Title: EFT of Gravity

Speakers: John Donoghue

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EFT Methods for General Relativity

John Donoghue
TRISEP 2023
June 19 - 21

Roughly 3 blocks:

1) GR as a QFT
   - how to think like a field theorist

2) Effective Field Theory
   - as a full QFT

3) GR as an EFT
   - tie together
   - limits

Along the way: Background Field Method

Feynman rules

2) Effective Field Theory

- as a <u>full</u> QFT

3) GR as an EFT

- tie together
- limits

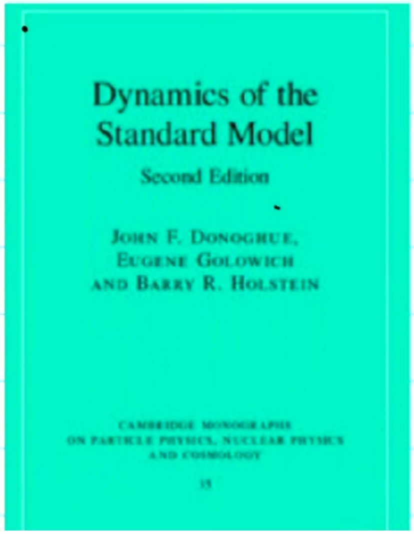
<u>Along the way:</u> Background Field Method
Path Integrals
Renormalization Theory
Heat Kernel Techniques

<u>Web site:</u> (Donoghue UMass or Donoghue GRQFT)

- References (EPFL lecture; Dynamics of SM ...)
- Notes, summaries, links

Web site: (Donoghue UMass or Donoghue GRQFT)

- References (EPFL lecture; Dynamics of SM ...)
- Notes, summaries, links
- also note ISQG lectures (~10 hours)
- plus EFT lectures at PI (~6 hrs)



Note: "Dynamics of Standard Model" is free
- JFD, Golowich + Holstein
- link is on webpage

Constructing theories

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\Psi$$

Symmetry $\Psi \to e^{i\theta}\Psi$

Current , $\bar{\Psi}\gamma_\mu\Psi = J_\mu$

Current $e\bar{\Psi}\gamma_\mu\Psi = eJ_\mu$

Gauge symmetry $\Psi \to e^{i\Theta(x)}\Psi$

New field $D_\mu\Psi \to e^{i\Theta(x)}D_\mu\Psi$

$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi \to \mathcal{L}$

Current $e\bar{\Psi}\gamma_\mu\Psi = eJ_\mu$

Gauge symmetry $\Psi \to e^{i\Theta(x)}\Psi$

New field $D_\mu\Psi \to e^{i\Theta(x)} D_\mu\Psi$

$\mathcal{L} = \bar{\Psi}(i\cancel{D} - m)\Psi \to \mathcal{L}$

$D_\mu = \partial_\mu + ieA_\mu$

$A_\mu \to A_\mu - \frac{1}{}$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$$

Now $\mathcal{L} = \bar{\psi} \ldots\ldots e\not{A}\psi$

$$\frac{\partial \mathcal{L}}{\partial A_\mu} \sim \ldots e\bar{\psi}\gamma_\mu\psi$$

✓

$$\frac{\partial \mathcal{L}}{\partial A_\mu} \sim \underbrace{\ldots e\,\bar{\psi}\,\gamma_\mu \psi}_{\text{source}} \quad \checkmark$$

Use

$$\underline{[D_\mu, D_\nu]}\,\psi = i\,\underbrace{\alpha F_{\mu\nu}}_{\text{invariant}}\,\psi$$

$$[D_\mu, D_\nu]\psi = i\,\underline{e F_{\mu\nu}}\,\psi$$

invariant

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\,i(\not{D} - m)\Psi$$

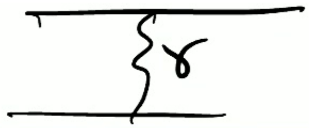
$$\Longrightarrow \underbrace{\partial^\mu F_{\mu\nu}} = e J_\mu$$

$O_{\mu\nu}\ O^{\nu}$

$$O_{\mu\nu}A^\nu = \underbrace{\Box A_\nu - \partial_\nu(\partial_\mu A^\mu)}_{\text{not invertabl}} = \cancel{e}$$

Choose gauge $\partial_\mu A^\mu = 0 \Longrightarrow \Box A = J$

QFT



$$J_\mu \frac{1}{q^2} J^\mu \xrightarrow[FT]{NR} e_1 \frac{1}{4\pi r} e_2$$

Non abelian

$$\psi \to U\psi$$

$$\psi \to U\psi \longrightarrow J_\mu^i = \bar{\psi} \ldots \frac{\lambda^i}{2} \psi$$

$$D_\mu = \partial_\mu + ig \frac{\lambda^A}{2} A_\mu^A = \partial_\mu + ig \underline{\underline{A}}_\mu$$

$$\underline{\underline{F}}_{\mu\nu} = \partial_\mu \underline{\underline{A}}_\nu - \partial_\nu \underline{\underline{A}}_\mu + ig \underbrace{\left[\underline{\underline{A}}_\mu , \underline{\underline{A}}_\nu \right]}_{\cancel{\star}\star}$$

$-$ ghosts $\cancel{\star}\star$

Gravity

Want energy as source
- Eötvös
- light bending

$\Bigg\} \Rightarrow T_{\mu\nu}$

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$T_{\mu\nu}$ is current for

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Ex: $\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\, \partial^\mu \phi - m^2 \phi\right)$

$$T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - \frac{\eta_{\mu\nu}}{2}\left(\partial_\lambda \phi\, \partial^\lambda \phi - m^2 \phi^2\right)$$

Gauge spacetime translation

$$Y_\mu \to Y_\mu + a_\mu(X)$$

Crazy May work!

Gravity

Want energy as source
- Eötvös
- light bending

$\Rightarrow T_{\mu\nu}$

$H = \int d^3x\, T_{00}$

$T_{\mu\nu}$ is current for

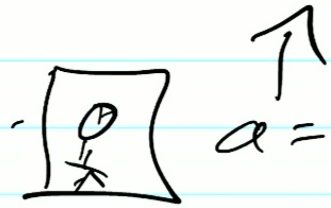
$\partial^\mu T_{\mu\nu} = 0$

$$x_\mu \to x_\mu + a_\mu$$

Ex: $\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\, \partial^\mu \phi - m^2 \phi\right)$

$$T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - \frac{\eta_{\mu\nu}}{2}\left(\partial_\lambda \phi\, \partial^\lambda \phi - m^2 \phi^2\right)$$

Crazy May work!

 $a=0$ $\sim$  $a=g$

 g

 $a=g$

~~$\mathcal{E}$~~ New field

$$ds = \eta_{\mu\nu}\, dy^\mu\, dy^\nu \rightarrow g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$$

$$dx'^\mu = J^\mu{}_\nu(x)\, dx^\nu \qquad J^\mu{}_\nu(x) = \frac{\partial x'^\mu}{\partial x^\nu}$$

$$g'_{\mu\nu}\, dx'^\mu\, dx'^\nu = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

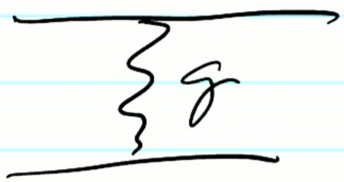
Success

$$S = \int \underbrace{d^4x \sqrt{-g}}_{\text{invariant}} \; \frac{1}{2}\left[g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - m^2 \phi^2 \right.$$

$$\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = \frac{\sqrt{-g}}{2}\left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\left(\partial_\alpha \phi\, \partial^\alpha \phi - m^2 \phi^2\right)\right]$$

$$= \frac{\sqrt{-g}}{2}\, \underline{\underline{T_{\mu\nu}}}$$

Success #2



$$= \frac{\kappa}{2}\, T_{\mu\nu}\, \frac{P_{\mu\nu\alpha\beta}}{q^2}\, T^{\alpha\beta}$$

$$\xrightarrow{NR} \frac{\kappa^2}{32\pi}\, \frac{m_1 m_2}{r}$$

Success # 3

$$\frac{\delta S}{\delta \phi} = \underbrace{\frac{1}{\sqrt{-g}} \partial_\mu \left(g^{\mu\nu} \sqrt{-g}\, \partial_\nu \right)}_{\partial^2} \phi + m^2 \phi = 0$$

$$\frac{\partial S}{\partial \phi} = \underbrace{\frac{1}{\sqrt{-g}} \partial_\mu \left(g^{\mu\nu} \sqrt{-g}\, \partial_\nu \right)}_{\partial^2} \phi + m^2 \phi = 0$$

N.R. reduction $\quad \phi(x,t) = e^{-imt}\, \psi(x,t)$

$$\frac{\partial S}{\partial \phi} = \underbrace{\frac{1}{\sqrt{-g}}\, \partial_\mu \left(g^{\mu\nu} \sqrt{-g}\, \partial_\nu \right)}_{\square^2} \phi + m^2 \phi = 0$$

N.R. reduction $\quad \phi(x,t) = \underline{\underline{e^{-imt}}}\, \overline{\psi}(x,t)$

Need $\quad g_{00} = 1 + 2\phi_g \quad$ ← grav pot $\qquad$ small t dep.

$\Rightarrow$

$$\frac{\partial S}{\partial \phi} = \underbrace{\frac{1}{\sqrt{-g}}\partial_\mu \left(g^{\mu\nu}\sqrt{-g}\,\partial_\nu \right)}_{D^2}\phi + m^2\,\phi = 0$$

N.R. reduction $\quad \phi(x,t) = \underline{\underline{e^{-imt}}}\;\overline{\psi(x,t)}$

small t dep.

Need $\quad g_{00} = 1 + 2\phi_g$

$\uparrow$ grav pot

$$\Rightarrow \quad i\frac{\partial}{\partial t}\psi = \left[-\frac{\nabla^2}{2m} + m\,\phi_g \right]\psi$$

N.R. reduction $\phi(x,t) = e^{i m t}\, \psi(x,t)$

small t dep.

Need $g_{00} = 1 + 2\phi_g$

grav pot

$$\Rightarrow \quad i\frac{\partial}{\partial t}\psi = \left[\frac{-\nabla^2}{2m} + m\phi_g \right]\psi$$

$m_I = m_g$

Force law $\dot{\vec{p}} = -i\,[H, \vec{p}\,] = -m\vec{\nabla}\phi = m\vec{a}$

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\mu\rho}{}^\nu V^\rho \qquad \partial_\mu + \underline{\underline{A_\mu}}$$

Could be new field

or composed field $\Gamma_{\mu\rho}{}^\nu$

Could be new field

or composed field $\Gamma_{\mu\nu}{}^{\lambda} = \frac{1}{2} g^{\lambda\sigma}\left[\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right]$

If use EP

$\Downarrow$

$$\partial_\mu \eta_{\alpha\nu} = 0$$

$$\underbrace{D_\mu g_{\alpha\beta} = 0}_{\Gamma = \Delta} \quad \leftarrow \text{metricity}$$

$$\breve{\Gamma} = A$$

Covariant

$$[D_\mu, D_\nu] V^\rho \qquad D_\mu' D_\nu' V^\rho$$

$$= \rho \qquad \rightarrow \underline{\underline{[\;]'[\;]'[\;][\;]V}}$$

$$R_{\mu\nu\alpha}{}^{\rho} = R_{\mu\nu}$$

$$\Gamma = A$$

Covariant

$$[D_\mu, D_\nu] V^\beta \qquad D'_\mu D'_\nu V^\beta$$

$$\equiv R_{\mu\nu\alpha}{}^\beta V^\alpha \qquad \Longrightarrow \Gamma'\Gamma'\Gamma\Gamma V$$

$$R_{\mu\nu\alpha}{}^\beta = \partial_\mu \Gamma_{\nu\alpha}{}^\beta - \partial_\nu \Gamma_{\mu\alpha}{}^\beta + \Gamma_\mu^\beta \Gamma_\nu - \Gamma_\nu \Gamma_\mu$$

$$\check{\Gamma} = \Delta$$

Covariant

$$[D_\mu, D_\nu] V^\rho \qquad D'_\mu D'_\nu V^\rho$$

$$= R_{\mu\nu\alpha}{}^{\rho} V^\alpha \qquad \longrightarrow \Sigma^{-1} \Sigma^{-1} \Sigma [\ldots]$$

$$R_{\mu\nu\alpha}{}^{\beta} = \partial_\mu \Gamma_{\nu\alpha}{}^{\beta} - \partial_\nu \Gamma_{\mu\alpha}{}^{\beta} + \Gamma_{\mu\rho}^{\beta} \Gamma_{\nu}^{\rho} - \Gamma_{\nu\rho}^{\beta} \Gamma_{\mu}^{\rho}$$

$$R_{\nu\alpha} = R_{\mu\nu\alpha}{}^{\mu} \quad , \quad R = g^{\nu\alpha} R_{\nu\alpha}$$

$$R_{\nu\alpha} = R_{\mu\nu\alpha}{}^{\mu} \quad , \quad R = g^{\nu\alpha} R_{\nu\alpha}$$

$$g^{\mu\nu} g_{\nu\alpha} = \delta^{\mu}_{\alpha}$$

EFT $\Gamma \Rightarrow \partial g$

$R \sim \partial^2 g$

<u>Invariant action</u>

$$\partial^2 g$$

$$S = \int d^4x \sqrt{-g} \left[-\Lambda - \frac{2}{\kappa^2} R + \underbrace{R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \cdots}_{} \right]$$

Weak field expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \qquad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \frac{1}{2} h^{\mu\lambda} h_{\lambda}{}^{\nu} + \cdots$$

Weak field expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \qquad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \frac{1}{2} h^{\mu\lambda} h_\lambda^{\ \nu} + \cdots$$

$$\underbrace{\left[-\Box h_{\mu\nu} - \partial_\mu \partial_\lambda h^\lambda_{\ \nu} \cdots \right]}_{\text{no inverse}} = \frac{\kappa^2}{4} T_{\mu\nu}$$

$$\mathcal{L} = \underbrace{-\tfrac{1}{\cdot}\left(h_{\mu\nu} - \partial_\mu \partial_\lambda h^\lambda{}_\nu \cdots \right)}_{\text{no inverse}} \quad J = \frac{\kappa'}{4} T_{\mu\nu}$$

$$O_{\mu\nu}{}^{\alpha\beta} G_{\alpha\beta\gamma\delta}(x-y) = \frac{1}{2} I_{\mu\nu\gamma\delta} \delta_D^{(4)}(x-y) ,$$

where

$$O^{\mu\nu}{}_{\alpha\beta} \equiv (\delta_\alpha^{(\mu} \delta_\beta^{\nu)} - \eta^{\mu\nu}\eta_{\alpha\beta})\Box - 2\delta^{(\mu}_{(\alpha}\partial^{\nu)}\partial_{\beta)} + \eta_{\alpha\beta}\partial^\mu\partial^\nu + \eta^{\mu\nu}\partial_\alpha\partial_\beta .$$

$$O^{\mu\nu}{}_{\alpha\beta} \equiv (\delta^{(\mu}_{\alpha}\delta^{\nu)}_{\beta} - \eta^{\mu\nu}\eta_{\alpha\beta})\Box - 2\delta^{(\mu}_{(\alpha}\partial^{\nu)}\partial_{\beta)} + \eta_{\alpha\beta}\partial^{\mu}\partial^{\nu} + \eta^{\mu\nu}\partial_{\alpha}\partial_{\beta}\,.$$

Gauge invariance $x^\mu \to x^\mu + \xi^\mu(x)$

$$h_{\mu\nu} \Rightarrow h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

Gauge fixing

harmonic gauge
de Donder gauge

$$\left[\partial_\mu h^\mu{}_\nu - \frac{1}{2}\partial_\nu h^\lambda{}_\lambda\right] = 0$$

Gauge fixing

harmonic gauge
de Donder gauge $\left[\partial_\mu h^\mu{}_\nu - \frac{1}{2}\partial_\nu h^\lambda{}_\lambda\right] = 0$

Then
$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^\lambda{}_\lambda\right) = \frac{\kappa^2}{4} T_{\mu\nu}$$

Invertable:

$$D_{\mu\nu\alpha\beta} = \frac{1}{q^2}\,\frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

$$h_{\mu\nu} = \epsilon \left[\epsilon_{\mu\nu}\, e^{ik\cdot x} + \dots \right]$$

$$\underline{\epsilon_\mu^{*} \epsilon_\nu^{*}}$$

Point mass

$$h_{\mu\nu} = \begin{pmatrix} 2\phi_g & & & 0 \\ & 2\phi_g & & \\ & & 2\phi_g & \\ 0 & & & 2\phi_g \end{pmatrix} \qquad \phi_g = -G$$

$$h_{\mu\nu} = C\left[\varepsilon_{\mu\nu}\, e^{ikx}\, e \qquad + \quad \ldots \right]$$

$$\leftarrow \underline{\varepsilon_\mu^{+}\, \varepsilon_\nu^{+}}$$

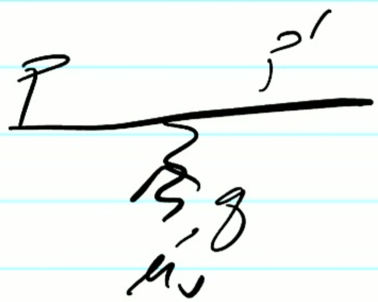
Point mass

$$h_{\mu\nu} = \begin{pmatrix} 2\phi_g & & & 0 \\ & 2\phi_g & & \\ & & 2\phi_g & \\ 0 & & & 2\phi_g \end{pmatrix} \qquad \phi_g = -\frac{GM}{r}$$

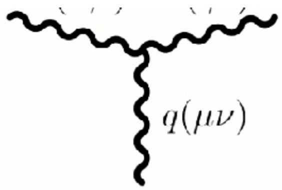
32 $G = \kappa^2$

Feynman rules

$$i D_{\mu\nu\alpha\beta}$$



$$= \frac{\kappa}{2}\left[\left(p_\mu p'_\nu + p'_\mu p_\nu\right) - \eta_{\mu\nu}\left(p\cdot p' - m^2\right)\right]$$



$$
\begin{aligned}
&= \frac{i\kappa}{2}\Bigg(P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu} q^2\Big] \\
&+ 2q_\lambda q_\sigma [I^{\lambda\sigma}{}_{\alpha\beta} I^{\mu\nu}{}_{\gamma\delta} + I^{\lambda\sigma}{}_{\gamma\delta} I^{\mu\nu}{}_{\alpha\beta} - I^{\lambda\mu}{}_{\alpha\beta} I^{\sigma\nu}{}_{\gamma\delta} - I^{\sigma\nu}{}_{\alpha\beta} I^{\lambda\nu}{}_{\gamma\delta}] \\
&+ [q_\lambda q^\mu (\eta_{\alpha\beta} I^{\lambda\nu}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu}{}_{\alpha\beta}) + q_\lambda q^\nu (\eta_{\alpha\beta} I^{\lambda\mu}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu}{}_{\alpha\beta}) \\
&- q^2 (\eta_{\alpha\beta} I^{\mu\nu}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu}{}_{\alpha\beta}) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma})] \\
&+ \Big[2q^\lambda \Big(I^{\sigma\nu}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu \\
&- I^{\sigma\nu}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \Big) \\
&+ q^2 (I^{\sigma\mu}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu}{}_{\alpha\delta}) + \eta^{\mu\nu} q^\lambda q_\sigma (I^{\rho\sigma}{}_{\gamma\delta} I_{\alpha\beta,\lambda\rho} + I^{\rho\sigma}{}_{\alpha\beta} I_{\gamma\delta,\lambda\rho}) \Big] \\
&+ \Big[(k^2 + (k-q)^2) \Big(I^{\sigma\mu}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2}\eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \Big) \\
&- k^2 \eta_{\gamma\delta} I^{\mu\nu}{}_{\alpha\beta} - (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu}{}_{\gamma\delta} \Big] \Bigg) .
\end{aligned}
$$

$$T_{\mu\nu\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}\right]$$

<u>Reflect</u>

① GR from

$$T_{\mu\nu\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}\right]$$

<u>Reflect</u>

① GR from symmetry

$$T_{\mu\nu\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}\right]$$

Reflect

- GR from symmetry
  It is all field.

GR from symmetry

It is all field.

$$\frac{2}{\kappa^2} R \longrightarrow -\frac{1}{4g^2} F^2$$

$$g = \eta + h$$

$$\longrightarrow \frac{1}{2} \partial h \, \partial h$$

 $\left(\frac{K}{2}\right)^2 \frac{1}{2} \mu^{4-d} \int \frac{d^d k}{(2\pi)^d}$

$$\left(\frac{K}{2}\right)^2 \frac{1}{2} \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{(k-\varepsilon)^2} \left[k \right.$$

$$\left(\frac{\kappa}{2}\right)^2 \frac{1}{2}\mu^{4-d}\int \frac{d^d k}{(2\pi)^d}\,\frac{1}{k^2}\,\frac{1}{(k-\varepsilon)^2}\left[k k \varepsilon \varepsilon\right]$$

$$\uparrow 4k$$

$$\sim \bigcirc \sim \left(\frac{K}{2}\right)^2 \frac{1}{2} \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{(k-q)^2} \left[k k q q \right]$$

$$\uparrow 4k$$

$$\sim \left[\frac{1}{\varepsilon} - \ln q^2/\mu^2\right] \left[q q q q \right]$$

$$\sim\!\bigcirc\!\sim \quad \left(\frac{\kappa}{2}\right)^2 \frac{1}{2}\mu^{4-d}\int \frac{d^d k}{(2\pi)^d}\,\frac{1}{k^2}\,\frac{1}{(k-\varepsilon)^2}\left[k k \varepsilon \varepsilon\right]$$

$$\uparrow 4k$$

$$\underbrace{\sim\left[\frac{1}{\varepsilon} - \ln q^2/\mu^2\right]\left[\varepsilon\varepsilon\varepsilon\varepsilon\right]}_{E^4}$$

Not like $R \sim \partial\partial \sim E^2$

R^2

Not like $R \sim \partial\partial \sim E^2$

$R^{\cancel{2}} \sim \partial\partial\partial\partial \sim E^4$

$$\Longrightarrow \Delta \mathcal{L} = \frac{1}{\epsilon}\left(a R^2 + b R_{\mu\nu} R^{\mu\nu}\right)$$

small g

R^2 terms will be generated in S_g

small g

R^2 terms will be generated in S_g

$$N_\mu' = A_\mu^{\ \nu}(x) N_\nu + a_\mu(x)$$