Title: Amplitudes

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Collection: TRISEP 2023

Date: June 27, 2023 - 11:00 AM

URL: https://pirsa.org/23060076

Mini-recap of where we got to yesterday:

$\mathcal{N}=4$ SYM w/ general ansatz for higher-derivative operators $\Rightarrow$ at low energy, the 4 pt amplitude is

$$A(s,u) \equiv A[z\,z\,\bar{z}\,\bar{z}] = -\frac{s}{u} + s^2 \sum_{0 \le q \le k} a_{k,q}\, s^{k-q}\, u^q$$

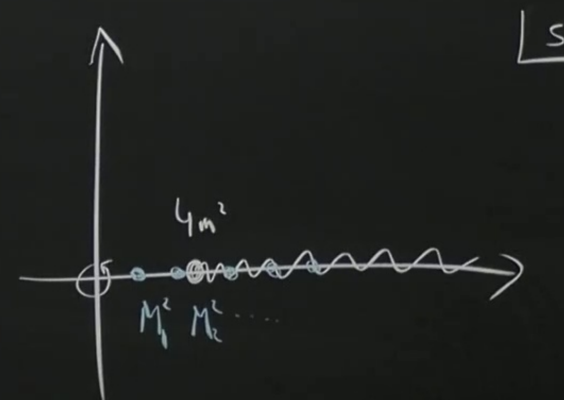
w/ $\mathcal{N}=4$ SUSY "crossing constraint" $a_{k,k-q} = a_{k,q}$

$a_{0,0}$ is $\mathrm{tr}\, F^4$ coupling

$a_{1,0} = a_{1,1}$ is $\mathrm{tr}\, D^2 F^4$ coupling etc.

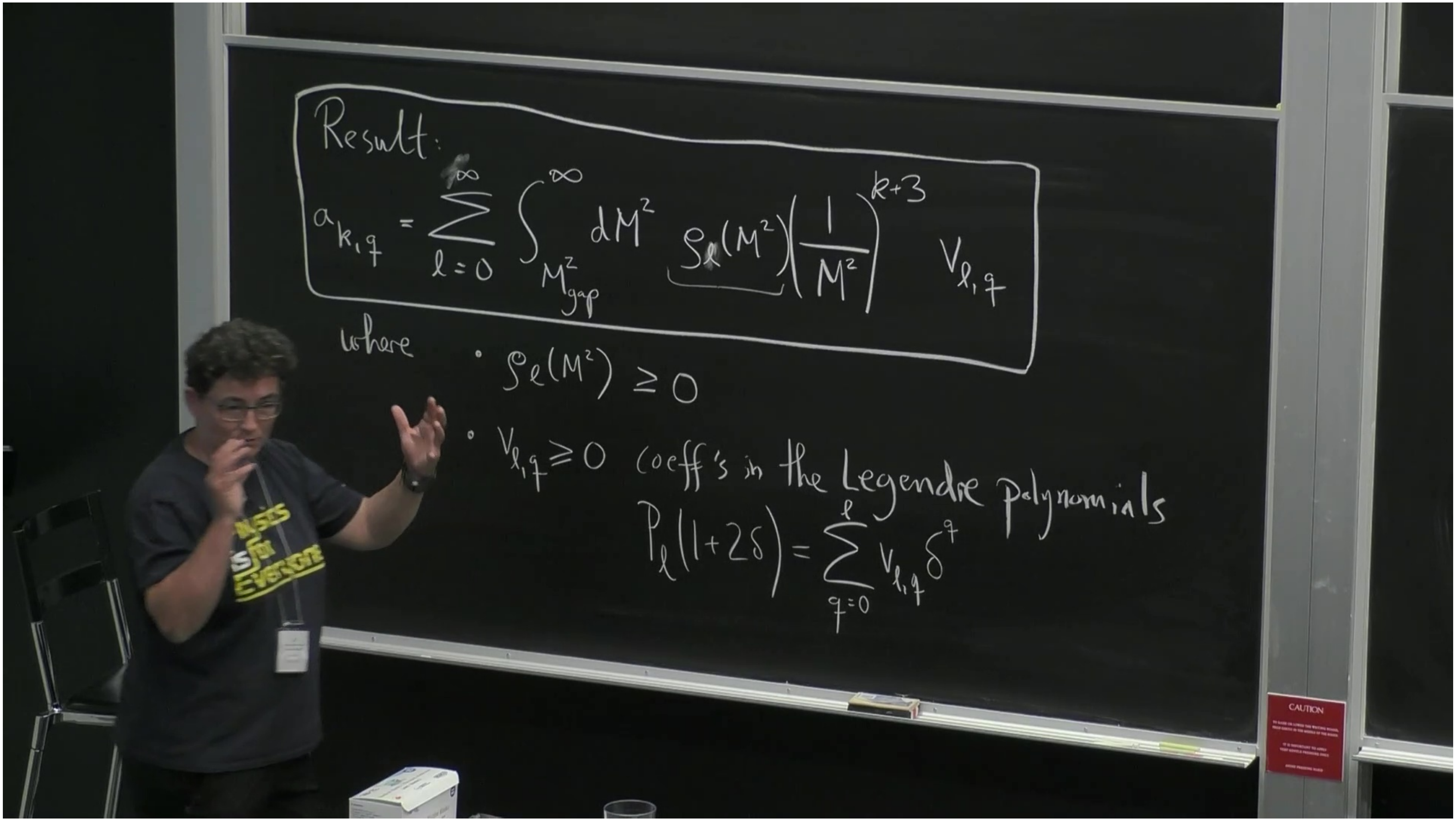
From general discussion, $A(s,u)$ is expected to have simple massive poles & branch cuts on the real s axis

Extracted $\boxed{a_{k,q}}$ from contour integral around $s=0$; deformed to pick up poles & discontinuity



assuming

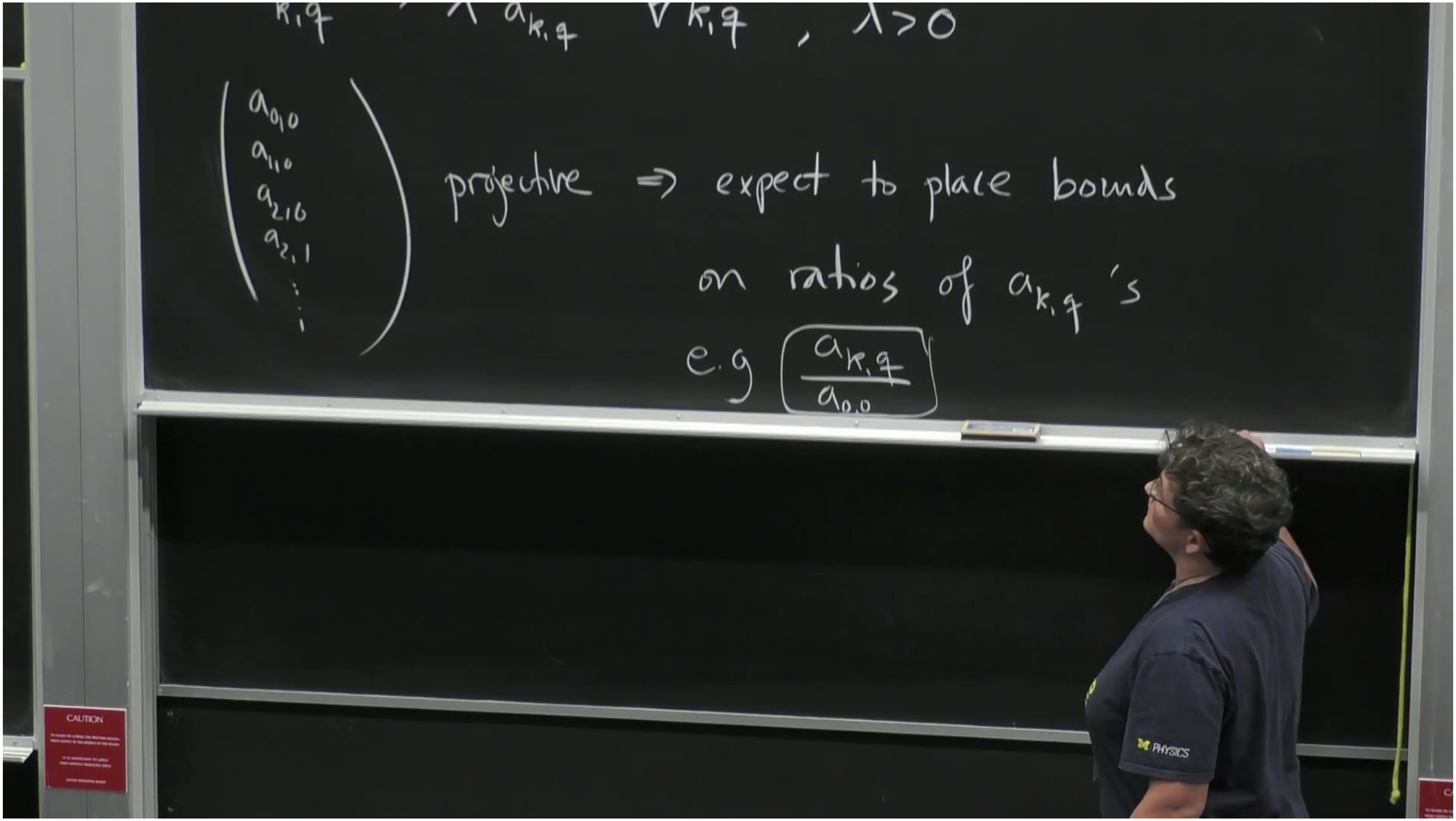
- Mass-gap $M_{gap} > 0$
- Froissart-Martin bound $\frac{A(s,u)}{s^2} \to 0$ as $|s| \to \infty$


Result:
$a_{k,q} = \sum_{\ell=0}^{\infty} \int_{M^2_{gap}}^{\infty} dM^2 \, \underbrace{\rho_\ell(M^2)} \left(\frac{1}{M^2}\right)^{k+3} v_{\ell,q}$
where
• $\rho_\ell(M^2) \geq 0$
• $v_{\ell,q} \geq 0$ coeff's in the Legendre polynomials
$P_\ell(1+2\delta) = \sum_{q=0}^{\ell} v_{\ell,q} \delta^q$
CAUTION

(1) $\boxed{a_{k,q} \geq 0}$

$$a_{k,q} \to \lambda\, a_{k,q} \quad \forall k,q \,, \quad \lambda > 0$$

$$\begin{pmatrix} a_{0,0} \\ a_{1,0} \\ a_{2,0} \\ a_{2,1} \\ \vdots \end{pmatrix}$$



$\lambda\, a_{k,q} \quad \forall k,q \,, \quad \lambda > 0$
$a_{0,0}$
$a_{1,0}$
$a_{2,0}$
$a_{2,1}$
projective ⇒ expect to place bounds
on ratios of $a_{k,q}$'s
e.g $\boxed{\frac{a_{k,q}}{a_{0,0}}}$
CAUTION
PHYSICS

Rescale $a_{k,q}\left(M_{gap}^2\right)^{k+2} \longrightarrow a_{k,q}$ dim'less.

New integration variable $0 \le x = \dfrac{M_{gap}^2}{M^2} \le 1$

Define $p_\ell(x) = x\,\rho_\ell\left(\dfrac{M_{gap}^2}{x}\right) \ge 0$

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Define $p_\ell(x) = x\,\rho_\ell\left(\dfrac{M_{gap}^2}{x}\right) \ge 0$

$s = M_{gap}^2$

$$\Rightarrow \boxed{a_{k,q} = \sum_{\ell=0} \int_0^1 dx\, p_\ell(x)\, x^k\, v_{\ell,q}\,, \quad p_\ell(x) \ge 0}$$

② $V_{\ell,0} = 1$, b/c $0 \le x \le 1$ $\rightarrow$ $\boxed{a_{k,0} \le a_{k',0} \text{ for } k \ge k'}$

also $\boxed{a_{k,q} \le a_{k',q} \text{ for } k \ge k'}$

$a_{0,0} \ge a_{1,0} \ge a_{2,0} \ge a_{3,0} \ge a_{4,0} \cdots$

② $V_{\ell,0} = 1$, b/c $0 \le x \le 1$ → $\boxed{a_{k,0} \le a_{k',0} \text{ for } k \ge k'}$

also $\boxed{a_{k,q} \le a_{k',q} \text{ for } k \ge k'}$

$$
\begin{array}{ccccccccc}
\boxed{a_{0,0}} \geq & a_{1,0} & \geq & a_{2,0} & \geq & a_{3,0} & \geq & a_{4,0} & \cdots \\
 & & & \| & & \| & & & \\
 & & & & & a_{3,3} & \geq & a_{4,3} & \geq \cdots \\
 & \| & & a_{2,2} & \geq & a_{3,2} & \geq & a_{4,2} & \geq \cdots \\
 & a_{1,1} & \geq & a_{2,1} & \geq & a_{3,1} & \geq & a_{4,1} & \cdots
\end{array}
$$

$\boxed{a_{0,0} \text{ is the biggest of all } a_{k,q}}$

Result:

$$\boxed{a_{k,q} = \sum_{\ell=0}^{\infty} \int_{M^2_{gap}}^{\infty} dM^2 \, \underbrace{\rho_\ell(M^2)} \left(\frac{1}{M^2}\right)^{k+3} v_{\ell,q}}$$

low E description

high E physics

where

- $\rho_\ell(M^2) \geq 0$
- $\underbrace{v_{\ell,q} \geq 0}$ (coeff's in the Legendre polynomials

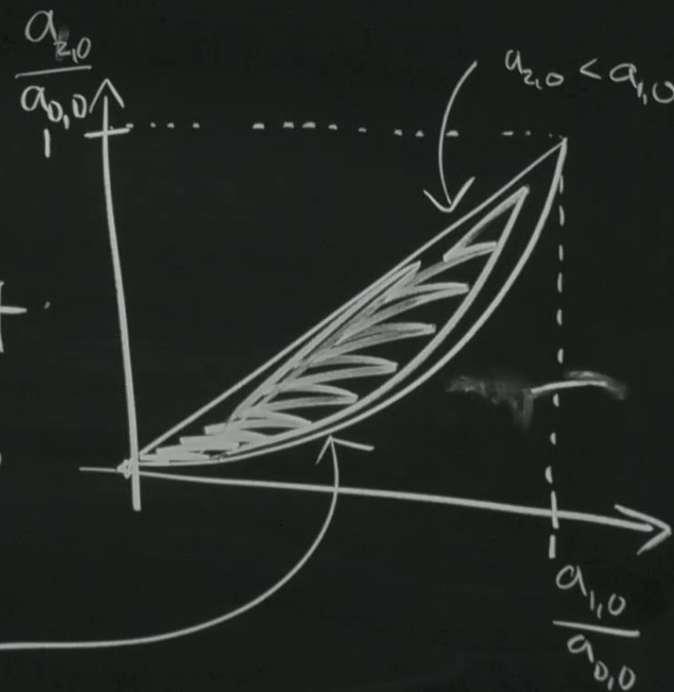
$$P_\ell(1+2\delta) = \sum_{q=0}^{\ell} v_{\ell,q}\,\delta^q$$

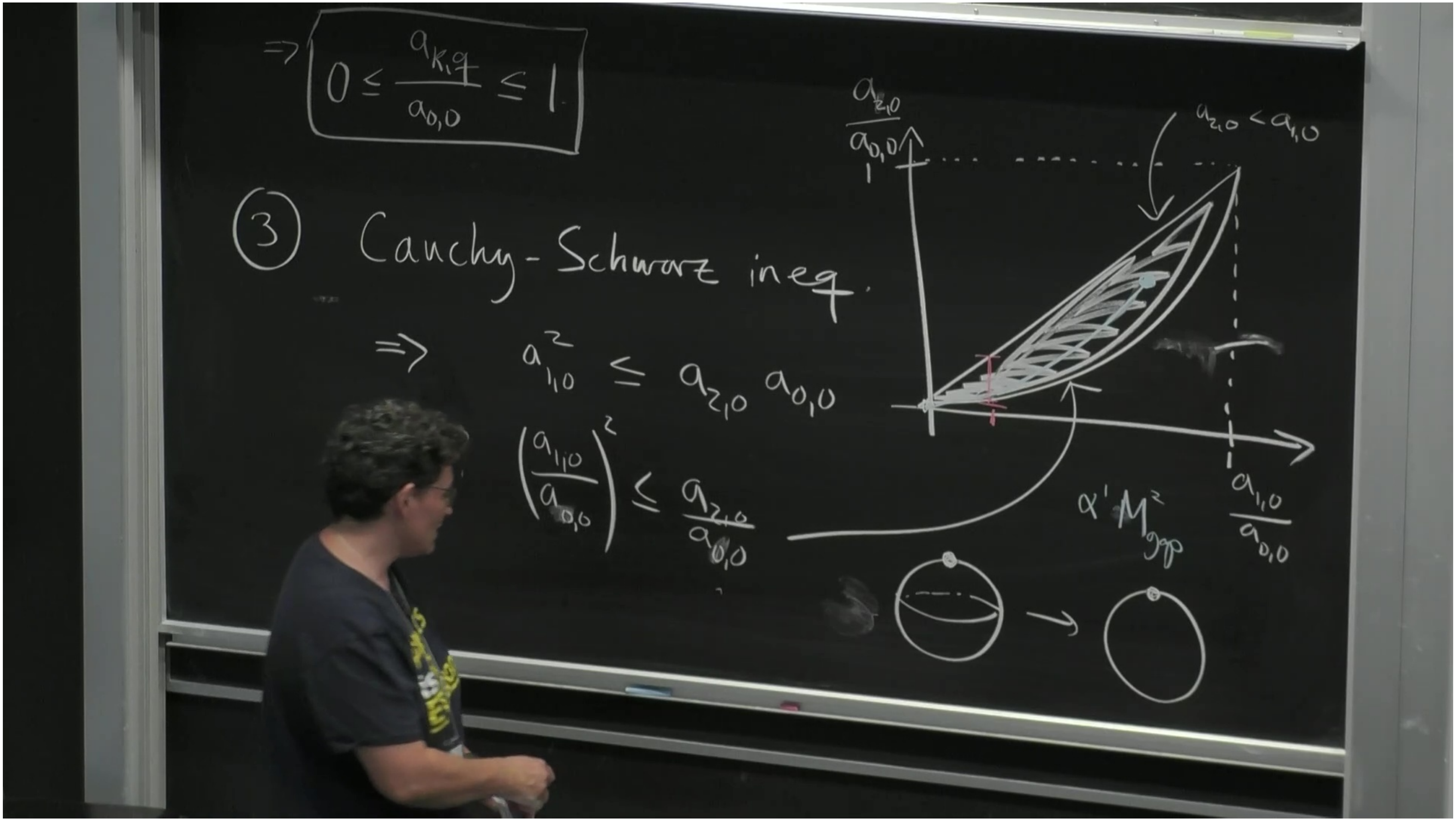
$$\Rightarrow \boxed{0 \le \frac{a_{k,q}}{a_{0,0}} \le 1.}$$

(3) Cauchy-Schwarz ineq.

$$\Rightarrow \quad a_{1,0}^2 \le a_{2,0}\, a_{0,0}$$

$$\Rightarrow \quad \left(\frac{a_{1,0}}{a_{0,0}}\right)^2 \le \frac{a_{2,0}}{a_{0,0}}$$




⇒ 0 ≤ a_{k,q} / a_{0,0} ≤ 1
③ Cauchy-Schwarz ineq.
⇒ a_{1,0}^2 ≤ a_{2,0} a_{0,0}
(a_{1,0} / a_{0,0})^2 ≤ a_{2,0} / a_{0,0}
a_{2,0} / a_{0,0}
1
a_{2,0} < a_{1,0}
a_{1,0} / a_{0,0}
α' M^2_{gap}

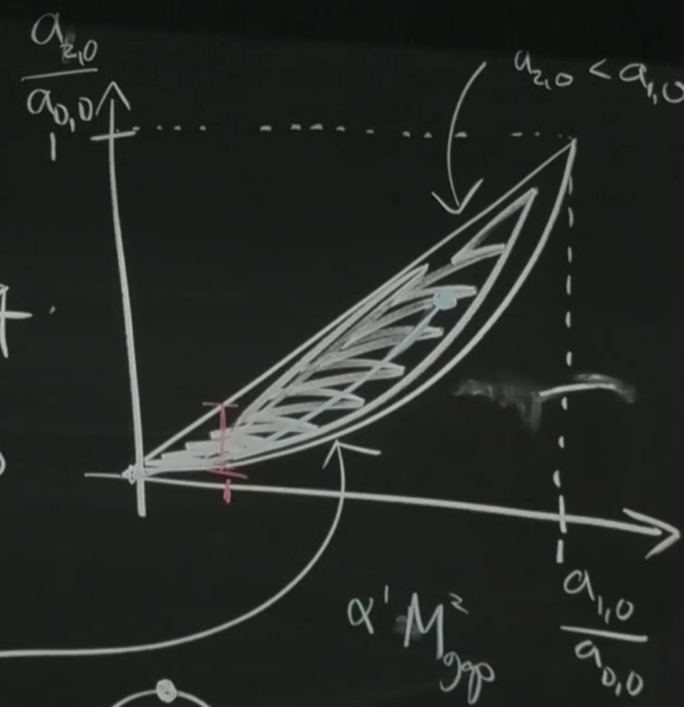
$$\Rightarrow \boxed{0 \leq \frac{a_{k,q}}{a_{0,0}} \leq 1}$$

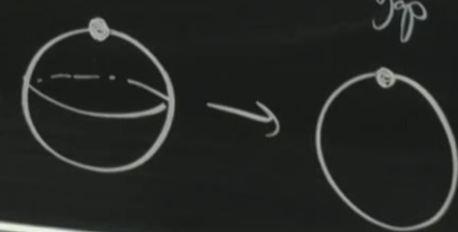
③ Cauchy-Schwarz ineq.

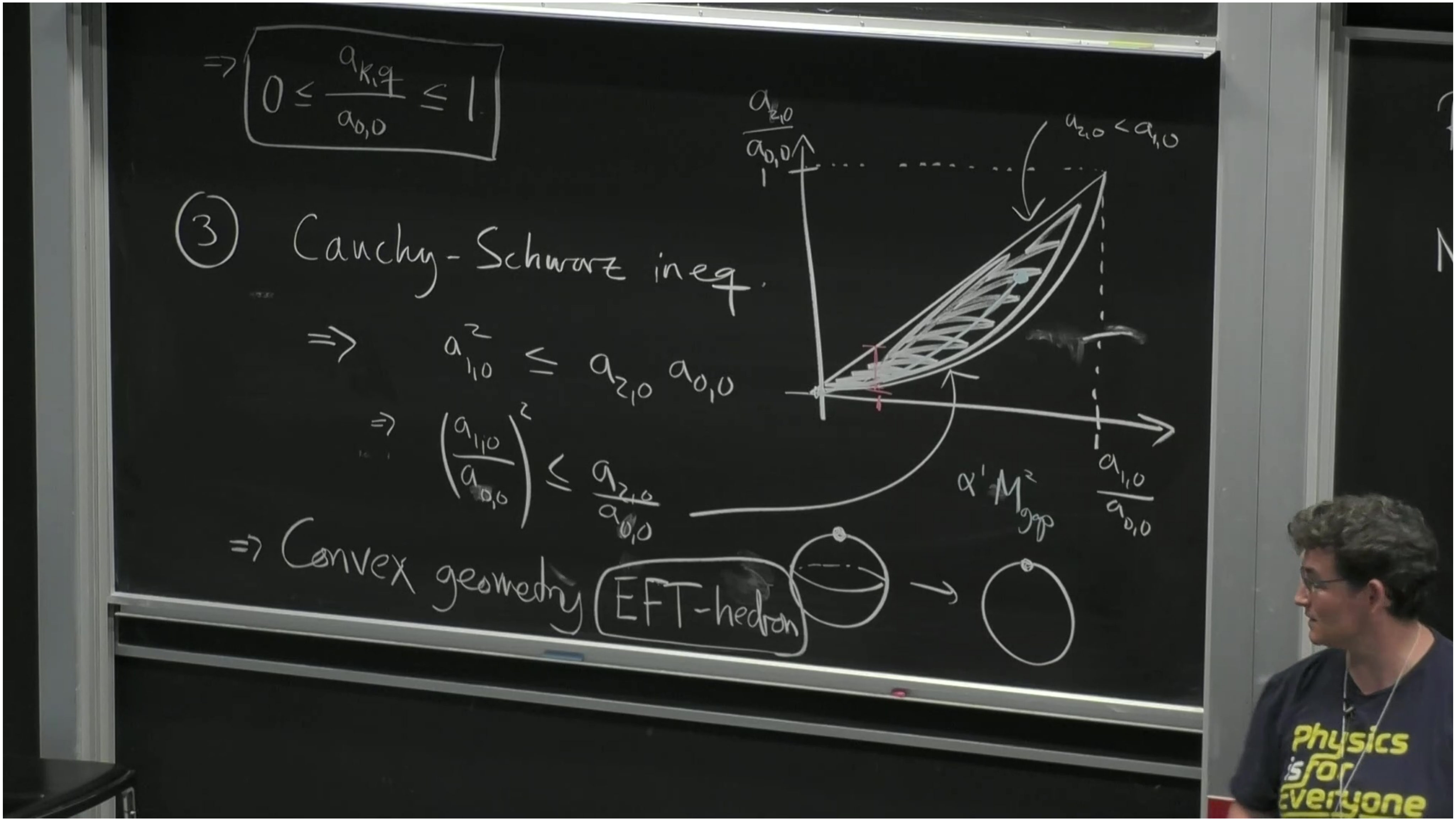
$$\Rightarrow \quad a_{1,0}^2 \leq a_{2,0}\, a_{0,0}$$

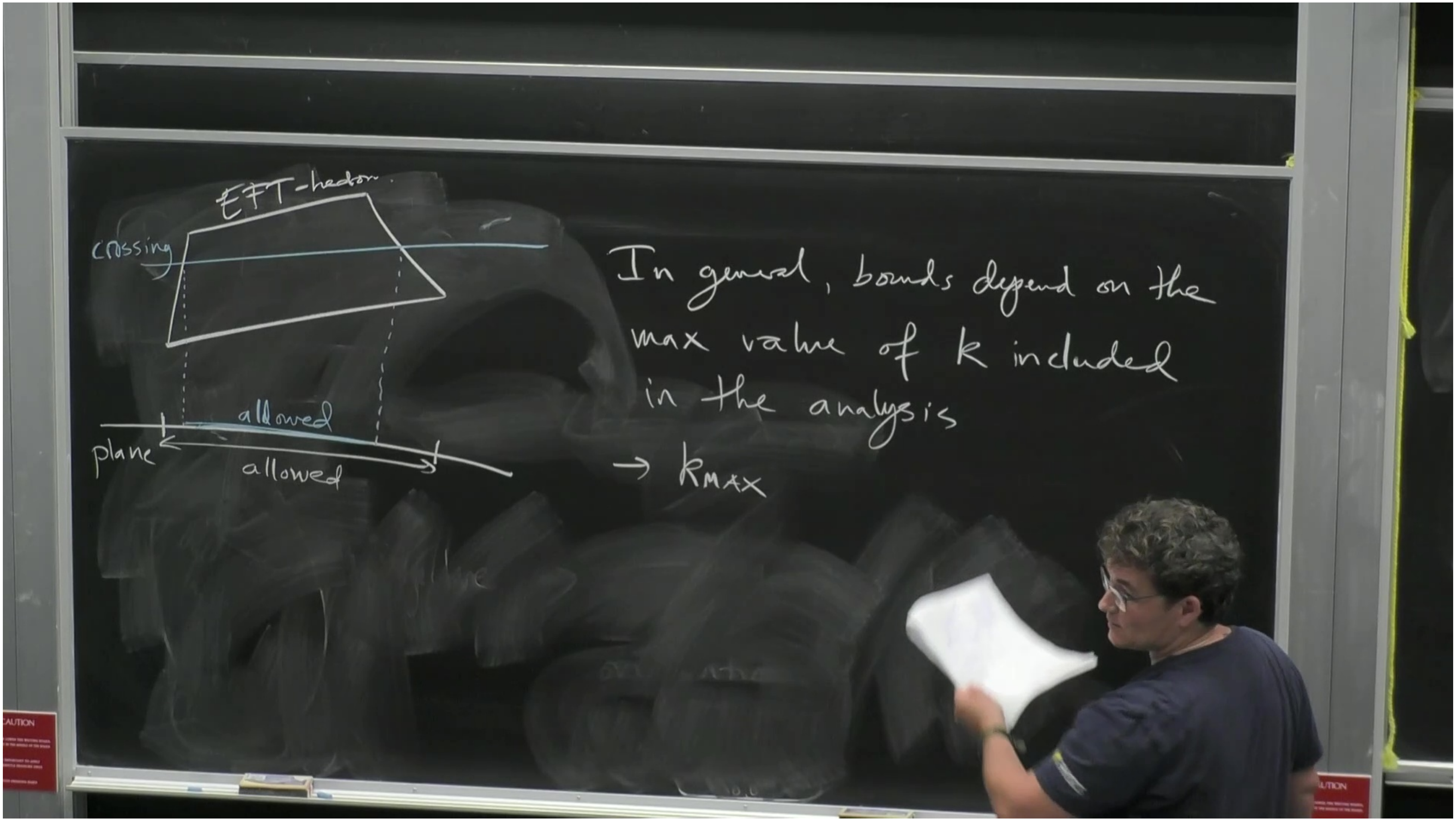
$$\Rightarrow \quad \left(\frac{a_{1,0}}{a_{0,0}}\right)^2 \leq \frac{a_{2,0}}{a_{0,0}}$$

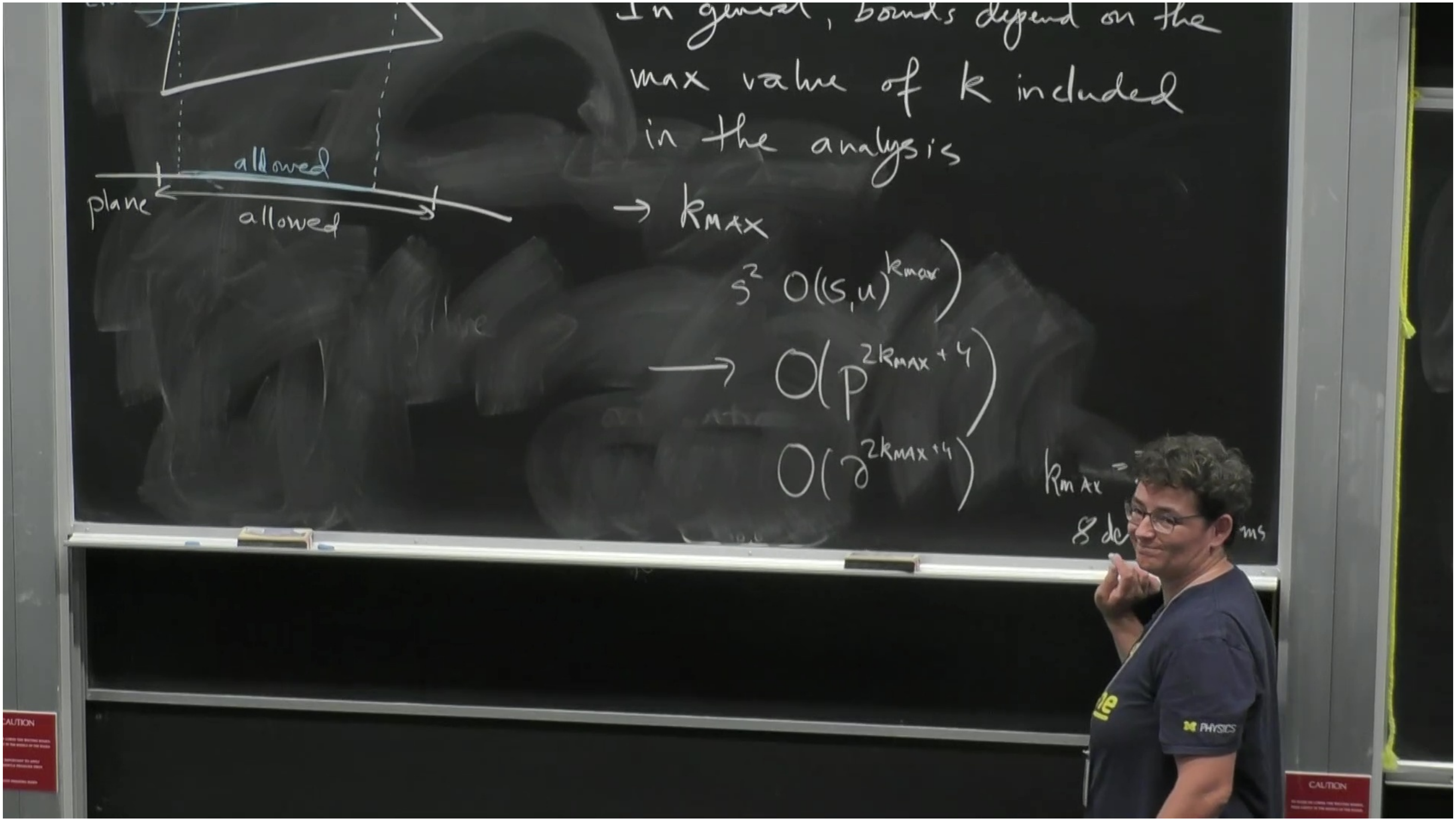
$\Rightarrow$ Convex geometry

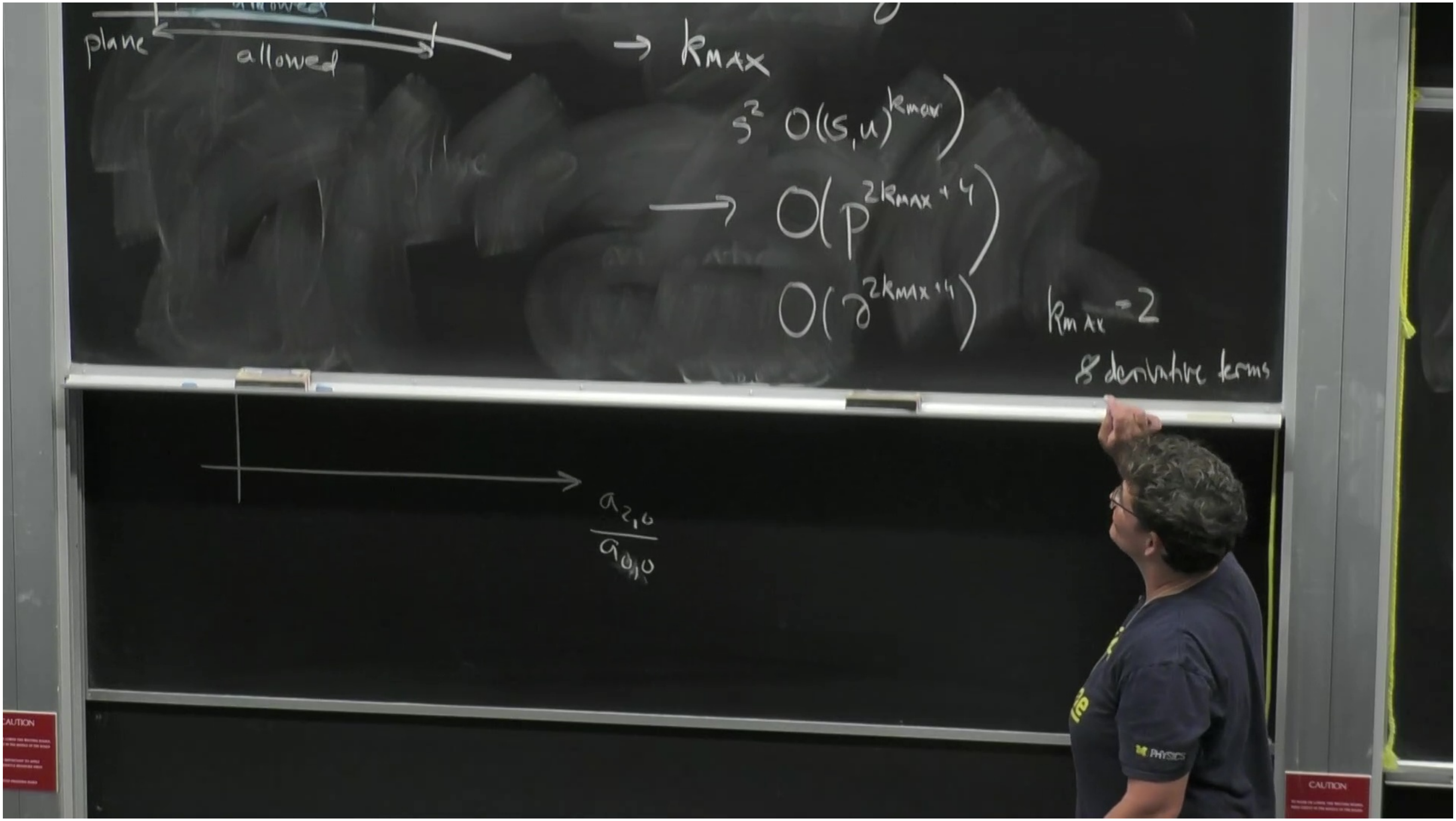


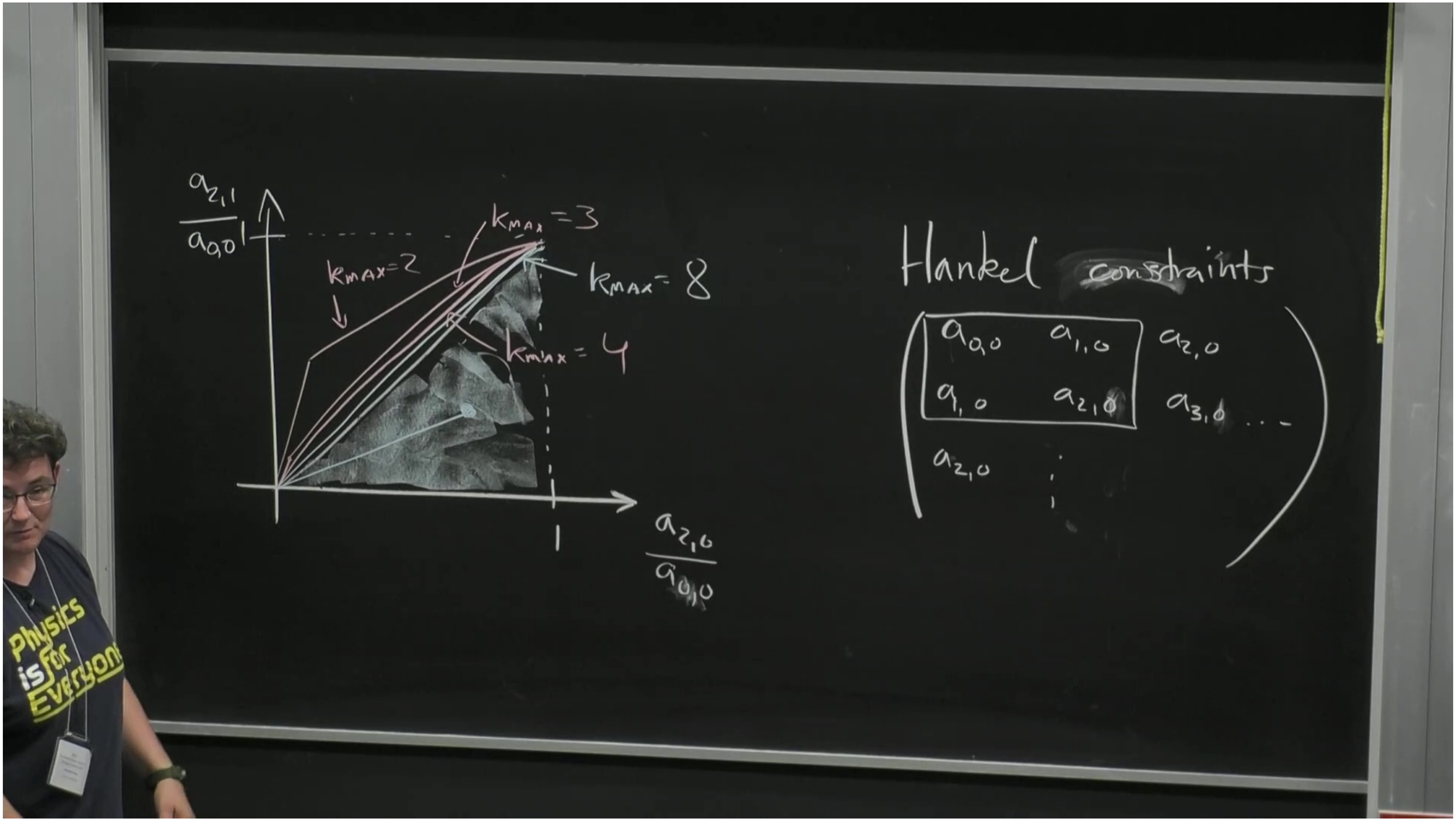



$\Rightarrow \boxed{0 \le \frac{a_{k,q}}{a_{0,0}} \le 1}$
③ Cauchy-Schwarz ineq.
$\Rightarrow a_{1,0}^2 \le a_{2,0}\, a_{0,0}$
$\Rightarrow \left(\frac{a_{1,0}}{a_{0,0}}\right)^2 \le \frac{a_{2,0}}{a_{0,0}}$
$\Rightarrow$ Convex geometry
EFT-hedron
$\frac{a_{2,0}}{a_{0,0}}$
1
$a_{2,0} < a_{1,0}$
$\frac{a_{1,0}}{a_{0,0}}$
$\alpha' M_{gap}^2$
Physics is for everyone

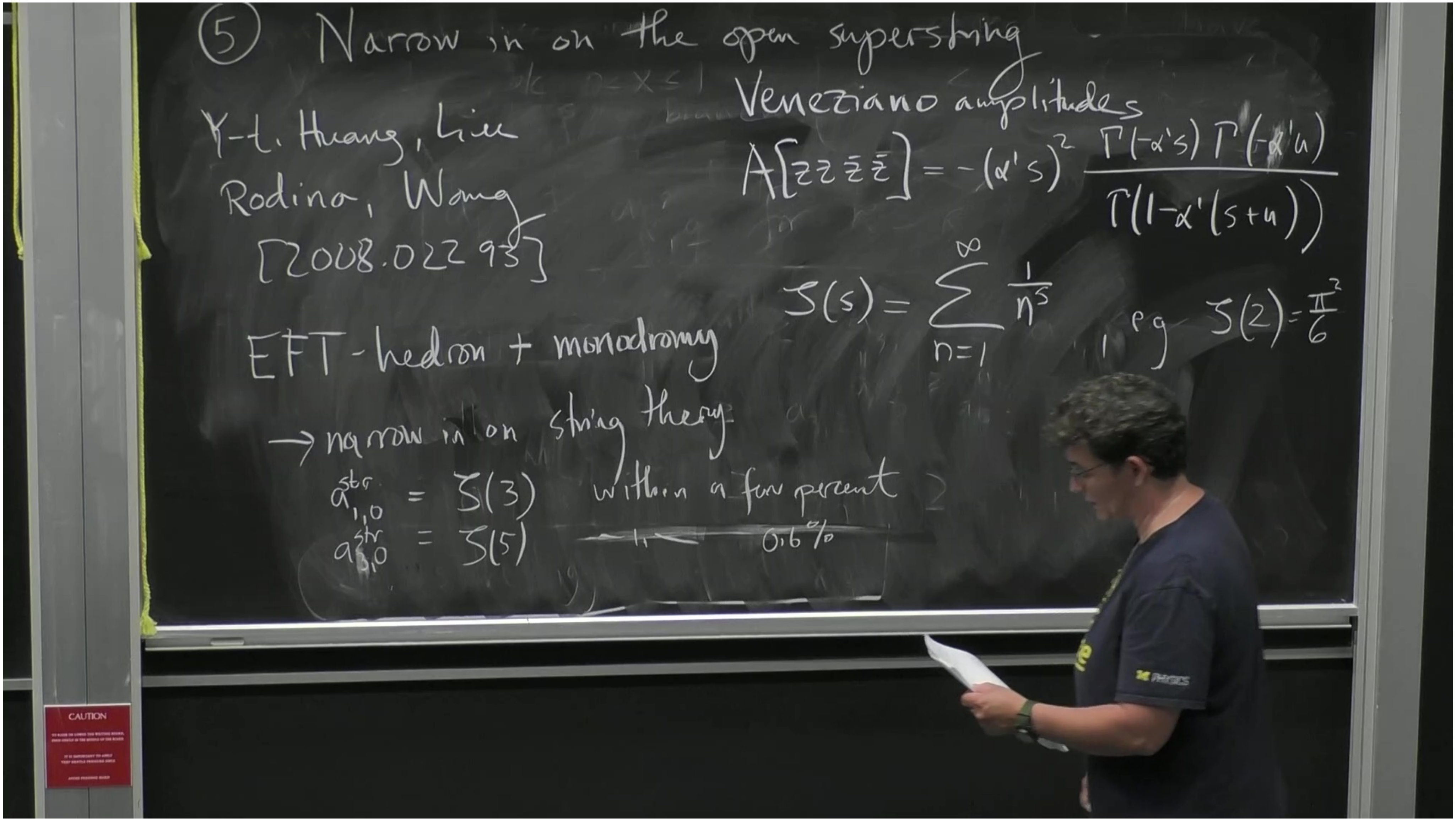

EFT-hedron
crossing
allowed
plane
allowed
In general, bounds depend on the
max value of k included
in the analysis
→ k_{MAX}


In general, bounds depend on the
max value of k included
in the analysis
→ k_MAX
s^2 O((s,u)^kmax)
→ O(p^(2kMAX+4))
O(∂^(2kMAX+4))
k_MAX =
allowed
plane
allowed


plane
allowed
→ k_MAX
s^2 O((s,u)^kmax)
→ O(p^(2kmax+4))
O(∂^(2kmax+4))
kmax = 2
8 derivative terms
a_2,0 / a_0,0
CAUTION
PHYSICS



a2,1 / a0,0
kMAX=2
kMAX=3
kMAX=8
kmax=4
1
a2,0 / a0,0
Hankel constraints
a0,0 a1,0 a2,0
a1,0 a2,0 a3,0 ...
a2,0
Physics is for Everyone



5 Narrow in on the open superstring
Veneziano amplitudes
Y-t. Huang, Liu
Rodina, Wang
[2008.02293]
$A[z z \bar{z} \bar{z}] = -(\alpha' s)^2 \frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' u)}{\Gamma(1-\alpha'(s+u))}$
$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$, e.g. $\zeta(2) = \frac{\pi^2}{6}$
EFT-hedron + monodromy
→ narrow in on string theory
$a^{str}_{1,0} = \zeta(3)$ within a few percent
$a^{str}_{3,0} = \zeta(5)$ — " — 0.6%

(5) Narrow in on the open superstring

Y-t. Huang, Liu
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Veneziano amplitudes

$$A[z z \bar{z} \bar{z}] = -(\alpha' s)^2 \frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' u)}{\Gamma(1-\alpha'(s+u))}$$

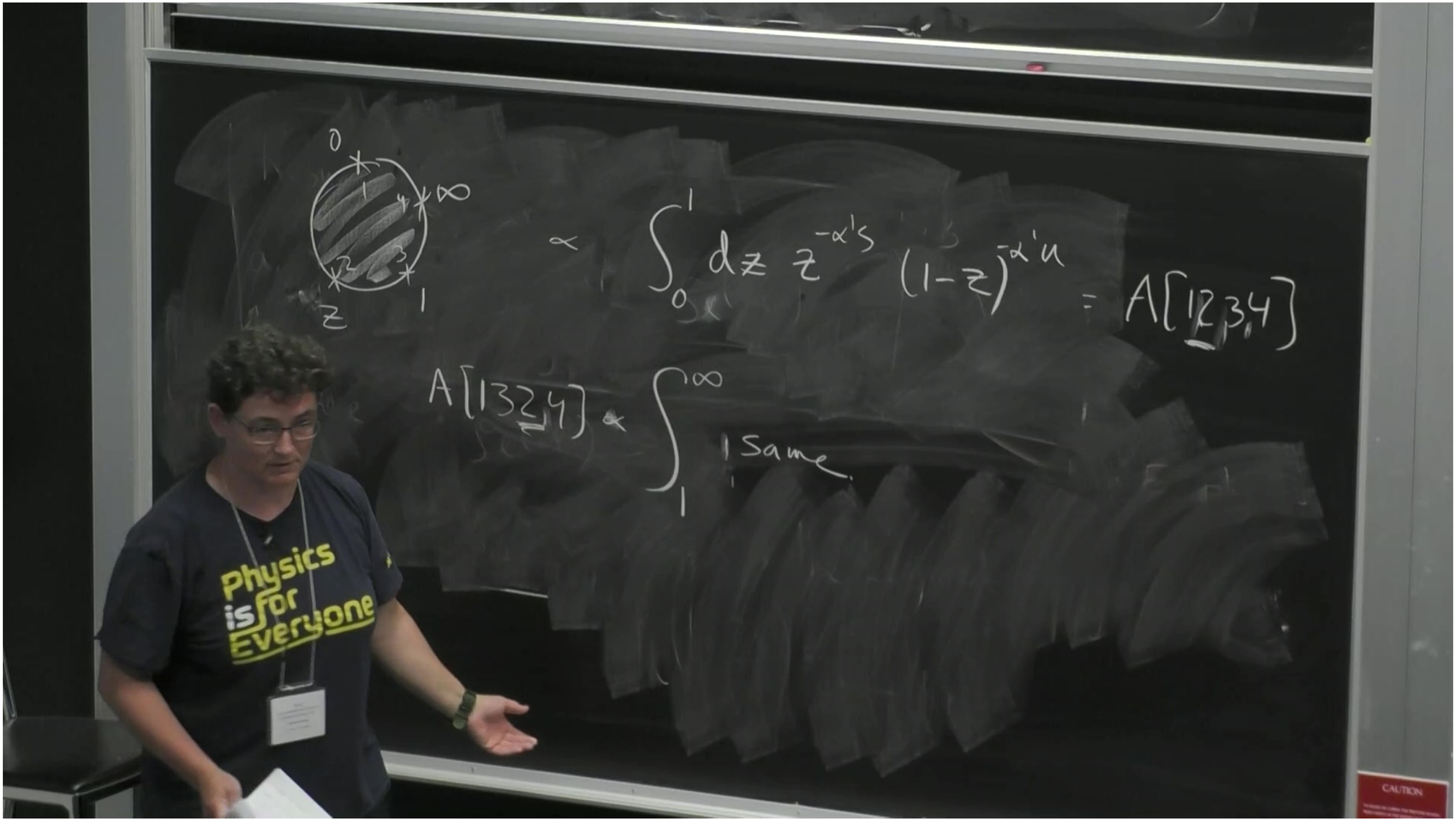
$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \text{e.g. } \zeta(2) = \frac{\pi^2}{6}$$

EFT-hedron + monodromy

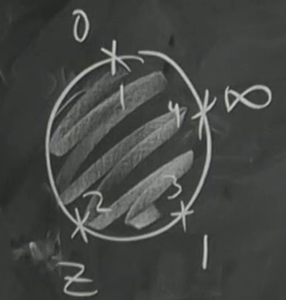
→ narrow in on string theory

$a^{str}_{1,0} = \zeta(3)$ within a few percent
$a^{str}_{3,0} = \zeta(5)$ — " — 0.6%
$a^{str}_{4,1}$ … — " — 50%

$\Big\}$ $k_{MAX} \approx 4$, + some constraints 6, 7, …



∝ ∫₀¹ dz z^(-α's) (1-z)^(-α'u) = A[1,2,3,4]
A[1324] ∝ ∫₁^∞ same
Physics is for Everyone

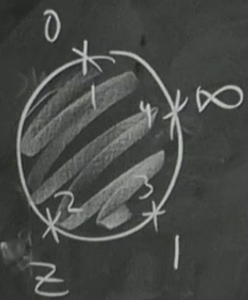


$$\propto \int_0^1 dz\, z^{-\alpha' s} (1-z)^{-\alpha' u} = A[1\underline{2}34]$$

$$A[13\underline{2}4] \propto \int_1^\infty \text{same}$$

linear relation string monodromy

$$0 = A[2134] + e^{i\pi\alpha' s} A[1234] + e^{-i\pi\alpha' t} A[1324]$$



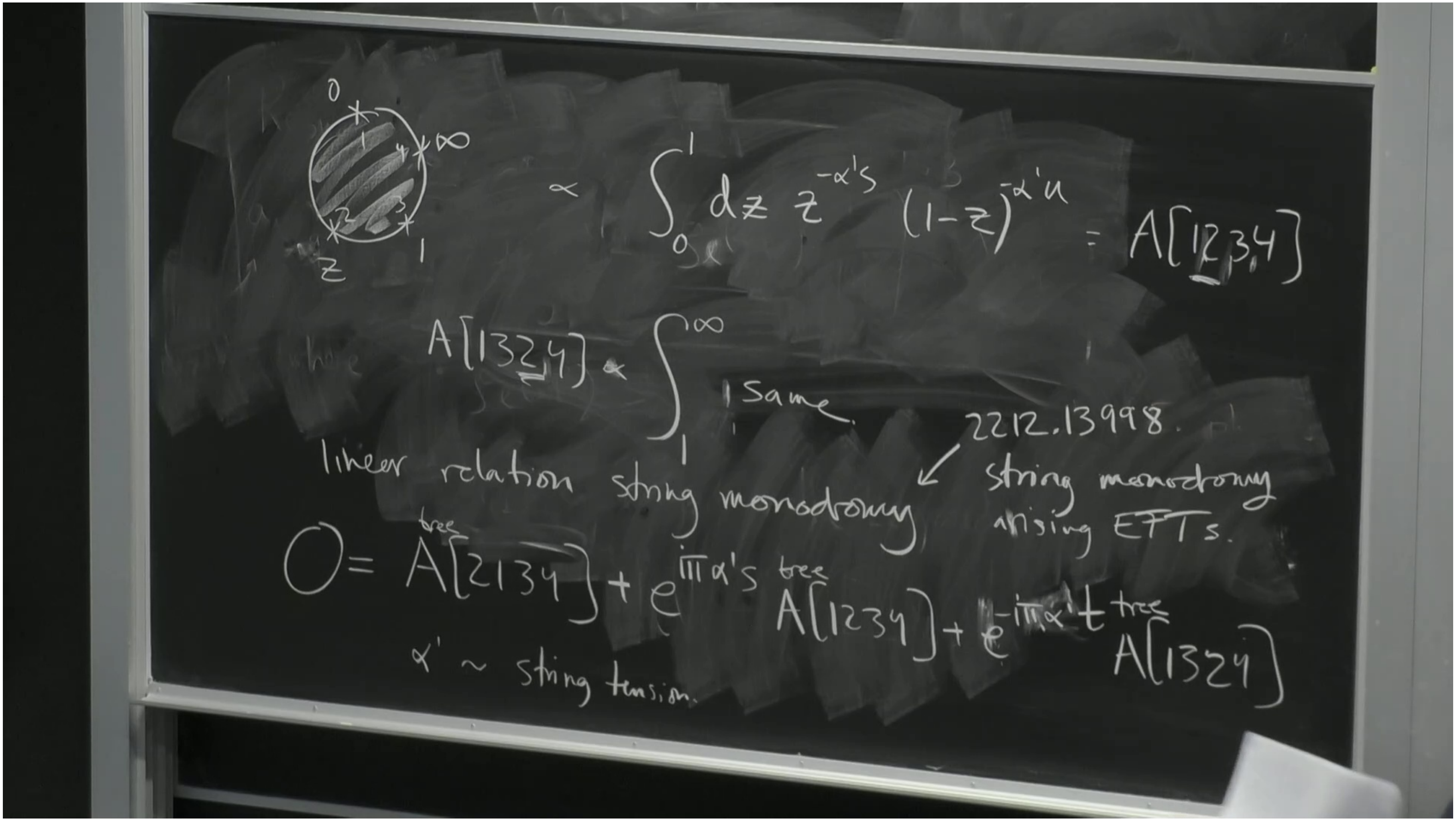
$$\propto \int_0^1 dz\, z^{-\alpha' s} (1-z)^{-\alpha' u} = A[1\underline{2}34]$$

$$A[13\underline{2}4] \propto \int_1^\infty \text{same}$$

linear relation string monodromy

$$0 = A^{tree}[2134] + e^{i\pi\alpha' s} A^{tree}[1234] + e^{-i\pi\alpha' t} A^{tree}[1324]$$

$\alpha' \sim$ string tension.


0
∞
1
z
$\propto \int_0^1 dz\, z^{-\alpha' s} (1-z)^{-\alpha' u} = A[1234]$
$A[1324] \propto \int_1^\infty$ same
2212.13998
string monodromy arising EFTs.
linear relation string monodromy
$0 = A^{tree}[2134] + e^{i\pi\alpha' s} A^{tree}[1234] + e^{-i\pi\alpha' t} A^{tree}[1324]$
$\alpha' \sim$ string tension.

monodromy only knows about π, not $\zeta(3)$, $\zeta(\text{odd})$

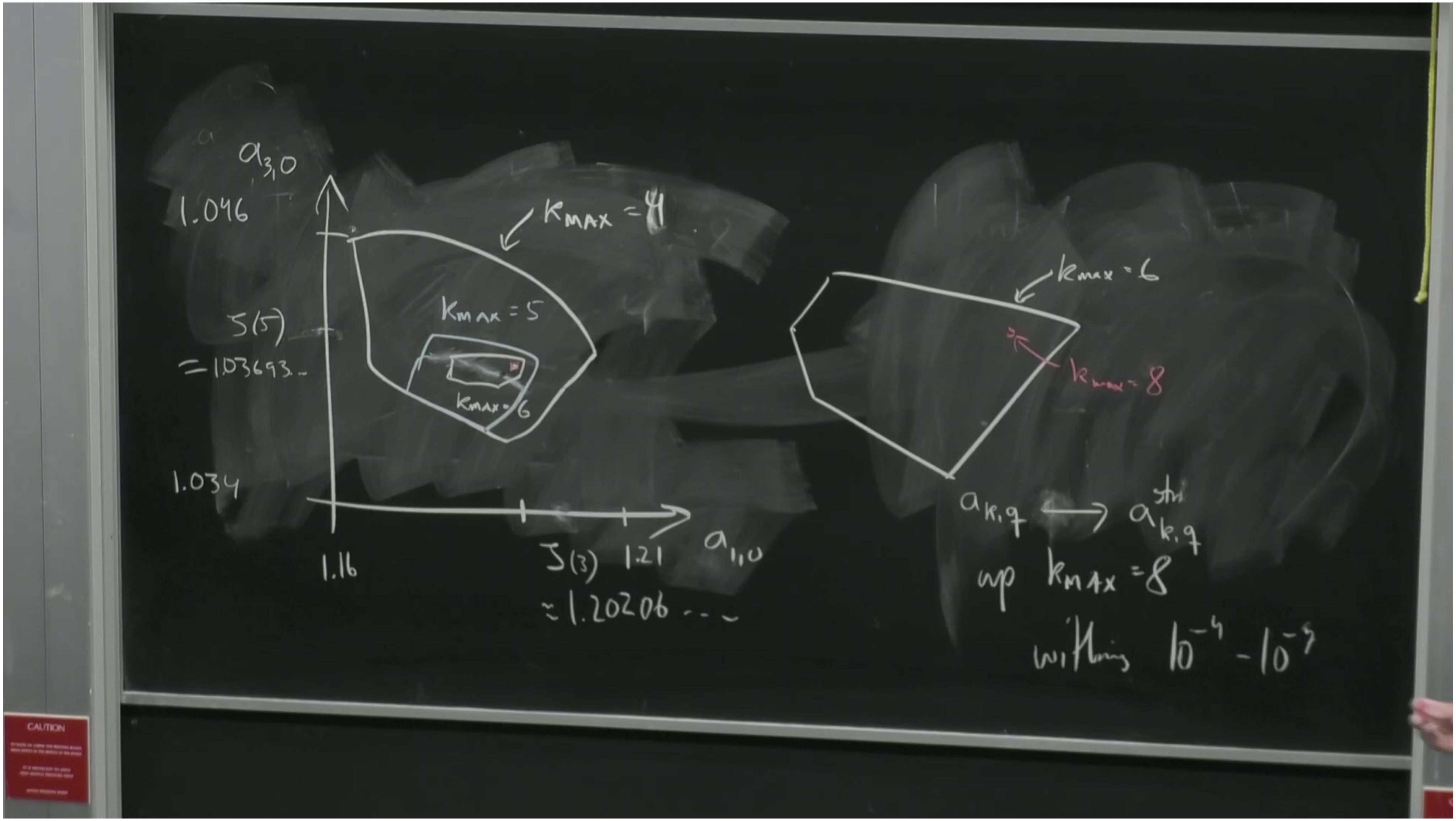
Monodromy Fixes

$$a_{0,0} = \zeta(2) = \frac{\pi^2}{6}$$

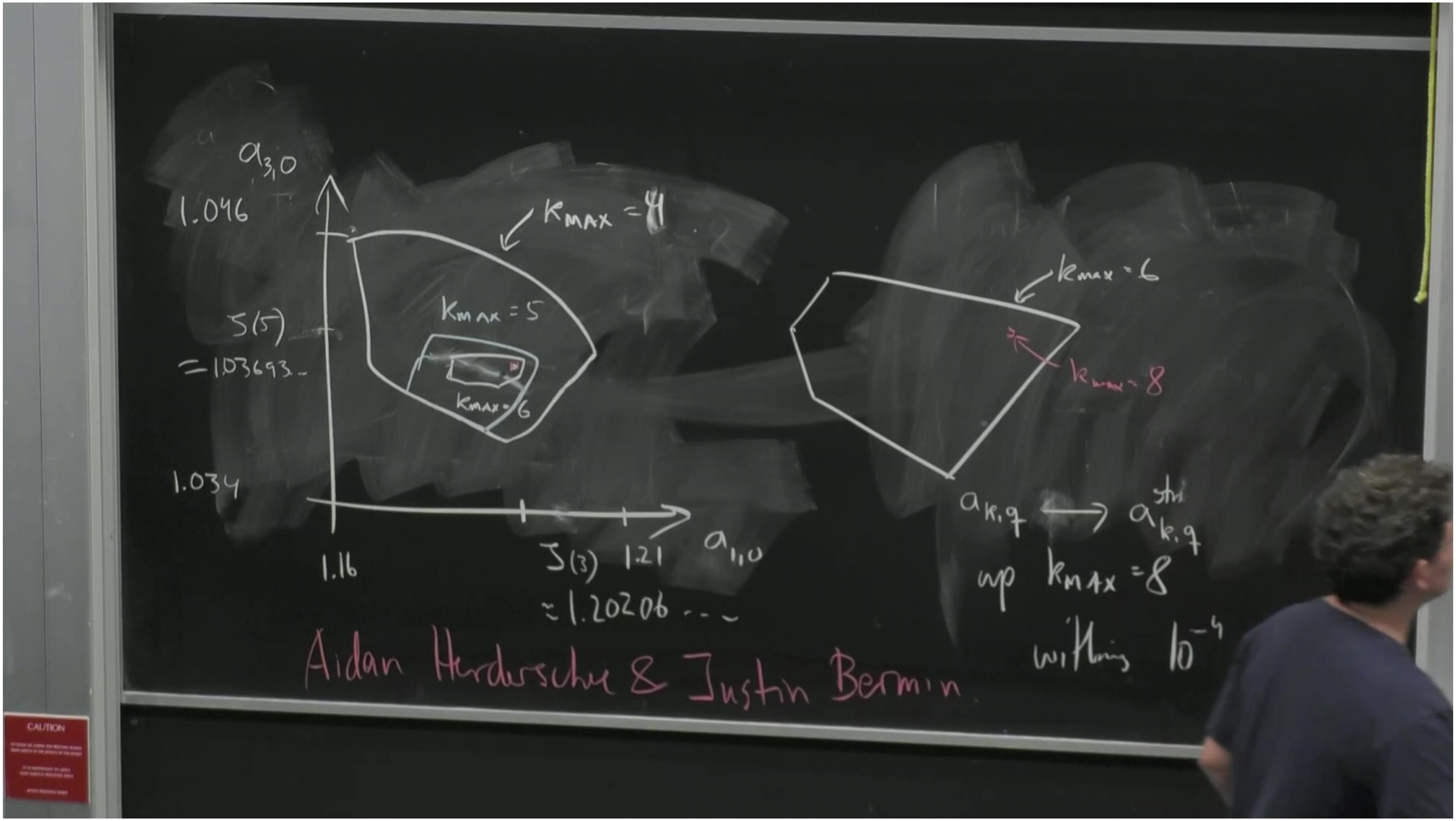
$$a_{2,0} = \zeta(4) = \frac{\pi^4}{90}$$

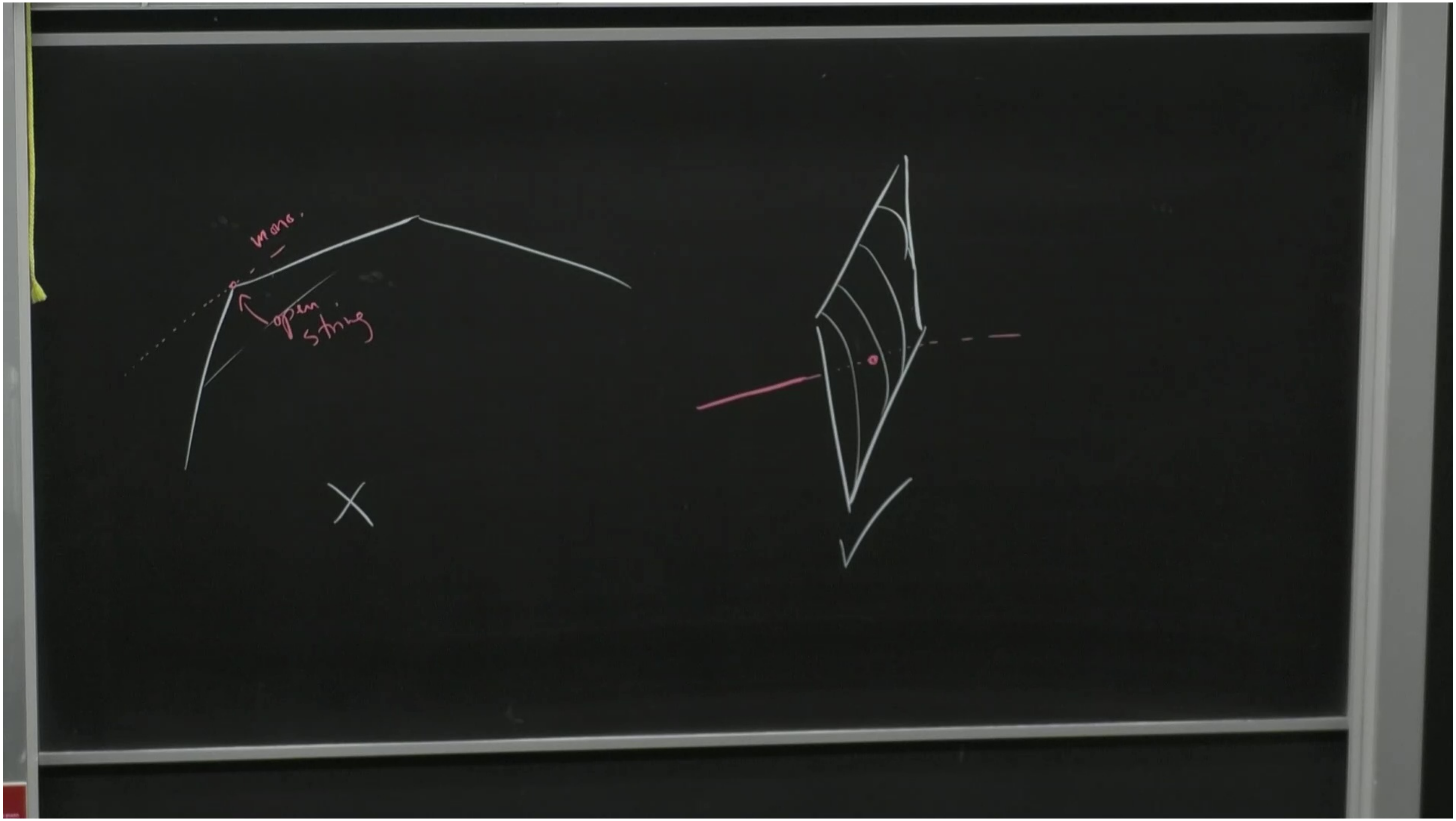
$$a_{3,1} - 2a_{3,0} = -\zeta(2)\, a_{1,0}, \quad a_{2,1} = \frac{1}{4}\zeta(4)$$

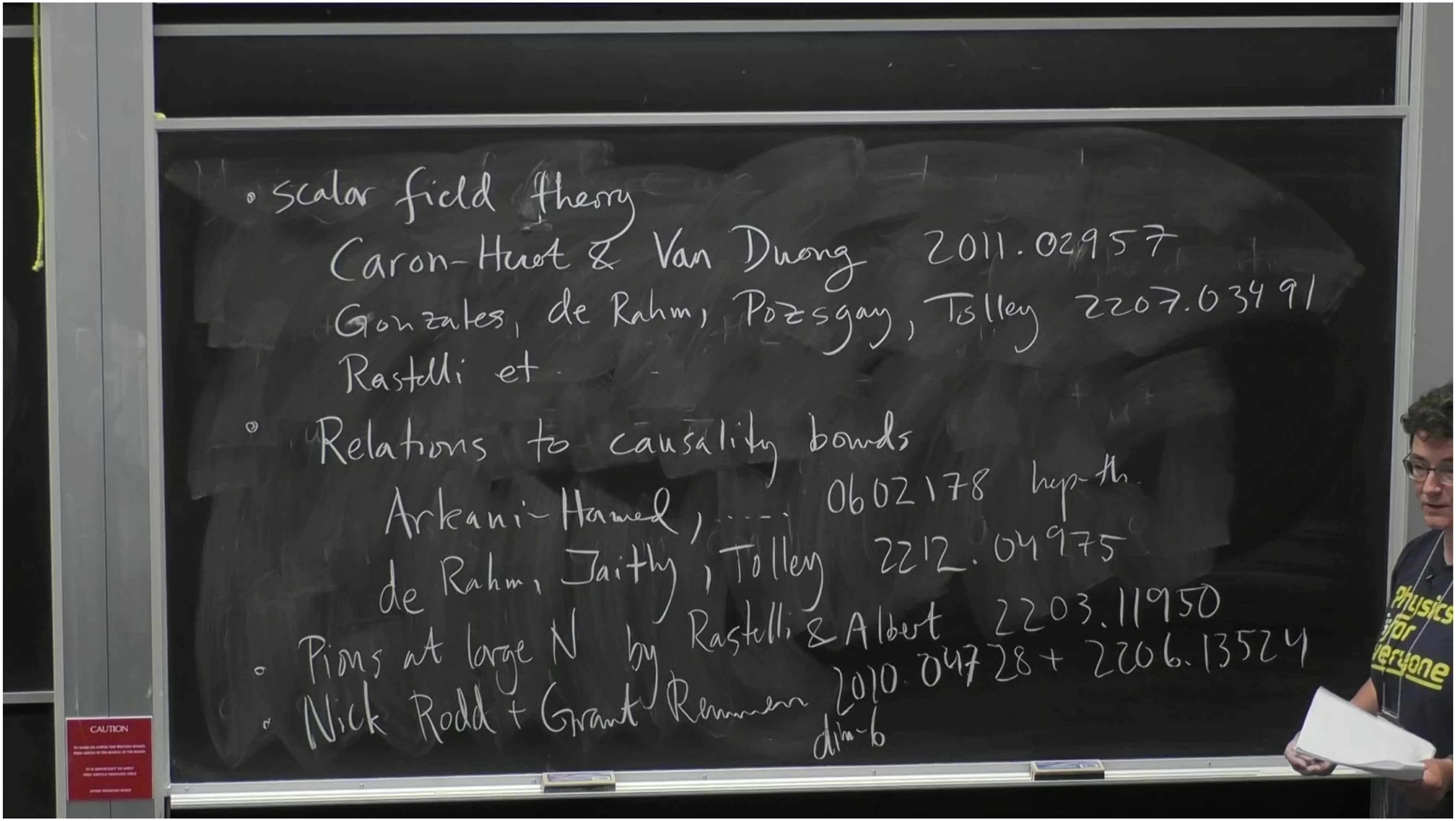
$$\vdots$$



a_{3,0}
1.046
k_MAX = 4
ζ(5) = 1.03693...
k_MAX = 5
k_MAX = 6
1.034
1.16
ζ(3) ≈ 1.20206...
1.21
a_{1,0}
k_max = 6
k_max = 8
a_{k,q} → a^{str}_{k,q}
up k_MAX = 8
within 10^{-4} - 10^{-9}


$a_{3,0}$
1.046
$k_{MAX} = 4$
$k_{max} = 5$
$\zeta(5) = 1.03693...$
$k_{max} = 6$
1.034
1.16
$\zeta(3)$
1.21
$a_{1,0}$
$= 1.20206...$
$k_{max} = 6$
$k_{max} = 8$
$a_{k,q} \longrightarrow a^{str}_{k,q}$
up $k_{MAX} = 8$
within 10^{-4}
Aidan Herderschee & Justin Berman


mono.
open string


• scalar field theory
Caron-Huot & Van Duong 2011.02957
Gonzales, de Rahm, Pozsgay, Tolley 2207.03491
Rastelli et
• Relations to causality bounds
Arkani-Hamed, ... 0602178 hep-th
de Rahm, Jaitly, Tolley 2212.04975
• Pions at large N by Rastelli & Albert 2203.11950
• Nick Rodd + Grant Remmen 2010.04728 + 2206.13524
dim-6
CAUTION

- scalar field theory
  - Caron-Huot & Van Duong 2011.02957
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- Relations to causality bounds
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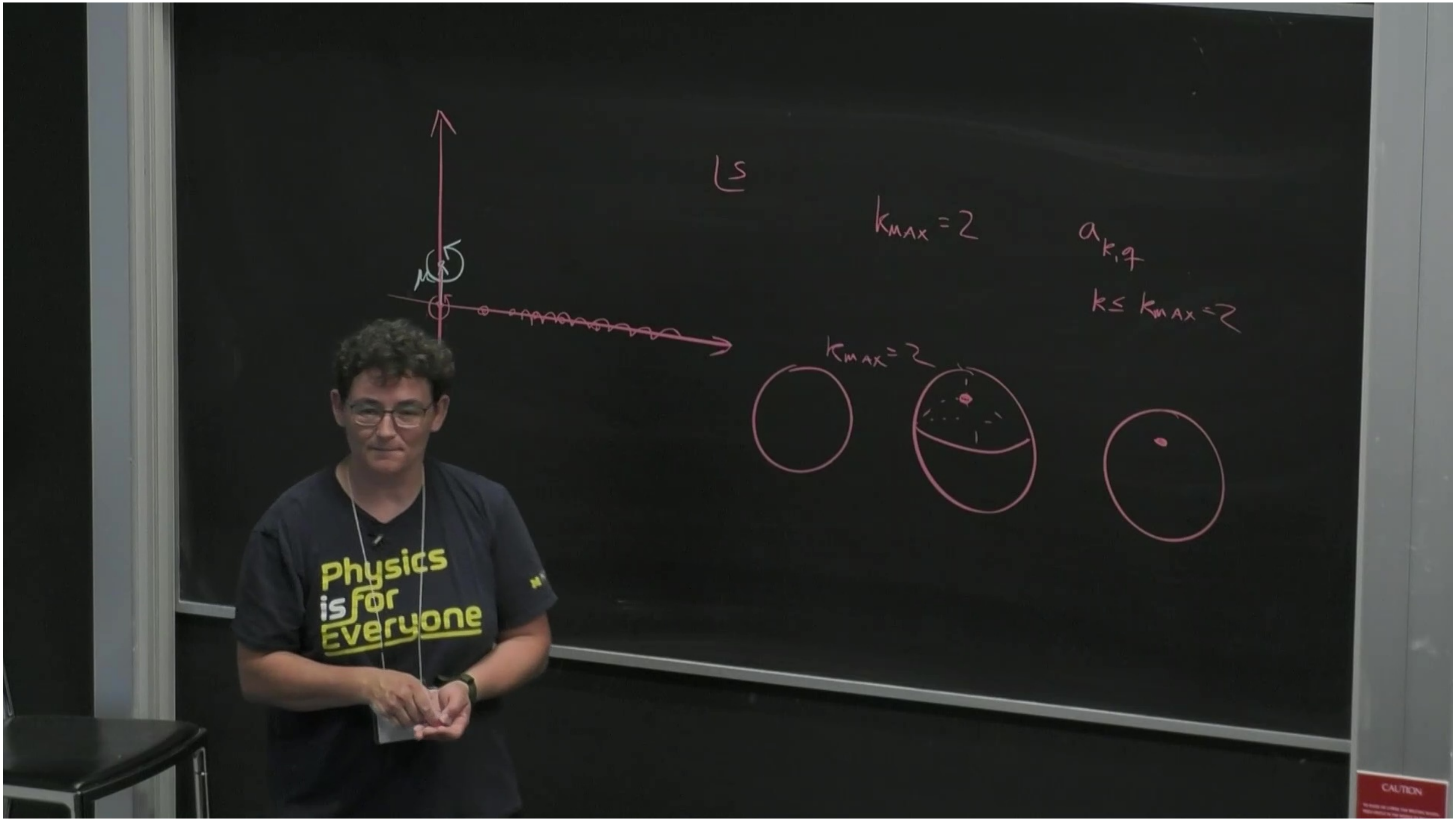
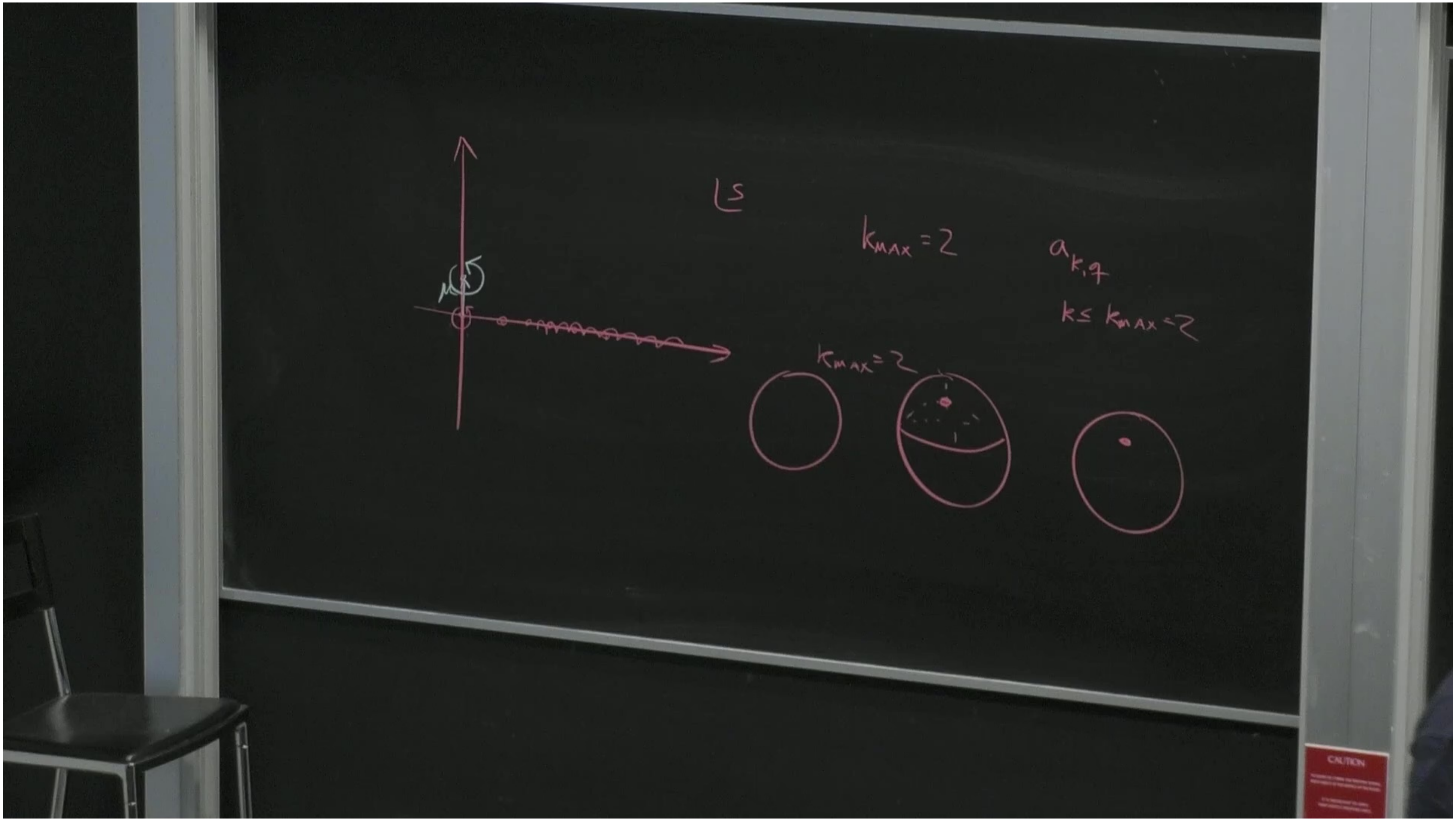
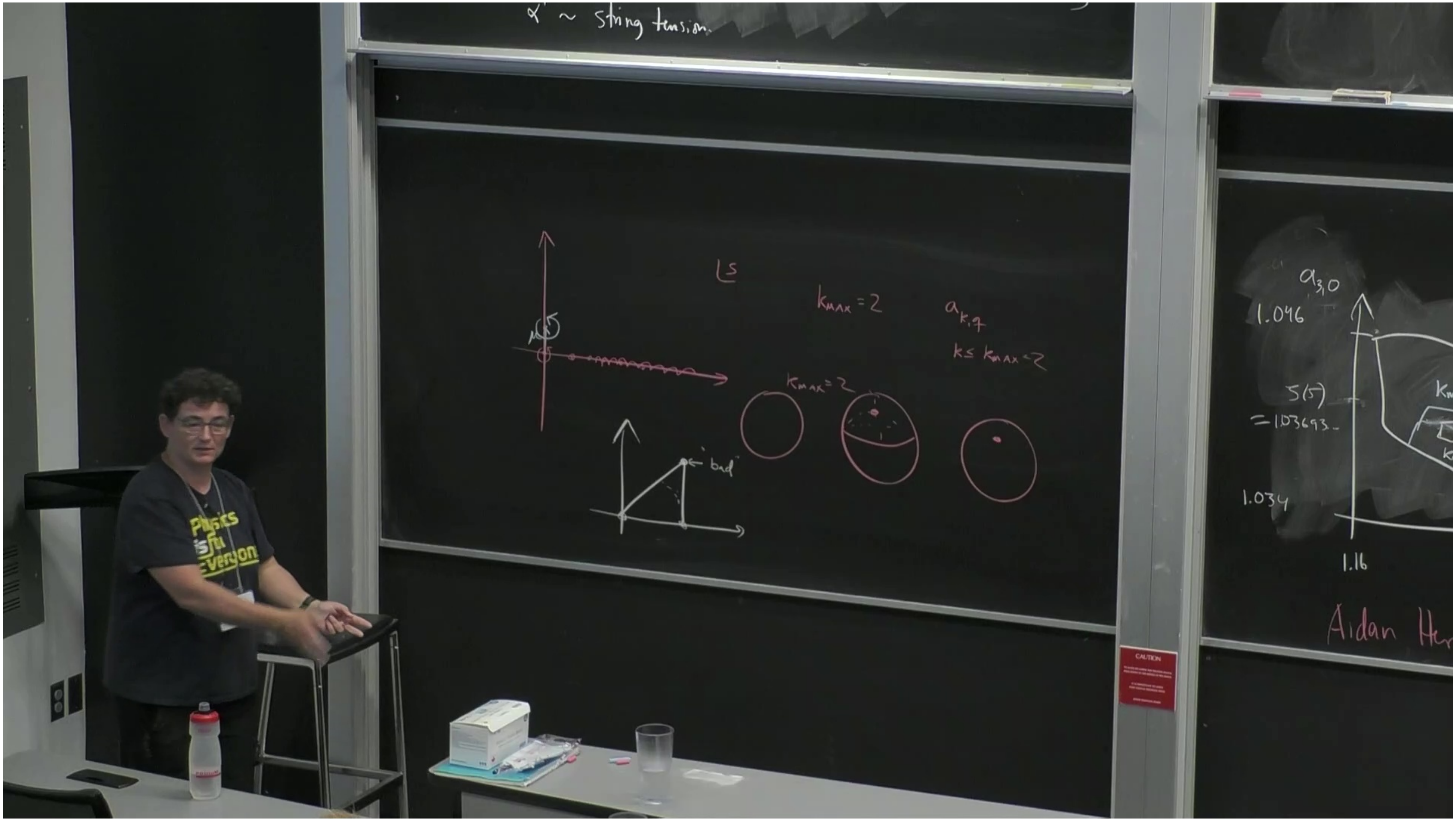
  - dim-6 Terong You.



$k_{MAX} = 2$
$a_{k,q}$
$k \leq k_{MAX} = 2$
$k_{MAX} = 2$
Physics is for Everyone



$k_{max} = 2$
$a_{k,q}$
$k \leq k_{max} = 2$
$k_{max} = 2$



α' ~ string tension
LS
k_max = 2
a_{k,q}
k ≤ k_max = 2
k_max = 2
"bad"
a_{3,0}
1.046
ζ(5)
= 1.03693...
1.034
1.16
Aidan