrow EEs. $G_{ab} = 8\pi T_{ab}$

$$R_{abcd} \overset{bd}{f} = R_{ac}$$

$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$- g_{ab} = \eta_{ab} + h_{ab} \quad ; \quad g^{ab} = \eta^{ab} - h^{ab} \quad h^{ab} = \eta^{ac} \eta^{bd} h_{cd}$$

$$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h_a^c + a \leftrightarrow b \right. \\ \left. - \partial_a \partial_b h - \square h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \square h \right)$$

$$4 \text{ coord choices } \partial_c h^c_a = 0$$

$$x^a \rightarrow x^a + \xi^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

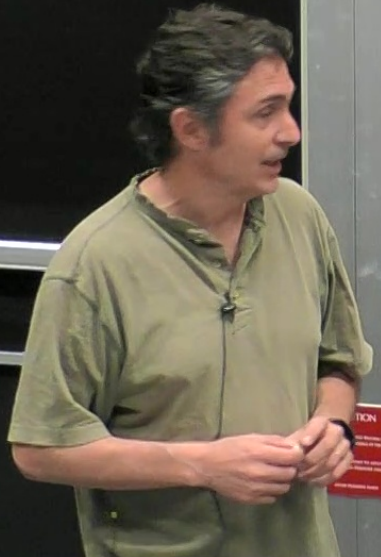
$$G_{ab} = \square \bar{h}_{ab}$$

$$\omega_{\alpha\beta} \begin{cases} u^a \partial_a p = -(\beta + p) \partial_a u^a \\ \dot{a}^a = -\frac{h^{ab} \partial_b p}{S + p} \end{cases}$$

$$u^a = u^b \partial_b u^a$$

$$h^{ab} = g^{ab} + u^a u^b$$

$$u^a u_a = -1$$



$$K_{ab} - \frac{1}{2} g_{ab} K = G_{ab}$$

$$h = \eta \quad h_{ab}$$

$$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h^c_a + \partial_c \partial_b h^c_a - \partial_c \partial_b h^c_a - \square h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \square h \right)$$

4 coord choices $\partial_c h^c_a = 0$

$$x^a \rightarrow x^a + \xi^a$$

$$G_{ab} = \square \bar{h}_{ab}$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$



$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h^c_a + a_{ab} - \partial_c \partial_b h - \square h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \square h \right)$$

$$h = \eta^{ab} h_{ab}$$

4 coord choices $\partial_c h^c_a = 0$ Lorentz / Harmonic / Hilbert gauge

$$x^a \rightarrow x^a + \xi^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$G_{ab} = \square \bar{h}_{ab}$$

$$\hat{h} \quad \bar{h}$$



$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h_a^c + \partial_c \partial_b h^c_a - \partial_c \partial_b h - \square h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \square h \right)$$

4 coord choices $\partial_c h^c_a = 0$

Lorentz / Harmonic / Hilbert gauge
De Dender

$$x^a \rightarrow x^a + \xi^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$x^e \rightarrow \square x^e = 0$$

$$G_{ab} = \square \bar{h}_{ab}$$

$$\hat{h} \quad \bar{h}$$



$$R_{ab} - \frac{1}{2} g_{ab} R = G_{ab}$$

$$G_{ab} = \frac{1}{2} \left(\partial_c \partial_b h^c_a + \partial_c \partial_a h^c_b - \partial_c \partial_b h^c_a - \square h_{ab} - \eta_{ab} \partial_c \partial_d h^{cd} + \eta_{ab} \square h \right)$$

4 coord choices $\partial_c h^c_a = 0$

Lorentz / Harmonic / Hilbert gauge / De Dender

$$x^a \rightarrow x^a + f^a$$

$$\bar{h}^{ab} = h^{ab} - \frac{1}{2} \eta^{ab} h$$

$$x^a \rightarrow \square x^a = 0$$

$$\square(x^a + y^a) = 0$$

$$G_{ab} = \square \bar{h}_{ab}$$

$$\hat{h} \quad \bar{h}$$

$$\square f^a$$



$$\begin{aligned}
 g_{as} &= \int_{as} + h_{as} & \nabla_a \bar{h}^{as} &= 0 \\
 \underbrace{\Gamma_{gB} \bar{h}_{as}} + 2 \underline{R_{acbd}} \bar{h}^{cd} + \sum_{a \leq c < d}^{(gB)} \bar{h}^{cd} &= -16\pi T_{as}
 \end{aligned}$$

$$\begin{aligned}
 g_{as} &= \bar{g}_{as} + h_{as} & \Delta_a \bar{h}^{as} & \rightarrow \\
 \underbrace{\Delta_{gB} \bar{h}_{as}} + 2 \underline{R_{acsb}} \bar{h}^{cd} & + \sum_{a,b,c,d}^{(gB)} \bar{h}^{cd} = -16\pi T_{as} \\
 & & 3G^{gB} & \text{ex}
 \end{aligned}$$



$$g_{as} = \bar{g}_{as} + h_{as} \quad \nabla_a \bar{h}^{as} = 0$$

$$\square_{g^B} \bar{h}_{as} + 2 \underline{R_{acsb}^{(g^B)}} \bar{h}^{cd} + \sum_{a \leq c < d}^{(g^B)} \bar{h}^{cd} = -16\pi T_{as}$$

$3G^{g^B}$

$z \uparrow$

$$\square_{\bar{g}} \bar{h}^{as} = 0$$

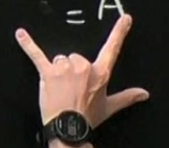
$$k^a = \left(\frac{\partial}{\partial t}\right)^a + \left(\frac{\partial}{\partial z}\right)^a$$

$$k_a = -dt_a + dz_a$$

$$\square \bar{h}_{as} \rightarrow \bar{h}_{as} = A_{as} e^{ik_\alpha x^\alpha}$$

$$\hookrightarrow k^a A_{as} = 0 \quad k_a A^{as} = 0$$

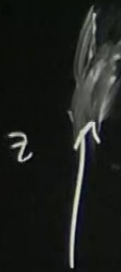
$$k^a k_a = 0 \quad A^{ta} = A^{za}$$



$$g_{as} = \bar{g}_{as} + h_{as}$$

$$\square_{g^0} \bar{h}_{as} + 2 \underline{K_{acba}}^{(g^0)} \bar{h}^{cd} + \overset{(g^0)}{S_{abcd}} \bar{h}^{cd}$$

$3G^{g^0}$



$$\square_{g^0} \bar{h}^{as} = 0$$

$$k^a = \left(\frac{\partial}{\partial t} \right)^a + \left(\frac{\partial}{\partial x^b} \right)^a$$

$$k_a = -dt_a + dx_a$$

$$\square \bar{h}_{as} \rightarrow \bar{h}_{as} = A_{as} e^{ik_c x^c}$$

$$\hookrightarrow k^a A_{as} = 0$$

$$k_a A^{as} = 0$$

$$k^a k_a = 0$$

$$A^{ta} = A^{za}$$

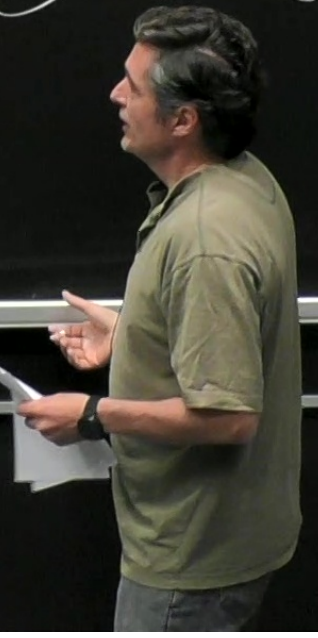
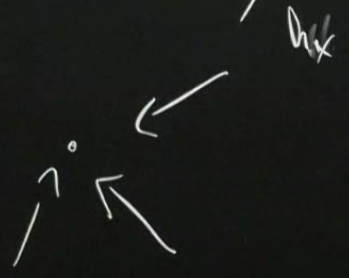
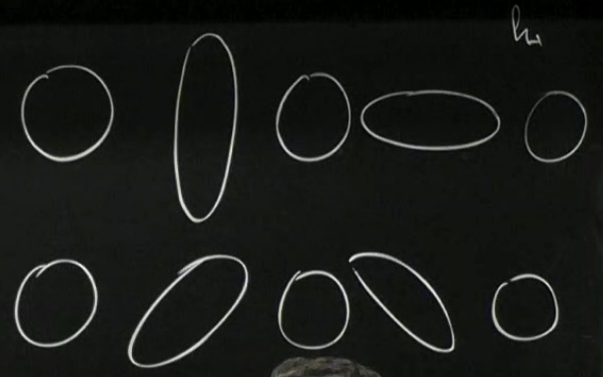
$$A^{ti} = 0$$

$$T_c(A) = 0$$

$h^{ab} = \eta^{ac} \eta^{bd} h_{cd}$
 $h = \eta^{ab} h_{ab}$
 $\Gamma(h)$
 Hilbert gauge
 De Dender
 0

$$A_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{tr}(A) = 0$$



CAUTION

$$g_{ab} = \eta_{ab} + h_{ab}$$

$$\frac{d^2 x^a}{d\tau^2} + \Gamma^a_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$$

$$\frac{dx}{d\tau} \rightarrow \frac{dx}{dt}$$

$$x^a = (t, x^i)$$

$$\frac{d^2 x^a}{dt^2} = -\Gamma^a_{bc} \dot{x}^b \dot{x}^c + \Gamma^0_{bc} \dot{x}^b \dot{x}^c \dot{x}^a$$

$$\dot{x}^a = \frac{dx^a}{dt}$$

$$v^i = \frac{dx^i}{dt} \ll c$$

$$L = \int_0^{L_{\text{com}}} dx \sqrt{1 + g_{xx}} \approx L_{\text{com}} \left[1 + \frac{1}{2} h_{xx}(t, x=0) \right]$$

$$\lambda \gg L_{\text{com}}$$

CAUTION
 TO AVOID INJURY OR PROPERTY DAMAGE,
 PLEASE HANDLE THE BOARD WITH CARE.
 IT IS IMPORTANT TO ALWAYS
 KEEP YOUR HANDS AND FEET
 AWAY FROM THE BOARD.

$$g_{ab} = \eta_{ab} + h_{ab}$$

$$\frac{d^2 x^a}{d\tau^2} + \Gamma^a_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0$$

$$\frac{dx}{d\tau} \rightarrow \frac{dx}{dt}$$

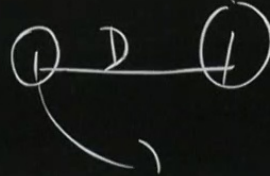
$$x^a = (t, x^i)$$

$$\frac{d^2 x^a}{dt^2} = -\Gamma^a_{bc} \dot{x}^b \dot{x}^c + \Gamma^0_{bc} \dot{x}^b \dot{x}^c \dot{x}^a$$

$$\dot{x}^a = \frac{dx^a}{dt}$$

$$v^i = \frac{dx^i}{dt} \ll c$$

$$L = \int_0^{L_{\text{com}}} dx \sqrt{1 + g_{xx}} \approx L_{\text{com}} \left[1 + \frac{1}{2} h_{xx}(t, x=0) \right]$$



$$\lambda > L_{\text{com}}$$

CAUTION
DO NOT TOUCH THE WHITEBOARD
OR THE SURFACE OF THE BOARD
IT IS NECESSARY TO AVOID
ANY DAMAGE TO THE BOARD
PLEASE REPORT DAMAGE

$$g_{as} = \eta_{as} + h_{as}^{\text{TT}}$$

$$ds^2 = -dt^2 + 2g_{jt} dx^j dt + g_{ij} dx^i dx^j$$

$$x^a = (t, x^i)$$

$$\frac{d^2 x^a}{dt^2} + \Gamma^a_{bc} \frac{dx^b}{dt} \frac{dx^c}{dt} = 0$$

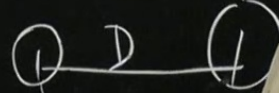
$$\frac{dx}{dz} \rightarrow \frac{dx}{dt}$$

$$\frac{d^2 x^a}{dt^2} = -\Gamma^a_{bc} \dot{x}^b \dot{x}^c + \Gamma^0_{bc} \ddot{x}^b \dot{x}^c \dot{x}^a$$

$$\dot{x}^a = \frac{dx^a}{dt}$$

$$v^i = \frac{dx^i}{dt} \ll c$$

$$L = \int_0^{L_{\text{conv}}} dx \sqrt{g_{xx}} \approx L_{\text{conv}} \left[1 + \frac{1}{2} h_{xx}(t, x) \right]$$

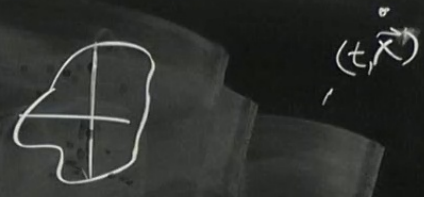


$x > L_{\text{conv}}$

CAUTION
Do not touch the surface of the screen.
Do not touch the screen.

$$\square \bar{h}_{ab} = -16\pi T_{ab}$$

$$\square G = \delta(t) \delta^3(\vec{x}) \quad h_{ab} = 4 \int \frac{T_{ab}(t - |\vec{x} - \vec{x}'|/c)}{|\vec{x} - \vec{x}'|} d^3x'$$



$$h_{ab} = \frac{4}{r} \int T_{ab}(t-r, x')$$

(1) Assume $|\vec{x}| \gg |\vec{x}'|$

(2) Slow dynamics. $(\frac{v}{c}) \ll 1$

$$x^i x^j \partial_t^2 T_{ij} = (\partial_t^2 T)$$

$$\int d^3x T^{ab} = \frac{1}{2} \int (\partial_t^2 T^{ab} x^a x^b)$$

$$\partial_a T^{ab} = 0 \rightarrow \partial_t T^{tb} = -\partial_i T^{ib}$$

$$\ddot{Q}$$

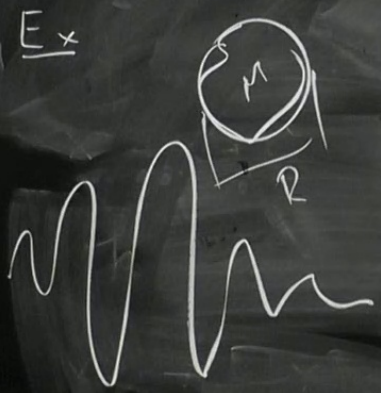
$$Q_{ij} = \frac{I_{ij}}{3} - \frac{1}{3} \delta_{ij} I$$

$$I_{ij} = \int (\rho x^i x^j) dV$$

(t, \mathbf{x})
 $\partial_t T^{t\alpha} = -\partial_i T^{i\alpha}$
 $\partial_j(x T)$
 $\int I$
 $\int dV$

$$ds^2 = -c^2 dt^2 + dx^2$$

$$\bar{h}_{ab} = \frac{2G}{c^4} \frac{\ddot{Q}_{ab}(t-r)}{r}$$



$\Omega \sim 100 \text{ Hz}$

$$Q \sim \frac{MR^2}{T^2} \sim MR^2 \Omega^2$$



CAUTION
 DO NOT lean on writing board
 DO NOT touch the board
 DO NOT touch the board
 DO NOT touch the board